Université de Montpellier
Faculté de Médecine
Montpellier-Nîmes

TOMOGRAHIE MEDICALE

❶ IMAGERIE DE PROJECTION ET EN COUPES
❷ TOMOGRAPHIE = PROBLEME LINEAIRE INVERSE MAL CONDITIONNE
❸ ALGORITHMES DE RECONSTRUCTION ANALYTIQUES ET ALGEBRIQUES
❹ SPECIFICITES EN TEP 3D
❺ UN EXEMPLE DE TRAVAUX EN COURS SUR LE SUJET…

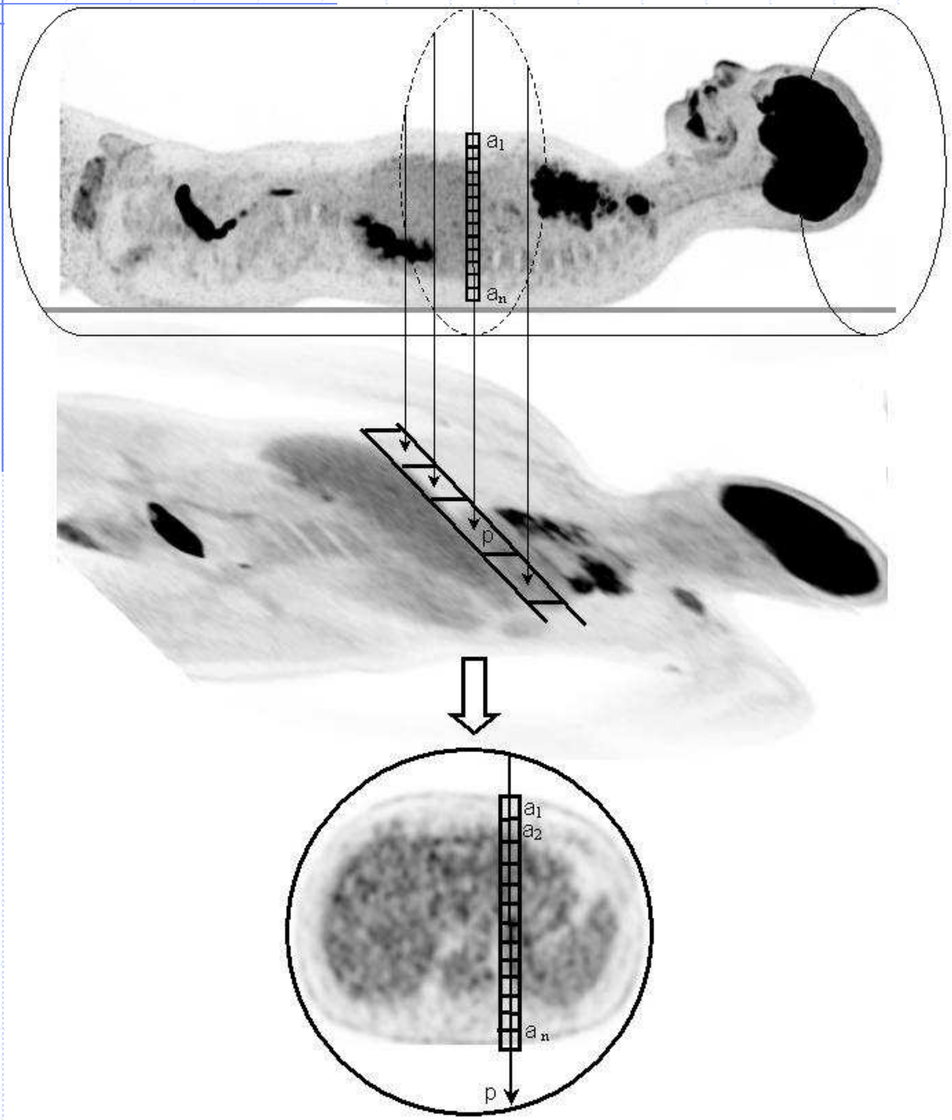
Nb: Les corrections des artefacts d'acquisition ne seront pas abordées.

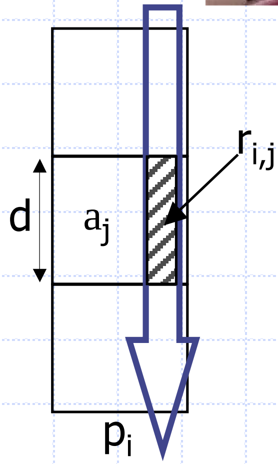
Denis Mariano-Goulart, http:\\scinti.edu.umontpellier.fr

= formalisme ou concepts sortant du programme du DFGSM

= notions d'un niveau 3° cycle ou recherches en cours

PROJECTION EN TE(M)P



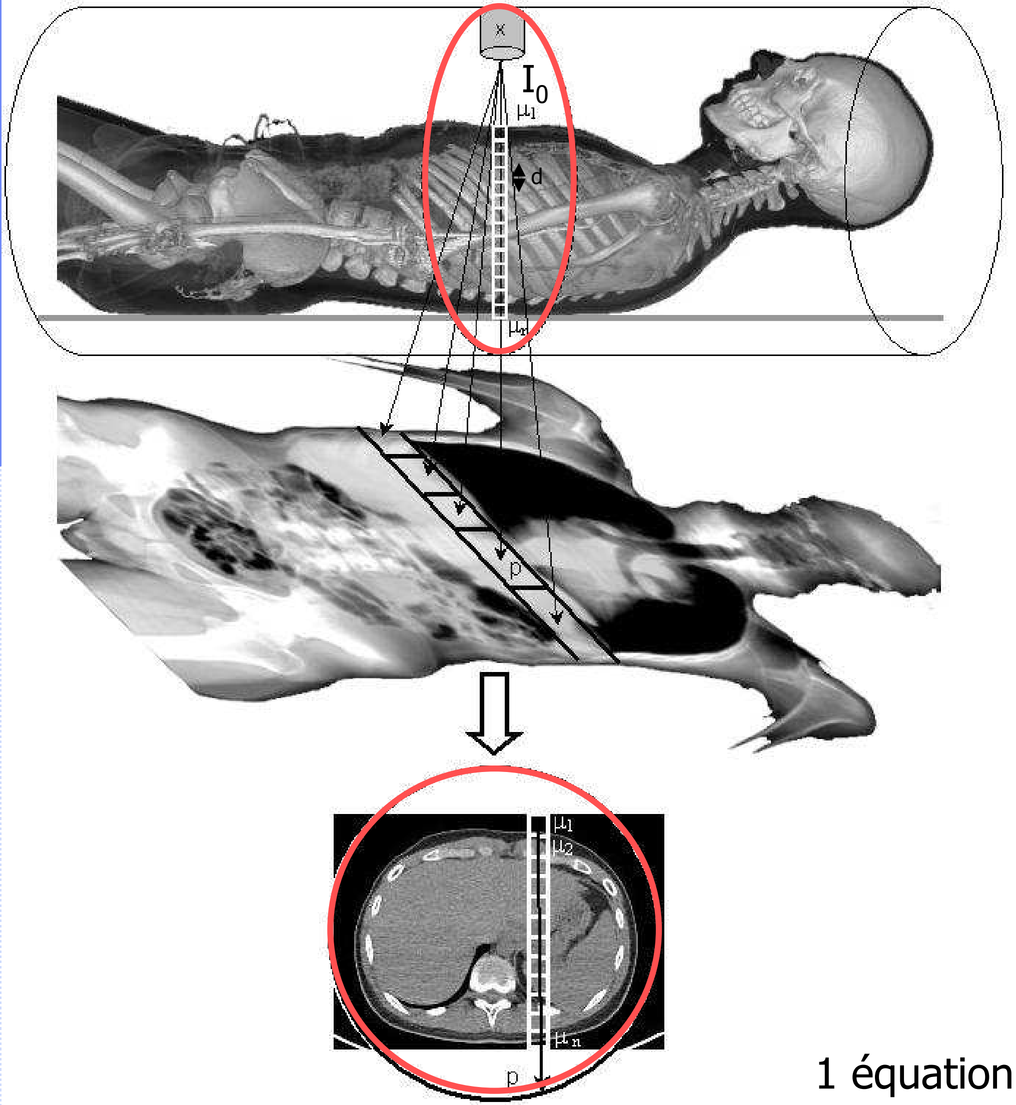


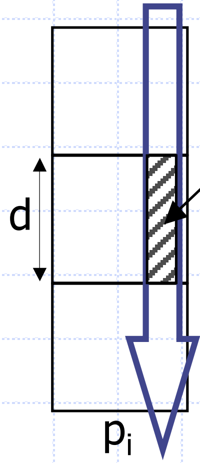
$$p_i = \sum_j r_{i,j} . a_j$$

$$\boxed{p_i = r_{i,1} . a_1 + r_{i,2} . a_2 + \ldots + r_{i,n} . a_n}$$

1 équation à n inconnues

PROJECTION EN TDM





$r_{i,j}$ = 0.25 ici

$$\mu_j = -\frac{1}{d}\frac{dI}{I}$$

$$I_i = I_0 . e^{-\sum_j r_{i,j}.(\mu_j.d)}$$

$$-\frac{1}{d}\ln\frac{I_i}{I_0} = p_i = \sum_j r_{i,j}.\mu_j$$

$$\boxed{p_i = r_{i,1}.\mu_1 + r_{i,2}.\mu_2 + \ldots + r_{i,n}.\mu_n}$$

1 équation à n inconnues $\Rightarrow$ P > n projections

MODELISATION ALGEBRIQUE

f_1	f_2
f_3	f_4

$P = 4$

$p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2$

$p_2 = r_{2,3}\, f_3 + r_{2,4}\, f_4$

$p_4 = r_{4,2}\, f_2 + r_{4,4}\, f_4$

$p_3 = r_{3,1}\, f_1 + r_{3,3}\, f_3$

$$\begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ r_{2,1} & r_{2,2} & r_{2,3} & r_{2,4} \\ r_{3,1} & r_{3,2} & r_{3,3} & r_{3,4} \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{pmatrix} . \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

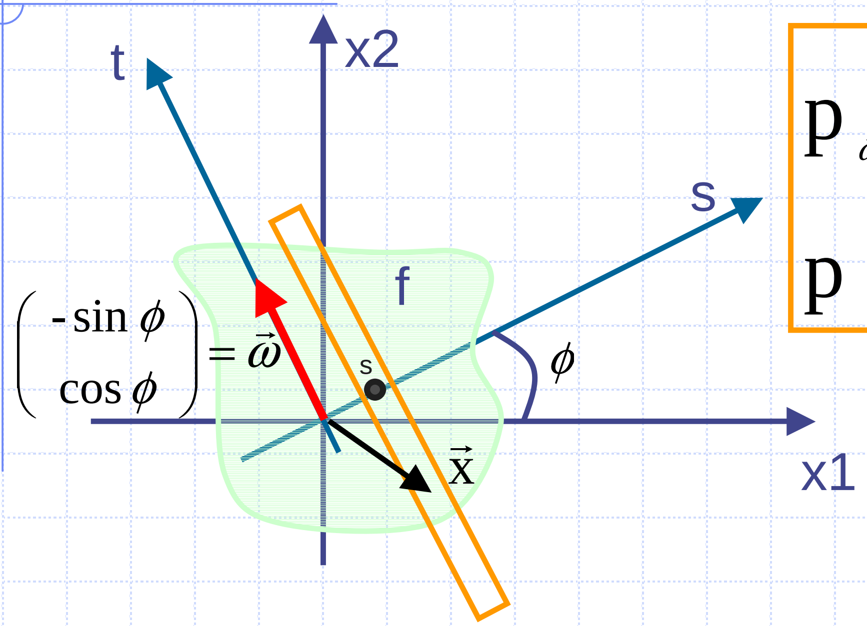
PROJECTION : $\boxed{R.\vec{f} = \vec{p}}$

$b_1 = r_{1.1}\, p_1 + r_{3,1}\, p_3$	$b_2 = r_{1.2}\, p_1 + r_{4,2}\, p_4$	$\leftarrow p_1$
$b_3 = r_{2.3}\, p_2 + r_{3,3}\, p_3$	$b_4 = r_{2,4}\, p_2 + r_{4,4}\, p_4$	$\leftarrow p_2$
$\uparrow p_3$	$\uparrow p_4$	

$$\begin{pmatrix} r_{1,1} & r_{2,1} & r_{3,1} & r_{4,1} \\ r_{1,2} & r_{2,2} & r_{3,2} & r_{4,2} \\ r_{1,3} & r_{2,3} & r_{3,3} & r_{4,3} \\ r_{1,4} & r_{2,4} & r_{3,4} & r_{4,4} \end{pmatrix} . \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

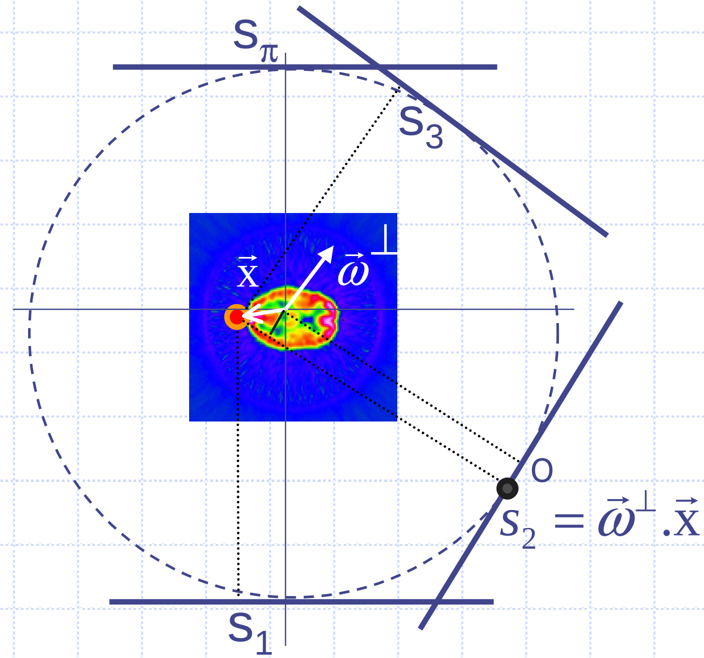
RETROPROJECTION : $\boxed{{}^{t}R.\vec{p} = \vec{b}}$

MODELISATION ANALYTIQUE



$$p_{\vec{\omega}}(s) = \int_t f(s\,\overrightarrow{\omega}^{\perp} + t\,\vec{\omega})\,dt$$

