Mathématiques appliquées & médecine nucléaire : quelques ponts

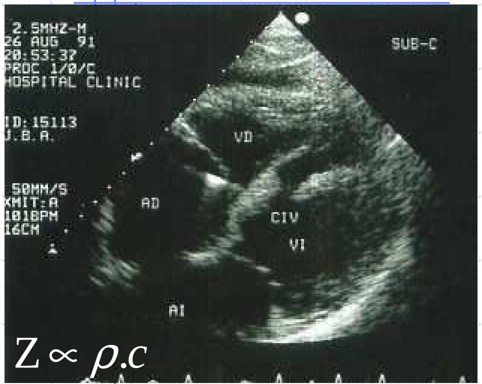
Denis Mariano-Goulart
Faculté de médecine et CHRU
Montpellier
d-mariano_goulart@chu-montpellier.fr

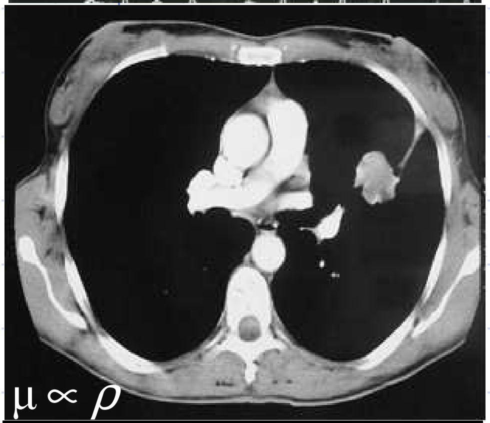
Résumé

Depuis la conception du premier "scanner", à la fin des années 1960, les techniques d'imagerie médicale ont grandement bénéficié de l'apport de nombreux outils de mathématiques appliquées et de traitement du signal numérique, tant dans le domaine de la reconstruction (tomographie, images paramétriques) que de l'analyse d'images (segmentation, modélisations). Après une brève présentation des diverses modalités de l'imagerie médicale et des bases de la reconstruction tomographique, l'exposé insistera sur quelques travaux récents menés lors de collaborations entre mathématiciens et médecins dans les domaines de la régularisation de problèmes inverses linéaires, de la modélisation de données bruitées ou de la segmentation d'images médicales. Quelques pistes d'éventuelles nouvelles collaborations seront suggérées au fil de l'exposé.

Imagerie médicale

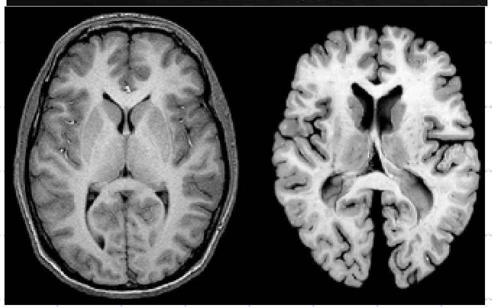
ANATOMIQUE



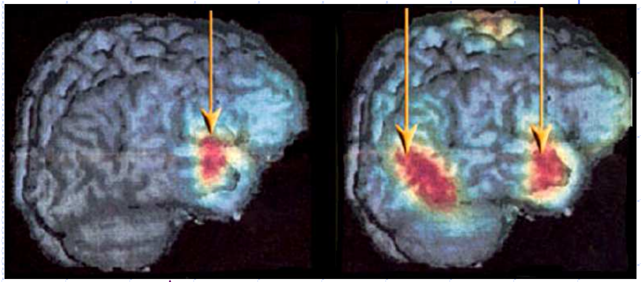


Radiologie & Imagerie médicale

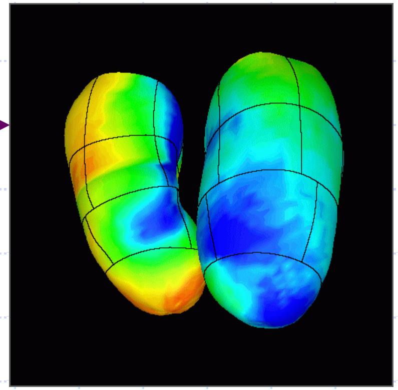
Imagerie médicale



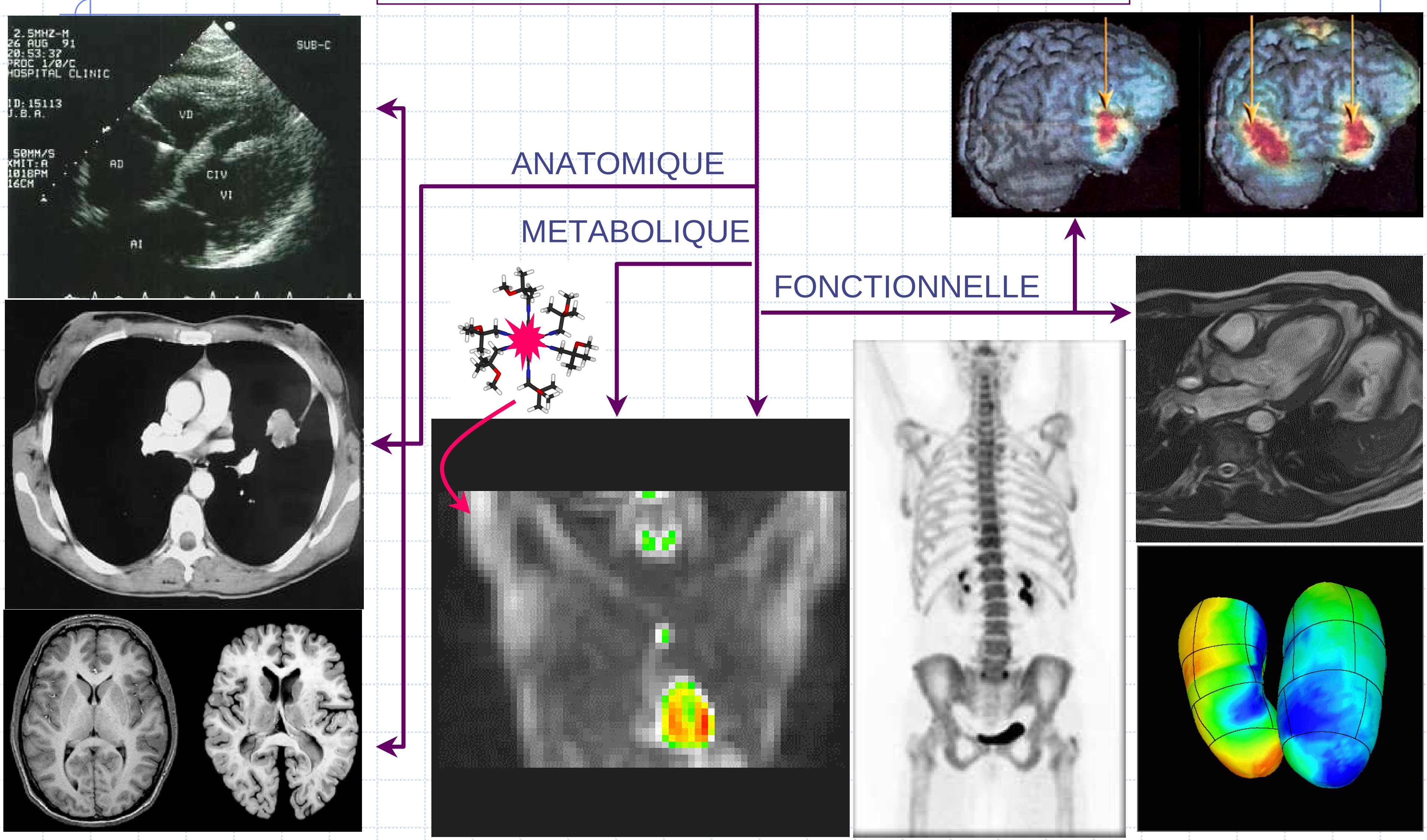
ANATOMIQUE



FONCTIONNELLE

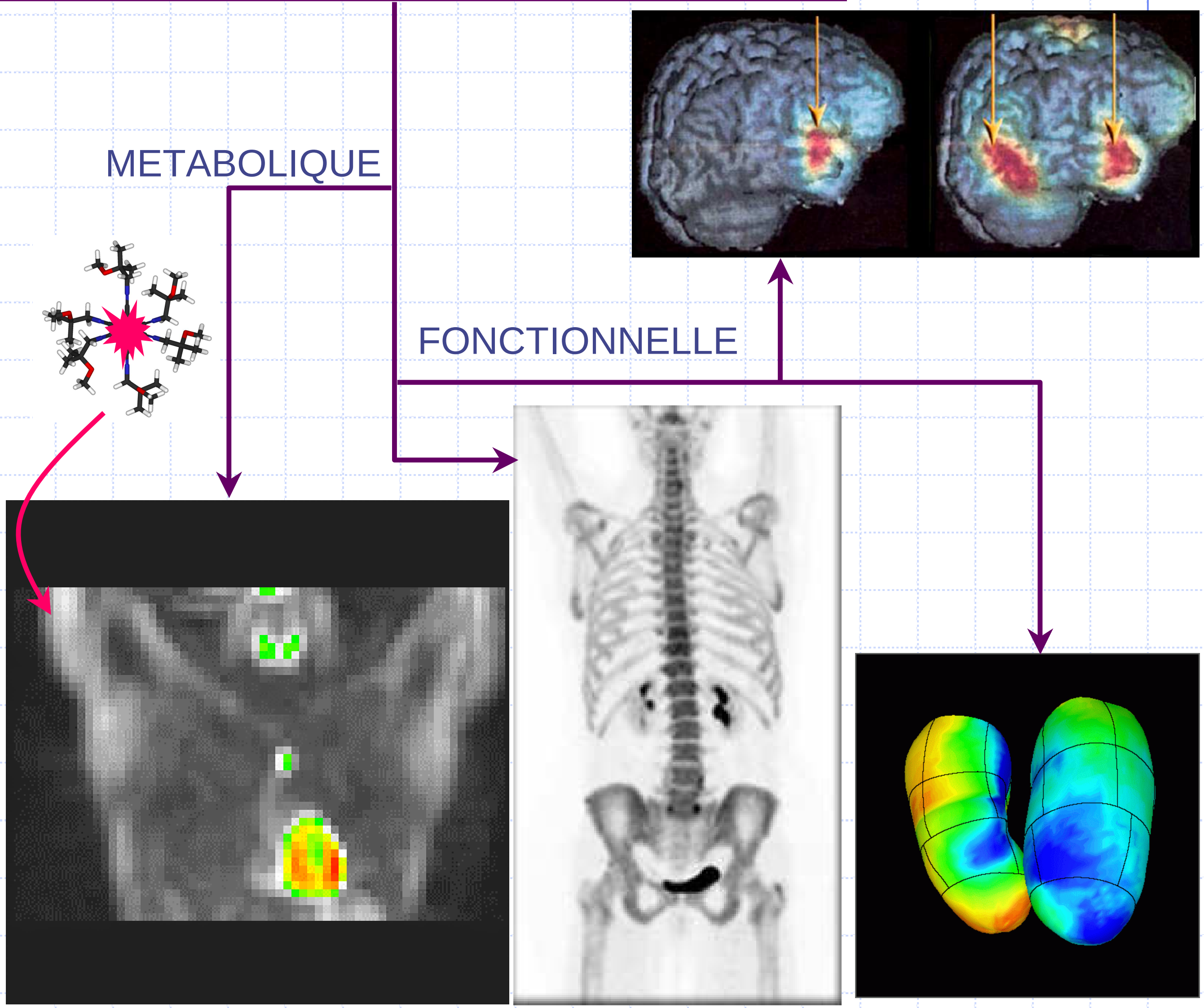


Imagerie médicale

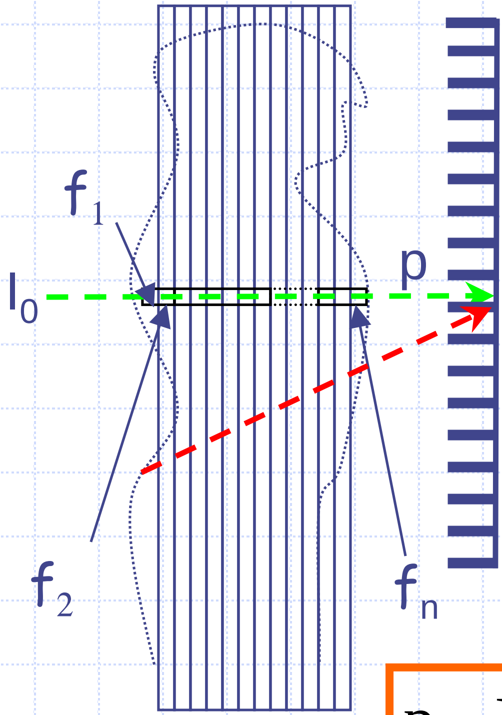


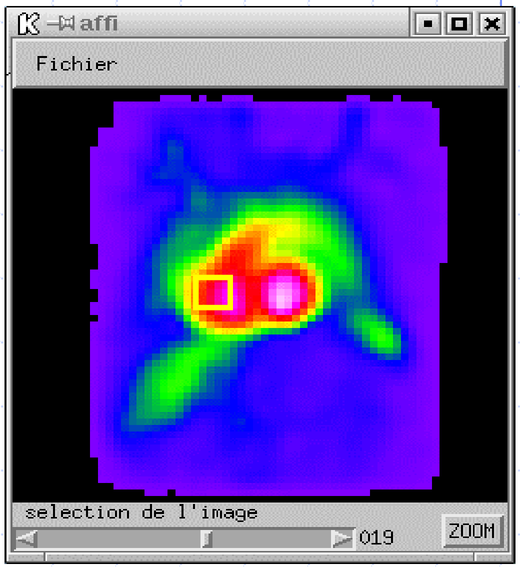
Imagerie médicale

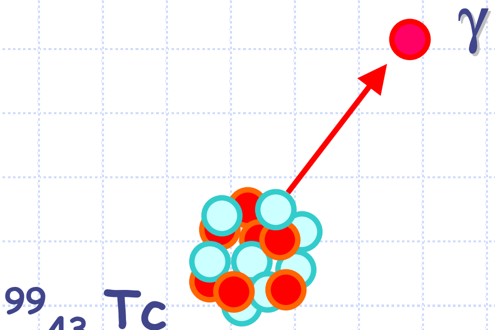
Biophysique
& médecine
nucléaire



TEMP (SPECT)





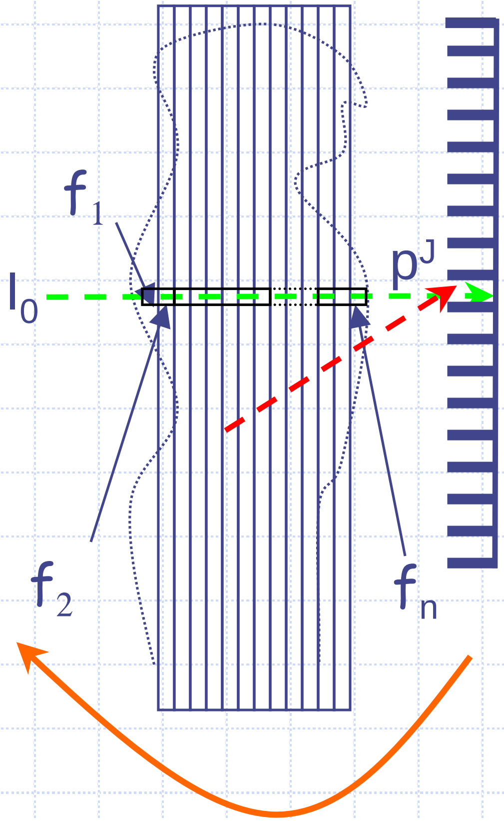


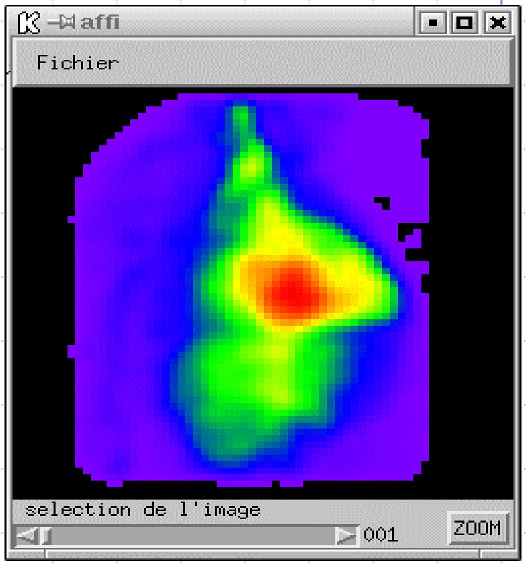
$$p = R_1 f_1 + R_2 f_2 + \ldots + R_n f_n$$

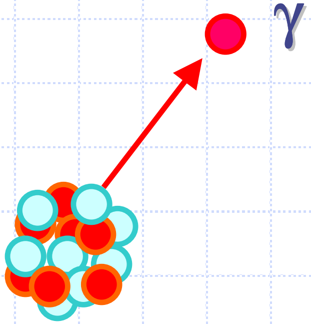
résolution ≈ cm

bruit de Poisson

TEMP (SPECT)

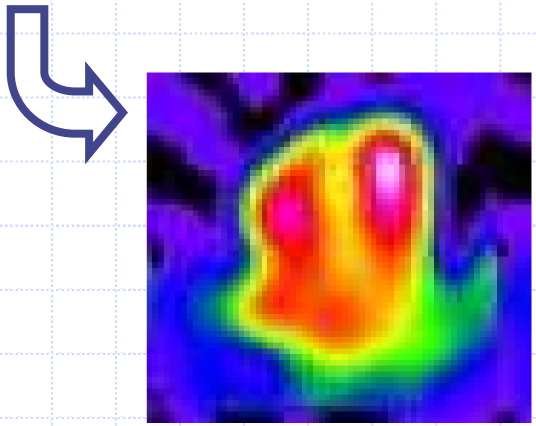




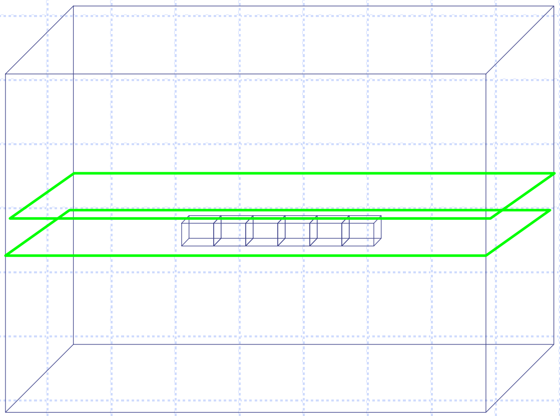


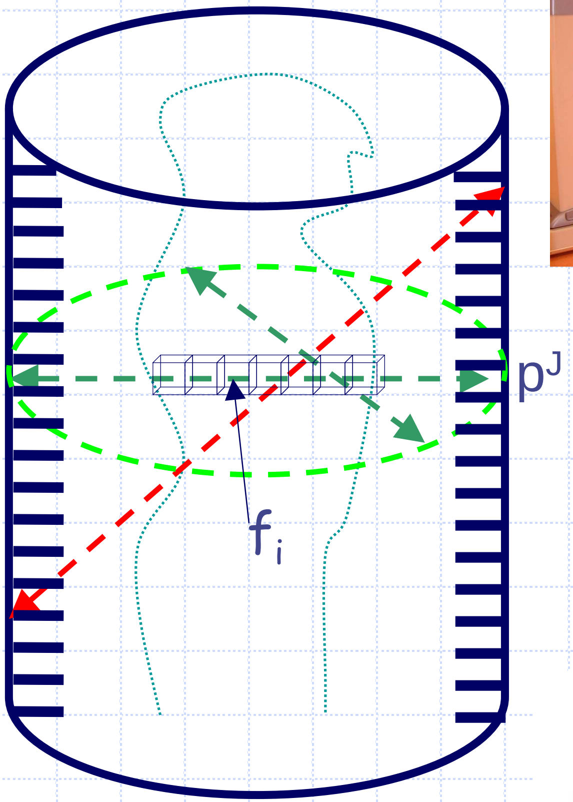
$^{99}_{43}Tc$

$$\boxed{p^j = \sum R_i^j f_i}$$

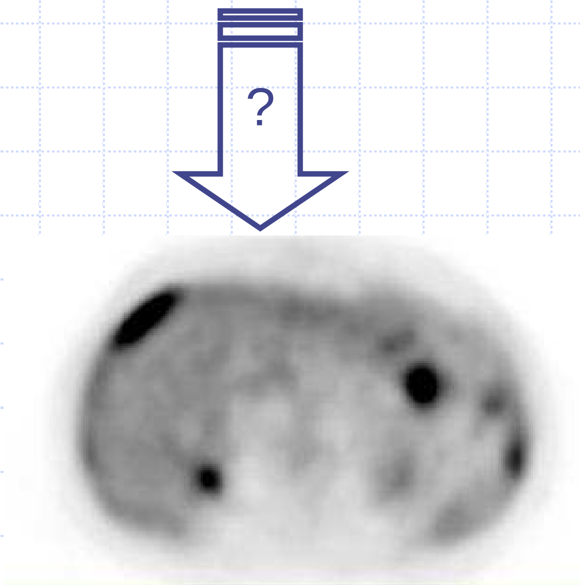


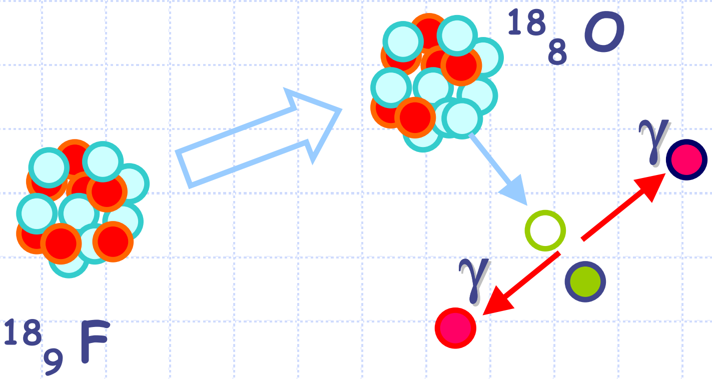
TEP (PET-SCAN)



