TRAITEMENT NUMERIQUE DES IMAGES MEDICALES (I)

NUMERISATION

FILTRAGE

SEGMENTATION

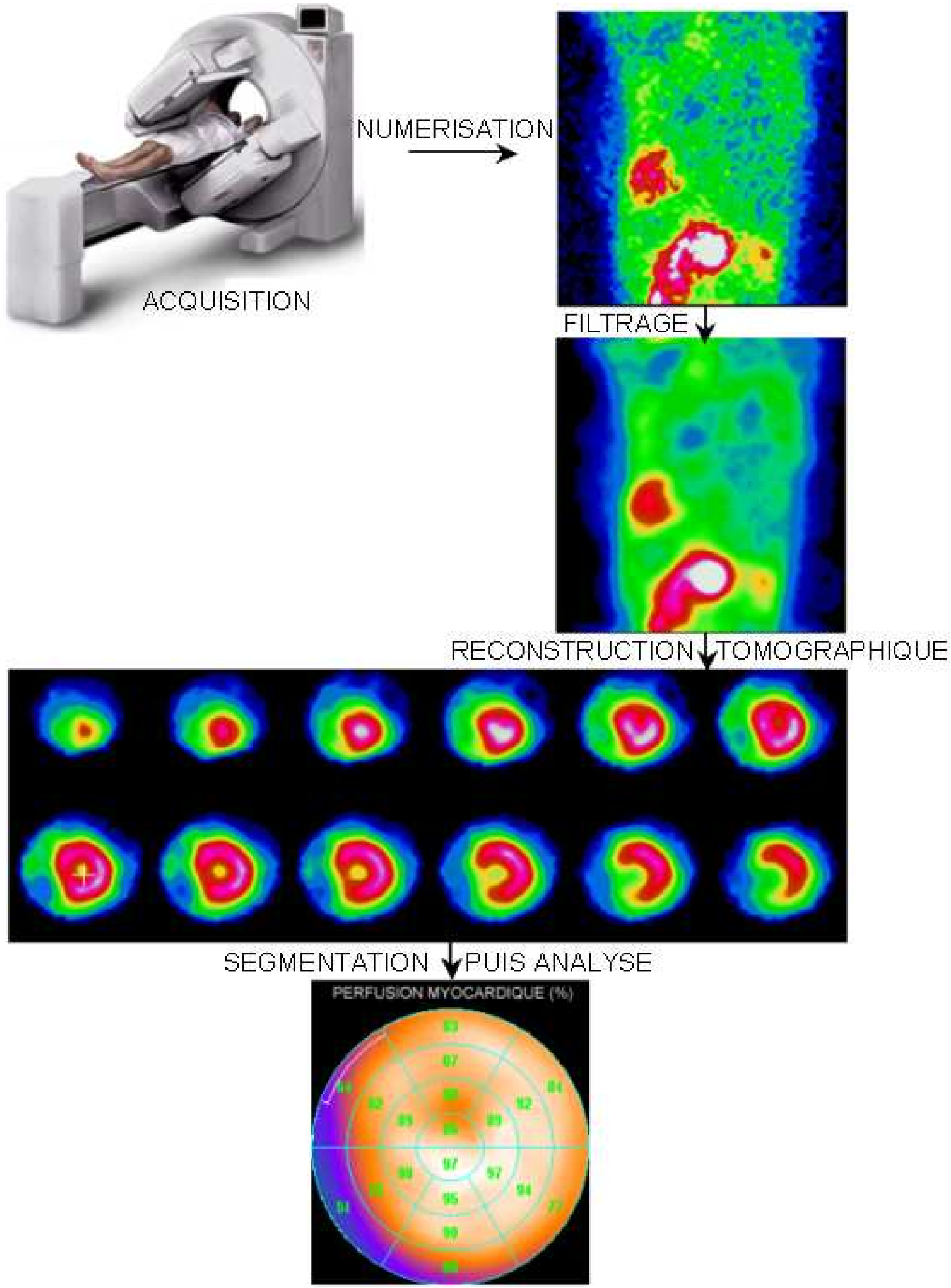
D. MARIANO-GOULART

PLAN

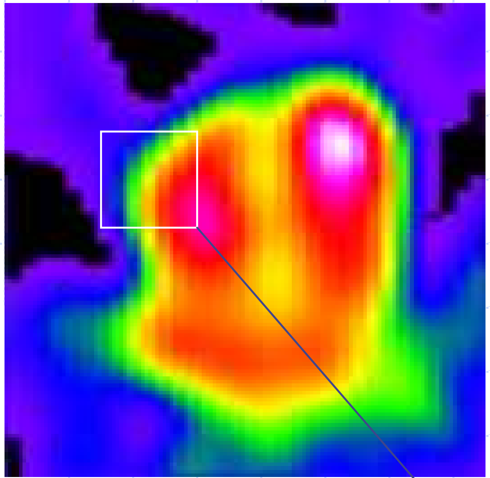
- **Images analogiques & numériques**
- Transformée de Fourier discrète
- Formation de l’image, résolution
- Théorème d’échantillonnage
- Filtrage linéaire
- Filtrage non linéaire