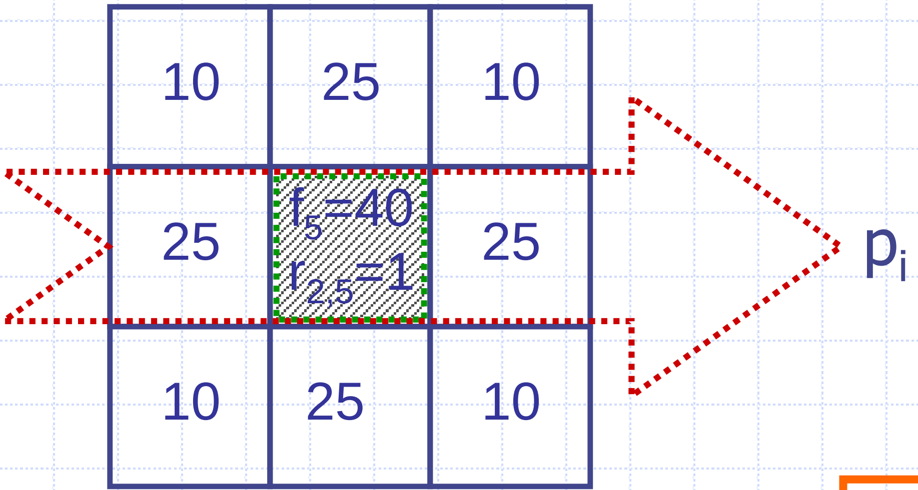
$p = Rf$ transformée de Radon



$$({}^tR\,p)(\vec{x}) = \int_{\phi=0}^{\pi} p_{\vec{\omega}}(\vec{\omega}^{\perp}.\vec{x})\,d\phi$$

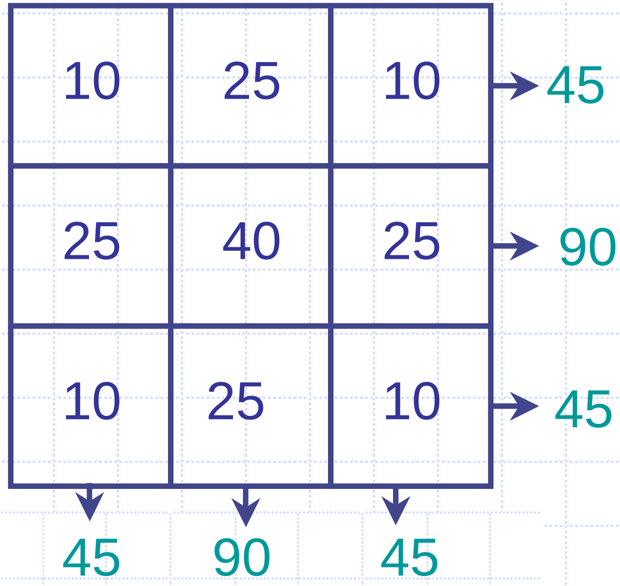
rétroprojection = épandage

PRO/RETROPROJECTION

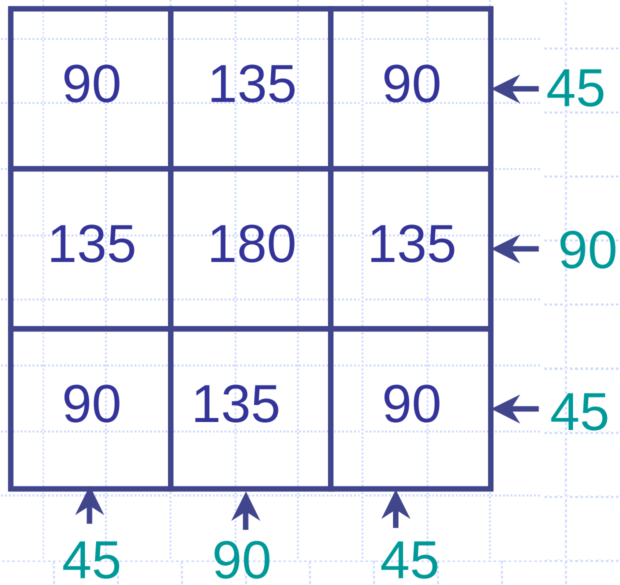


Hypothèse:
$r_{i,j}$=1 si le pixel j se projette dans la raie i; $r_{i,j}$=0 sinon.

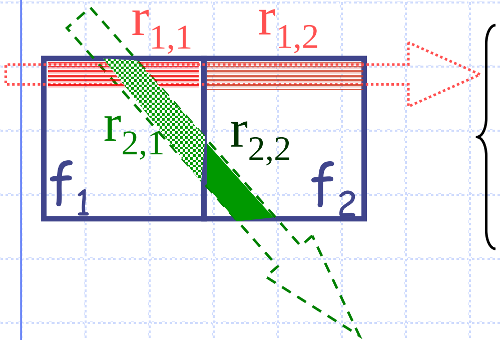
$$R.\vec{f} = \vec{p}$$



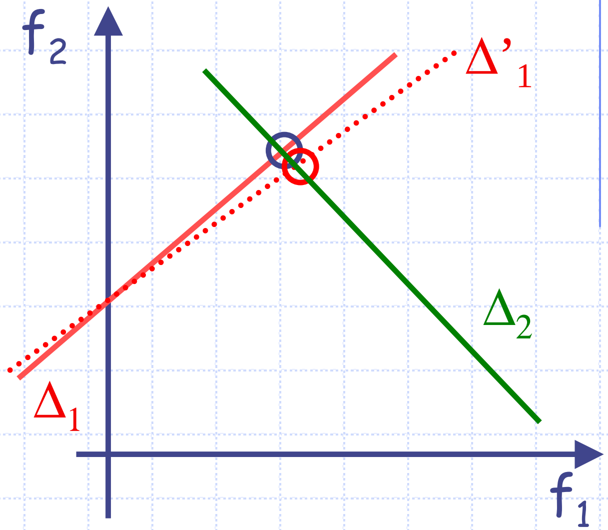
$$^{t}R.\vec{p} = \vec{b}$$



Pb INVERSE LINEAIRE

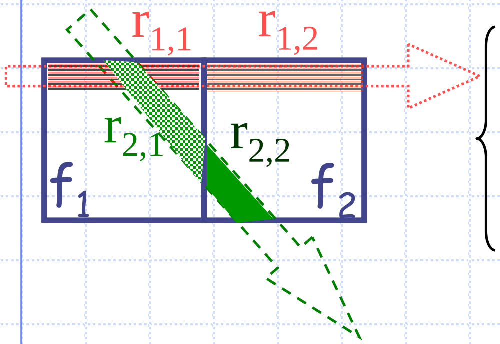


$$\begin{cases} \Delta_1 : \ p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2 \\ \Delta_2 : \ p_2 = r_{2,1}\, f_1 + r_{2,2}\, f_2 \end{cases}$$

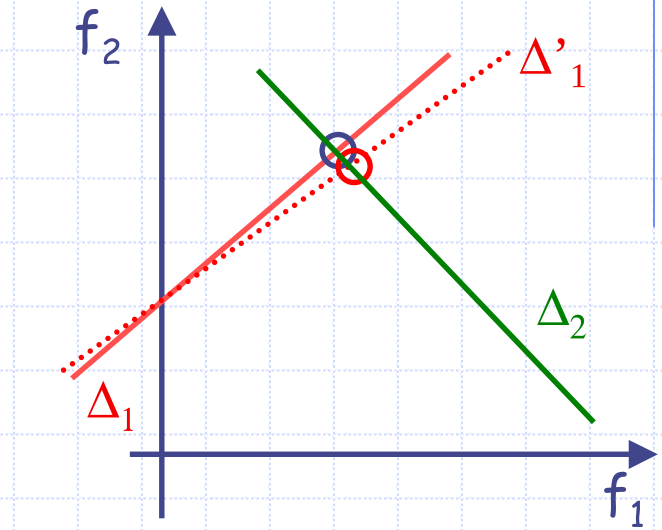


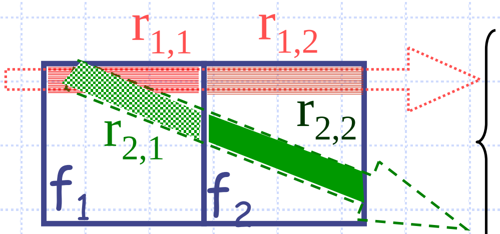
Reconstruire une coupe
=
Résoudre un système linéaire
de n² équations ($p_i = \ldots$)
et n² inconnues (f_j)

Pb INVERSE LINEAIRE MAL CONDITIONNE

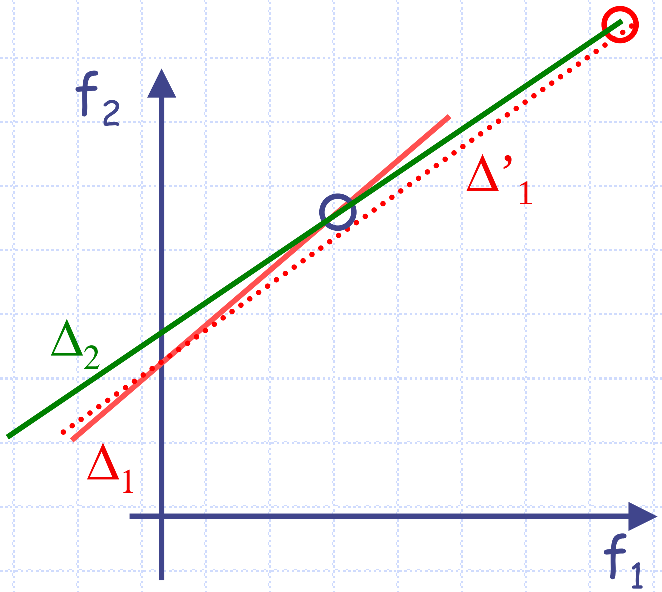


$$\begin{cases} \Delta_1 : \; p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2 \\ \Delta_2 : \; p_2 = r_{2,1}\, f_1 + r_{2,2}\, f_2 \end{cases}$$





$$\begin{cases} \Delta_1 : \; p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2 \\ \Delta_2 : \; p_2 = r_{2,1}\, f_1 + r_{2,2}\, f_2 \end{cases}$$



CONDITIONNEMENT

$$R.\begin{pmatrix}1\\1\\1\\1\end{pmatrix}=\begin{pmatrix}10&7&8&7\\7&5&6&5\\8&6&10&9\\7&5&9&10\end{pmatrix}\boxed{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}}=\boxed{\begin{pmatrix}32\\23\\33\\31\end{pmatrix}}$$

coupe
projections

$$\begin{pmatrix}10&7&8&7\\7&5&6&5\\8&6&10&9\\7&5&9&10\end{pmatrix}\boxed{\begin{pmatrix}9,2\\-12,6\\4,5\\-1,1\end{pmatrix}}=\begin{pmatrix}32,\boxed{1}\\22,\boxed{9}\\33,\boxed{1}\\30,\boxed{9}\end{pmatrix}$$

$$\begin{pmatrix}10&7&8&7\\7&5&6&5\\8&6&10&9\\7&5&9&10\end{pmatrix}\boxed{\begin{pmatrix}-7,2\\14,6\\-2,5\\3,1\end{pmatrix}}=\begin{pmatrix}31,\boxed{9}\\23,\boxed{1}\\32,\boxed{9}\\31,\boxed{1}\end{pmatrix}$$

$$\kappa(R)=\|R\|.\|R^{-1}\|=\frac{\mu_{\max}}{\mu_{\min}}$$

$$\text{où } \mu=\sqrt{\text{valeurs propres de } R^{t}R}$$

$$Sp(R)\approx\{0{,}01;\ 0{,}84;\ 3{,}86;\ 30{,}29\}\Rightarrow\kappa(R)\approx\frac{30{,}29}{0.01}\approx 3029>>1$$

$$\frac{\|\delta\vec{f}\|}{\|\vec{f}\|}\leq\frac{\kappa(R)}{1-\kappa(R)\frac{\|\delta R\|}{\|R\|}}\left[\frac{\|\delta\vec{p}\|}{\|\vec{p}\|}+\frac{\|\delta R\|}{\|R\|}\right]$$

$$\|\delta R\|=0\Rightarrow\frac{\|\delta\vec{f}\|}{\|\vec{f}\|}\leq\kappa(R)\frac{\|\delta\vec{p}\|}{\|\vec{p}\|}$$

10 = 3000 0.1/30

SOLUTION

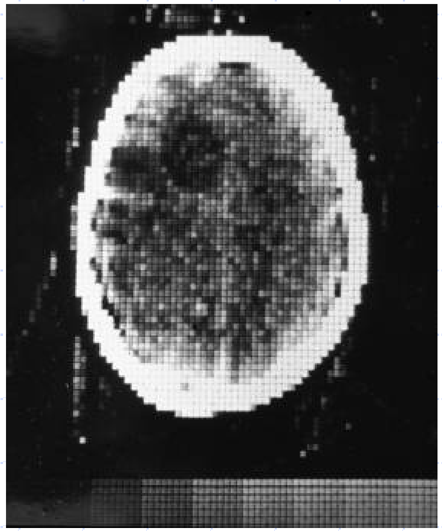
£

G. Hounsfield 1919-2004

J. Radon
1887-1956

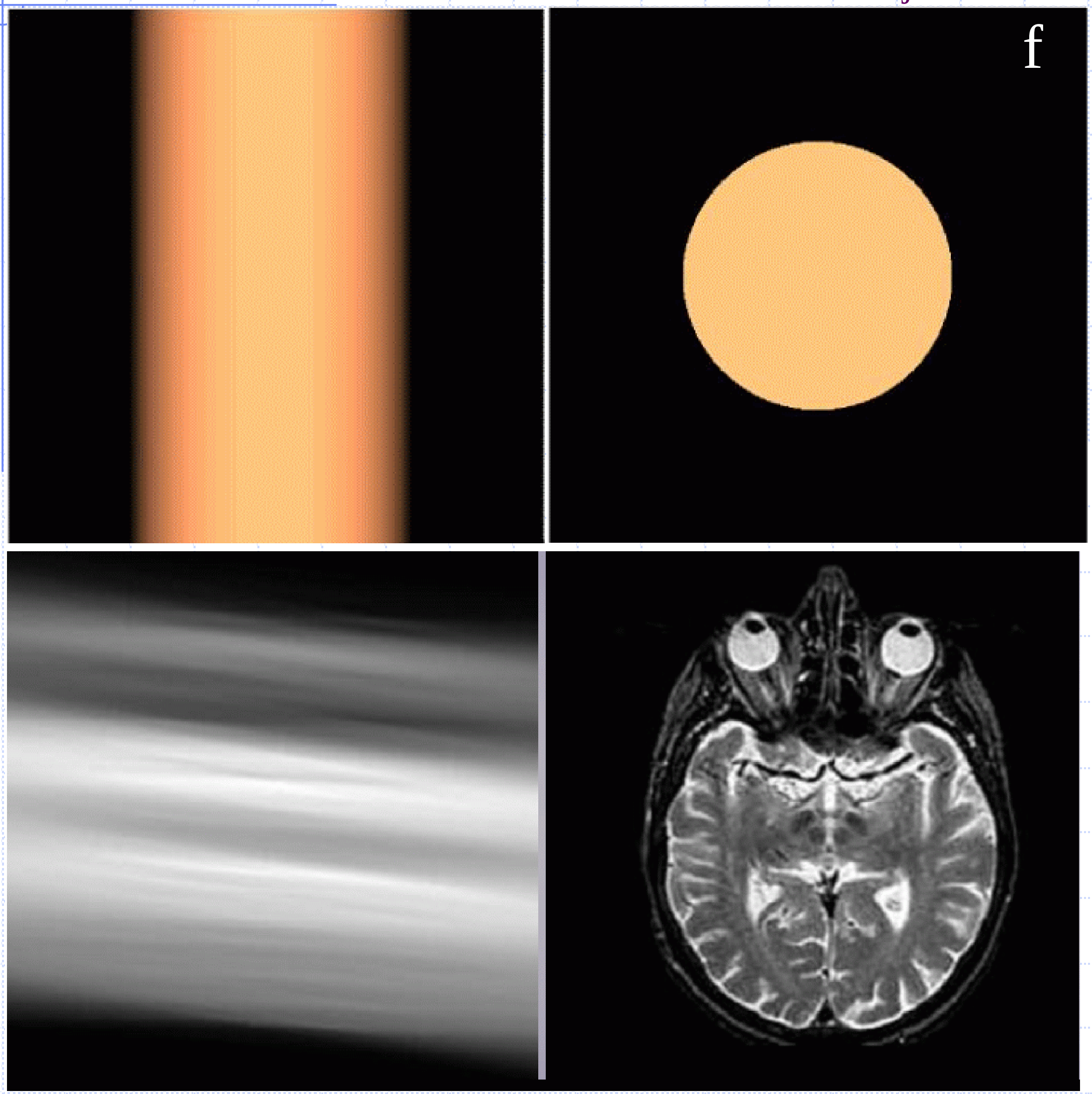
S. Kaczmarz
1895-1940

$$\hat{p}_{\vec{\theta}}(\sigma)=\hat{f}(\sigma.\vec{\theta}) \qquad \partial\vec{f}^{\,n} = \frac{p_j - p_j^{n-1}}{\|\vec{\omega}_j\|^2}\vec{\omega}_j$$



$$^*R = \text{RETROPRO}/(P.\sum_j r_{i,j}^2)$$

$^*R\,p$



Hypothèse:
$r_{i,j}=1$ si le pixel j se projette dans la raie i; $r_{i,j}=0$ sinon.