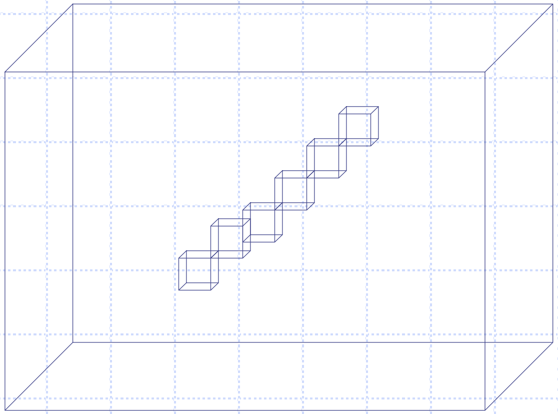


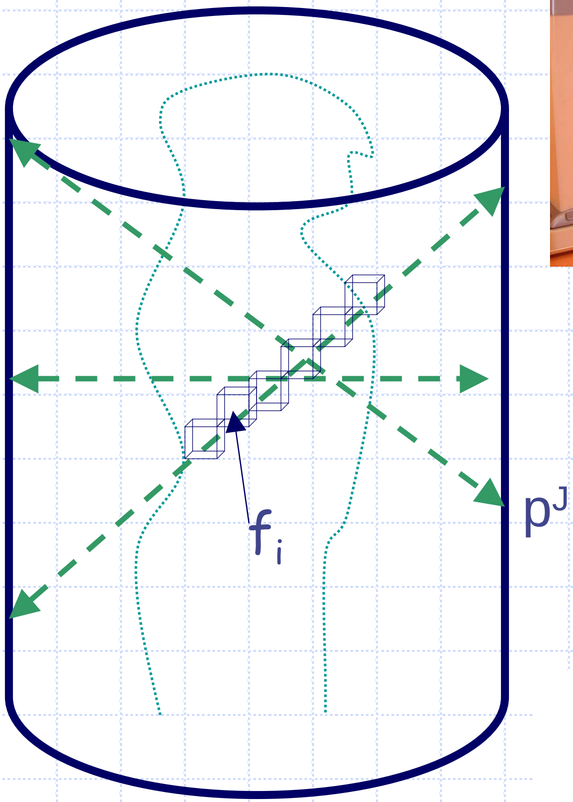
$$p^j = \sum R_i^j f_i$$



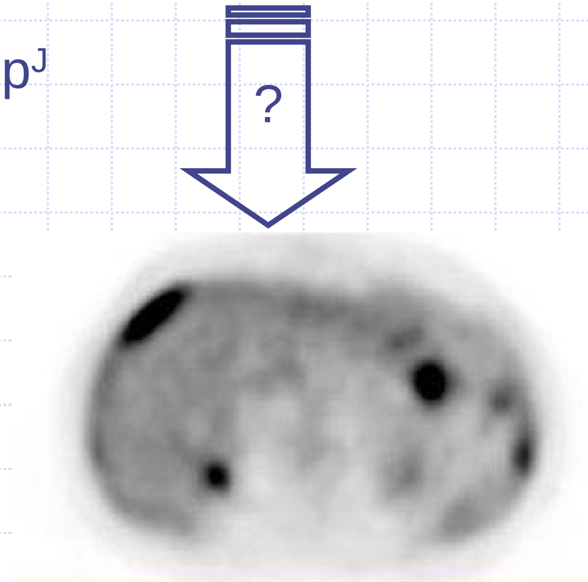


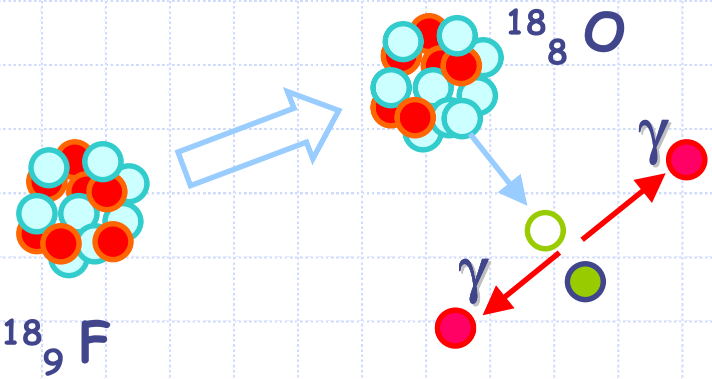
TEP



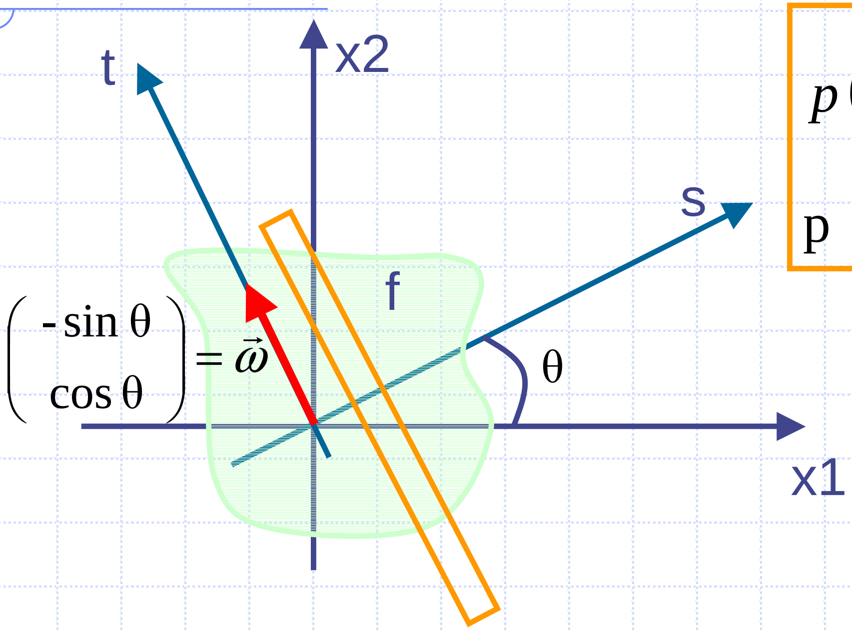


$$p^j = \sum R_i^j f_i$$





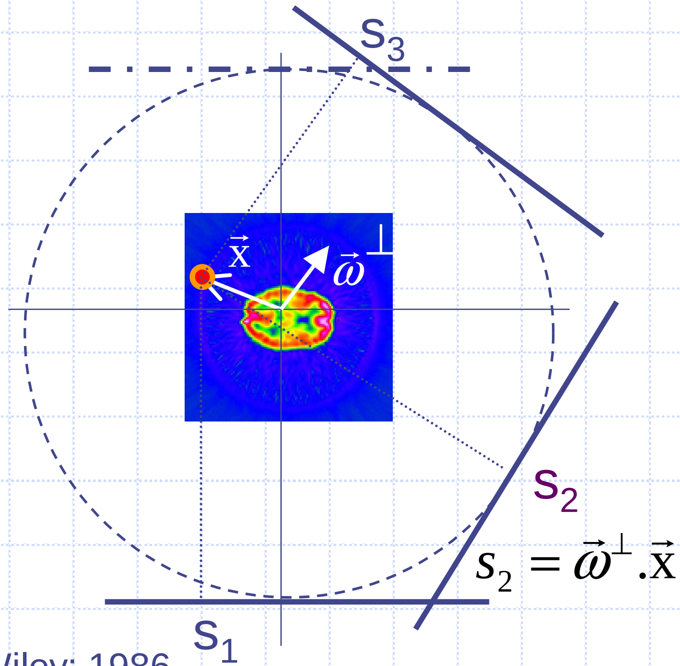
Modélisation analytique



$$p(\vec{\omega}, s) = p_{\vec{\omega}}(s) = \int_t f(s\,\vec{\omega}^{\perp} + t\vec{\omega})\,dt$$

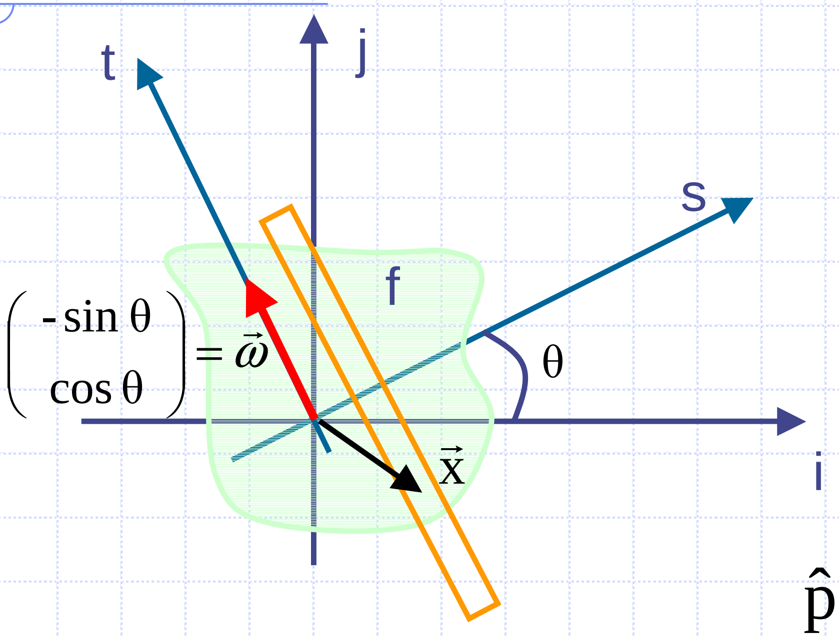
$$p = Rf$$

$$(R^* p)(\vec{x}) = \int_{\theta=0}^{\pi} p\left(\vec{\omega}, \vec{\omega}^{\perp}.\vec{x}\right) d\theta$$



Natterer F. The mathematics of computerized tomography. New York: Wiley; 1986.

Théorème de la projection



$$\begin{cases} p_{\vec{\omega}}(s) = \int_t f(s\vec{\omega}^{\perp} + t\vec{\omega})\,dt \\ \hat{p}_{\vec{\omega}}(\sigma) = \int_s p_{\vec{\omega}}(s).e^{-i.s.\sigma} ds \end{cases}$$

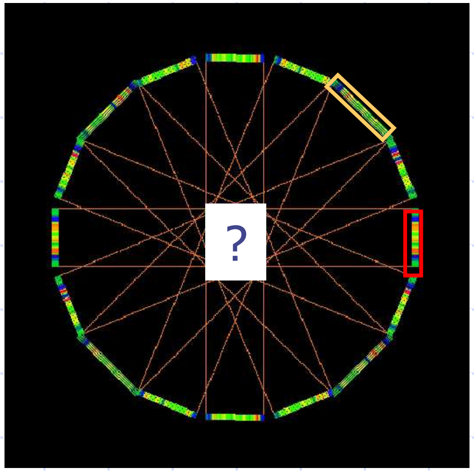
$$\hat{p}_{\vec{\omega}}(\sigma) = \int_s \int_t f(s\vec{\omega}^{\perp} + t\vec{\omega})\, e^{-i.s.\sigma}\, dt\, ds$$

$$\hat{p}_{\vec{\omega}}(\sigma) = \iint f(\vec{x})\, e^{-i.\sigma\, \vec{x}.\vec{\omega}^{\perp}} d\vec{x}$$

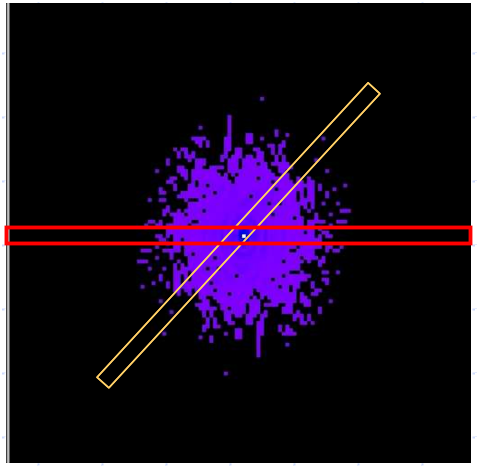
$$\boxed{\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}(\sigma.\cos\theta\,,\, \sigma.\sin\theta) = \hat{f}\left(\sigma.\vec{\omega}^{\perp}\right)}$$

Johann Radon, ¨ Uber die Bestimmung von Funktionen durch ihre Integralwerte langs gewisser Mannigfaltigkeiten, Ber. Verh. Sach. Akad. 69 (1917), 262–77.

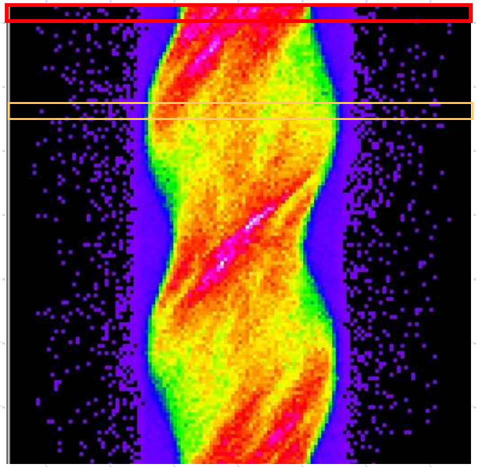
Théorème de la projection

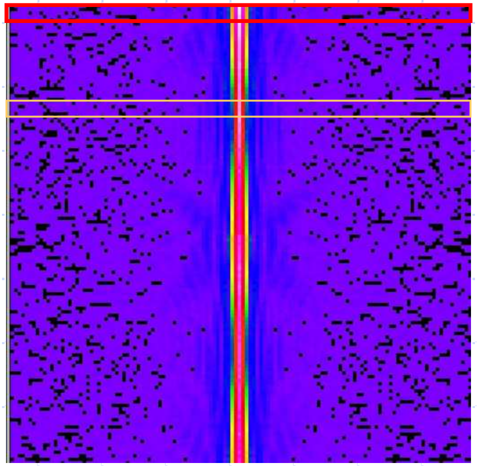


$$\boxed{\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}\left(\sigma.\vec{\omega}^{\perp}\right)}$$

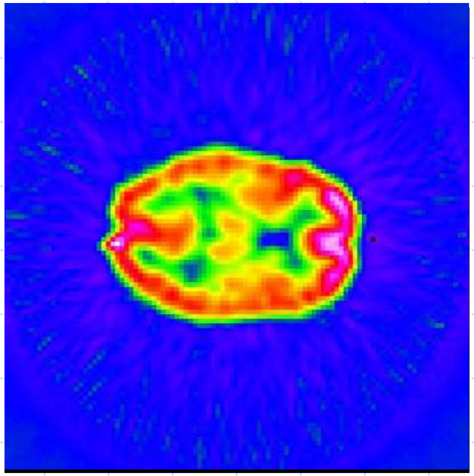


$$\begin{tikzcd}
f \arrow[r, leftrightarrow, "TF"] \arrow[d, "R"] & \hat{f} \arrow[d, phantom, "\|"] \\
p_{\vec{\omega}}(s) \arrow[r, leftrightarrow, "TF"] & \hat{p}_{\vec{\omega}}(\sigma)
\end{tikzcd}$$

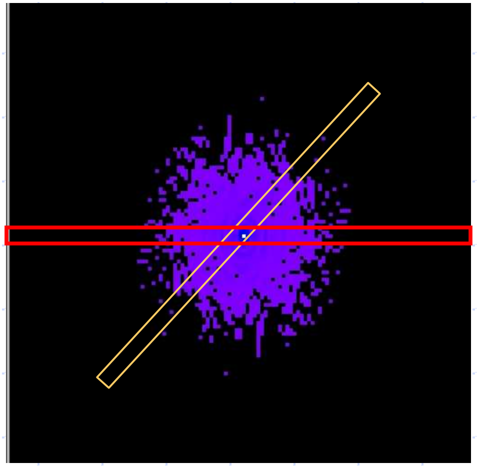




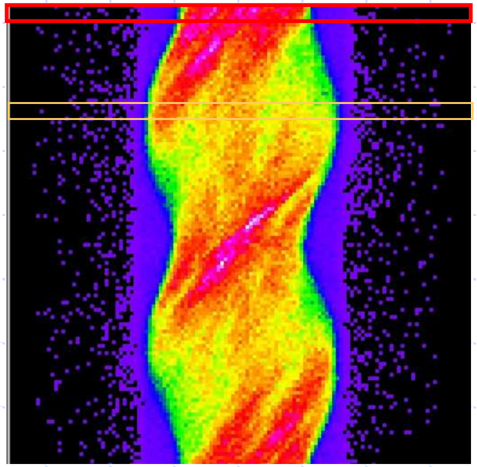
Théorème de la projection

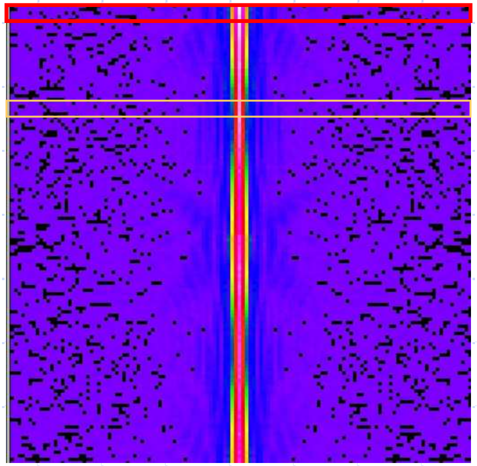


$$\boxed{\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}\left(\sigma.\vec{\omega}^{\perp}\right)}$$

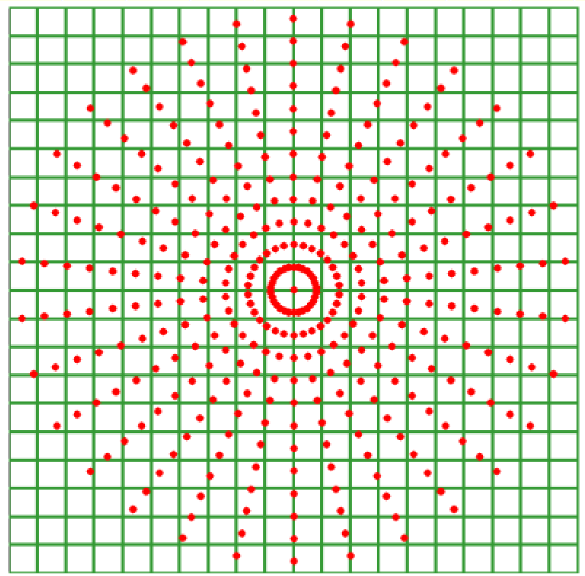


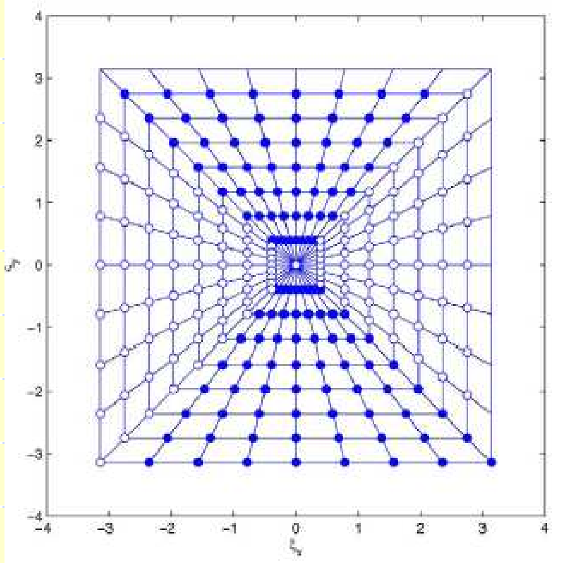
$$\begin{tikzcd}
f \arrow[rr, leftrightarrow, "\mathrm{TF}"] \arrow[d, leftrightarrow, "R"'] & & \hat{f} \arrow[d, leftrightarrow, "\|"] \\
p_{\vec{\omega}}(s) \arrow[rr, leftrightarrow, "\mathrm{TF}"] & & \hat{p}_{\vec{\omega}}(\sigma)
\end{tikzcd}$$

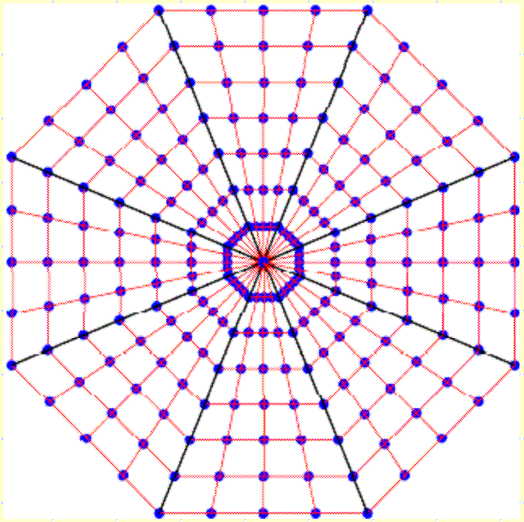




FFT polaire ?