① Notion d'image

CHAINE D'ACQUISITION,
DE TRAITEMENT,
DE RECONSTRUCTION
ET D'ANALYSE
D'IMAGE



Notion d'image



MESURE D'UN SIGNAL PHYSIQUE 2D

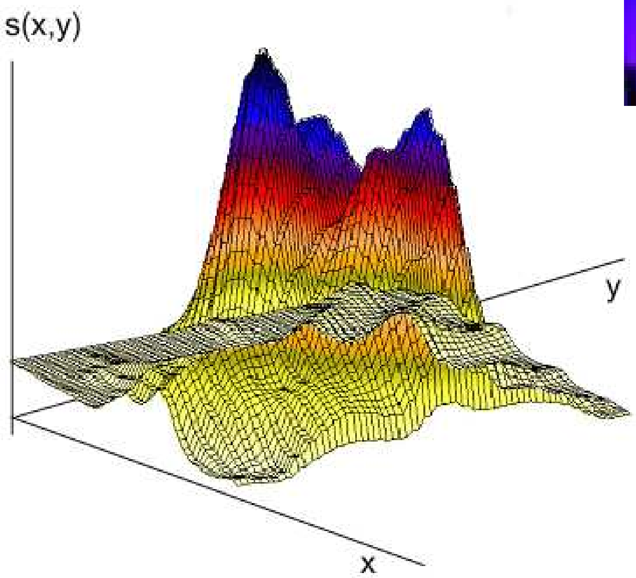
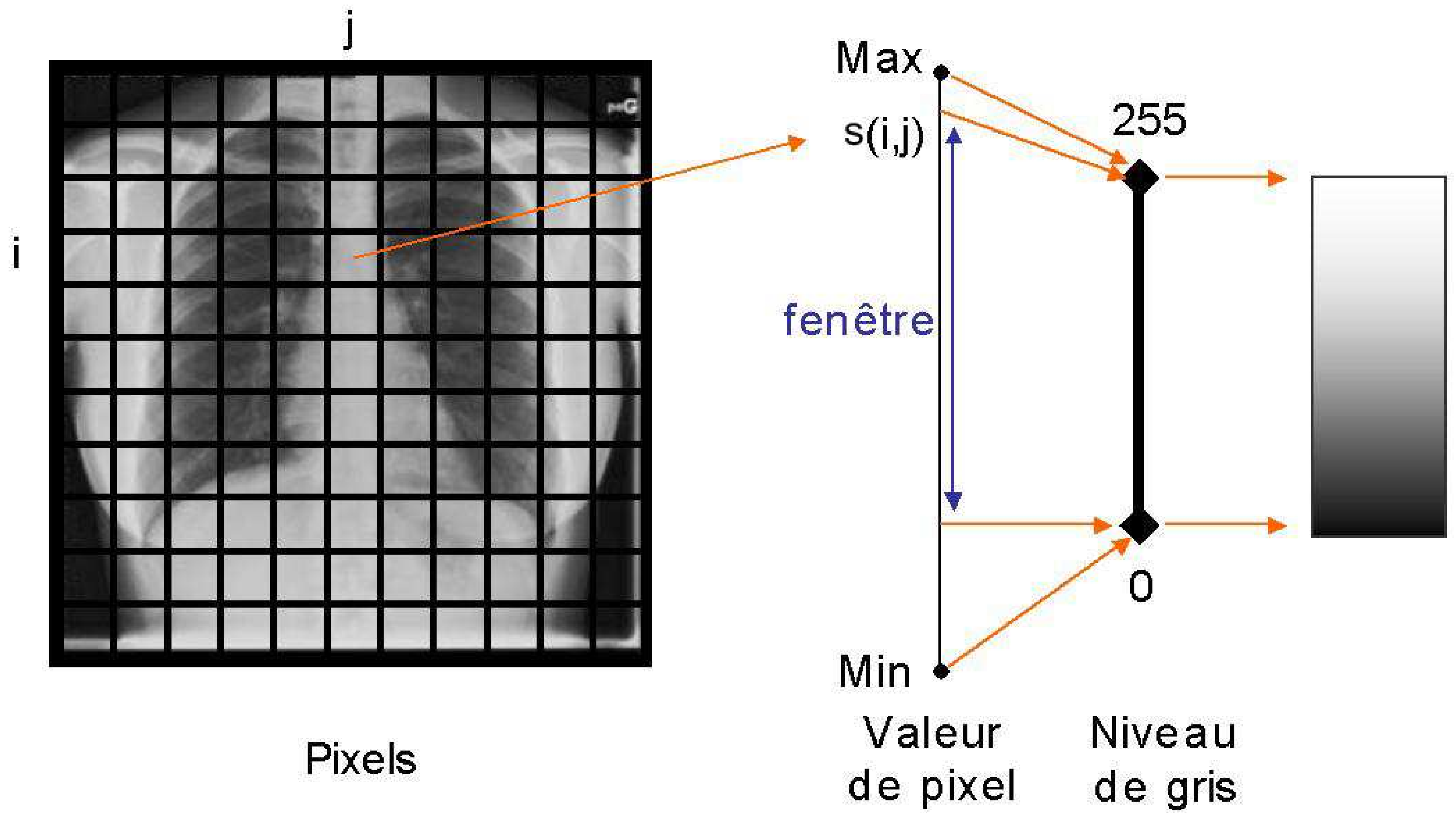


IMAGE ANALOGIQUE

10	12	15	15	13	15	23	24
9	15	19	18	15	16	22	25
5	16	25	22	18	18	22	29
4	15	28	32	23	21	25	32
2	7	21	23	25	22	22	25
1	5	15	21	22	21	19	19
2	6	13	16	18	18	18	18
9	9	10	15	16	15	16	16