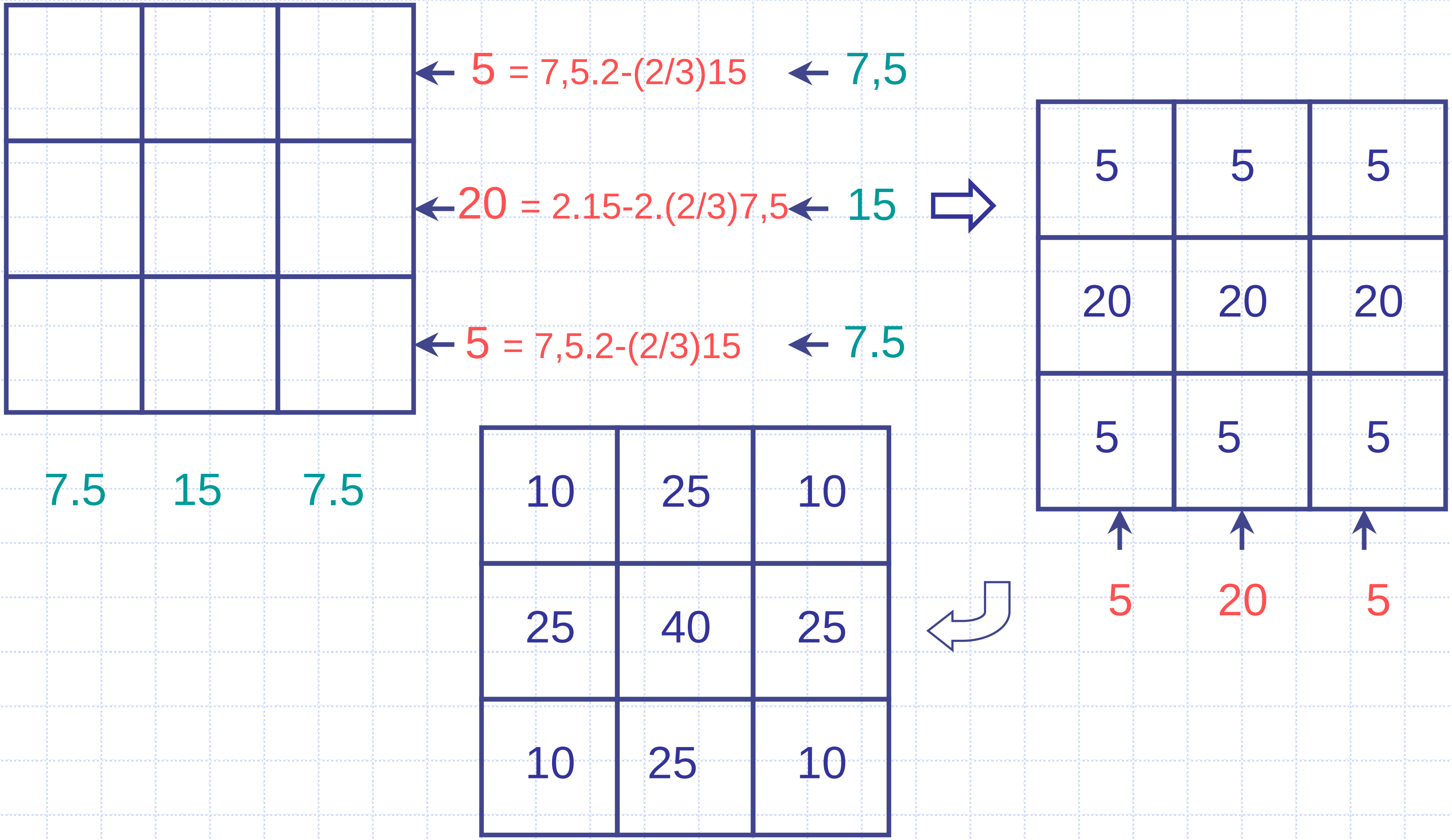
10	25	10	→ 45
25	40	25	→ 90
10	25	10	→ 45
↓ 45	↓ 90	↓ 45	

15	22,5	15	← $\frac{45}{2.3}=7.5$
22,5	30	22,5	← $\frac{90}{2.3}=15$
15	22,5	15	← $\frac{45}{2.3}=7.5$
↑ 7.5	↑ 15	↑ 7.5	$\boxed{^*R.\vec{p}}$

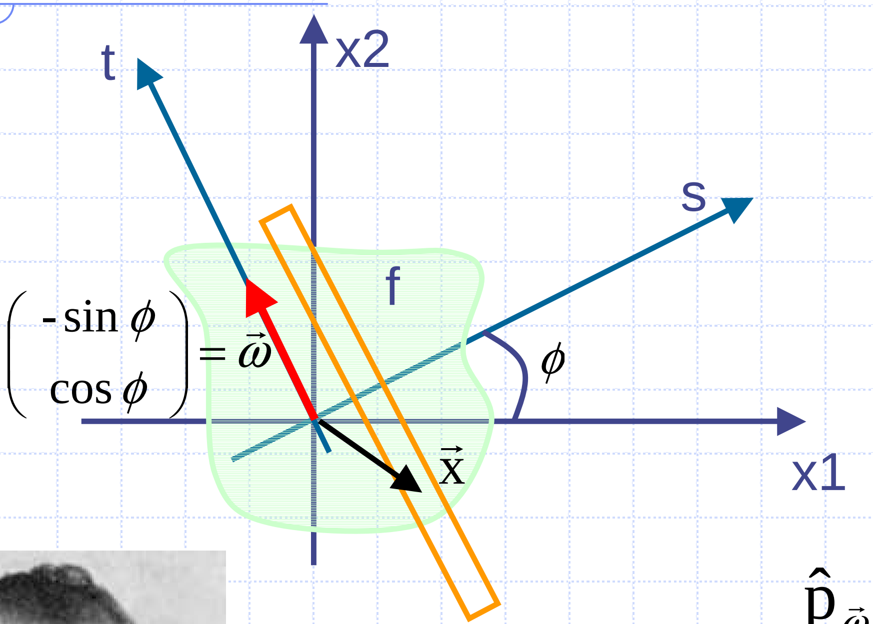
RETROPROJECTION FILTREE

Hypothèse: $r_{i,j}=1$ si le pixel j se projette dans la raie i; $r_{i,j}=0$ sinon.

$$Filtre = \begin{pmatrix} -\frac{2}{3} & 2 & -\frac{2}{3} \end{pmatrix}$$



THEOREME DE RADON



$$\begin{cases} p_{\vec{\omega}}(s) = \int_t f(s\vec{\omega}^{\perp} + t\vec{\omega})\, dt \\ \hat{p}_{\vec{\omega}}(\sigma) = \int_s p_{\vec{\omega}}(s).e^{-i.s.\sigma} ds \end{cases}$$

$$\hat{p}_{\vec{\omega}}(\sigma) = \int_s \int_t f(s\vec{\omega}^{\perp} + t\vec{\omega})\, e^{-i.s.\sigma}\, dt\, ds$$

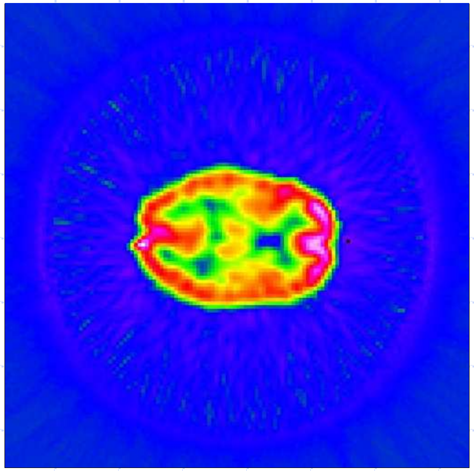
$$\hat{p}_{\vec{\omega}}(\sigma) = \iint f(\vec{x})\, e^{-i.\sigma\, \vec{x}.\vec{\omega}^{\perp}} d\vec{x}$$

$$\boxed{\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}(\sigma.\cos\phi, \sigma.\sin\phi) = \hat{f}\left(\sigma.\vec{\omega}^{\perp}\right)}$$

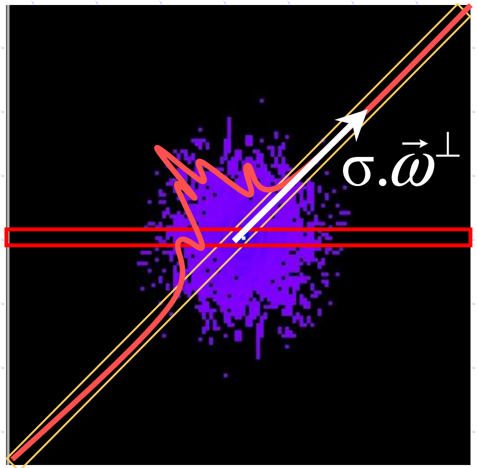
J. Radon
1887-1956

Johann Radon, ¨ Uber die Bestimmung von Funktionen durch ihre Integralwerte langs gewisser Mannigfaltigkeiten, Ber. Verh. Sach. Akad. 69 (1917), 262–77.

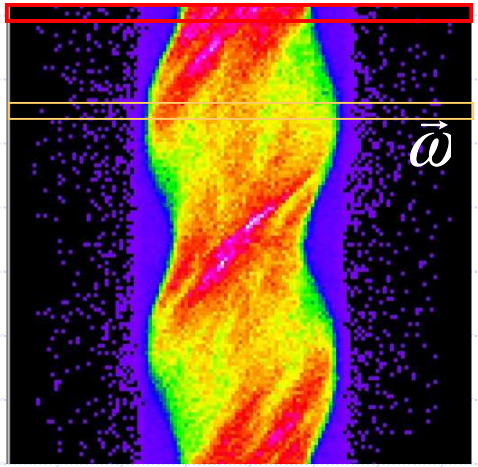
THEOREME DE RADON

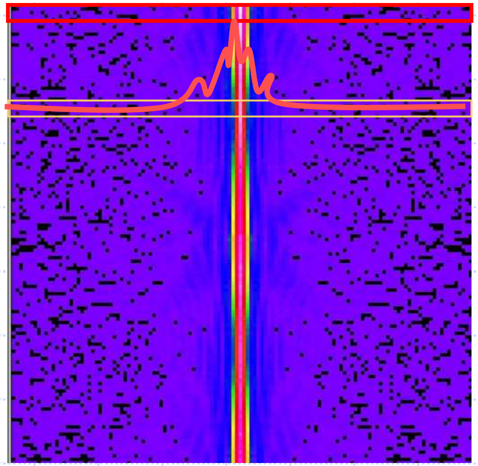


$$\boxed{\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}\left(\sigma.\vec{\omega}^{\perp}\right)}$$

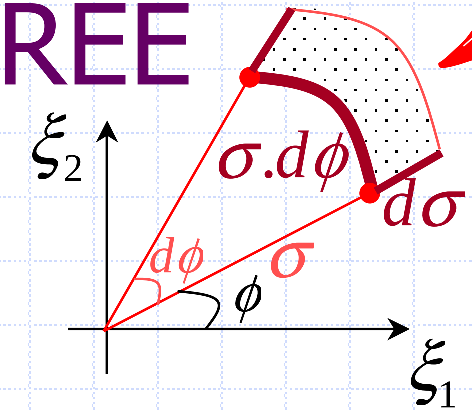


$$\begin{array}{ccc} f & \xleftarrow{TF^{-1}} & \hat{f} \\ \downarrow R & & \Updownarrow \, \| \\ p_{\vec{\omega}}(s) & \xrightarrow{TF} & \hat{p}_{\vec{\omega}}(\sigma) \end{array}$$





RETROPROJECTION FILTREE



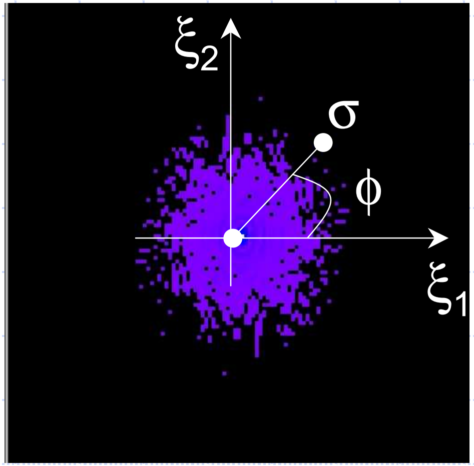
$$f(\vec{x}) = \iint \hat{f}(\vec{\xi})\, e^{i\vec{x}.\vec{\xi}}\, d\vec{\xi}$$

$$f(\vec{x}) = \int_{\phi=0}^{\pi} \int_{\sigma=-\infty}^{\sigma=+\infty} \hat{f}(\sigma\,\vec{\omega}^{\perp})\, e^{i\sigma\vec{\omega}^{\perp}.\vec{x}}\, |\sigma|\, d\sigma\, d\phi$$

$$f(\vec{x}) = \int_{\phi=0}^{\pi} \boxed{\int_{\sigma=-\infty}^{\sigma=+\infty} \hat{p}_{\vec{\omega}}(\sigma)\, |\sigma|\, e^{i\sigma\vec{\omega}^{\perp}.\vec{x}}\, d\sigma}\, d\phi$$

$$\| $$

$$\underbrace{TF_s^{-1}\left[\hat{p}_{\vec{\omega}}\, .\, \mathrm{abs}\right]}_{p^f_{\vec{\omega}}}(\vec{\omega}^{\perp}.\vec{x})$$

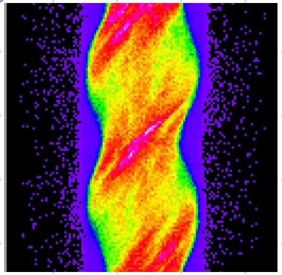


1887-1956
J. Radon

$$\boxed{f(\vec{x}) = ({}^{t}R\, p^{f})(\vec{x})}$$

Johann Radon, ¨ Uber die Bestimmung von Funktionen durch ihre Integralwerte langs gewisser Mannigfaltigkeiten, Ber. Verh. Sach. Akad. 69 (1917), 262–77.