FFT-

A Averbuch, RR Coifman, DL Donoho, M Elad, M Israeli. *Appl Comput Harmon Anal*. 2006; 21:145-167

O Levi, O Harari, Ronen Peretz. 2007. The Fast Octa-Polar Fourier Transform and its expansion to an accurate discrete radon transform

Fast Fourier Transform (FFT)

$$\hat{s}(\nu) = \sum_{k=0}^{N-1} s(k).e^{-j.(k\omega_0)\nu} = \sum_{k=0}^{N-1} s(k).W_N^{k\nu}$$

$$W_N = e^{-j.\frac{2.\pi}{N}} = e^{-j.\omega_0}$$

$$W_N = \cos(\frac{2\pi}{N}) - j.\sin(\frac{2\pi}{N})$$

$$\hat{s}(\nu) = \sum_{k=0}^{\frac{N}{2}-1} s(2k).W_N^{2.k.\nu} + \sum_{k=0}^{\frac{N}{2}-1} s(2k+1).W_N^{(2.k+1).\nu}$$

$$= \sum_{k=0}^{\frac{N}{2}-1} s(2k).W_N^{2.k.\nu} + W_N^{\nu} \sum_{k=0}^{\frac{N}{2}-1} s(2k+1).W_N^{2.k.\nu}$$

$$= \sum_{k=0}^{\frac{N}{2}-1} s(2k).W_{N/2}^{k.\nu} + W_N^{\nu} \sum_{k=0}^{\frac{N}{2}-1} s(2k+1).W_{N/2}^{k.\nu}$$

$$= G(\nu) + W_N^{\nu}.H(\nu)$$

Fast Fourier Transform (FFT)

$$\cos(\frac{2\pi.\nu}{N}) - j.\sin(\frac{2\pi.\nu}{N})$$

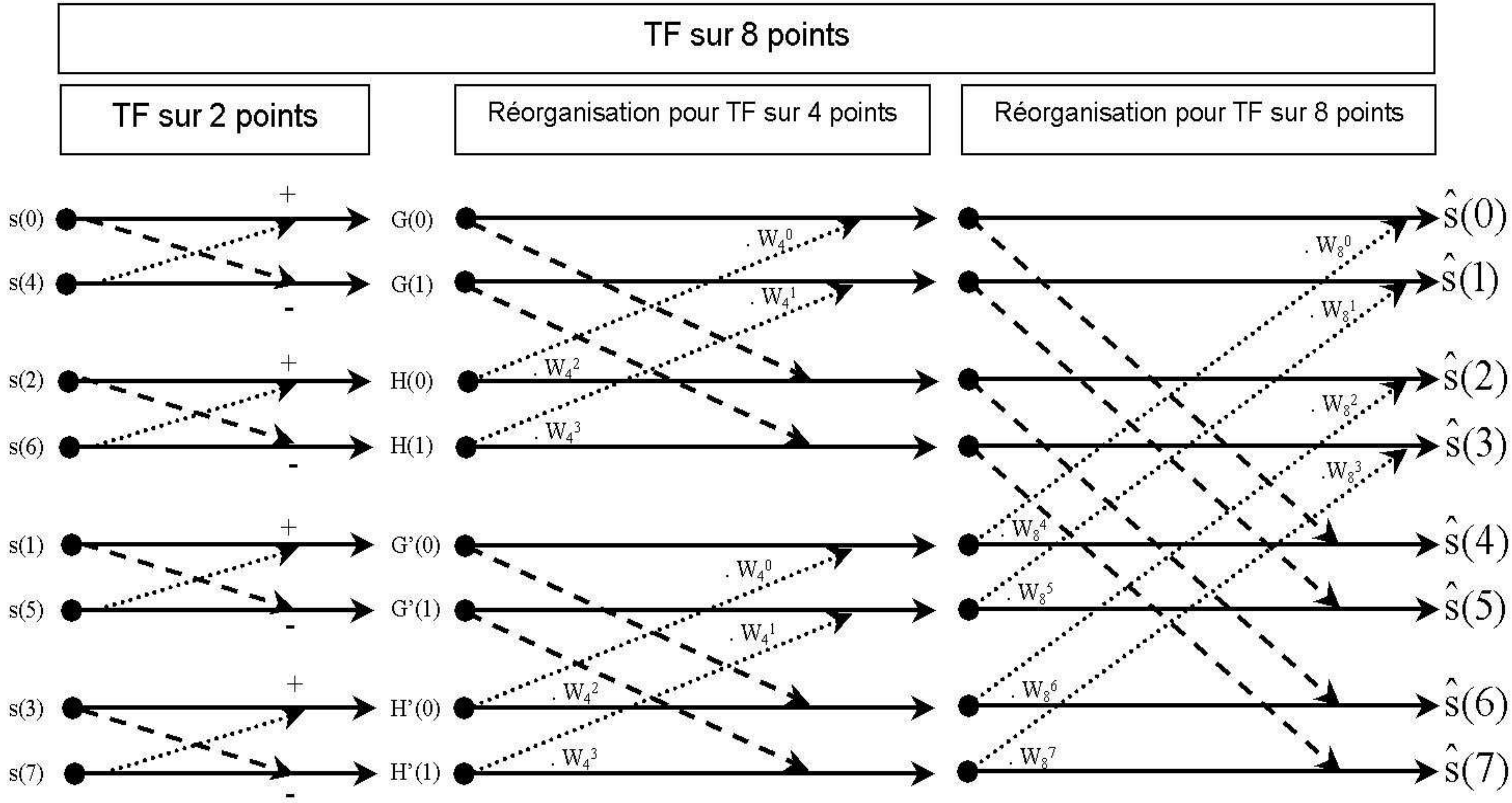
$$\hat{s}(\nu) = G(\nu) + W_N^{\nu}.H(\nu)$$

TF sur N points TF sur N/2 points

TF sur 2 points: $\hat{s}(\nu) = \sum_{k=0}^{1} s(k).e^{-j.(k\frac{2\pi}{2})\nu} = s(0) + (-1)^{\nu} s(1)$

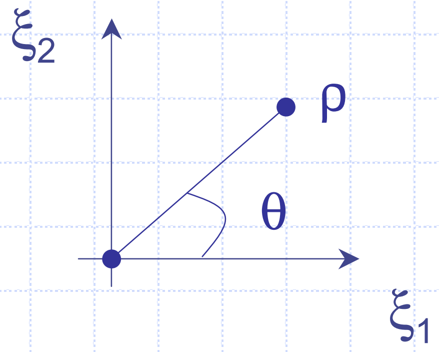
Complexité N² $\rightarrow$ N.$\log_2$ N (512² $\rightarrow$ 512x9 i.e 57 fois moins)

Algorithme FFT



Rétroprojection filtrée

$$f(\vec{x}) = \iint \hat{f}(\vec{\xi})\, e^{i\,\vec{x}.\vec{\xi}}\, d\vec{\xi}$$



$$f(\vec{x}) = \int_{\theta=0}^{\pi} \int_{\sigma=-\infty}^{\sigma=+\infty} \hat{f}\left(\sigma\, \vec{\omega}^{\perp}\right) e^{i\,\sigma\, \vec{\omega}^{\perp}.\vec{x}}\, |\sigma|\, d\sigma\, d\theta$$

$$f(\vec{x}) = \int_{\theta=0}^{\pi} \boxed{\int_{\sigma=-\infty}^{\sigma=+\infty} \hat{p}_{\vec{\omega}}(\sigma)\, |\sigma|\, e^{i\,\sigma\, \vec{\omega}^{\perp}.\vec{x}}\, d\sigma}\, d\theta$$

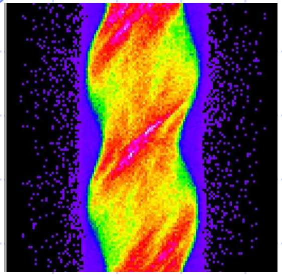
$$\| $$

$$\underbrace{TF_s^{-1}\left[\hat{p}_{\vec{\omega}}\, .\, \mathrm{abs}\right]}_{p'_{\vec{\omega}}}\left(\vec{\omega}^{\perp}.\vec{x}\right)$$

1887-1956

$$\boxed{f(\vec{x}) = (R^{*}\, p')\,(\vec{x})}$$

Rétroprojection filtrée



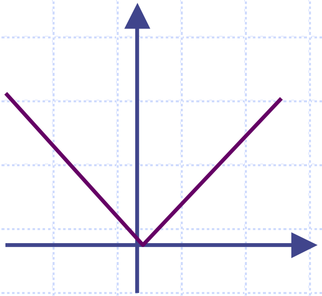
$$\boxed{f(\vec{x}) = (R^{*}\, p')(\vec{x})}$$

Projections sur 180°

$p_{\vec{\omega}}$

$\downarrow$

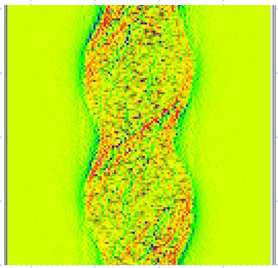
$\hat{p}_{\vec{\omega}}$ abs

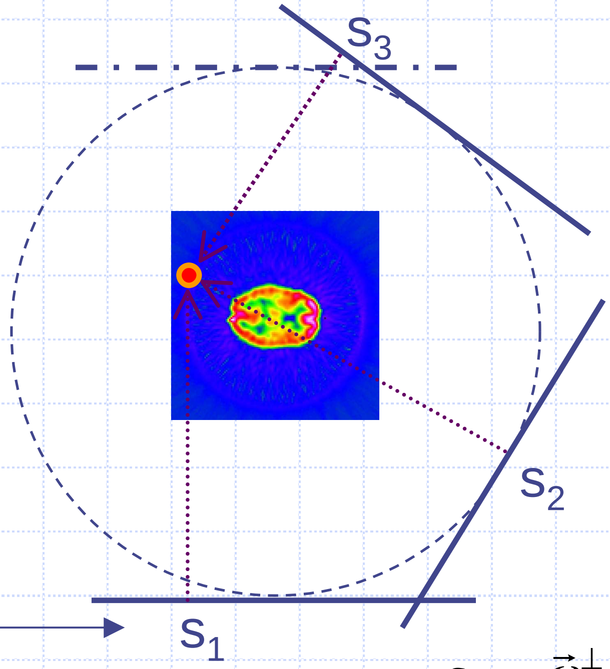


x

$\downarrow$

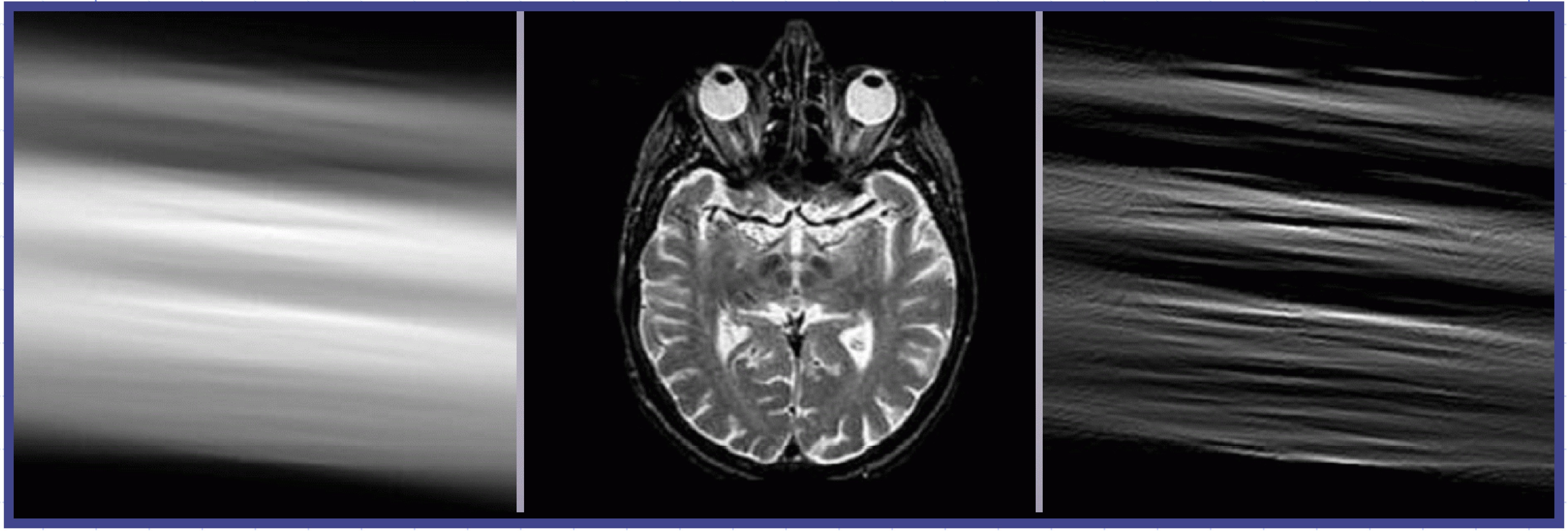
$$TF_s^{-1}\left[\hat{p}_{\vec{\omega}} \,.\, \mathrm{abs}\right] = p'_{\vec{\omega}}$$





$$s_2 = \vec{\omega}^{\perp} . \vec{x}$$

Rétroprojection filtrée



Modélisation algébrique

f_1	f_2
f_3	f_4

$p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2$

$p_2 = r_{2,3}\, f_3 + r_{2,4}\, f_4$

$p_4 = r_{4,2}\, f_2 + r_{4,4}\, f_4$

$p_3 = r_{3,1}\, f_1 + r_{3,3}\, f_3$

$$\begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ r_{2,1} & r_{2,2} & r_{2,3} & r_{2,4} \\ r_{3,1} & r_{3,2} & r_{3,3} & r_{3,4} \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{pmatrix} . \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

$r_{i,j}$ = % du pixel j intersecté par la projection i

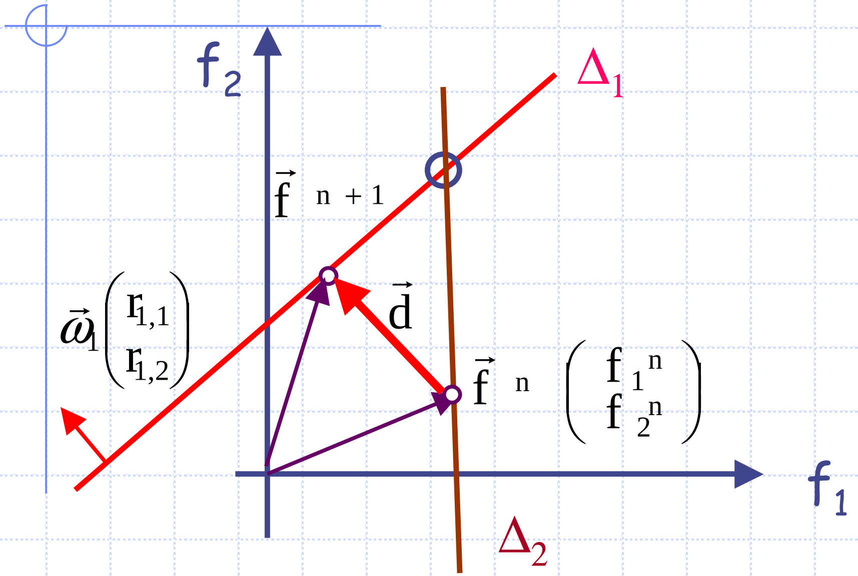
$$\boxed{R.\vec{f} = \vec{p}}$$

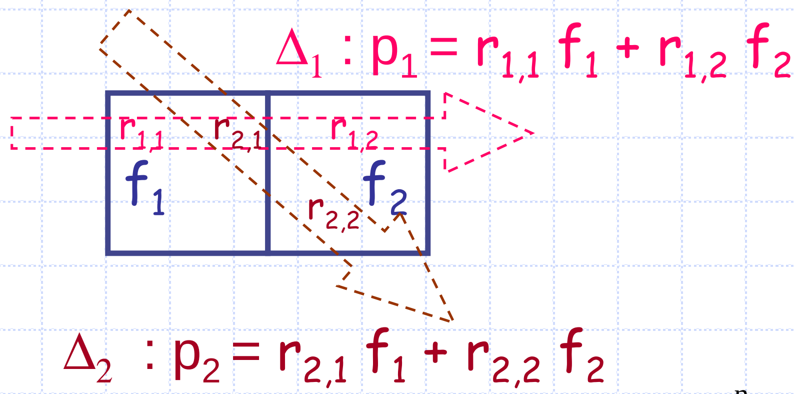
$$\begin{pmatrix} r_{1,1} & r_{2,1} & r_{3,1} & r_{4,1} \\ r_{1,2} & r_{2,2} & r_{3,2} & r_{4,2} \\ r_{1,3} & r_{2,3} & r_{3,3} & r_{4,3} \\ r_{1,4} & r_{2,4} & r_{3,4} & r_{4,4} \end{pmatrix} . \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$$\boxed{{}^tR.\vec{p} = \vec{b}}$$

$b_1 = r_{1.1}\, p_1 + r_{3,1}\, p_3$	$b_2 = r_{1.2}\, p_1 + r_{4,2}\, p_4$	$\leftarrow p_1$
$b_3 = r_{2.3}\, p_2 + r_{3,3}\, p_3$	$b_4 = r_{2,4}\, p_2 + r_{4,4}\, p_4$	$\leftarrow p_2$
$\uparrow p_3$	$\uparrow p_4$	

Algebraic Reconstruction Technique





p_1^n

$$\vec{f}^{\,n+1} = \vec{f}^{\,n} + d\,\frac{\vec{\omega}_1}{\left\|\vec{\omega}_1\right\|} \qquad d = \frac{p_1 - \vec{f}^{\,n}\cdot\vec{\omega}_1}{\left\|\vec{\omega}_1\right\|}$$

$$\vec{f}^{\,n+1} = \vec{f}^{\,n} + \frac{p_1 - p_1^n}{\left\|\vec{\omega}_1\right\|^2}\,\vec{\omega}_1$$

$$\boxed{\vec{f}^{\,n+1} = \vec{f}^{\,n} + R * (p_1 - p_1^n)}$$

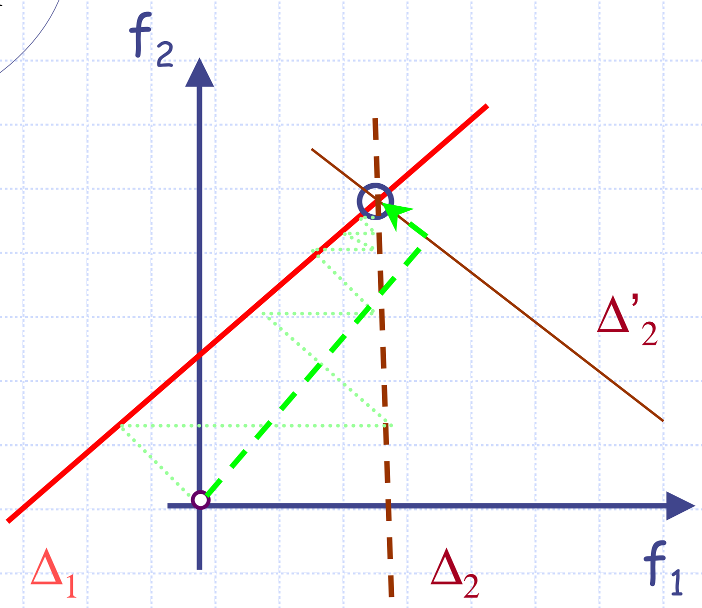
Kaczmarz S. Angenährte Auflösung von Systemen linearer Gleichungen. *Bull Int Acad Pol Sci Lett A* 1937;35:355-7.

MLEM et OSEM

- Maximiser $\log\left[P(\mathrm{p}'/\mathrm{f})\right] = \log\left[\prod_{\mathrm{i}=1}^{\mathrm{K}} \frac{\mathrm{e}^{-\mathrm{p}_\mathrm{i}}\,\mathrm{p}_\mathrm{i}^{\mathrm{p}'_\mathrm{i}}}{\mathrm{p}'_\mathrm{i}\,!}\right]$

$$\mathrm{f}_\mathrm{i}^{\mathrm{n}+1} = \mathrm{f}_\mathrm{i}^{\mathrm{n}} \cdot \frac{1}{\sum_{l'=1}^{K} r_{l',i}} \sum_{l=1}^{K} r_{l,i} \frac{p_l}{\sum_{s=1}^{N} r_{l,s} f_s^{\,n}}$$

$$= R^{*}\left[\frac{\mathrm{p}_{\mathrm{l}}}{\mathrm{p}_{\mathrm{l}}^{\mathrm{n}}}\right]$$



Problème bien conditionné ?

- ✓ En continu : R opérateur bijectif d'inverse continue (Hadamard).

- ✓ En discret :
  - surjectivité $\Leftarrow {}^{t}R.R\vec{f} = A\vec{f} = {}^{t}R.\vec{p} = \vec{q}$
    - qui revient à minimiser $\left\|R\vec{f} - \vec{p}\right\|^2$
  - R injectif ? choix parmi le solutions (initialisation)
  - R^{-1} continue mais $||R^{-1}||$ grande : $\kappa(R) = \|R\| \left\|R^{-1}\right\| = \frac{\lambda_{max}}{\lambda_{min}}$

$$\frac{\left\|\delta\vec{f}\right\|}{\left\|\vec{f}\right\|} \leq \frac{\kappa(R)}{1 - \kappa(R)\frac{\|\delta R\|}{\|R\|}} \left[\frac{\left\|\delta\vec{p}\right\|}{\left\|\vec{p}\right\|} + \frac{\|\delta R\|}{\|R\|}\right]$$

Régularisation

- $\vec{f} = \arg\min_{\vec{f}} \left\{ \underbrace{\left\| \vec{p} - R\vec{f} \right\|^2}_{\text{surjectivité}} + \underbrace{\alpha . \left\| \vec{f} \right\|^2}_{\text{injectivité}} \right\} \Leftrightarrow \left(R^* R + \alpha I \right) f = R^* p$

  $\vec{f} = \left(R^* R + \alpha I \right)^{-1} R^* p$

- $P(\vec{f} / \vec{p}) = P(\vec{p}/\vec{f}) . P(\vec{f}) / P(\vec{p}) = P(\vec{p}/\vec{f}) . P(\vec{f})$

$$\tilde{\vec{f}} = \arg\min_{\vec{f}} \left[-\log P(\vec{p}/\vec{f}) - \log P(\vec{f}) \right]$$

Adéquation aux données

régularisation

Gradient conjugué

$$\boxed{\bar{f} = \arg\min_{f \in C} \left\| A\vec{f} - \vec{q} \right\|^2}$$

$$\vec{d}^0 = \vec{r}^0 = A^*.\vec{q}$$

$$\left\{\begin{array}{l} \boxed{\omega^j = \dfrac{\left\|\vec{r}^j\right\|^2}{\left\langle \vec{d}^j \middle| A^*.A.\vec{d}^j \right\rangle}} \\ \vec{r}^{j+1} = \vec{r}^j - \omega^j.A^*.A.\vec{d}^j \\ \vec{d}^{j+1} = \vec{r}^{j+1} + \boxed{\dfrac{\left\|\vec{r}^{j+1}\right\|^2}{\left\|\vec{r}^j\right\|^2}}.\vec{d}^j \\ \vec{f}^{j+1} = \vec{f}^j + \omega^j.\vec{d}^j \end{array}\right.$$

Matrice de Galerkine

$$\omega^j = \frac{\left\| \vec{r}^{\,j} \right\|^2}{\left\langle \vec{d}^{\,j} \middle| R^*.R.\vec{d}^{\,j} \right\rangle}$$

$$\beta^j = \frac{\left\| \vec{r}^{\,j+1} \right\|^2}{\left\| \vec{r}^{\,j} \right\|^2}$$

$$G^j = \begin{pmatrix} \frac{1}{\omega^0} & -\frac{\sqrt{\beta^0}}{\omega^0} & 0 & 0 \\ -\frac{\sqrt{\beta^0}}{\omega^0} & \frac{1}{\omega^1}+\frac{\beta^0}{\omega^0} & \ddots & 0 \\ 0 & \ddots & \ddots & -\frac{\sqrt{\beta^{j-1}}}{\omega^{j-1}} \\ 0 & 0 & -\frac{\sqrt{\beta^{j-1}}}{\omega^{j-1}} & \frac{1}{\omega^j}+\frac{\beta^{j-1}}{\omega^{j-1}} \end{pmatrix}$$

Analyse numérique matricielle appliquée à l'art de l'ingénieur. P. Lascaux & R. Théodor (II). MASSON

Régularisation statistique

$$P(\vec{f}/\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})/P(\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})$$

$$\tilde{\vec{f}} = \arg\min_{\vec{f}} \left[-\log P(\vec{p}/\vec{f}) - \log P(\vec{f}) \right]$$

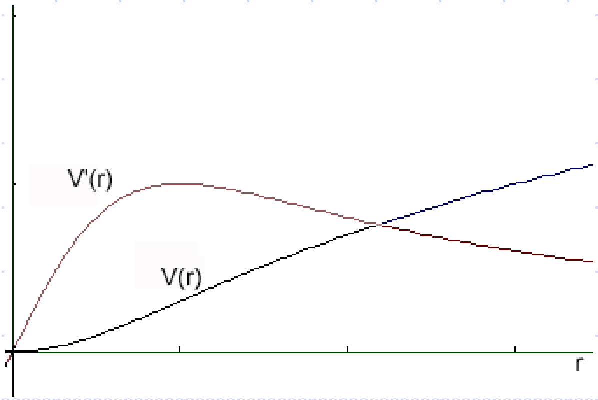
Adéquation aux données

régularisation

Distribution de Gibbs : $P(\vec{f}) = \frac{1}{K} e^{-\beta.\sum_{i,j} w_{i,j}.V\left(f_i - f_j\right)}$

$$\tilde{\vec{f}} = \arg\min_{\vec{f}} \left[-\log P\left(\vec{p}/\vec{f}\right) + \beta \sum_{i,j} w_{i,j} V(f_i - f_j) \right]$$

MAP-EM-OSL



Contrôle de convergence

Evaluation de la stabilité : $\kappa(R) = \dfrac{\lambda_{max}}{\lambda_{min}}$

Erreur inverse (backward error) :

$$\varepsilon(\vec{x}) = \min_{\delta A,\, \delta\vec{q}} \left\{ \mathrm{Max}\left(\frac{\|\delta A\|}{\alpha}, \frac{\|\delta\vec{q}\|}{\beta} \right) / \ (A+\delta A)\ \vec{x} = \vec{q} + \delta\vec{q} \right\}$$

$$\varepsilon(\vec{x}) = \frac{\|A\ \vec{x} - \vec{q}\|}{\alpha \|\vec{x}\| + \beta}$$

$$\beta = 0,\ \|\delta\vec{q}\| = 0 \quad \text{et} \quad \alpha = \|A\| \Rightarrow \boxed{\varepsilon^j = \frac{1}{\lambda_{max}} \frac{\|\vec{r}^{\,j}\|}{\|\vec{f}^{\,j}\|}}$$

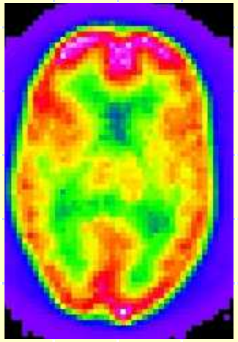
D Mariano-Goulart, P Maréchal, S Gratton, L Giraud, M Fourcade. *Comput Med Imaging Graph* 2007; 31:502-9

Régularisation ?

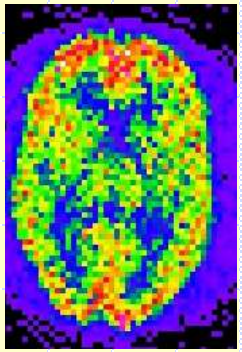
$$\vec{f} = \underset{\vec{f} \in C}{\arg\min} \left\{ \left\| L(\vec{p}) - R\vec{f} \right\|^2 + \left\| H(\vec{f}) \right\|^2 \right\}$$

$$\vec{f} = \arg\min_{\vec{f}} \left[-\log P\left(\vec{p}/\vec{f}\right) + \beta \sum_{i,j} w_{i,j} V(f_i - f_j) \right]$$

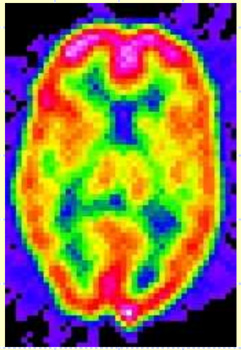
- **Estimation du spectre et de l'erreur possibles.**
- **Choix optimal de L, H, β, w, V ?**
- **Pour ce choix, fréquences à l'itération i ?**



MLEM 6



MLEM 200

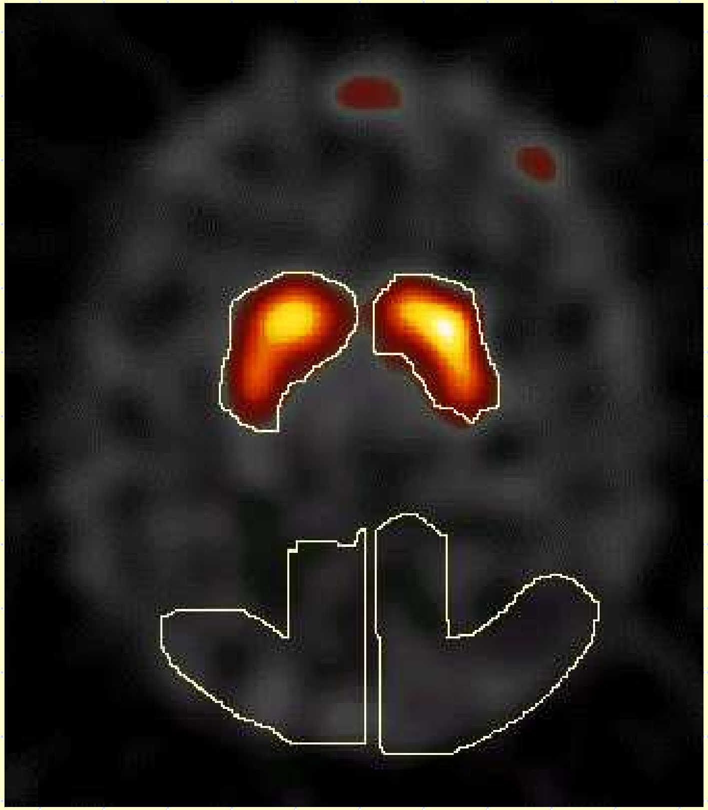


FRECT

D Mariano-Goulart, P Maréchal, S Gratton, L Giraud, M Fourcade. *Comput Med Imaging Graph* 07 & *Lect Notes Comput Sci*. 04

Statistiques dans les coupes ?

$$\boxed{\mathbf{S = (R^*R + H^*H)^{-1}\, R^*\, L}}$$



$$d\vec{f} = S\, d\vec{p} \Rightarrow \left\| d\vec{f} \right\| \leq \|S\| \,.\, \left\| d\vec{p} \right\|$$

$$\mathrm{Cov}\left(d\vec{f}\right) = S \,.\, \mathrm{Cov}\left(d\vec{p}\right).\, S^*$$

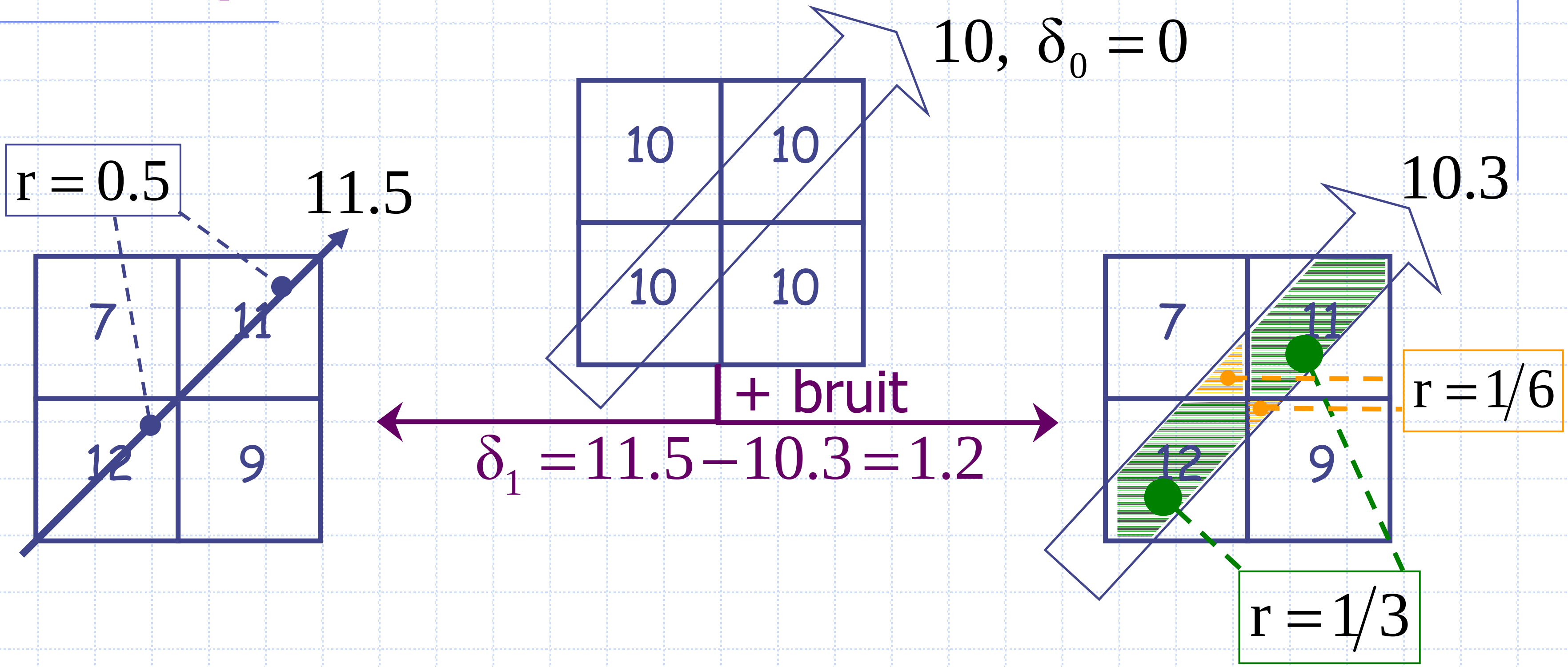
Calculs inextricables…

Alternatives ? :

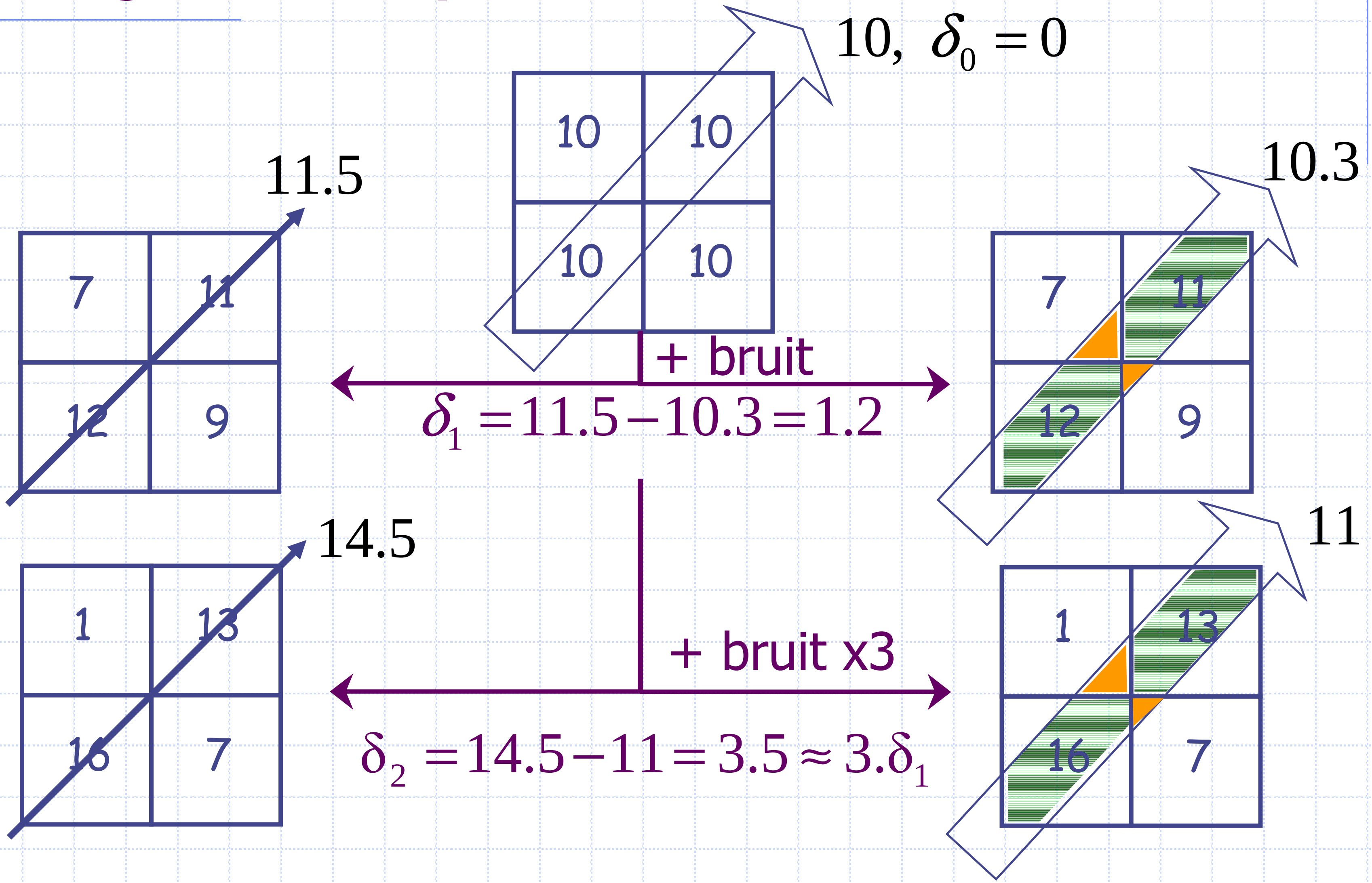
- tomographie par intervalles (intégrales de Choquet).
- rapports de vraissemblances
- …