RETROPROJECTION FILTREE



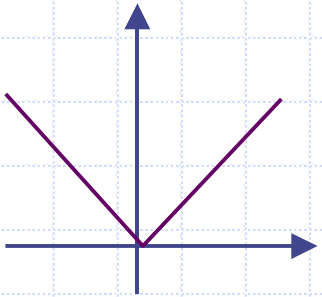
$$\boxed{f(\vec{x}) = ({}^{t}R\, p^{f})(\vec{x})}$$

Projections sur 180°

$p_{\vec{\omega}}$

$\downarrow$

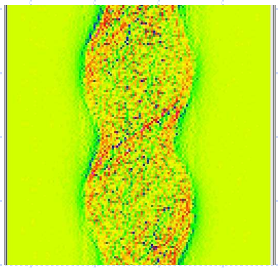
$\hat{p}_{\vec{\omega}}$ abs



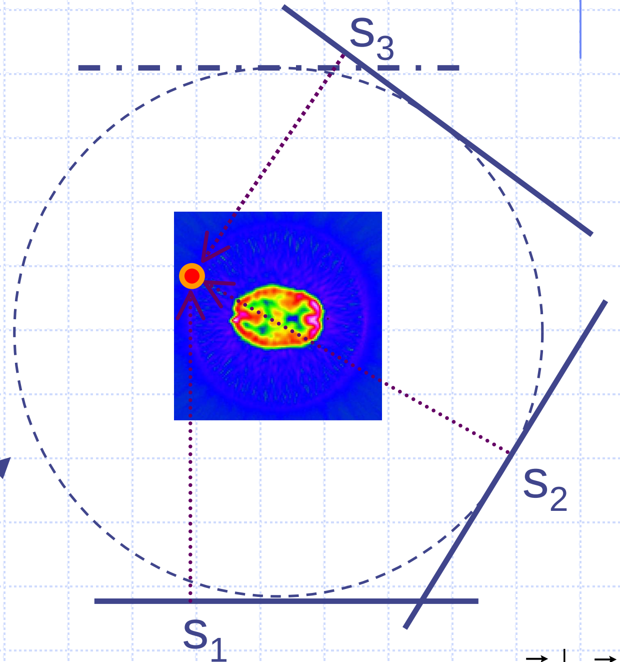
x

$\downarrow$

$$TF_s^{-1}\left[\hat{p}_{\vec{\omega}} \,.\, abs\right] = p^{f}_{\vec{\omega}}$$



${}^{t}R$

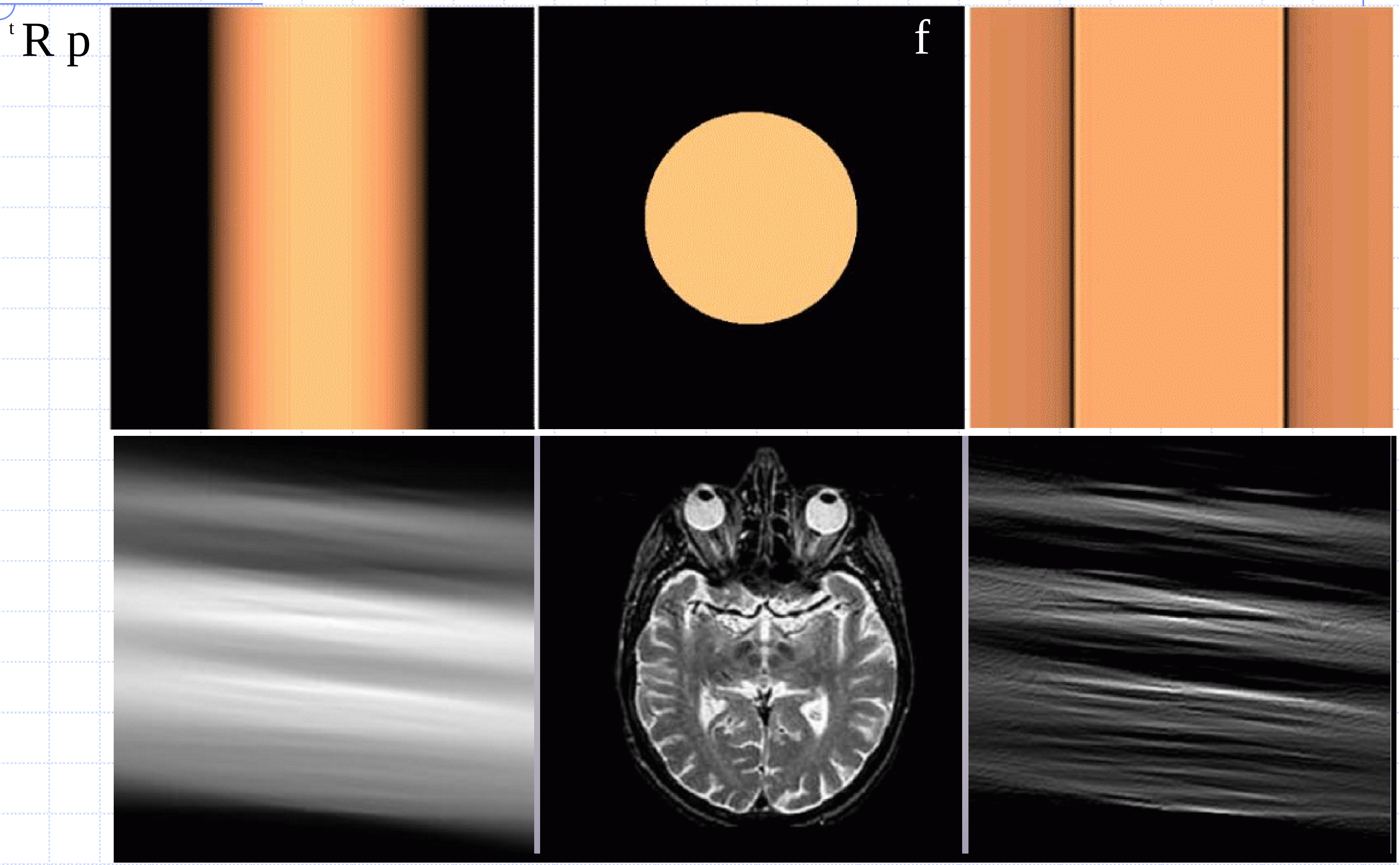


$$s_2 = \vec{\omega}^{\perp}.\vec{x}$$

$$TF^{-1}[abs] = 1/(4d^2).\begin{cases} 1 \ \text{si} \ \ i = 0 \\ -4/(\pi.i)^2 \ \text{si} \ \ i \ \text{impair} \\ 0 \ \text{si} \ \ i \ \text{pair} \end{cases}$$

$$d = FWHM/2$$

RETROPROJECTION FILTREE



LIMITES DE LA RPF

- Nécessité de données sur 180° ou sur une hémisphère
  - Problème important en TEP 3D (détecteur cylindrique)
- Prise en compte des atténuations en SPECT et PET :
  - Difficulté majeure d'introduire des facteurs du type $\exp(-\mu.L_{x,s,\phi})$

  $\rightarrow$ problème pour corriger les artefacts d'atténuation (Compton et PE)
- En revanche la correction de la réponse est possible
- Nécessité d'un filtre passe-bas supplémentaire
  - Ajustement de la fréquence de coupure en fonction du bruit dans les données
  - Dosimétrie en TDM où f_{max} = 1/LMH est élevée

TECHNIQUE DE RECONSTRUCTION ALGEBRIQUE

Hypothèse: $r_{i,j}=1$ si le pixel j se projette dans la raie i; $r_{i,j}=0$ sinon.

$$P.^{*}\mathrm{R}(\mathrm{erreur})$$

0	0	0
0	0	0
0	0	0

45 - 0 = 15 + 15 + 15

90 - 0 = 30 + 30 + 30

45 - 0

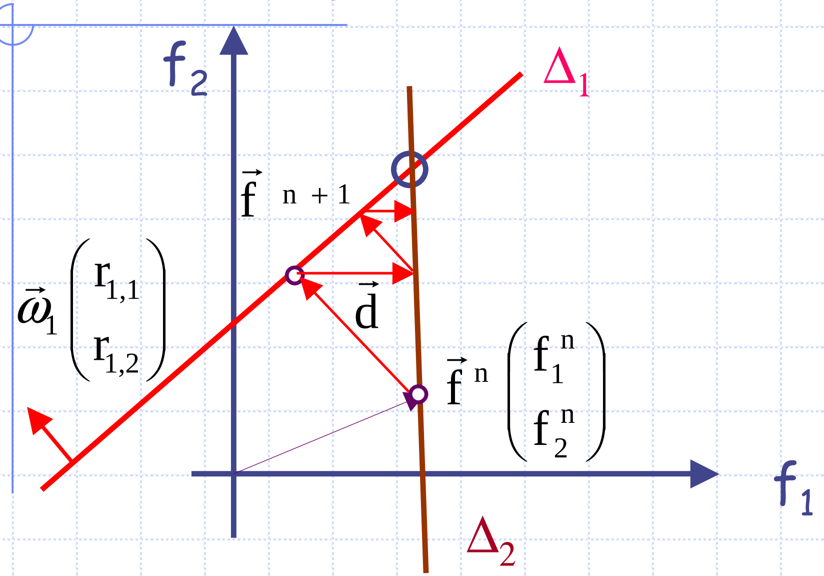
45 90 45

15	15	15
30	30	30
15	15	15

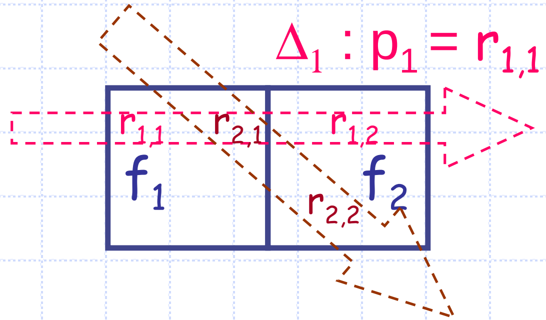
	45	90	45
-	60	60	60
	-15	30	-15

10	25	10
25	40	25
10	25	10

TECHNIQUE DE RECONSTRUCTION ALGEBRIQUE



$\Delta_1 : p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2$



$\Delta_2 : p_2 = r_{2,1}\, f_1 + r_{2,2}\, f_2$

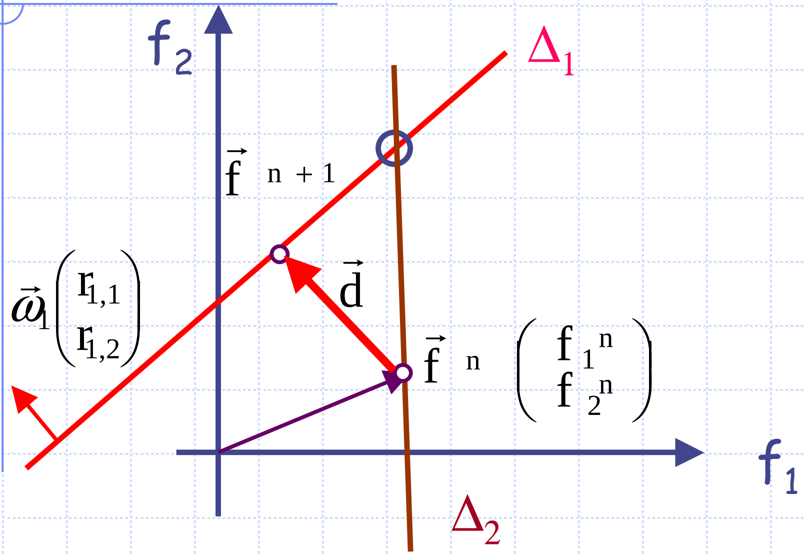
S. Kaczmarz
1895-1940

$$d = \frac{p_1 - \vec{f}^{\,n} \cdot \vec{\omega}_1}{\|\vec{\omega}_1\|} = \frac{p_1 - p_1^n}{\|\vec{\omega}_1\|}$$

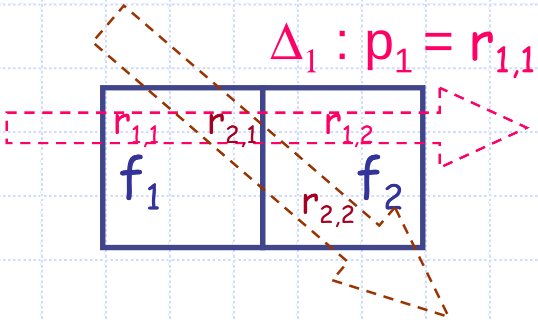
$p_1^n = r_{1,1}\, f_1^n + r_{1,2}\, f_2^n$, projection qui serait mesurée si f^n était la solution

Kaczmarz S. Angenährte Auflösung von Systemen linearer Gleichungen. *Bull Int Acad Pol Sci Lett A* 1937;35:355-7.

TECHNIQUE DE RECONSTRUCTION ALGEBRIQUE



$\Delta_1 : p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2$



$\Delta_2 : p_2 = r_{2,1}\, f_1 + r_{2,2}\, f_2$

S. Kaczmarz
1895-1940

$$\vec f^{\,n+1} = \vec f^{\,n} + d\,\frac{\vec\omega_1}{\|\vec\omega_1\|} = \vec f^{\,n} + \frac{p_1 - p_1^n}{\|\vec\omega_1\|^2}\,\vec\omega_1$$

$$f_1^{n+1} = f_1^n + \left(p_1 - [r_{1,1} f_1^n + r_{1,2} f_2^n]\right)\frac{r_{1,1}}{r_{1,1}^2 + r_{1,2}^2}$$

$$\boxed{\vec f^{\,n+1} = \vec f^{\,n} + P.^{*} R\ (p_1 - p_1^n)}$$

Kaczmarz S. Angenährte Auflösung von Systemen linearer Gleichungen. *Bull Int Acad Pol Sci Lett A* 1937;35:355-7.