A. Rico, O. Strauss, D. Mariano-Goulart. *Fuzzy Sets and Systems* 2009; 160(2):198-211

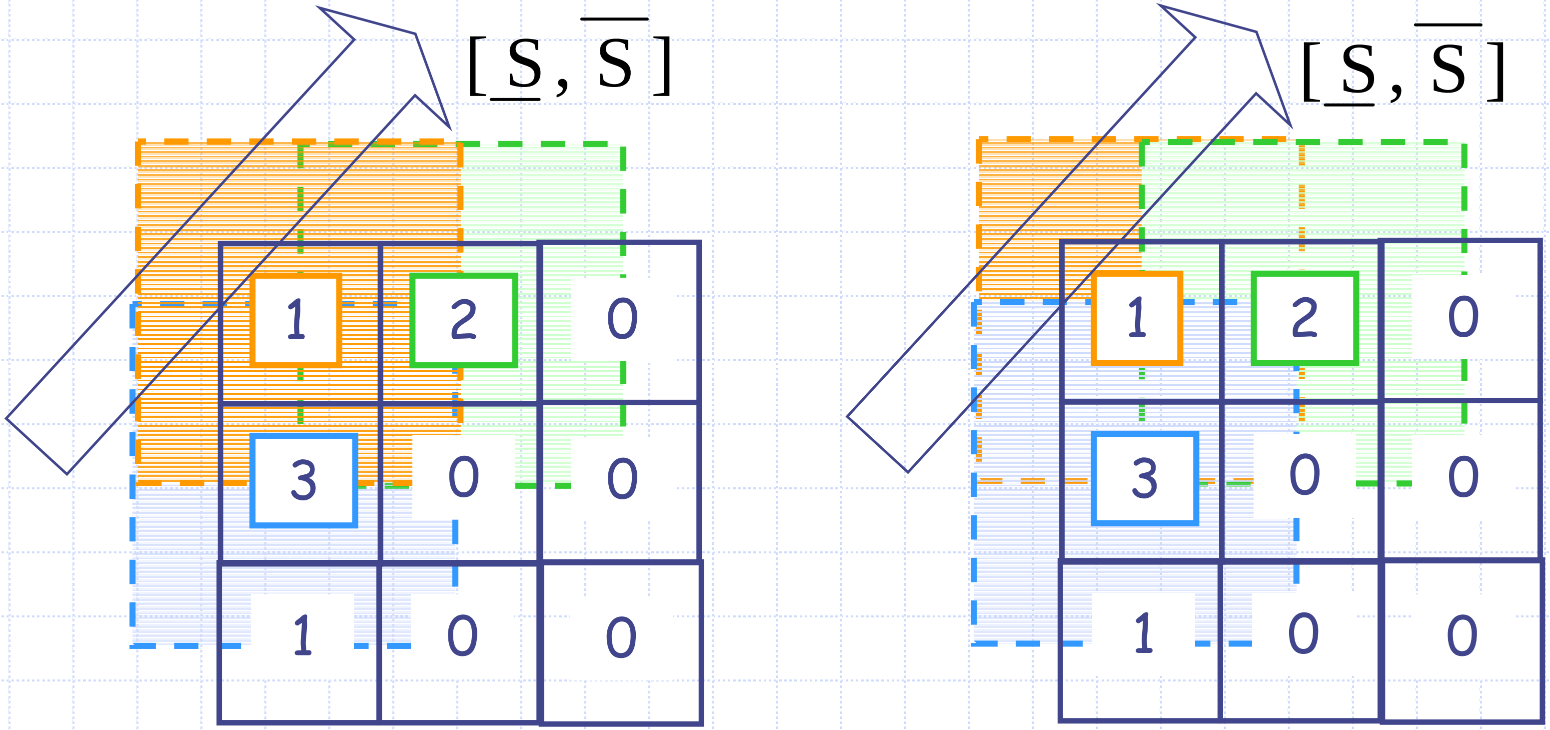
Une piste ?...


10, $\delta_0 = 0$
10
10
10
10
$r = 0.5$
11.5
7
11
12
9
+ bruit
$\delta_1 = 11.5 - 10.3 = 1.2$
10.3
7
11
12
9
$r = 1/6$
$r = 1/3$

Projection par intervalle & bruit



Projection par intervalle



$\underline{S} = (1.4+2.2+3.2)/8=1.8$

$\overline{S} = (3.4+2.3+1.1)/8=2.4$

$$[S]=[1.8\,,\,2.4]$$

Modélisation rigoureuse

- Intégrale de Choquet asymétrique par rapport à une capacité
- Nos résultats montrent un lien fort entre le bruit dans la coupe et le diamètre de l'intervalle reconstruit
- La prise de décision à un niveau *p* donné reste un problème ouvert…

A. Rico, O. Strauss, D. Mariano-Goulart. *Fuzzy Sets and Systems* 2009; 160(2):198-211

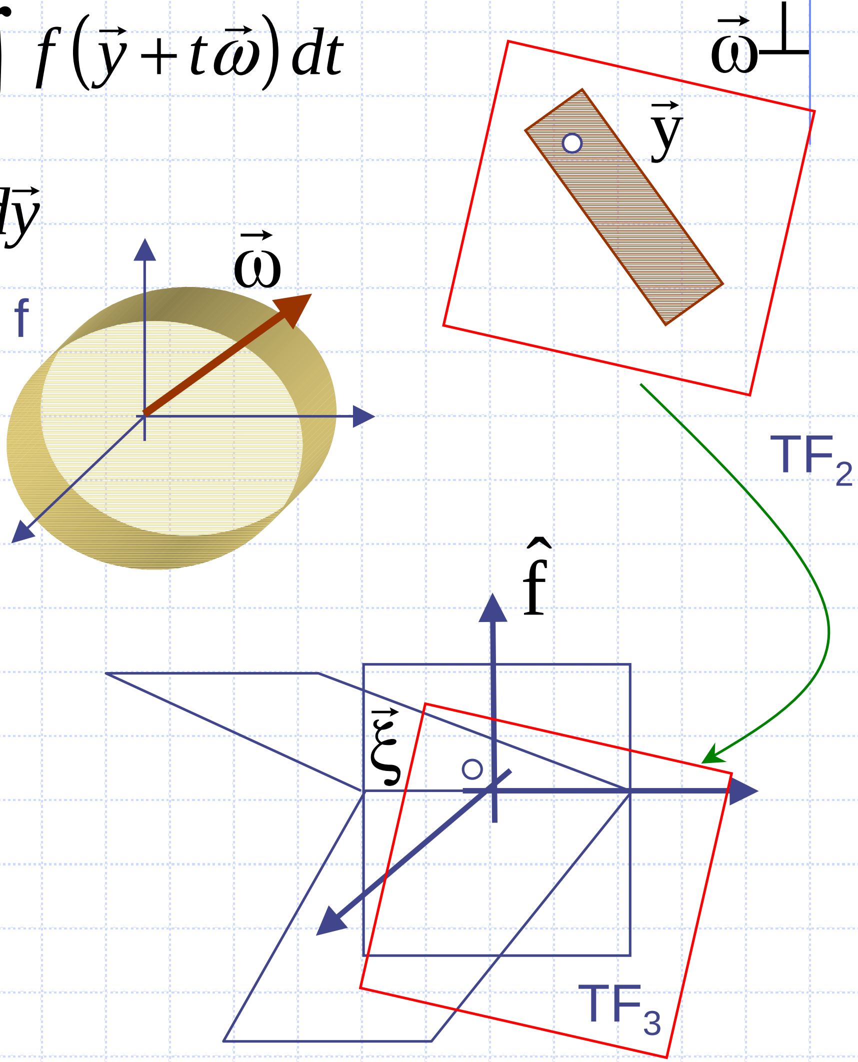
Un théorème de Radon 3D…

$$\forall\, \vec{\omega} \in S, \qquad \forall\, \vec{y} \in \vec{\omega}^{\perp},\; p_{\vec{\omega}}(\vec{y}) = \int f(\vec{y} + t\vec{\omega})\, dt$$

$$\hat{p}_{\vec{\omega}}\left(\vec{\xi}\right) = \iint_{\vec{\omega}^{\perp}} \int f(\vec{y} + t\vec{\omega})\, e^{-2i\pi\, \vec{y}.\vec{\xi}}\; dt\, d\vec{y}$$

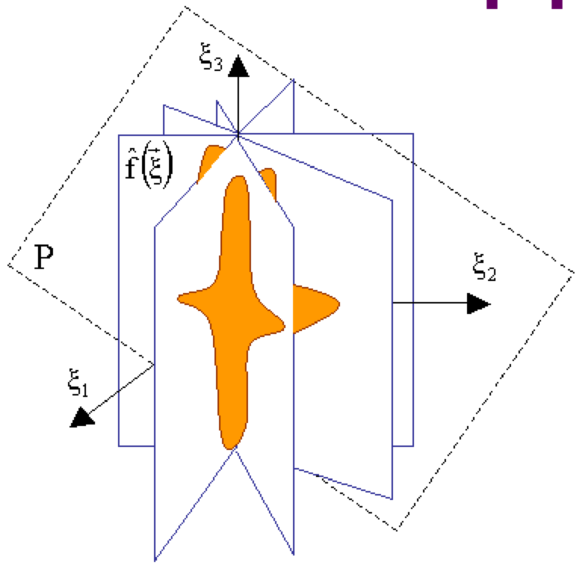
$$\hat{p}_{\vec{\omega}}\left(\vec{\xi}\right) = \iiint f(\vec{x})\, e^{-2i\pi\, \vec{x}.\vec{\xi}}\, d\vec{x} = \hat{f}\left(\vec{\xi}\right)$$

$$\boxed{\forall \vec{\xi} \in \vec{\omega}^{\perp},\; \hat{p}_{\vec{\omega}}\left(\vec{\xi}\right) = \hat{f}\left(\vec{\xi}\right)}$$



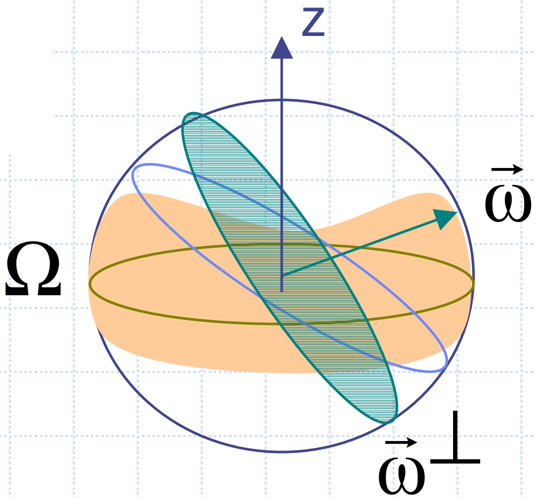
… un peu difficile à appliquer…

1- Condition d'Orlov :



Si Ω contient au moins un cercle équatorial de S

(ou si Ω intersecte tout cercle équatorial de S)



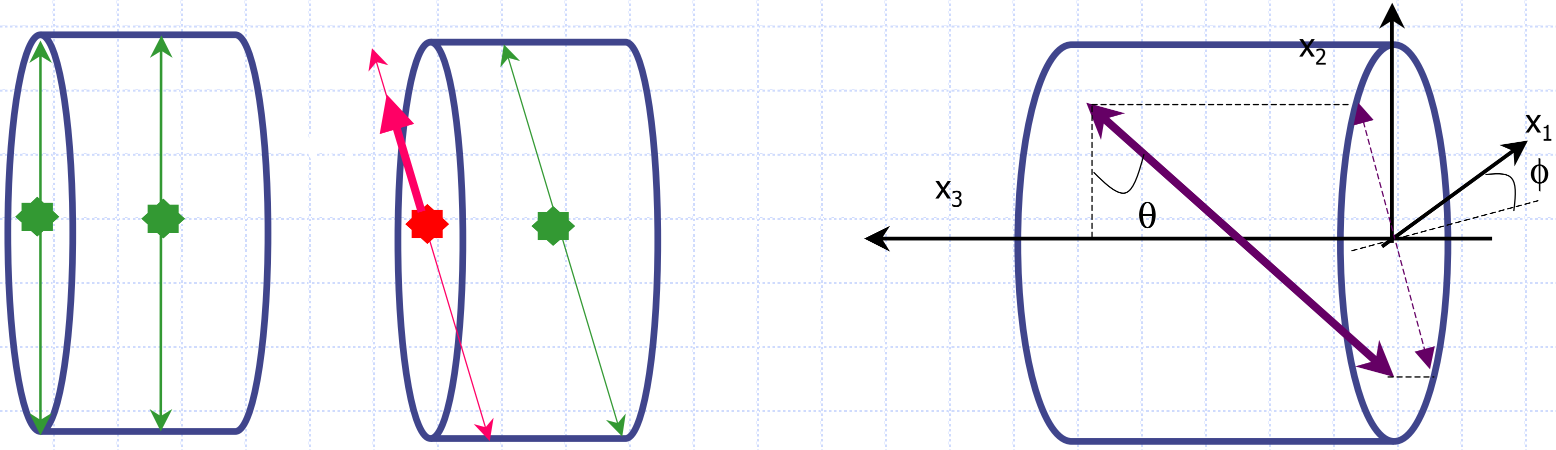
$$\forall \vec{\omega} \in \Omega \subset S, \quad \boxed{\forall \eta \in \vec{\omega}^{\perp}, \hat{P}_{\vec{\omega}}(\eta) = \hat{f}(\eta)}$$

SS. Orlov. Sov.Phys. Crystallogr., Vol 20, 3:312-4 et 4:429-433

… plutôt difficile à appliquer…

1- Condition d'Orlov

2 - si les projections ne sont pas tronquées (projections complètes)

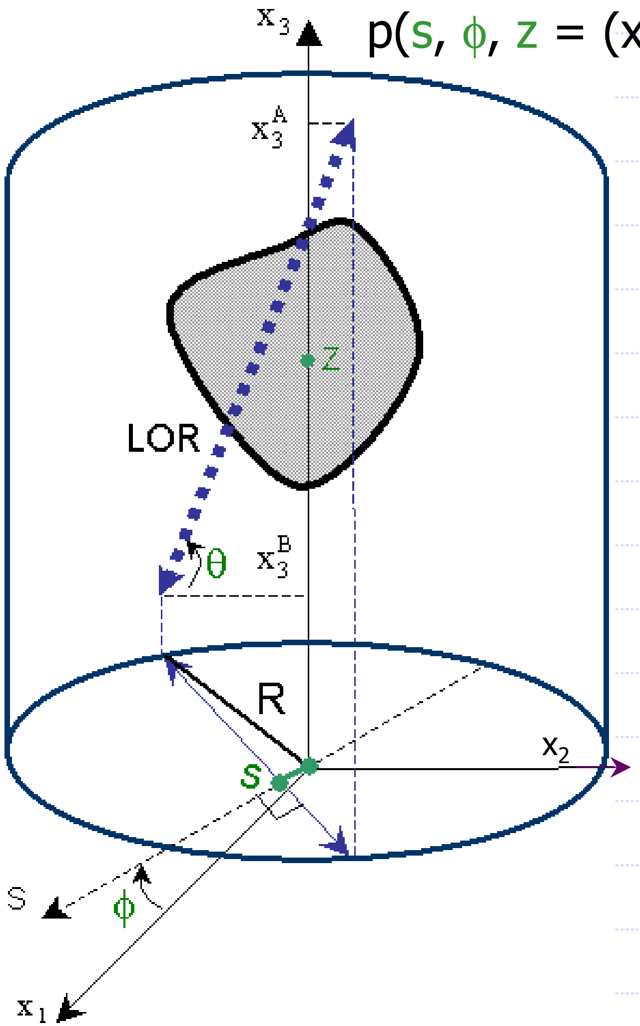


3 – moyennant une interpolation 3D dans le domaine des fréquences

$$\hat{p}(\xi_1, \xi_2) = \hat{f}(\xi_1 \cos\theta \sin\phi - \xi_2 \sin\theta, \xi_1 \sin\theta \sin\phi + \xi_2 \cos\theta, -\xi_1 \cos\phi)$$

Synthèse de projections complètes

$p(s, \phi, z = (x^A_3+z^B_3)/2, \delta = \mathrm{tg}\ \theta)$: 4 paramètres, 3 ddl

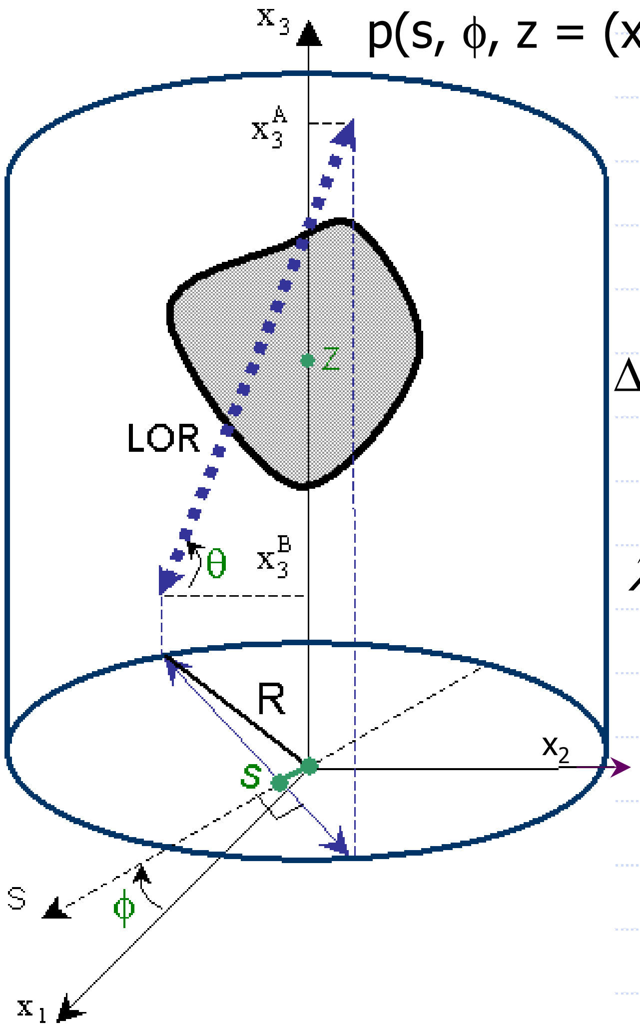


F Ben Bouallègue, JF Crouzet & D Mariano-Goulart, IEEE Trans Med Imaging 07 & Ann Nucl Med 09

Synthèse de projections complètes

$p(s, \phi, z = (x^A_3+z^B_3)/2, \delta= \mathrm{tg}\ \theta)$

TF3



$$\boxed{\hat{p}\left(\sigma, k, \zeta, \delta\right) = e^{-i\Delta\Phi}\ \hat{p}\left(\chi\sigma, k, \zeta, \delta'\right)}$$

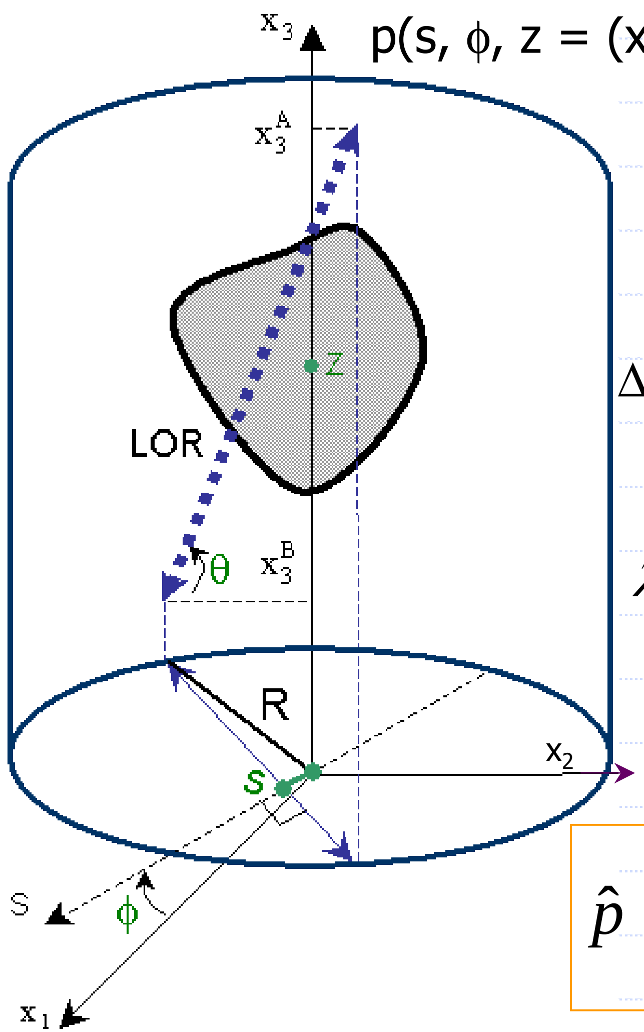
$$\Delta\Phi = k\left[\ arctg\left(\frac{\delta}{\sigma}\zeta\right) - arctg\left(\frac{\delta'}{\chi\sigma}\zeta\right)\right],\quad \delta > \delta'$$

$$\chi = \sqrt{1 + \left(\delta^2 - \delta'^2\right)\frac{\zeta^2}{\sigma^2}}$$

F Ben Bouallègue, JF Crouzet & D Mariano-Goulart, IEEE Trans Med Imaging 07 & Ann Nucl Med 09

Synthèse de projections complètes

$p(s, \phi, z = (x^A_3+z^B_3)/2, \delta = \mathrm{tg}\ \theta)$ TF3



$$\boxed{\hat{p}\left(\sigma, k, \zeta, \delta\right) = e^{-i\Delta\Phi}\ \hat{p}\left(\chi\sigma, k, \zeta, \delta'\right)}$$

$$\Delta\Phi = k\left[\ arctg\left(\frac{\delta}{\sigma}\zeta\right) - arctg\left(\frac{\delta'}{\chi\sigma}\zeta\right)\right],\quad \delta > \delta'$$

$$\chi = \sqrt{1 + \left(\delta^2 - \delta'^2\right)\frac{\zeta^2}{\sigma^2}}$$

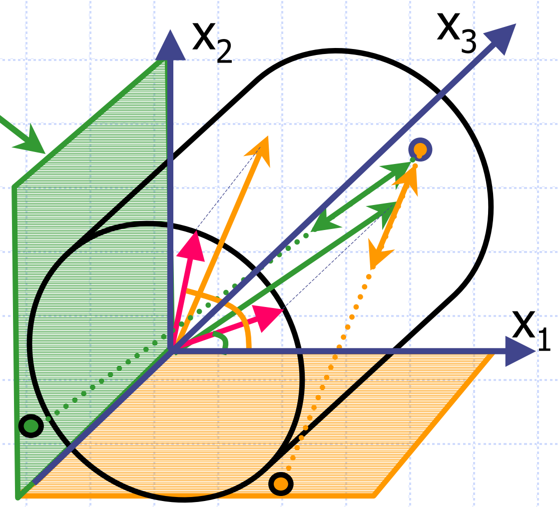
DL en $O\left(\left(\frac{\delta}{\sigma}\zeta\right)^2\right)$

$$\boxed{\hat{p}\left(\sigma, k, z, \delta\right) \approx \hat{p}\left(\sigma, k, z - \mathrm{k}\frac{\left(\delta - \delta'\right)}{\sigma}, \delta'\right)}$$

F Ben Bouallègue, JF Crouzet & D Mariano-Goulart, IEEE Trans Med Imaging 07 & Ann Nucl Med 09

Optimisation de l’interpolation

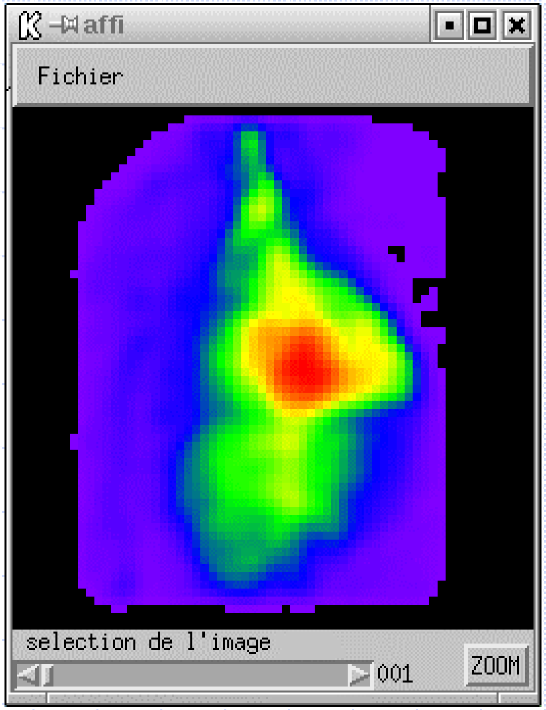
$$\hat{p}_w(\xi_1,\xi_2)=\frac{1}{|\sin\theta\,\sin\varphi|}\,\hat{f}\Big(\underbrace{\xi_1}_{},-\frac{\xi_1}{\tan\theta}-\frac{\xi_2}{\sin\theta\,\tan\varphi},\underbrace{\xi_2}_{}\Big)$$

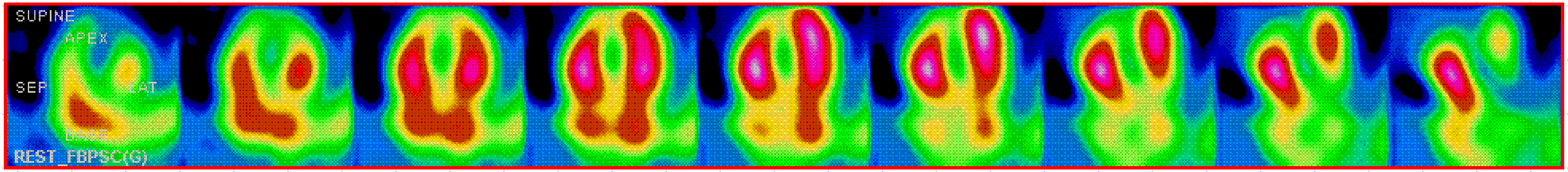


$$\hat{p}_w(\xi_1,\xi_2)=\frac{1}{|\cos\theta\,\sin\varphi|}\,\hat{f}\Big(-\xi_1\tan\theta-\frac{\xi_2}{\tan\varphi\,\cos\theta},\underbrace{\xi_1,\xi_2}_{}\Big)$$

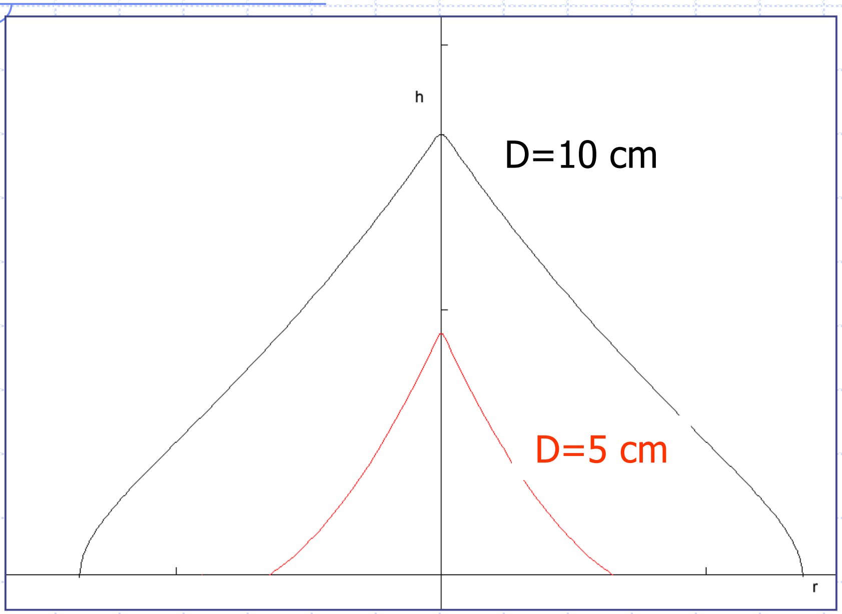
F Ben Bouallègue, JF Crouzet & D Mariano-Goulart, CR Physique 05 & Comput Med Imag. Graphics 08.

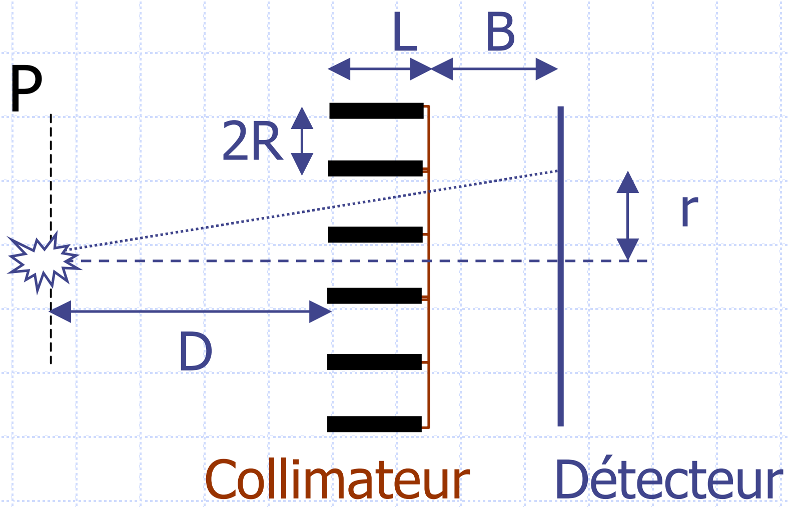
TOMO-VENTRICULOGRAPHIE





Réponse d'un collimateur



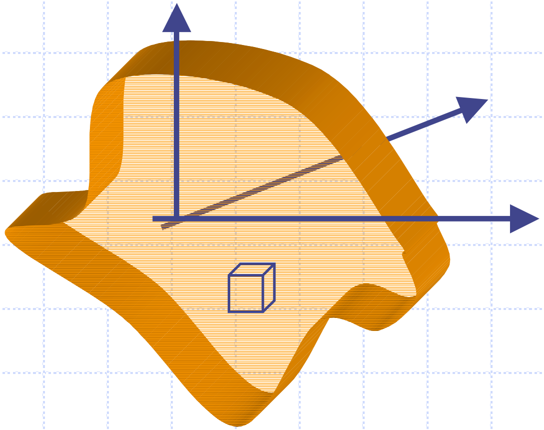


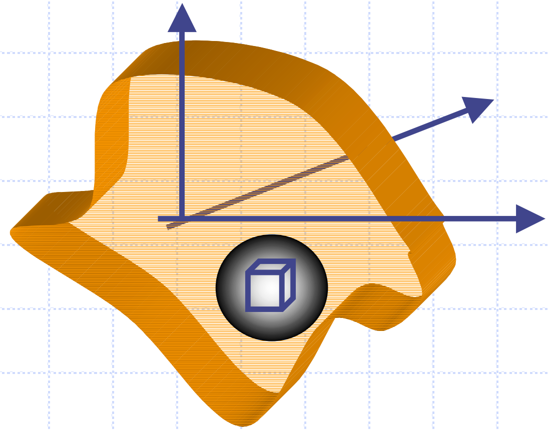
$$h(r) = \frac{\varepsilon}{\pi}\left[2.\arccos\left(\frac{\lambda.r}{2}\right) - \lambda.r.\sqrt{1-\left(\frac{\lambda.r}{2}\right)^2}\right] \qquad \lambda = \frac{L}{R.(L+D+B)}$$

LEHR : L = 4,1 cm ; B = 0,64 cm ; R = 0,19 cm ; ε = 0,065

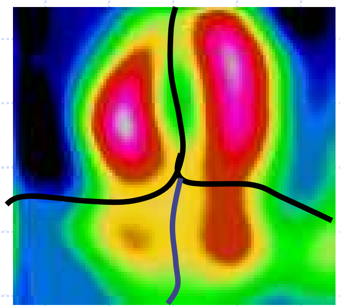
Quantitative Analysis in Nuclear Medicine Imaging, Springer US. 2006; CE Metz. Phys Med Biol 1990; 35.

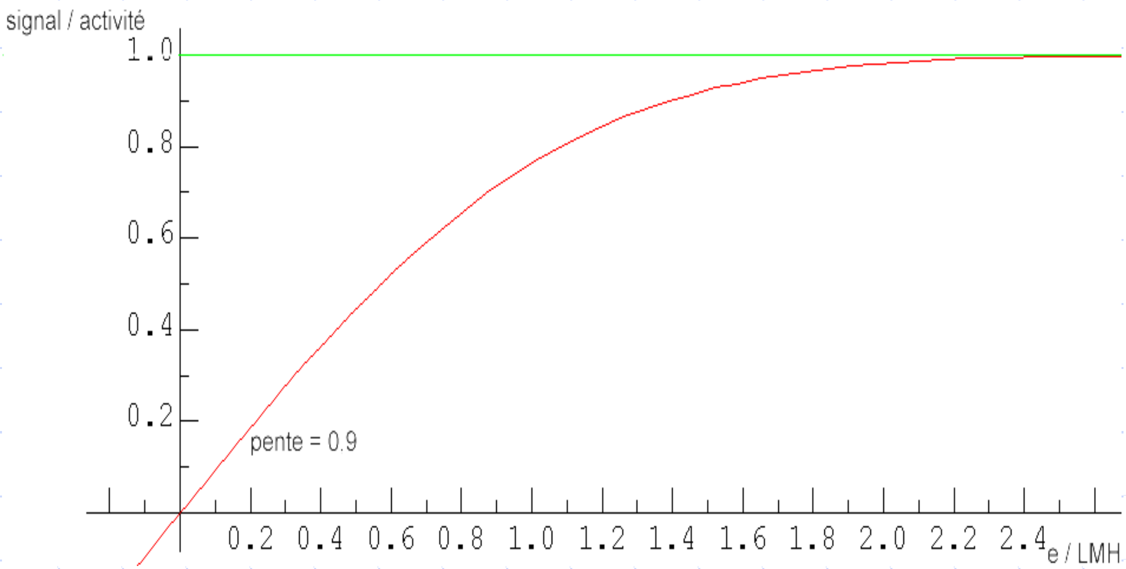
« Effet de volume partiel »





$$S = \frac{2\sqrt{\frac{\ln 2}{\pi}}}{LMH} \int_{-e/2}^{e/2} e^{-\frac{4.\ln 2}{LMH^2} z^2} \, dz$$





JR Galt et al. «Effects of myocardial wall thichness on SPECT quantification». IEEE TMI 1990;9

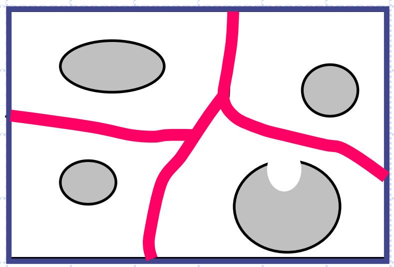
Squelette

Soit $X = \cup X_i$ une réunion de régions compactes disjointes.

On définit la zone d'influence de X_i :

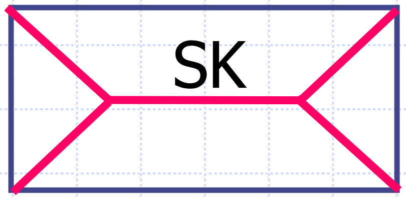
$$IZ(X_i) = \left\{ x, d(x, X_i) < d(x, X \setminus X_i) \right\}$$

SKIZ



le squelette par zones d'influences de X :

$$SKIZ(X) = X \setminus \bigcup_i IZ(X_i)$$



et le squelette de X :

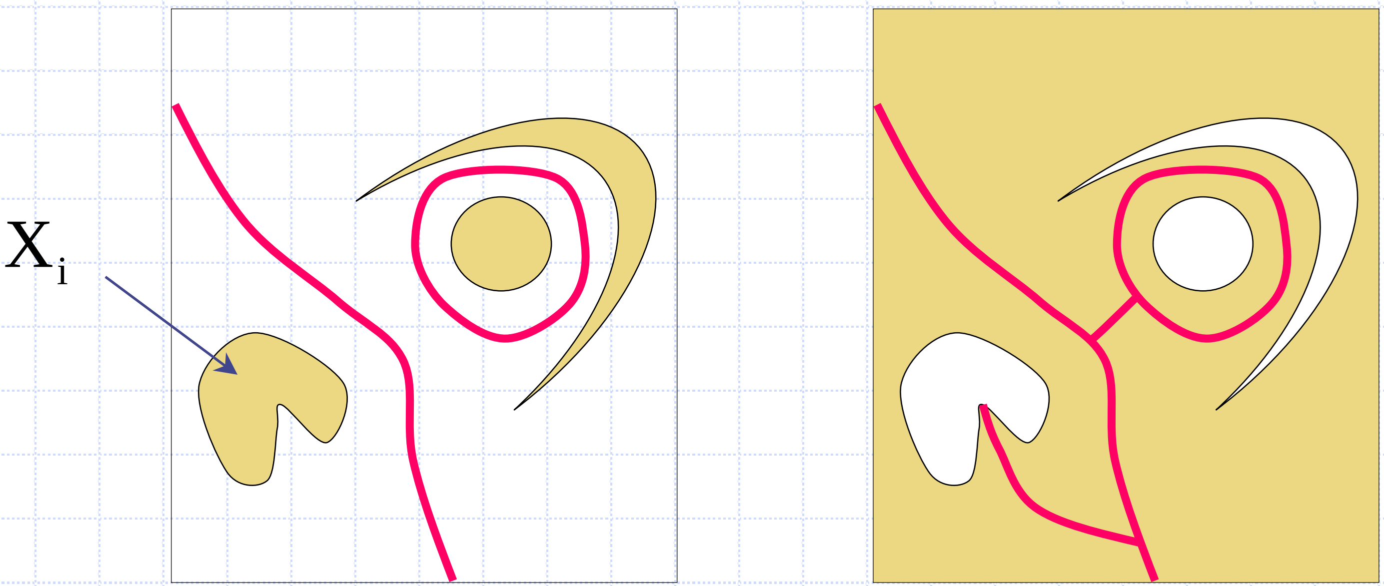
$$SK(X) = \left\{ x \in X, \exists (p,p') \in \partial X^2, p \neq p' / d(x, \partial X) = d(s,p) = d(s,p') \right\}$$

J. Serra. Image Analysis & Mathematical Morphology. Academic Press. Vol 1(1982) et 2 (1988).
M. Schmidt & J. Mattioli. « Morphologie Mathématique ». 1994. Masson.