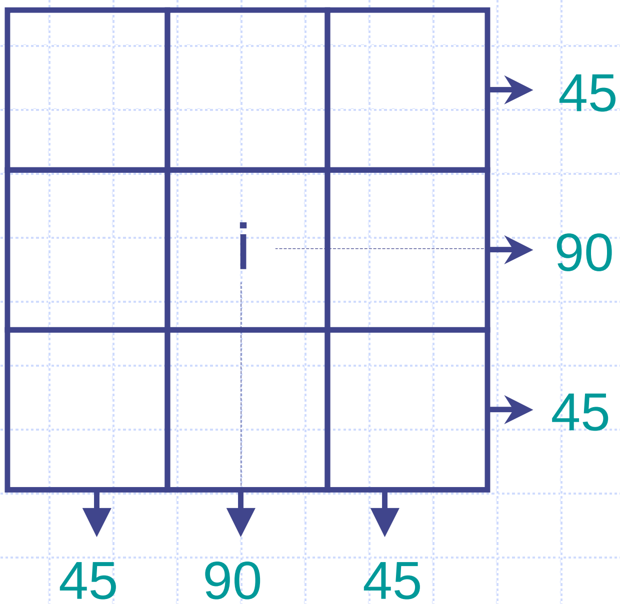
MLEM et OSEM

Hypothèse:

6 projections

$r_{i,j}=1$ si j se projette en i

$r_{i,j}=0$ sinon



$$f_i^{n+1} = f_i^n \cdot \frac{1}{\sum_{l'=1}^{P} r_{l',i}} \sum_{l=1}^{P} r_{l,i} \frac{p_l}{\sum_{s=1}^{N} r_{l,s} f_s^n}$$

$$\frac{1}{\sum_{l'=1}^{P} r_{l',i}} = \boxed{\text{Nombre de raies passant par i}}$$

$$\sum_{l=1}^{P} r_{l,i} \frac{p_l}{\sum_{s=1}^{N} r_{l,s} f_s^n} = \boxed{\sum_{Raies \supset i} \frac{p_{mesurée}}{p_{calculée}}}$$

$$f_i^{n+1} = f_i^n \cdot \frac{1}{2} \left(\frac{p^H_{mesurée}}{p^H_{calculée}} + \frac{p^V_{mesurée}}{p^V_{calculée}} \right)$$

MLEM

Hypothèse:
$r_{i,j}=1$ si le pixel j se projette dans la raie i;
$r_{i,j}=0$ sinon.

Itération 1: initialisation

			Projections mesurées	Projections estimées 1	Rapport
1	3	1	45	5	9
3	4	3	90	10	9
1	3	1	45	5	9

Itération 2

9	27	9
27	36	27
9	27	9

$$f_{i=1}^{iter2}=1.\frac{1}{2}\left(\frac{45}{5}+\frac{45}{5}\right)=9$$

$$f_{i=2}^{iter2}=3.\frac{1}{2}\left(\frac{45}{5}+\frac{90}{10}\right)=27$$

$$f_{i=5}^{iter2}=4.\frac{1}{2}\left(\frac{90}{10}+\frac{90}{10}\right)=36$$

45	90	45	Projections mesurées
45	90	45	Projections estimées 2
1	1	1	Rapports = 1: Convergence atteinte

$$f_i^{n+1}=f_i^n.\frac{1}{2}\left(\frac{p^H_{mesurée}}{p^H_{calculée}}+\frac{p^V_{mesurée}}{p^V_{calculée}}\right)$$

MLEM (Maximum likelihood Expectation Maximization)

Bayes : $P(\vec{f}\,/\,\vec{p}) = P(\vec{p}/\vec{f}).\,P(\vec{f})/P(\vec{p}) = P(\vec{p}/\vec{f}).\,P(\vec{f})$

$$\boxed{\tilde{\vec{f}} = \arg\min_{\vec{f}}\left[-\log P(\vec{p}/\vec{f}) - \cancel{\log P(\vec{f})}\right]}$$

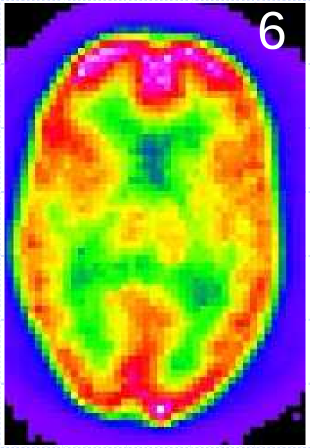
régularisation

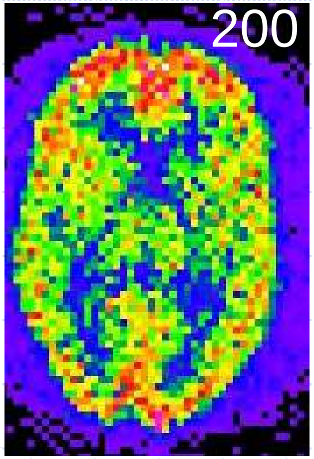
Vraisemblance = adéquation aux données

$$\log \prod_i \frac{e^{-\tilde{p}_i}\,\tilde{p}_i^{\,p_i}}{p_i!} \quad \text{où} \quad \tilde{p}_i = \sum_s r_{i,s}.f_s$$

$$\Downarrow$$

$$f_i^{n+1} = f_i^n \cdot \frac{1}{\sum_{l'=1}^{P} r_{l',i}} \sum_{l=1}^{P} r_{l,i} \frac{p_l}{\sum_{s=1}^{N} r_{l,s} f_s^n} = f_i^n . R^{*'}\left[\frac{p_l}{p_l^n}\right]$$





Dempster A et al. Maximum likelihood from incomplete data via the EM algorithm. J R Stat Soc 1977;39:1-38.

OSEM (Ordered Subsets Expectation Maximization)

Bayes : $P(\vec{f} / \vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})/P(\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})$

$$\tilde{\vec{f}} = \arg\min_{\vec{f}} \left[-\log P(\vec{p}/\vec{f}) - \cancel{\log P(\vec{f})} \right]$$

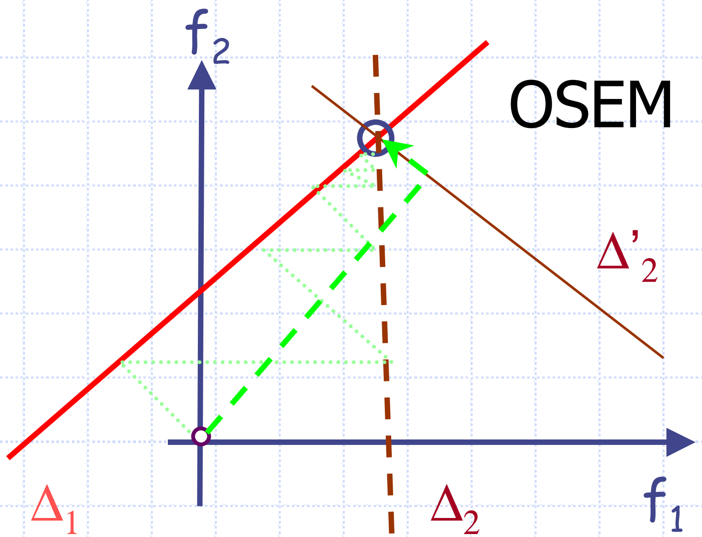
régularisation

Vraisemblance = adéquation aux données

$$\log \prod_i \frac{e^{-\tilde{p}_i}\, \tilde{p}_i^{\,p_i}}{p_i!} \quad \text{où} \quad \tilde{p}_i = \sum_s r_{i,s}.f_s$$

$$\Downarrow$$

$$f_i^{n+1} = f_i^n . \frac{1}{\sum_{l'=1}^{P} r_{l',i}} \sum_{l=1}^{P} r_{l,i} \frac{p_l}{\sum_{s=1}^{N} r_{l,s} f_s^n} = f_i^n . R^{*'}\left[\frac{p_l}{p_l^n} \right]$$



Dempster A et al. Maximum likelihood from incomplete data via the EM algorithm. J R Stat Soc 1977;39:1-38.
Hudson H et al. Accelerated image reconstruction using ordered subsets of projection data. IEEE Trans Med Imaging 1994;13:601-9.

REGULARISATION MAP-EM-OSL

Bayes : $P(\vec{f}\,/\,\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})/P(\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})$

$$\boxed{\tilde{\vec{f}} = \arg\min_{\vec{f}} \left[-\log P(\vec{p}/\vec{f}) - \log P(\vec{f}) \right]}$$

Adéquation aux données

$$\log \prod_i \frac{e^{-\tilde{p}_i}\,\tilde{p}_i^{\,p_i}}{p_i!}$$

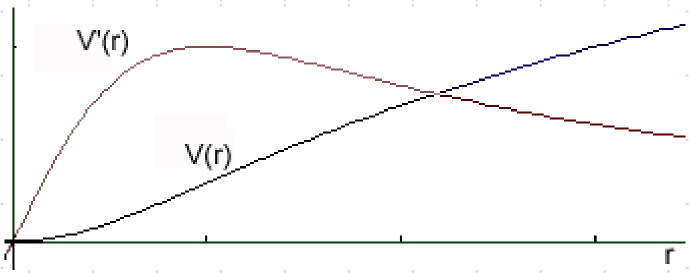
régularisation

Gibbs : $P(\vec{f}) = \frac{1}{K} e^{-\beta.\sum_{i,j} w_{i,j}.V\left(f_i - f_j\right)}$

$\Downarrow$

$$f_i^{n+1} = f_i^n . \frac{1}{\sum_{l'=1}^{P} r_{l',i} + \boxed{\beta.\partial U}} . \sum_{l=1}^{P} r_{l,i} \frac{p_l}{\sum_{s=1}^{N} r_{l,s} f_s^n}$$

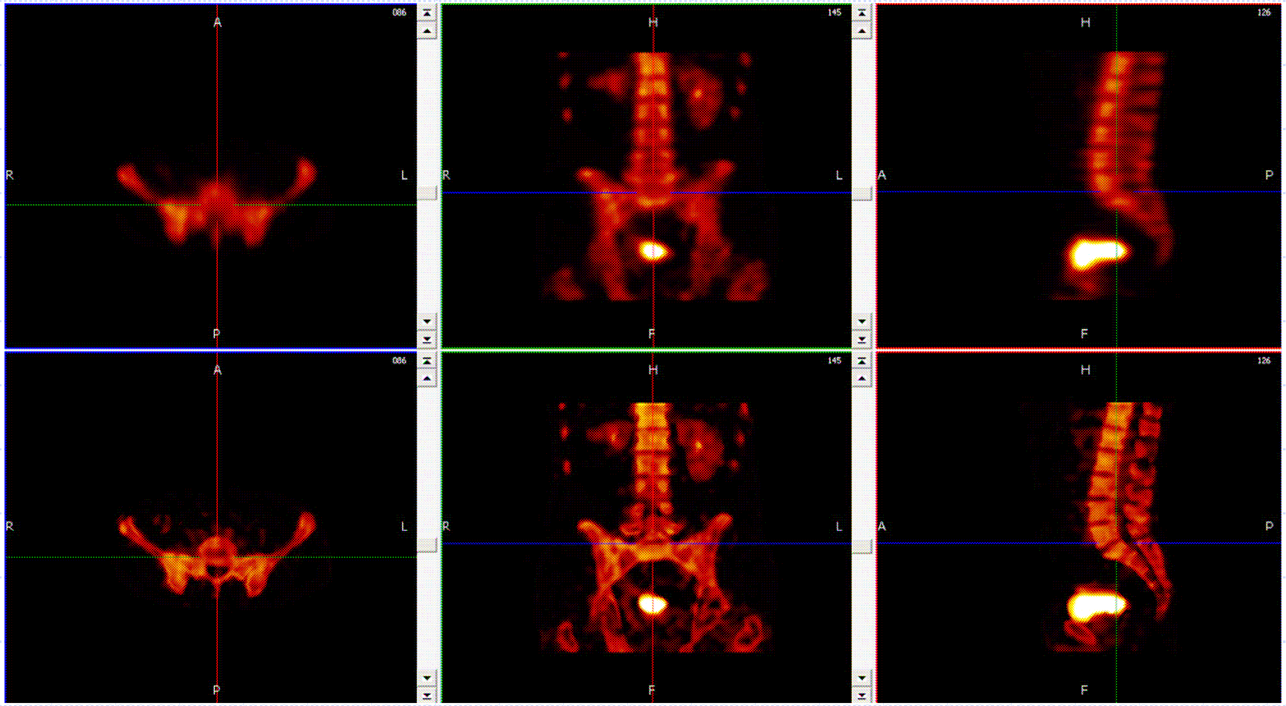
$$\partial U = \sum_{f_k \in \text{voisin}(f_i)} w_{i,k}.\frac{\partial V}{\partial r}(f_i - f_k)$$



Green PJ. Bayesian reconstructions from emission tomography data using a modified EM algorithm. IEEE Trans Med Imaging 1990;9:84-93

ALGORITHME DU GRADIENT

$$\tilde{\vec{f}} = \arg\min_{\vec{f}} \left[\sum_j \frac{(p_j - [R\vec{f}\,]_j)^2}{\sigma_{p_j}^2} \right] = \arg\min_{\vec{f}} \left\| \vec{p} - R\vec{f} \right\|^2$$



xSPECT-Bone (Siemens)

$n \in$ { air, gras, mou, os médullaire et cortical }

$\forall$ n, $I_n = z_n(\mathrm{TDM}).\, I$
$p_n = \mathrm{PRO}(I_n)$
$p = \Sigma_n\, c_n\, p_n$

$\mathrm{GC}(p^{\text{mesurées}}, p) \rightarrow c_n > 0$

$p = \Sigma_n\, c_n\, p_n$
$I = \mathrm{RETROPRO}(p)$

A. Hans Vija, Introduction to xSPECT* Technology. Siemens White Paper. 2013

ALGORITHME DU GRADIENT

$$\tilde{\vec{f}} = \arg\min_{\vec{f}} \left[\sum_j \frac{(p_j - [R\vec{f}\,]_j)^2}{\sigma_{p_j}^2} \right] = \arg\min_{\vec{f}} \left\| \vec{p} - R\vec{f} \right\|^2$$

$$\vec{d}_0 = \vec{r}_0 = {}^tR\ \vec{p}$$

$$\omega_n = \frac{\left\|\vec{r}_n\right\|^2}{\left\langle \vec{d}_n \,\middle|\, {}^tR.R.\vec{d}_n \right\rangle}$$

$$\vec{r}_{n+1} = \vec{r}_n - \omega_n.{}^tR.R.\vec{d}_n$$

$$\beta_n = \frac{\left\|\vec{r}_{n+1}\right\|^2}{\left\|\vec{r}_n\right\|^2}$$

$$\vec{d}_{n+1} = \vec{r}_{n+1} + \frac{\left\|\vec{r}_{n+1}\right\|^2}{\left\|\vec{r}_n\right\|^2}.\vec{d}_n$$

$$\vec{f}_{n+1} = \vec{f}_n + \omega_n.\vec{d}_n$$

$$G_n = \begin{pmatrix} \frac{1}{\omega_0} & -\frac{\sqrt{\beta_0}}{\omega_0} & 0 & 0 \\ -\frac{\sqrt{\beta_0}}{\omega_0} & \frac{1}{\omega_1} + \frac{\beta_0}{\omega_0} & \ddots & 0 \\ 0 & \ddots & \ddots & -\frac{\sqrt{\beta_{n-1}}}{\omega_{n-1}} \\ 0 & 0 & -\frac{\sqrt{\beta_{n-1}}}{\omega_{n-1}} & \frac{1}{\omega_n} + \frac{\beta_{n-1}}{\omega_{n-1}} \end{pmatrix}$$

$$\boxed{\kappa(G_n) \xrightarrow{n\to\infty} \kappa(R)}$$

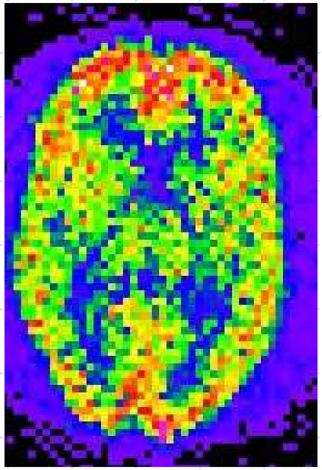
www.inma.ucl.ac.be/~vdooren/Krylov.pdf

Analyse numérique matricielle appliquée à l'art de l'ingénieur. P. Lascaux & R. Théodor (II), p. 516, 1987. MASSON
D. Mariano-Goulart, P. Maréchal, S. Gratton, et al. Comput. Med. Imaging & Graphics 2007; 31 : 502-509