Squelette : propriété

$$SK(X) = \left\{ x \in X, \exists (p,p') \in \partial X^2, p \neq p' / d(x,\partial X) = d(s,p) = d(s,p') \right\}$$

$$SKIZ(X) = X \setminus \bigcup_i \left\{ x, d(x, X_i) < d(x, X \setminus X_i) \right\}$$



$$SKIZ(X) \quad \subset \quad SK(X^C)$$

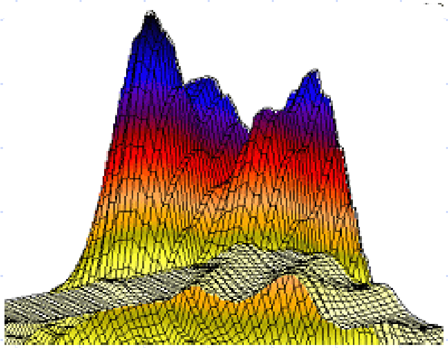
Ligne de partage des eaux

Soit f de classe C^1 / f(m)= 0 si m est un minimum local.
On définit :

$$LPE(f) = \{x, \exists (m, m')\text{minima locaux}/\mathrm{d_f}(x,m) = \mathrm{d_f}(x,m')\}$$

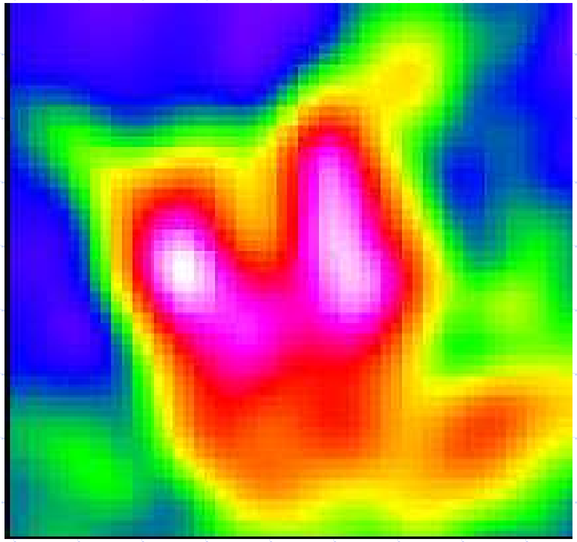
où :

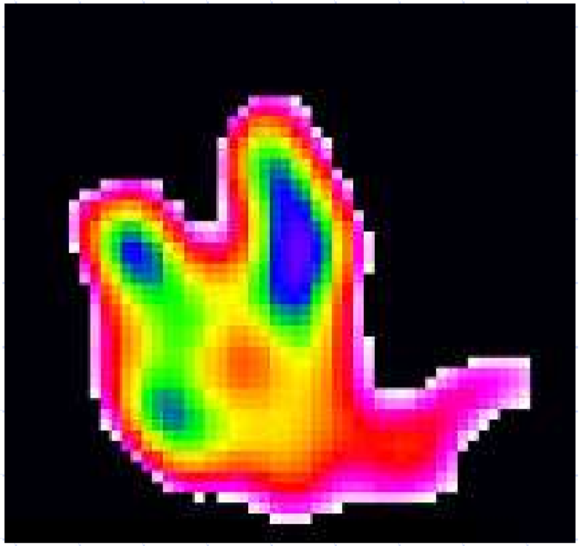
$$\mathrm{d_f}(a,b) = \inf_{\gamma_{a,b}} \int_a^b \left\| \nabla f\left(\gamma_{a,b}(s)\right)\right\| ds$$

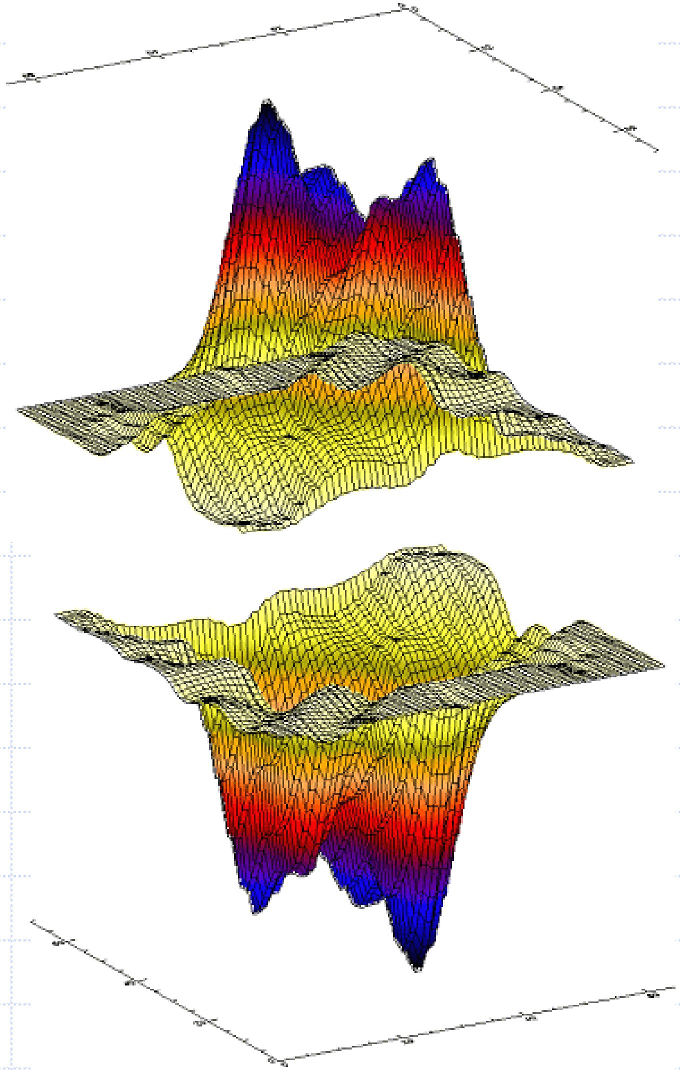


Propriété : la LPE est un SKIZ(U{m_i}) pour d_f

Intérêt en segmentation



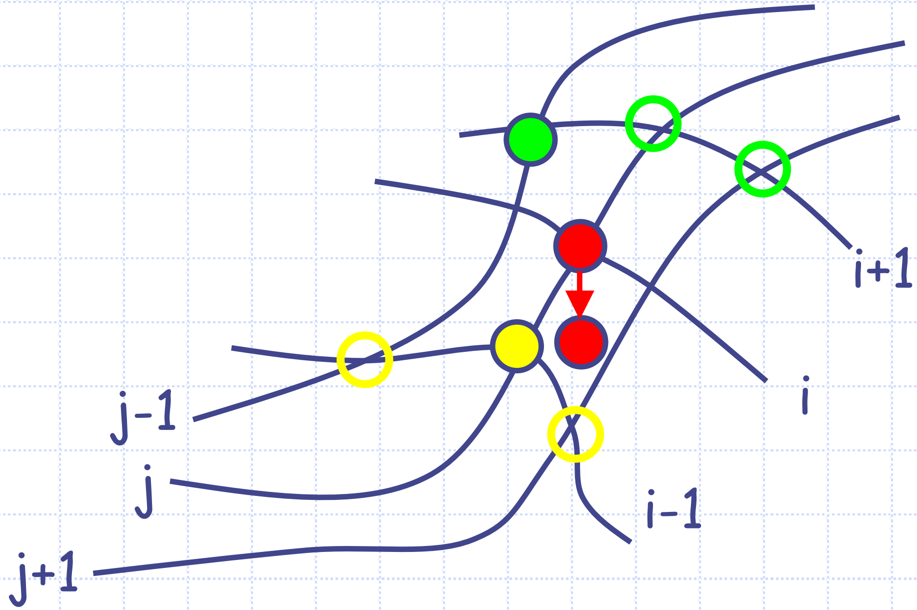




Sq(f^c) amincissements…

$$\left(f \circ \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \right)(i,j) = f_{max} \quad \text{si} \quad f_{max} < f(i,j) \leq f_{min}$$

$$L = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

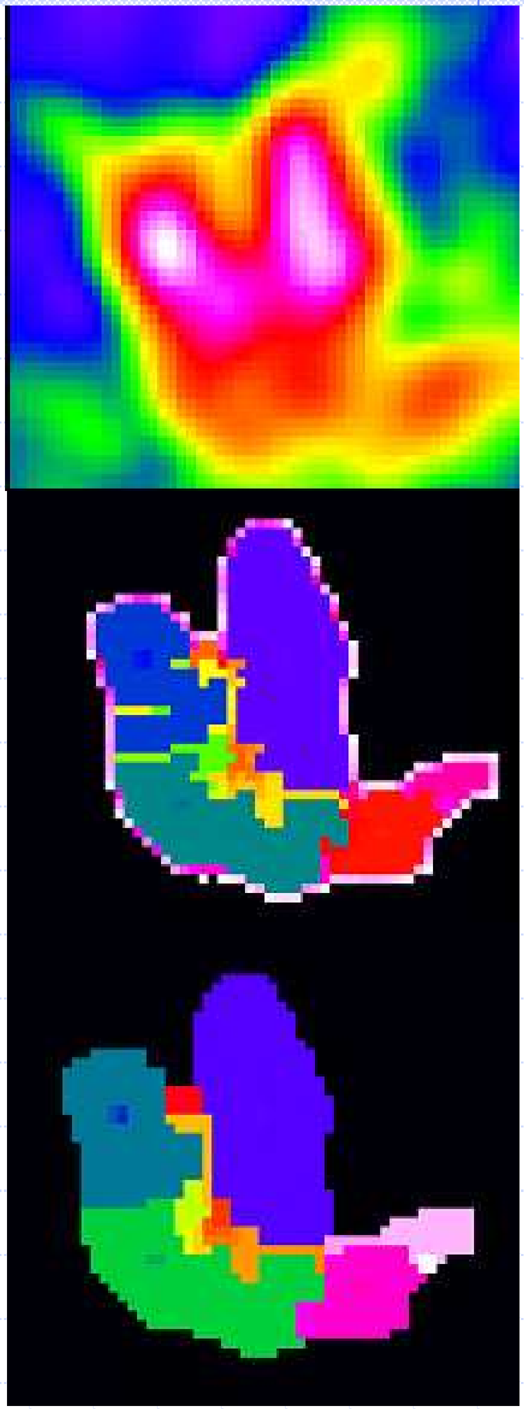


Mariano-Goulart et al. Eur J Nucl Med 1998; 22:1300-07

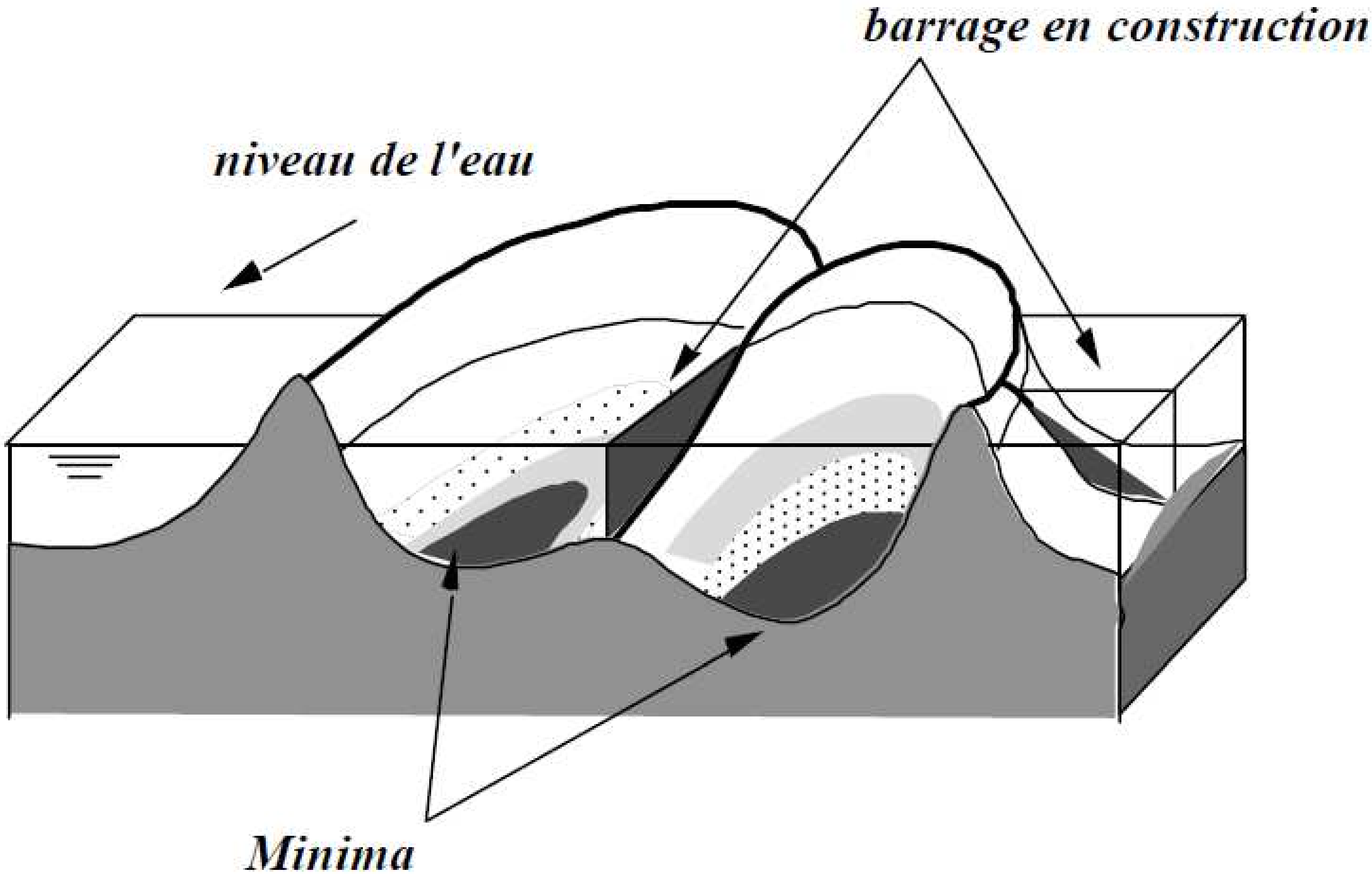
puis LPE par ébarbulage

$$Sq = \left(f \circ L_i\right)^{\infty}$$

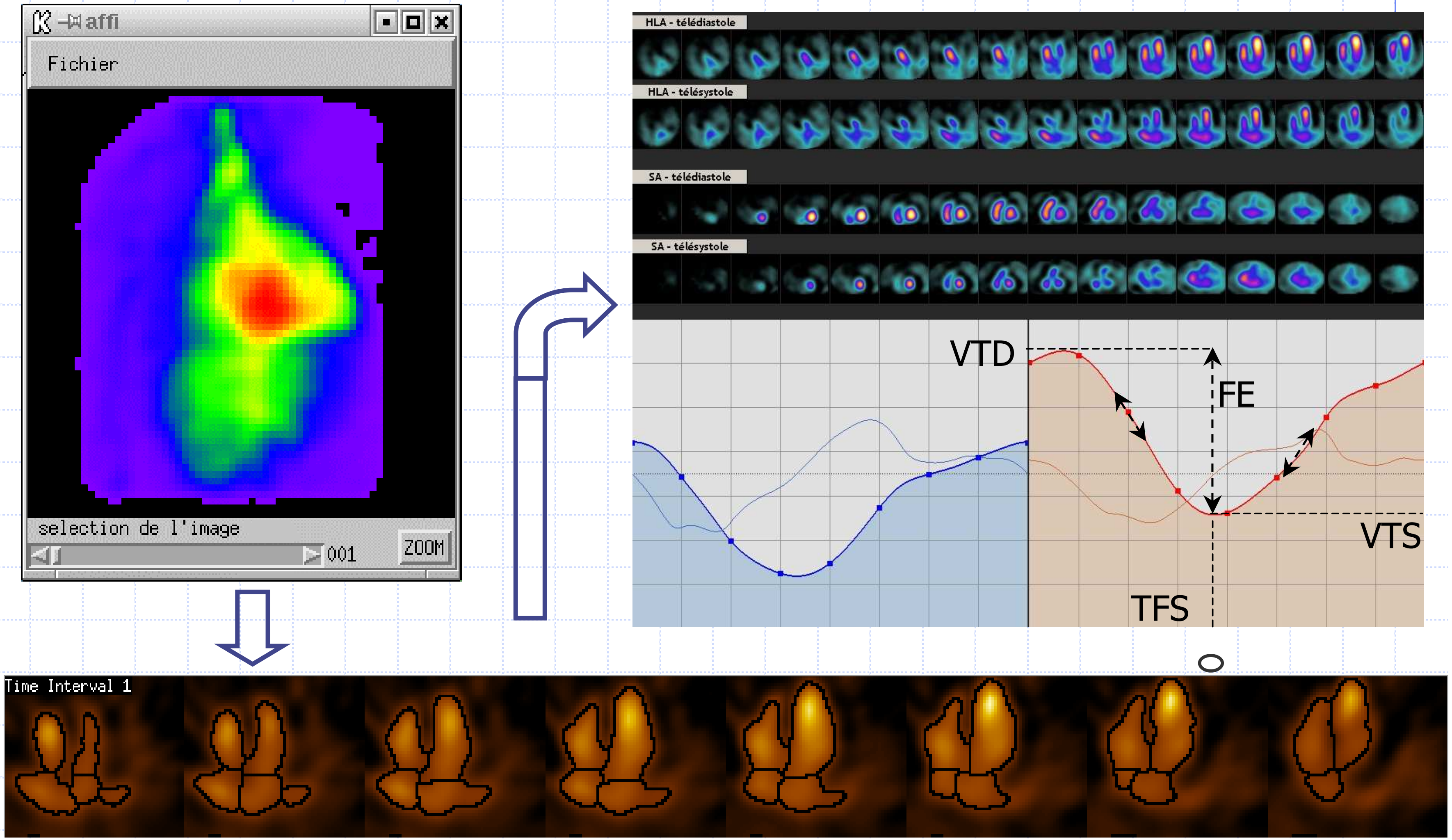
$$LPE = \left(f \circ \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_i \right)^{\infty}$$



Alternative: immersion

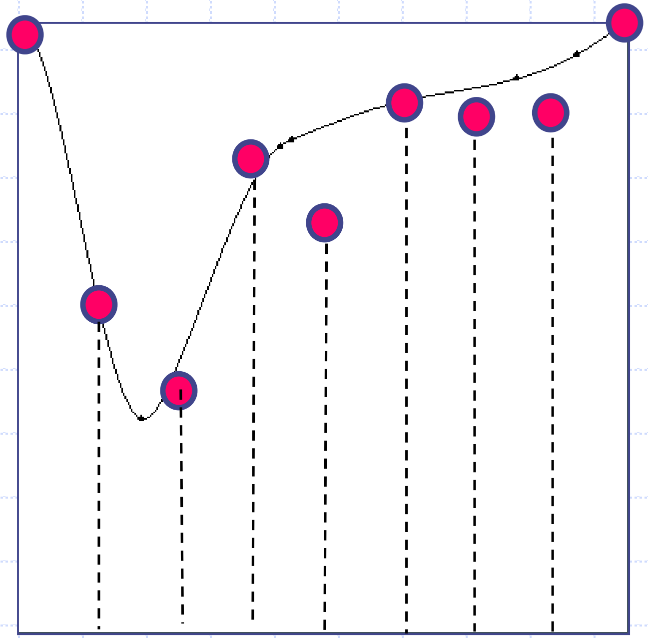


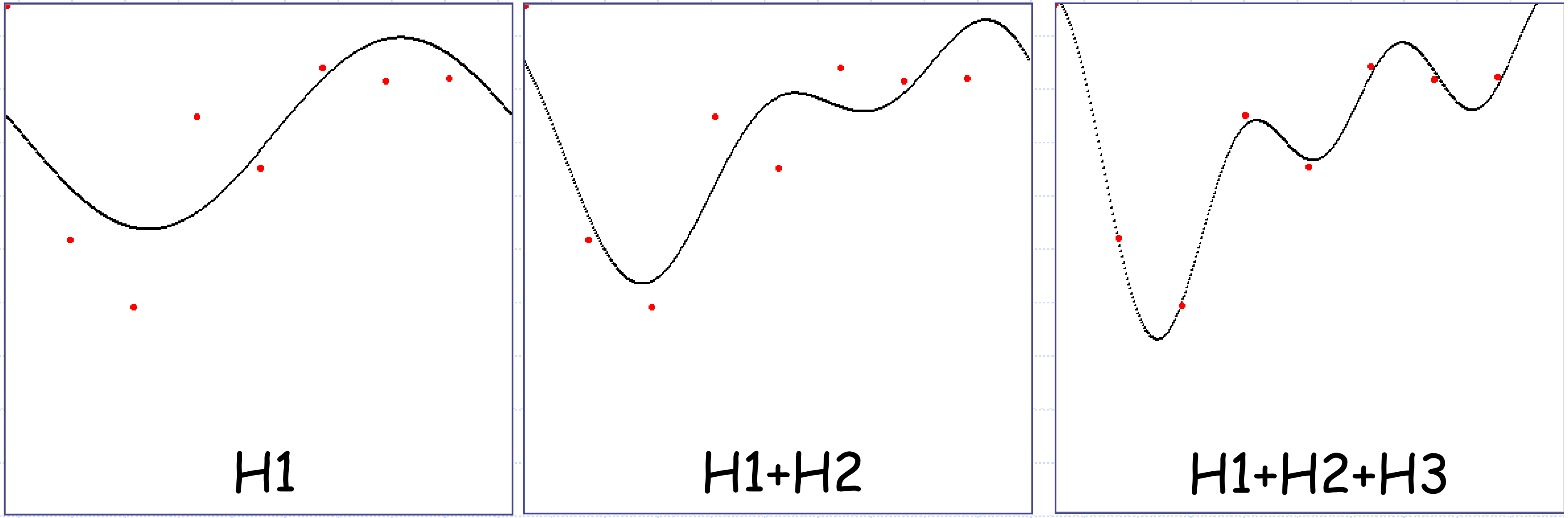
TOMO-VENTRICULOGRAPHIE



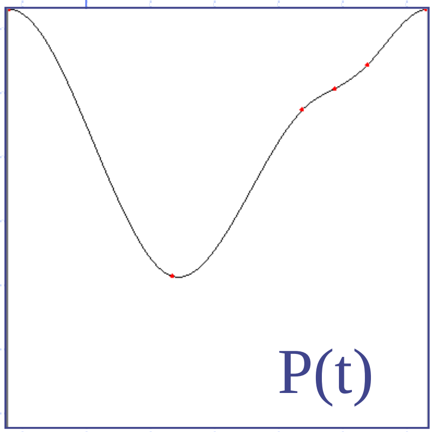
F. Ben Bouallègue et D.Mariano-Goulart. Eur J Nucl Med 1998-2001-J Nucl Med 2001-2007

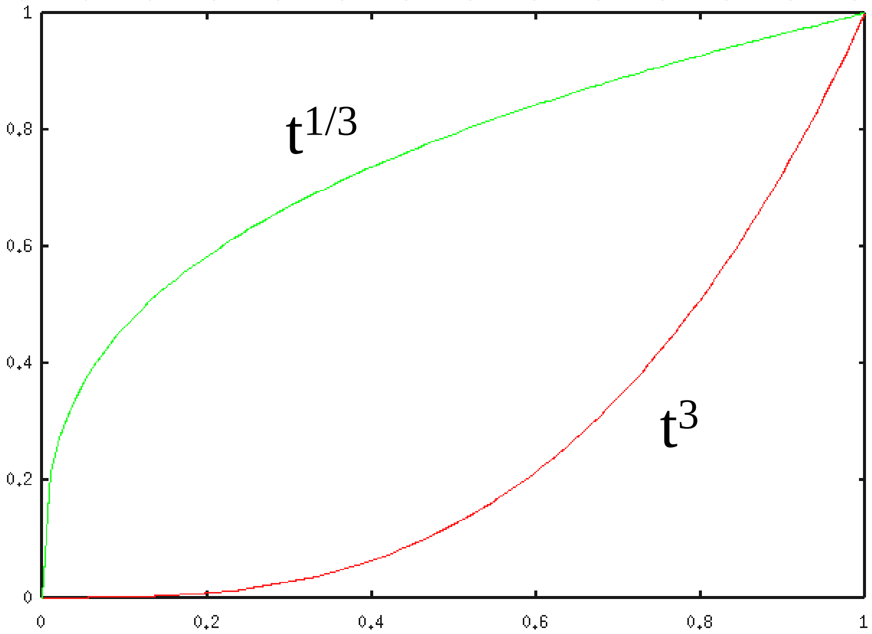
DEBRUITAGE

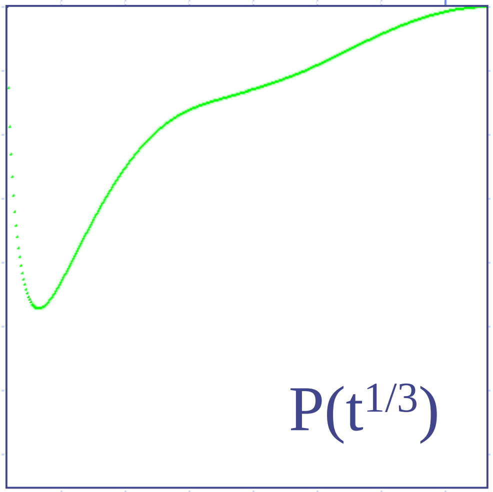


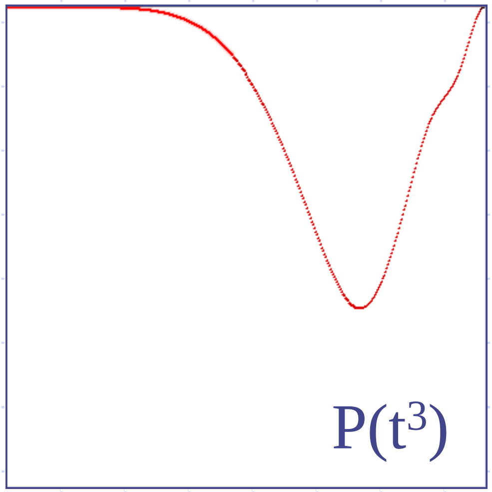


AJUSTEMENT D'UN MODELE

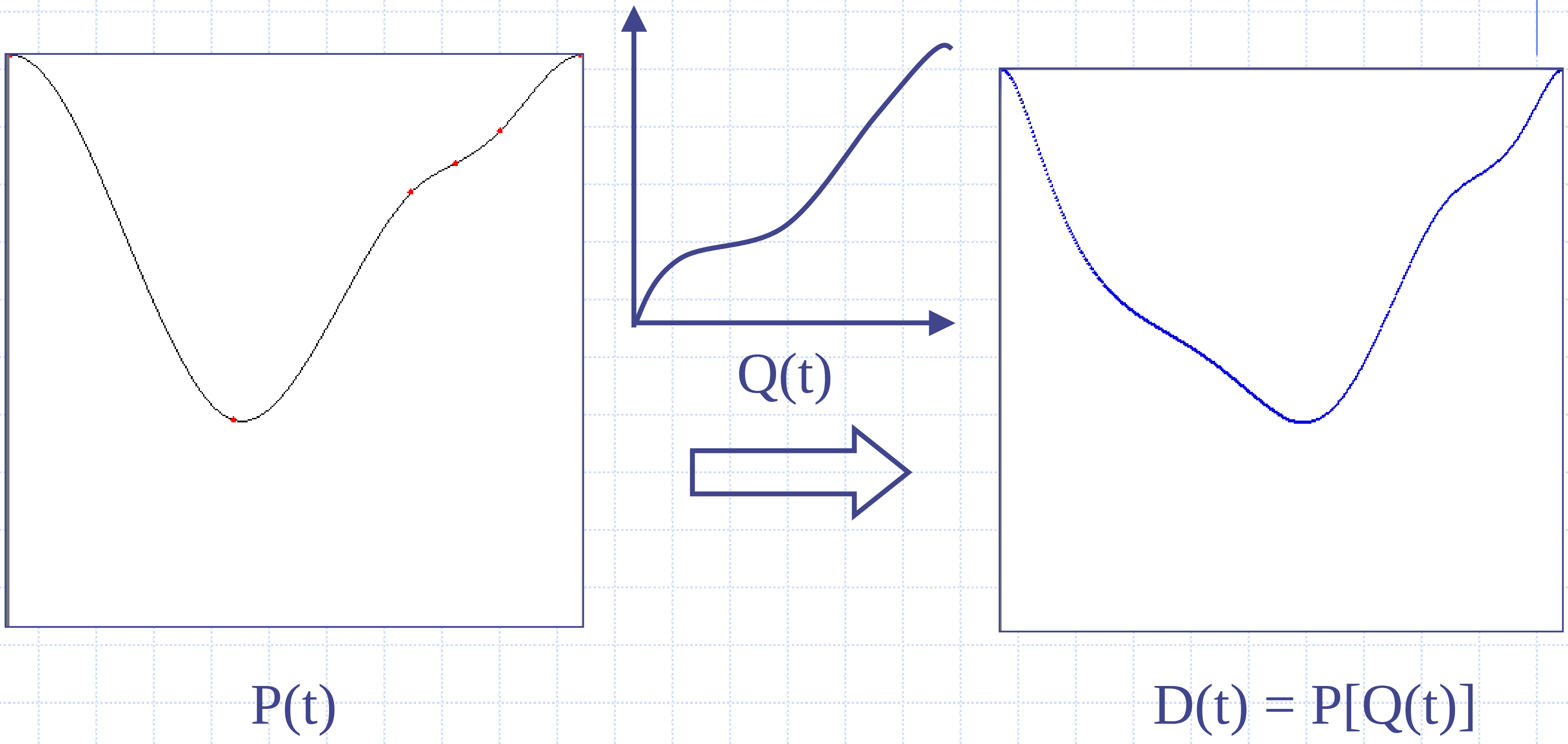




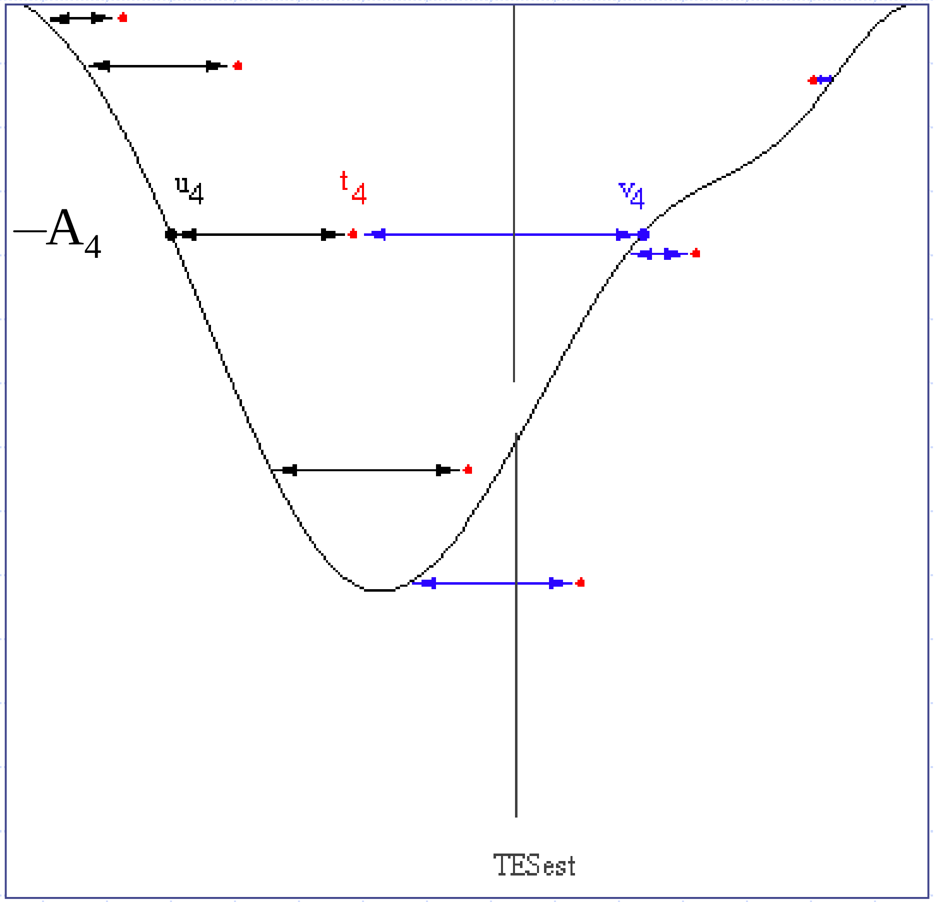




AJUSTEMENT D'UN MODELE

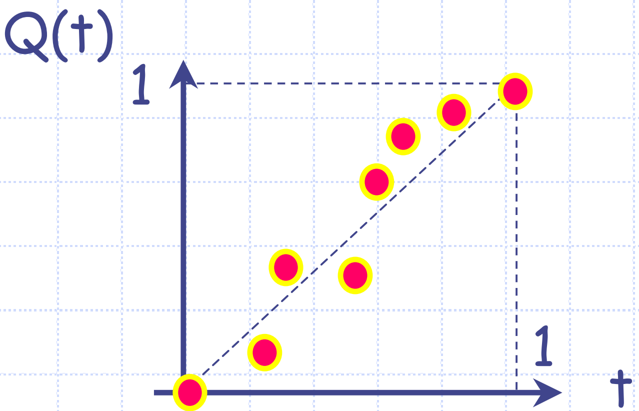


AJUSTEMENT D'UN MODELE

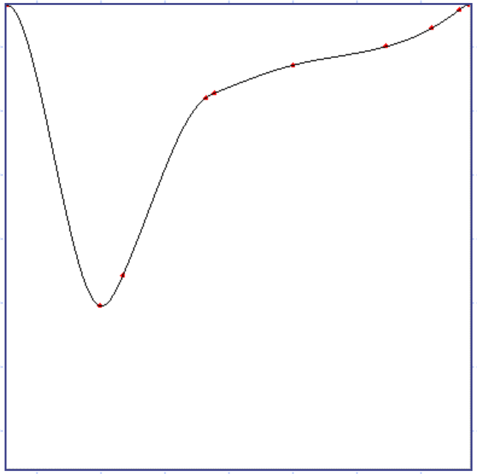


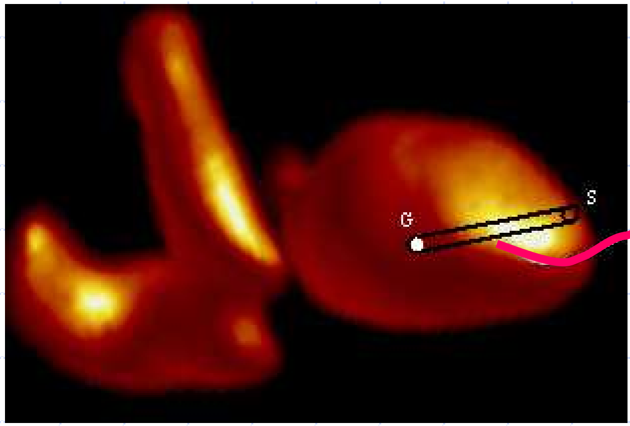
Acquisition bruitée (t_4 , A_4)

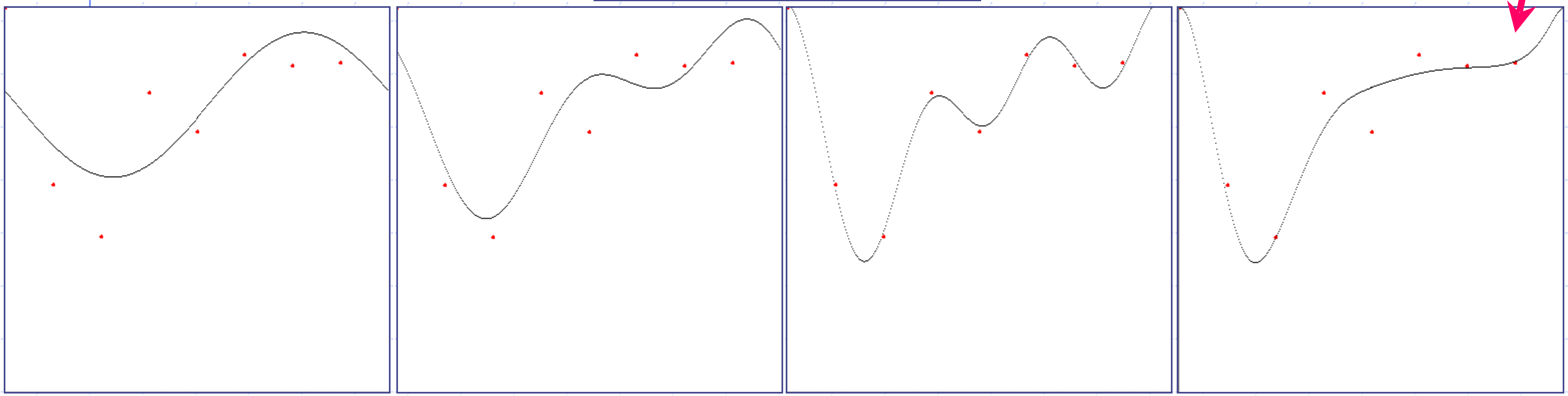
$$A_4 = P(u_4)$$
$$A_4 = D(t_4) = P[\, Q(t_4)]$$
$$\boxed{Q(t_4) = u_4}$$



RESULTATS



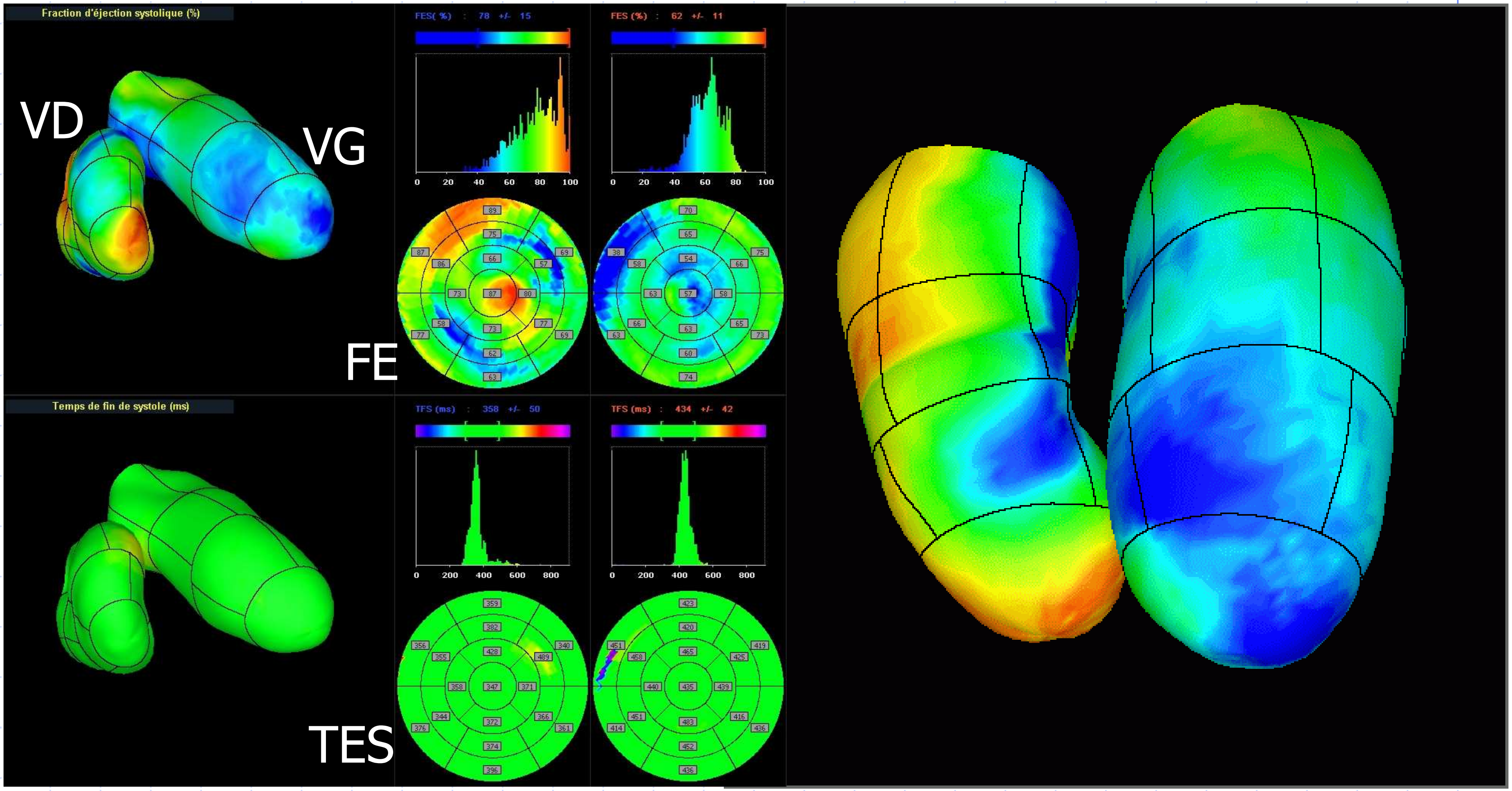




H1 H1+H2 H1+H2+H3 D(t)

C. Caderas de Kerleau et D. Mariano-Goulart, *IEEE Trans Med Imaging* 2004

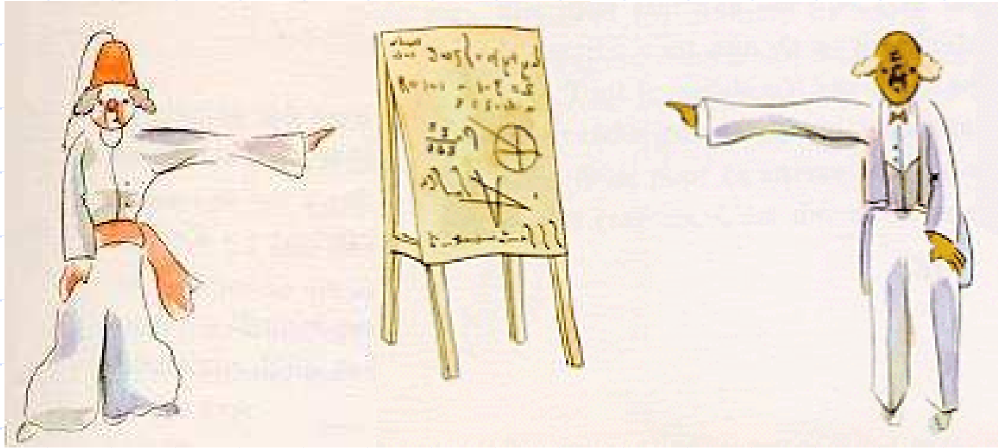
ANALYSE 3D DE CTA LOCALES



D.Mariano-Goulart et al. J. Nucl Med 2007

La collaboration Médecine-I3M

- Depuis 2003
  - JF Crouzet, F Ben Bouallègue, B Mohammadi
  - O Strauss, P. Maréchal, D Mariano-Goulart
- 8 articles internationaux (IF=1-4)
- 3 conférences internationales
- 5 thèses de sciences
  - JL Bernon (2000), C Caderas de Kerleau (2003)
  - Y Saesor(2007), F Ben Bouallègue (2009), D Hoa…



M. Fourcade
D. Mariano-Goulart
M. Rossi
M. Zanca

F. Ben Bouallègue
JF. Crouzet
B. Mohammadi

Merci pour votre attention…