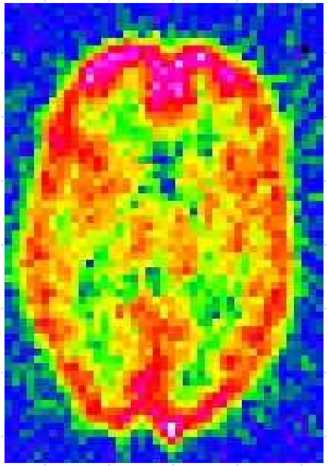
REGULARISATION FRECT

Tikhonov: $\vec{f} = \underset{\vec{f}}{\arg\min}\ [\ \|\vec{p} - R\vec{f}\|^2 + \alpha.\|\vec{f}\|^2\ ]$

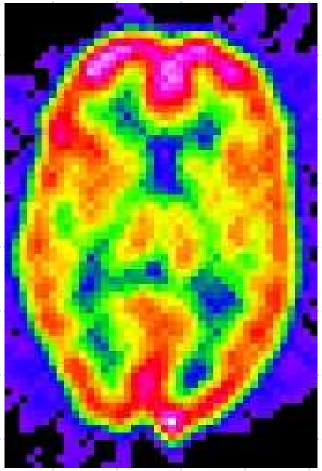
FRECT : $\vec{f} = \underset{\vec{f}}{\arg\min}\ [\ \|\mathbf{PB}(\vec{p}) - R\vec{f}\|^2 + \|\mathbf{PH}(\vec{f})\|^2\ ]$



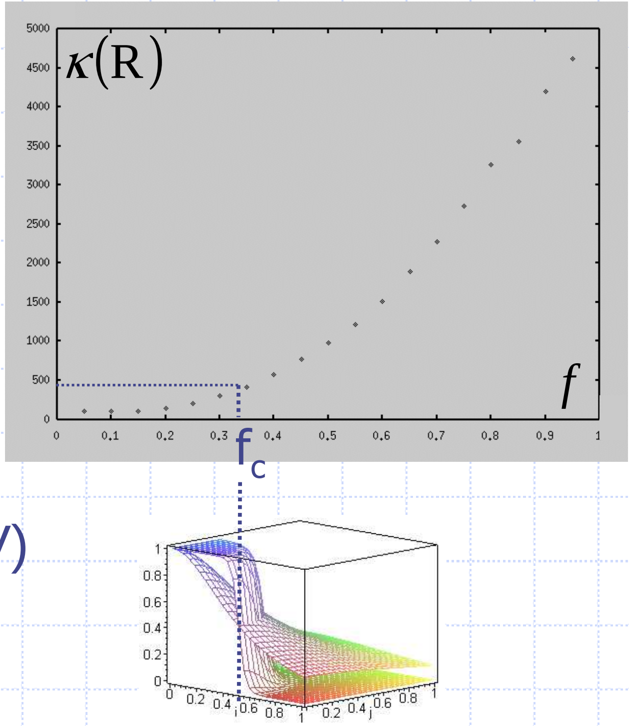
MLEM 200



GC 16



FRECT 34 (CV)



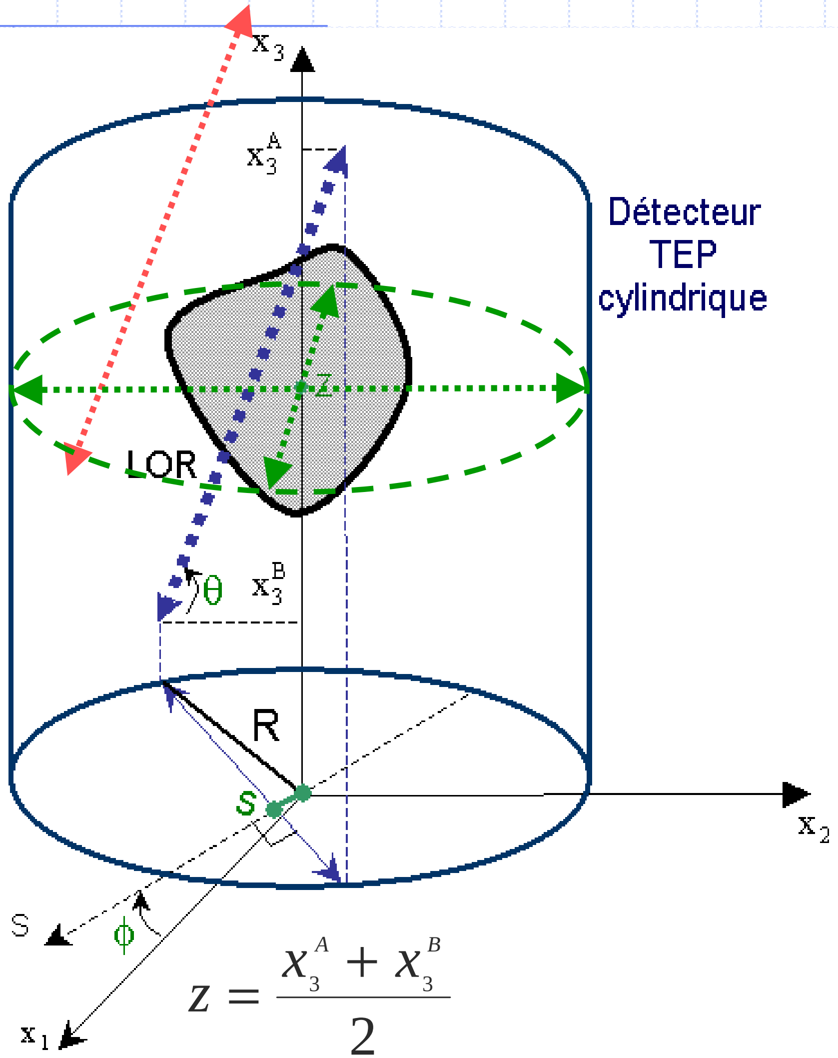
D Mariano-Goulart, P Maréchal, S Gratton, L Giraud, M Fourcade. *Comput Med Imaging Graph* 07 & *Lect Notes Comput Sci*. 04

AVANTAGES DES ALGO ALGEBRIQUES

- Ne nécessitent pas de projections complètes (sur 180° ou une hémisphère)
  - essentiel en PET 3D
- Permettent une modélisation dans l'opérateur R des artefacts d'atténuation
  - Important en SPECT-CT, essentiel et PET-CT
- Permettent une régularisation plus sophistiquée (non linéaire) et plus ou moins facilement paramétrable

Dempster A et al. Maximum likelihood **from incomplete data** via the EM algorithm. J Royal Stat Soc. **1977**; 39:1-38.

SPECIFICITES EN TEP 3D



Projections 3D redondantes

- ? f(x,y,z) connaissant p(s,ϕ,z,δ)
- 1 DDL de plus (δ = tgθ) dans p
- Les données transverses sont complètes et suffisent

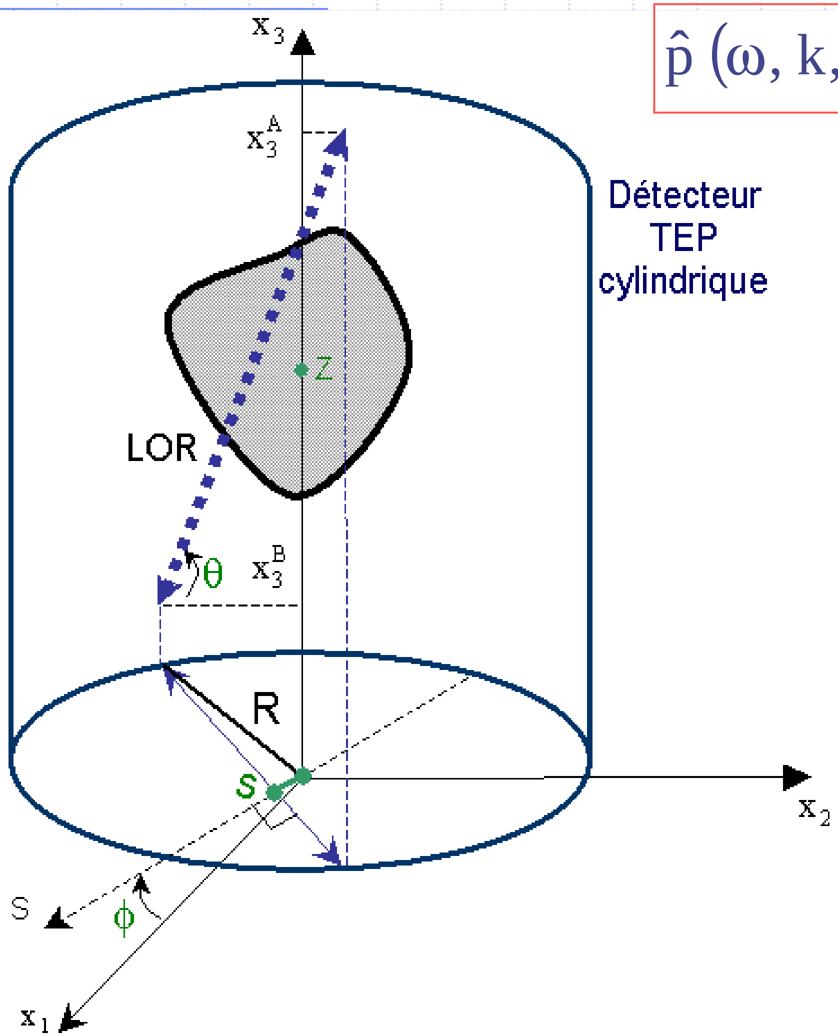
et incomplètes

- certaines projections obliques ne sont pas enregistrées si θ ≠ 0
- RPF 3 D impossible directement

Il faut donc :

- soit opter pour OSEM3D
- soit estimer les projections manquantes pour reconstruire en RPF 2D ou 3D

PROJECTIONS MANQUANTES



$$\hat{p}(\omega, k, \zeta, \delta) = e^{-ik \arctan(\alpha)}\, \hat{p}\left(\omega\sqrt{1+\alpha^2}, k, \zeta, 0\right)$$

DL à l'ordre 1 sur $\alpha = \dfrac{\delta\zeta}{\omega}$

$$\hat{p}(\omega, k, \zeta, \delta) \approx e^{-ik\alpha}\, \hat{p}(\omega, k, \zeta, 0)$$

$$\hat{p}(\omega, k, z, \delta) \approx \hat{p}\left(\omega, k, z\text{-}k\frac{\delta}{\omega}, 0\right)$$

SYNTHESE DE DONNEES 2D à S/B ↑ :

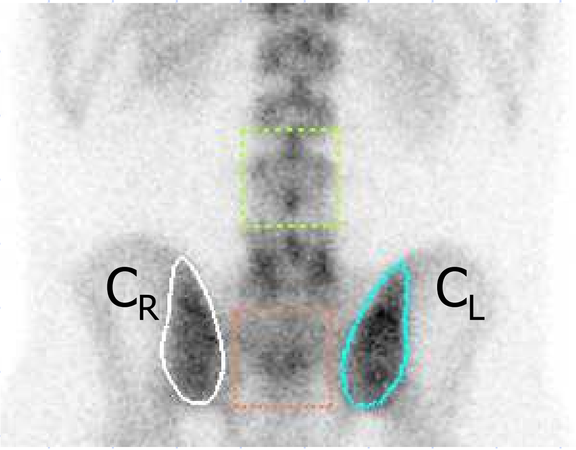
$$\boxed{\hat{p}(\omega, k, z, 0) \approx \hat{p}\left(\omega, k, z+k\frac{\delta}{\omega}, \delta\right)}$$

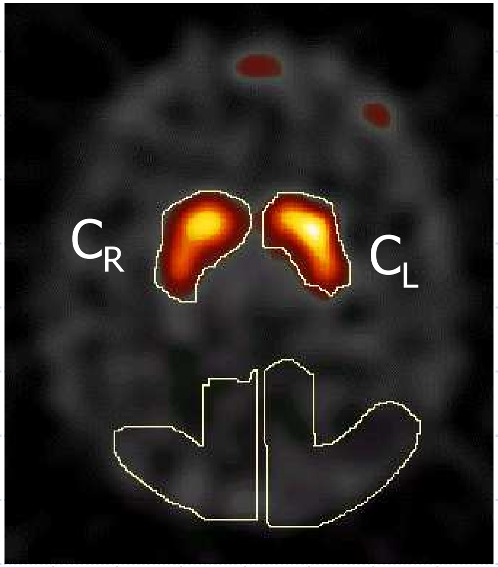
SYNTHESE DE DONNEES MANQUANTES :

$$\boxed{\hat{p}(\omega, k, z, \delta) \approx \hat{p}\left(\omega, k, z-k\frac{(\delta-\delta')}{\omega}, \delta'\right)}$$

Defrise M, Kinahan PE, Townsend DW, Michel C, Sibomana M, Newport DF. IEEE Trans Med Imaging 1997;16:145-58.
Ben Bouallègue F, Crouzet JF, Comtat C, Fourcade M, Mohammadi B, Mariano-Goulart D.IEEE Trans Med Imaging 2007;26:1001-9.

UN PEU DE RECHERCHE…





$$C_R \overset{?}{=} C_L$$

Planaire :

- Bruit de Poisson: $\sigma_R^2 = C_R$

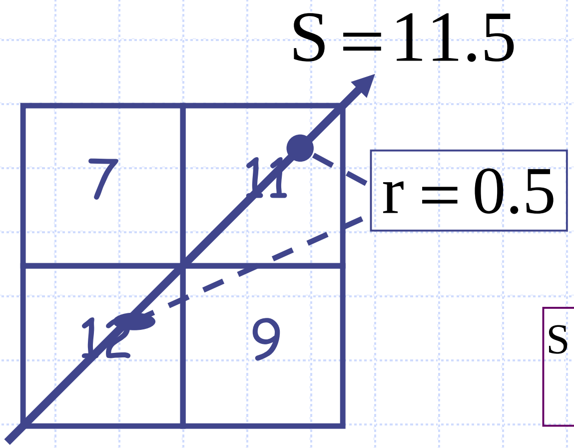
Tomograhie :

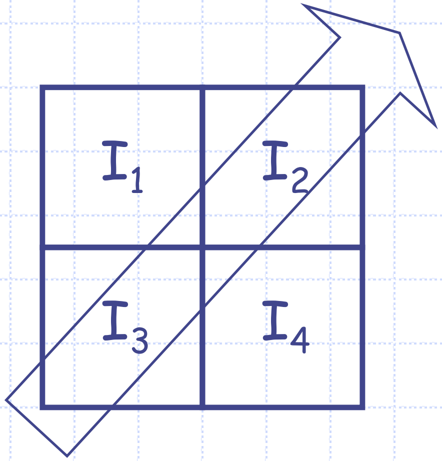
- σ n'est pas connue
- Transfert de variance (Fessler)

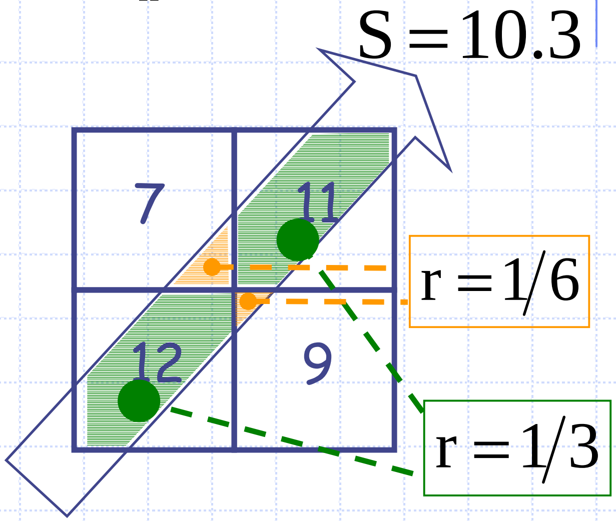
T Budinger et al. J Comp Assist Tomog 1977;1 - JA Fessler. IEEE Tr. Im Proc. 1996;5(3):493-506 – D. Mariano-Goulart EANM 2009

MODELES DE PROJECTEURS

$$S_k = \sum_n r_{k,n} I_n$$

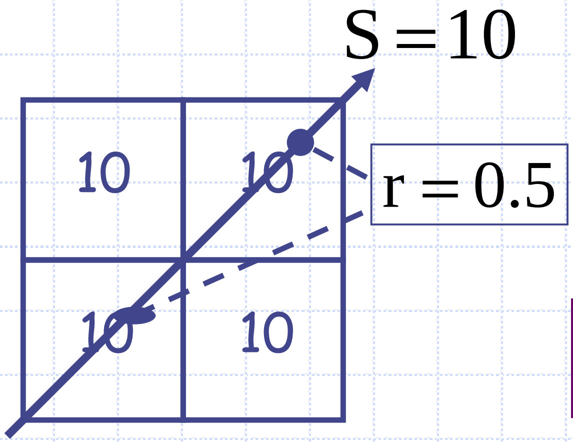


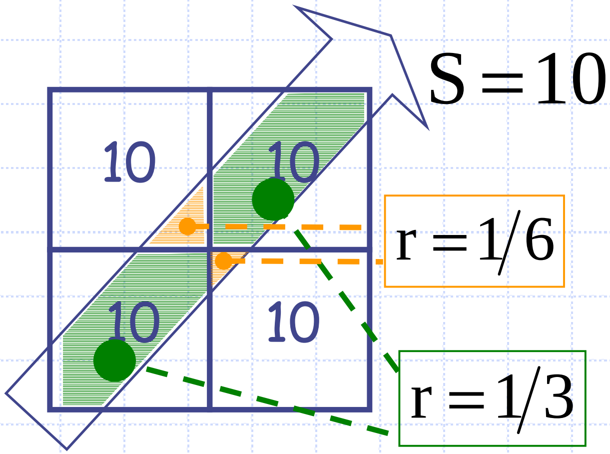




$S = [S] = [\underline{S}, \overline{S}] = [10.3 ; 11.5]$

$\delta = 11.5 - 10.3 = 1.2$



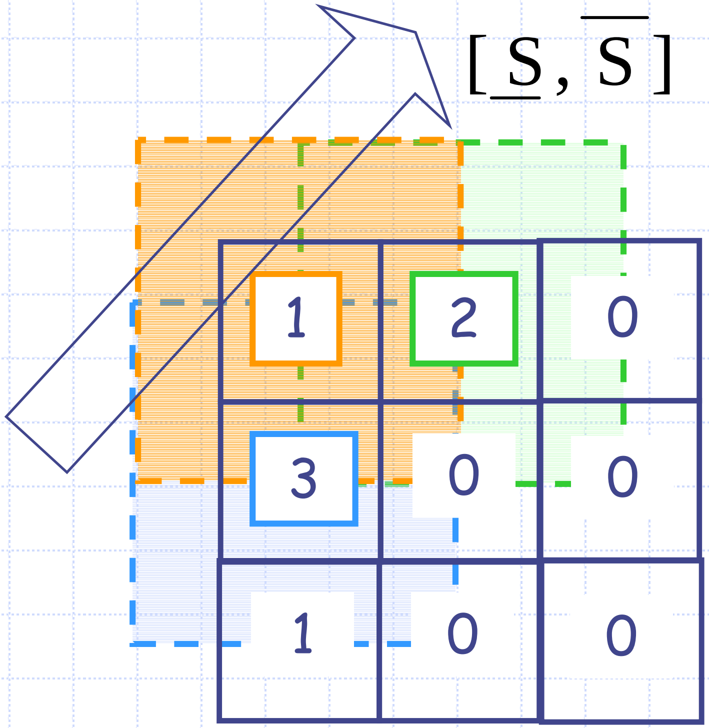


$S = [S] = [\underline{S}, \overline{S}] = [10 ; 10]$

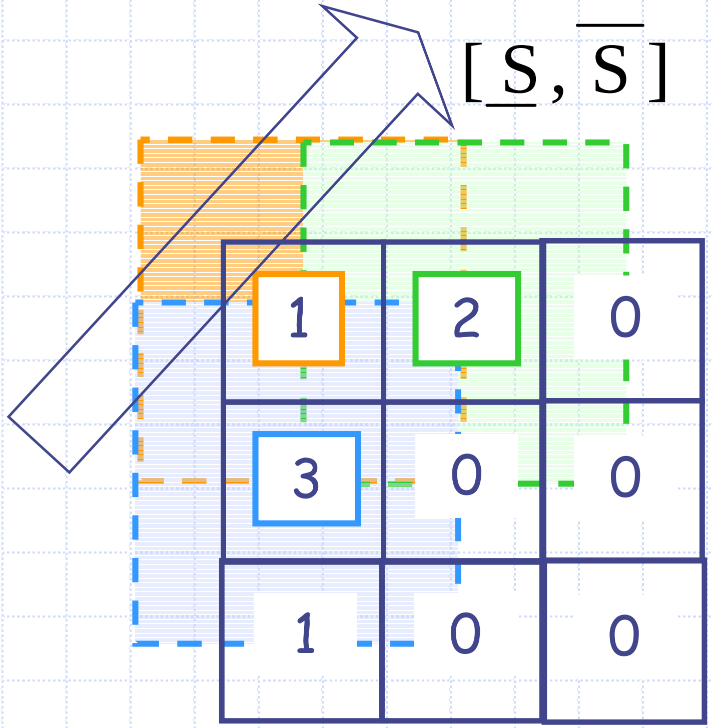
$\delta = 0$

PROJECTION PAR INTERVALLE

$[\underline{S}, \overline{S}]$



$[\underline{S}, \overline{S}]$



$\underline{S} = (1.4+2.2+3.2)/8=1.8$ $\overline{S} = (3.4+2.3+1.1)/8=2.4$

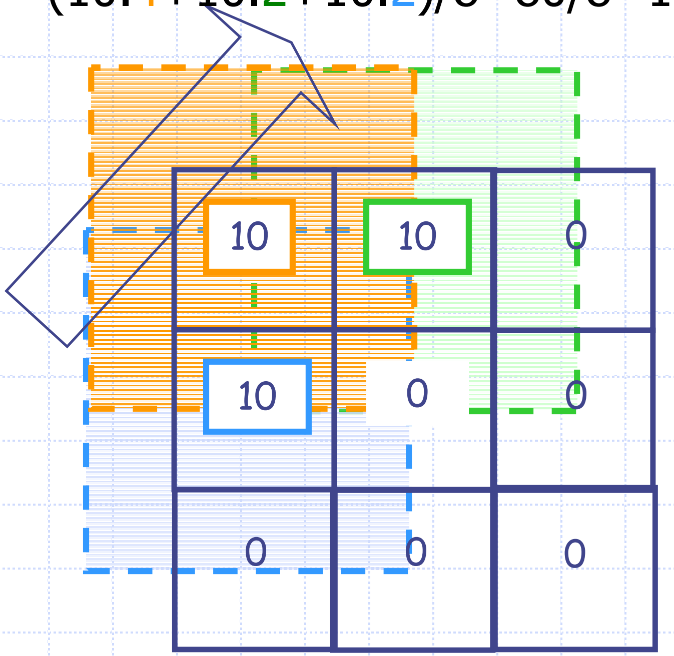
$$[S]=[1.8;2.4]$$

Choquet, G. (1953). "Theory of capacities". Annales de l'Institut Fourier **5**: 131–295.
A. Rico, **O. Strauss**, D. Mariano-Goulart. *Fuzzy Sets and Systems* 2009; 160(2):198-211

PROJECTION PAR INTERVALLE

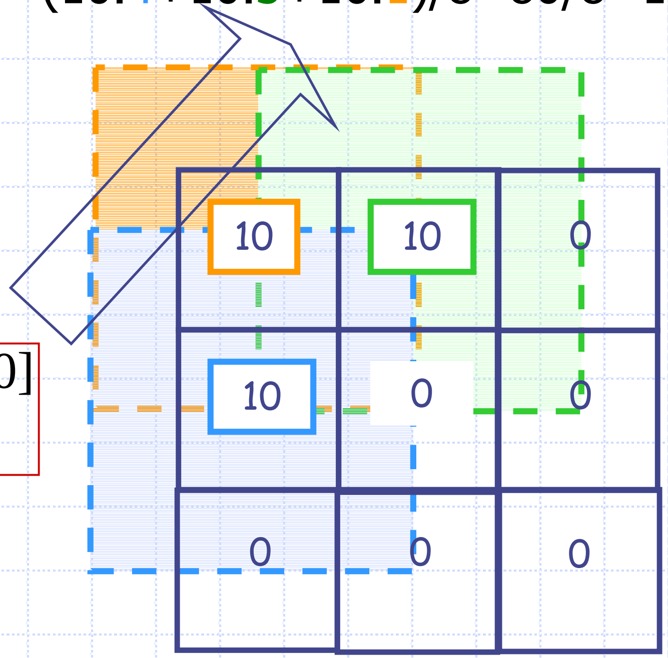
$\underline{S} = (10.4+10.2+10.2)/8=80/8=10$

$\overline{S} = (10.4+10.3+10.1)/8=80/8=10$



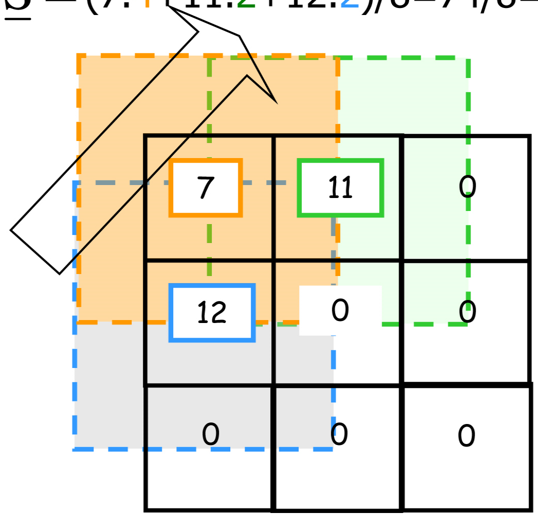
$[S]=[10,10]$

$\delta = 0$

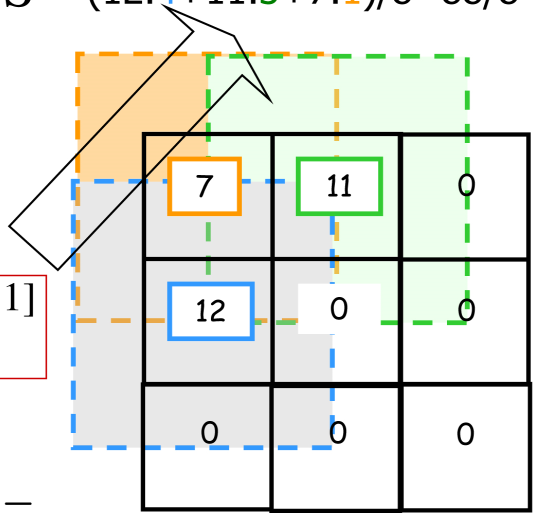


PROJECTION PAR INTERVALLE & BRUIT

$\underline{S} = (7.4+11.2+12.2)/8=74/8=9.25$



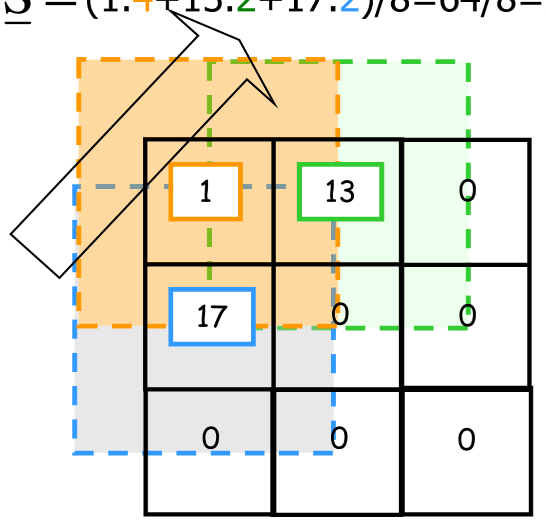
$\overline{S} = (12.4+11.3+7.1)/8=88/8=11$



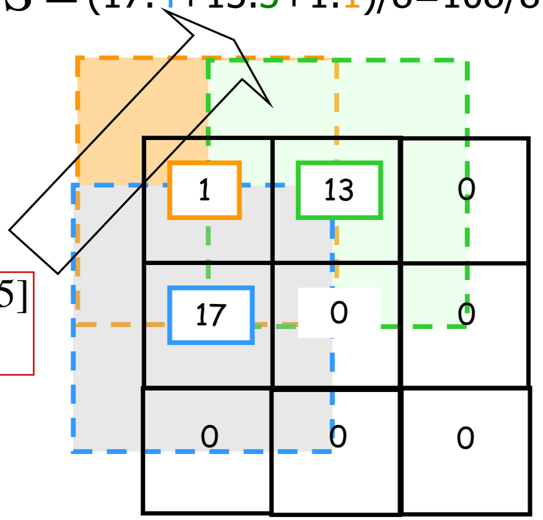
[S]=[9.25; 11]

$\delta=1.75$

$\underline{S} = (1.4+13.2+17.2)/8=64/8=8$



$\overline{S} = (17.4+13.3+1.1)/8=108/8=13.5$



[S]=[8 ; 13.5]

$\delta=5.5$

ART INTERVALLISTE (NIBART)

- Modélisation: logique floue (capacités)
- Pro/Rétroprojection: Intégrale de Choquet
- SIRT avec des opérateurs de Minkowski:

$$\boxed{[I^{n+1}] = [I^{n}] \oplus \mathbf{B}[\,S - \mathbf{P}([I^{n}])\,]}$$

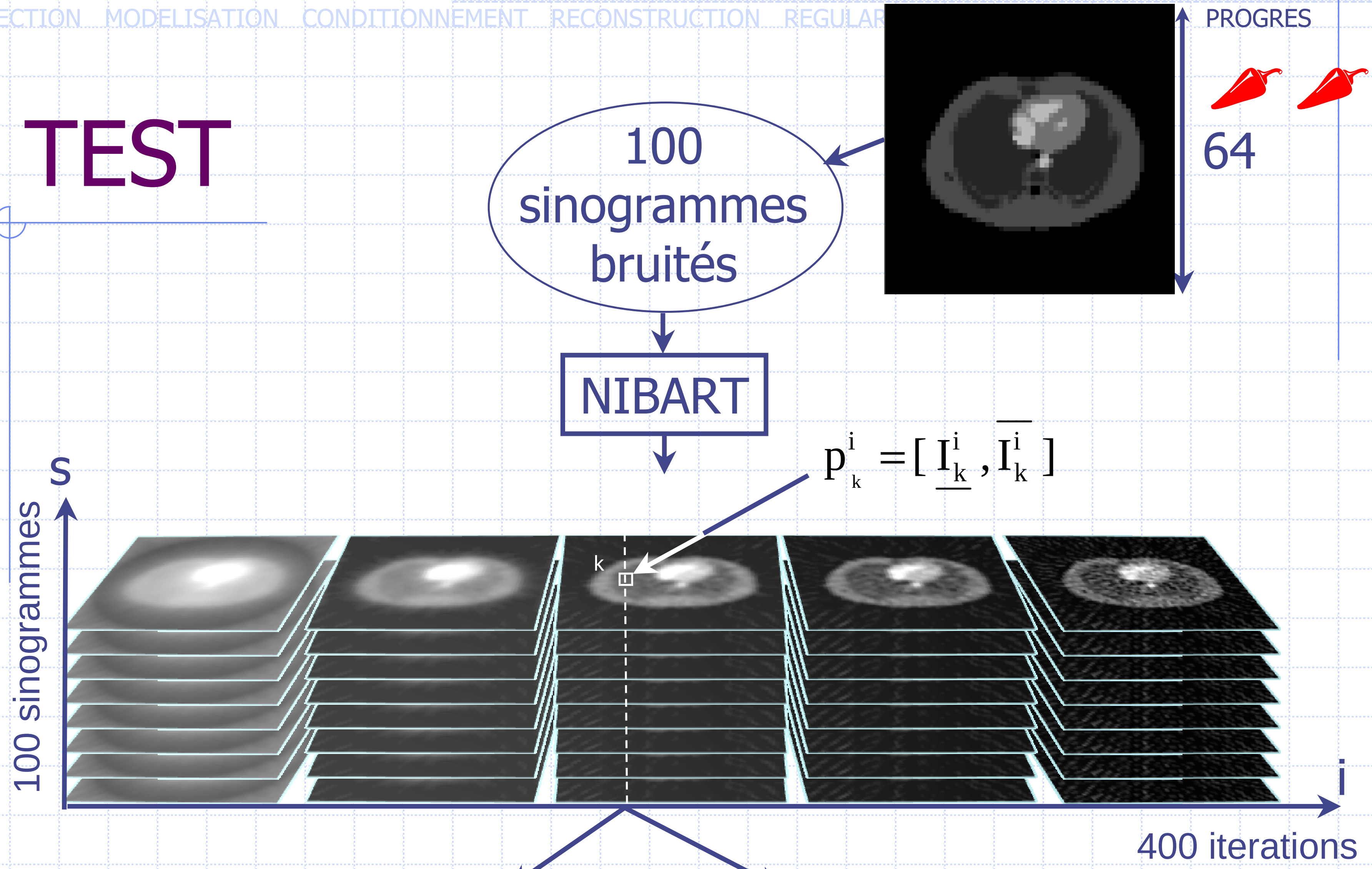
$$[\underline{a},\overline{a}] \oplus [\underline{b},\overline{b}] = [\underline{a}+\overline{b},\ \overline{a}+\underline{b}] \text{ or } [\overline{a}+\underline{b},\underline{a}+\overline{b}]$$

$$[\underline{a},\overline{a}] - [\underline{b},\overline{b}] = [\underline{a}-\overline{b},\overline{a}-\underline{b}]$$

- NIBEM en cours de publication…

O. Strauss, A Lahrech, A Rico, D. Mariano-Goulart, B Telle. MICCAI 2009, Part 1, LNCS 5761, p. 148-155

TEST

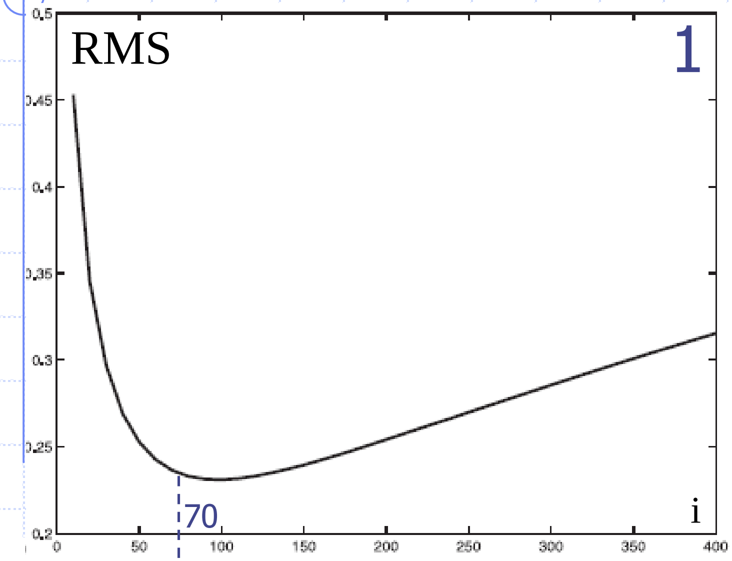


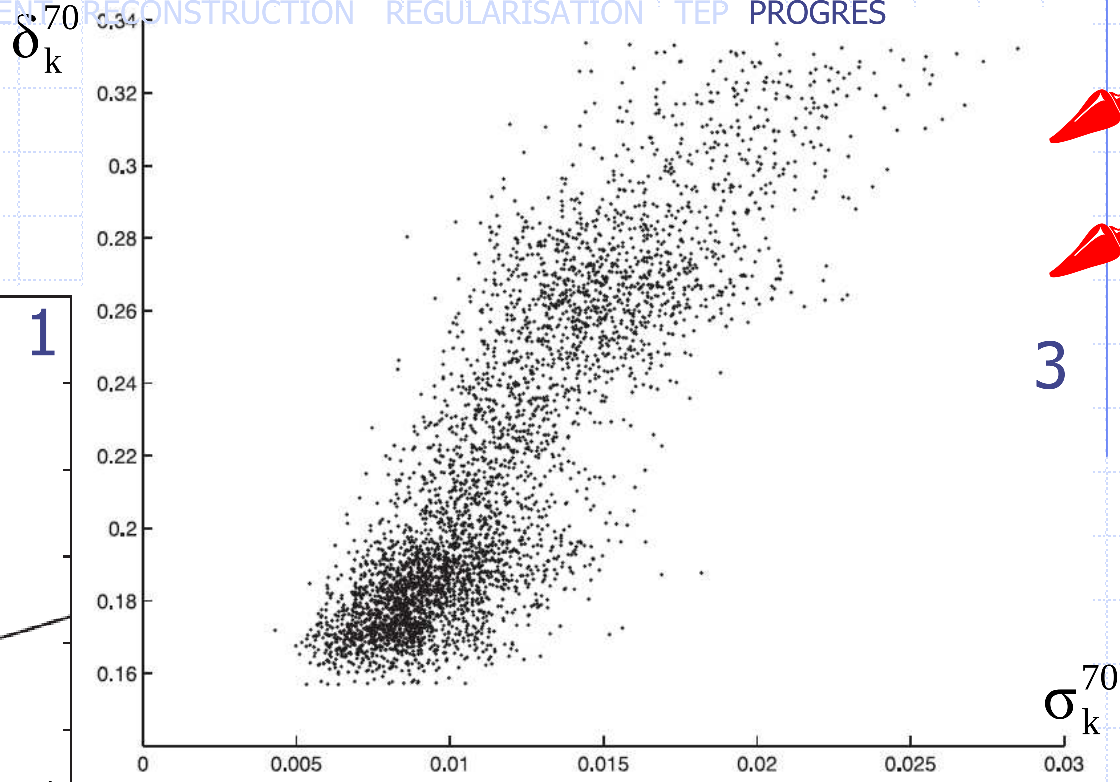
$$\sigma^i_k = SD_s \left[\frac{\underline{I^{i,s}_k} + \overline{I^{i,s}_k}}{2} \right]$$

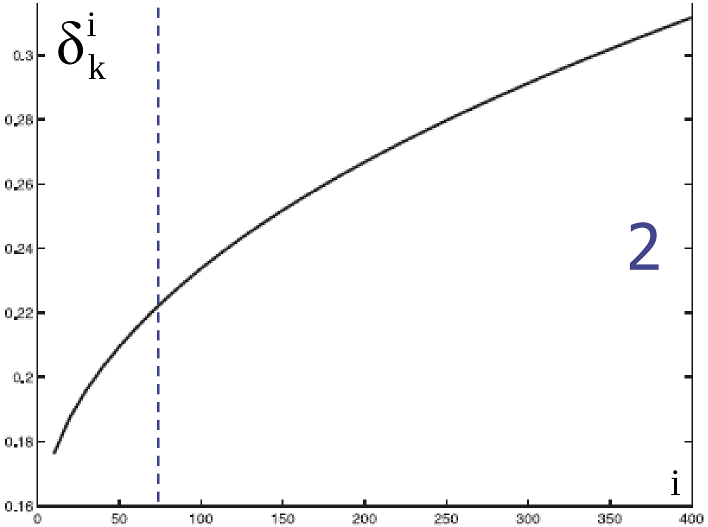
?

$$\delta^i_k = \frac{1}{100} \sum_{s=1}^{100} \left[\overline{I^{i,s}_k} - \underline{I^{i,s}_k} \right]$$

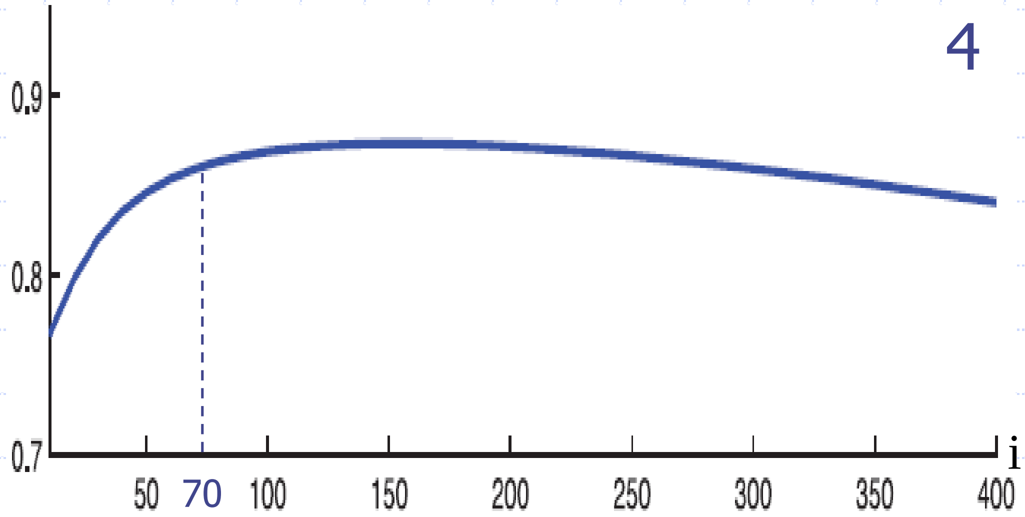
Results



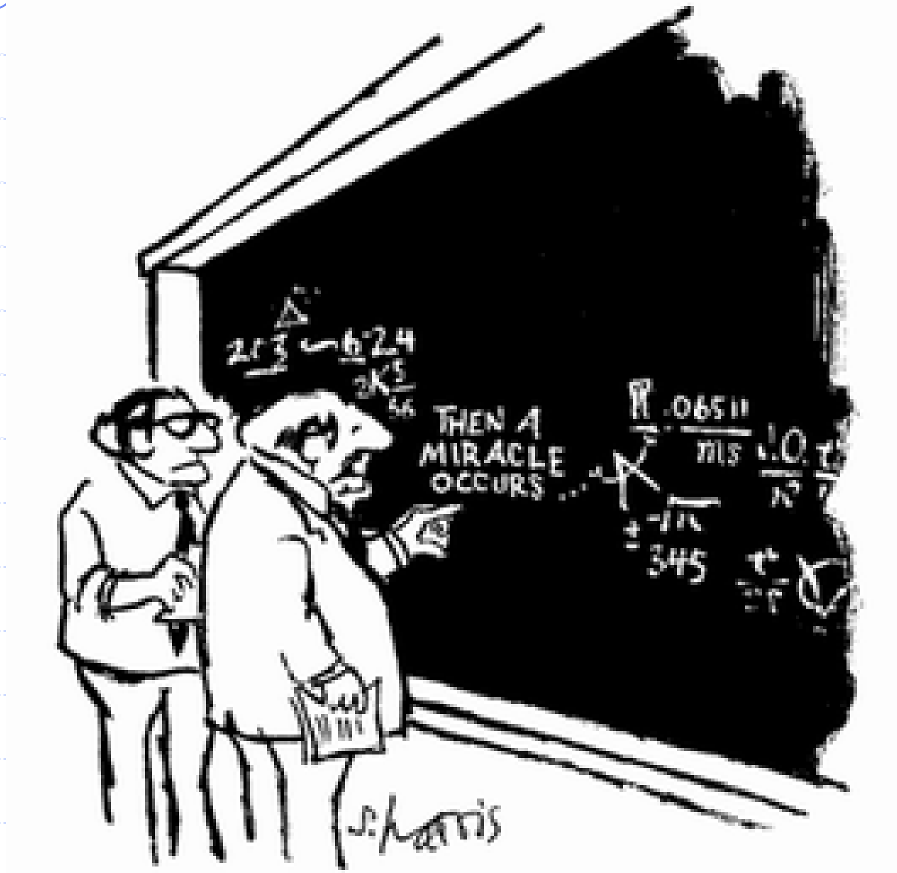




Pearson



Merci de votre attention…

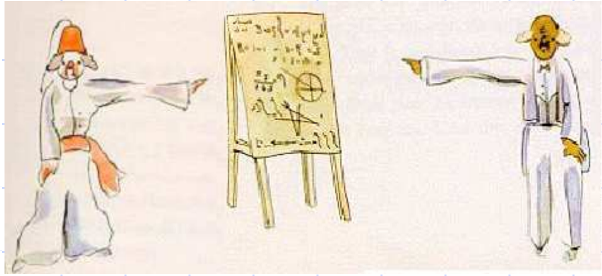


"I think you should be more explicit here in step two."

The Mathematics of Computerized Tomography.
F. Natterer. 2001. SIAM.

Reconstruction tomographique en imagerie médicale. D. Mariano-Goulart Encyclopédie Médico-chirurgicale, 35-105-A-10, 2015.

Reconstruction tomographique
Cours rédigé pour étudiants en 2° année de médecine
http://scinti.edu.umontpellier.fr/files/2016/06/Reconstruction_tomographique.pdf



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