IMAGE NUMERIQUE

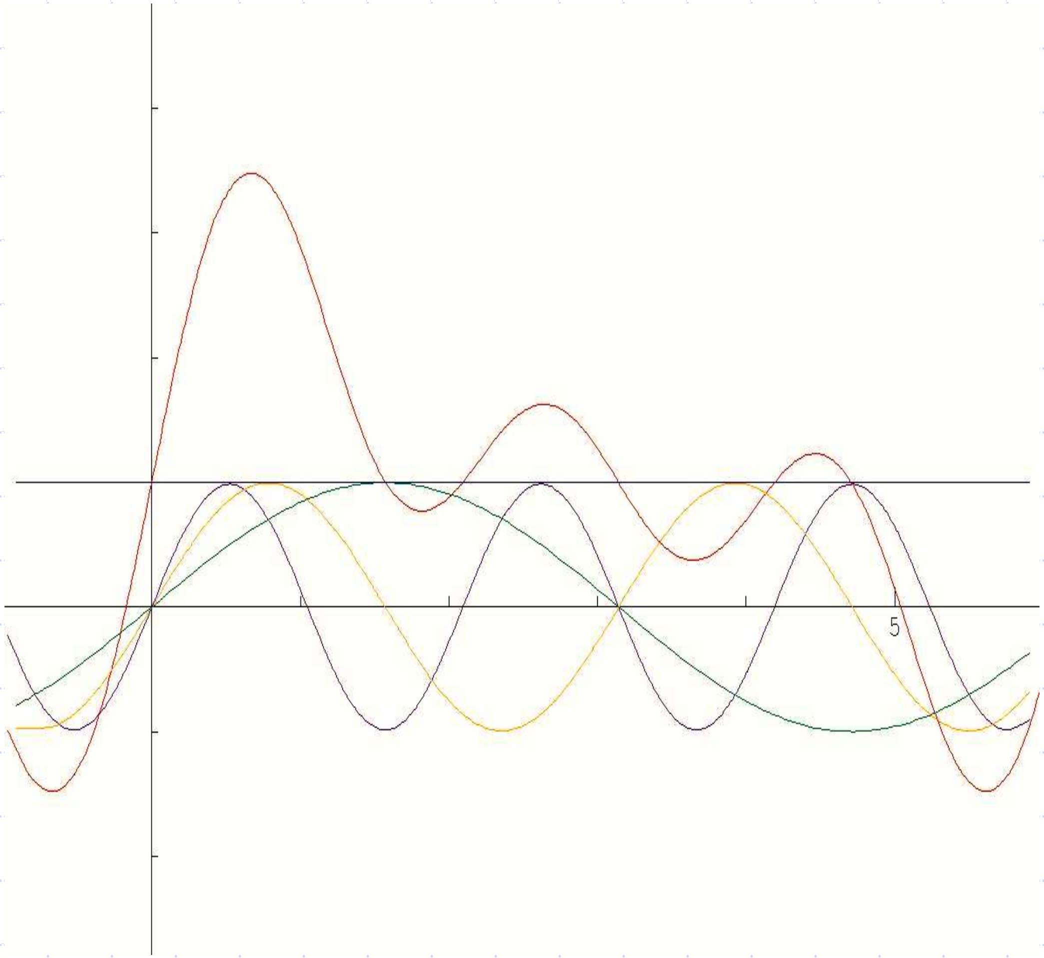
Numérisation

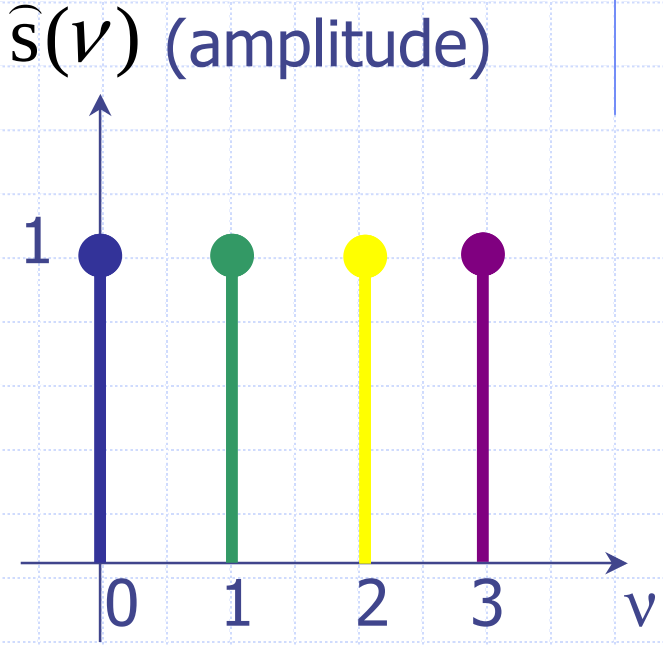


REPRESENTATION EN FREQUENCES

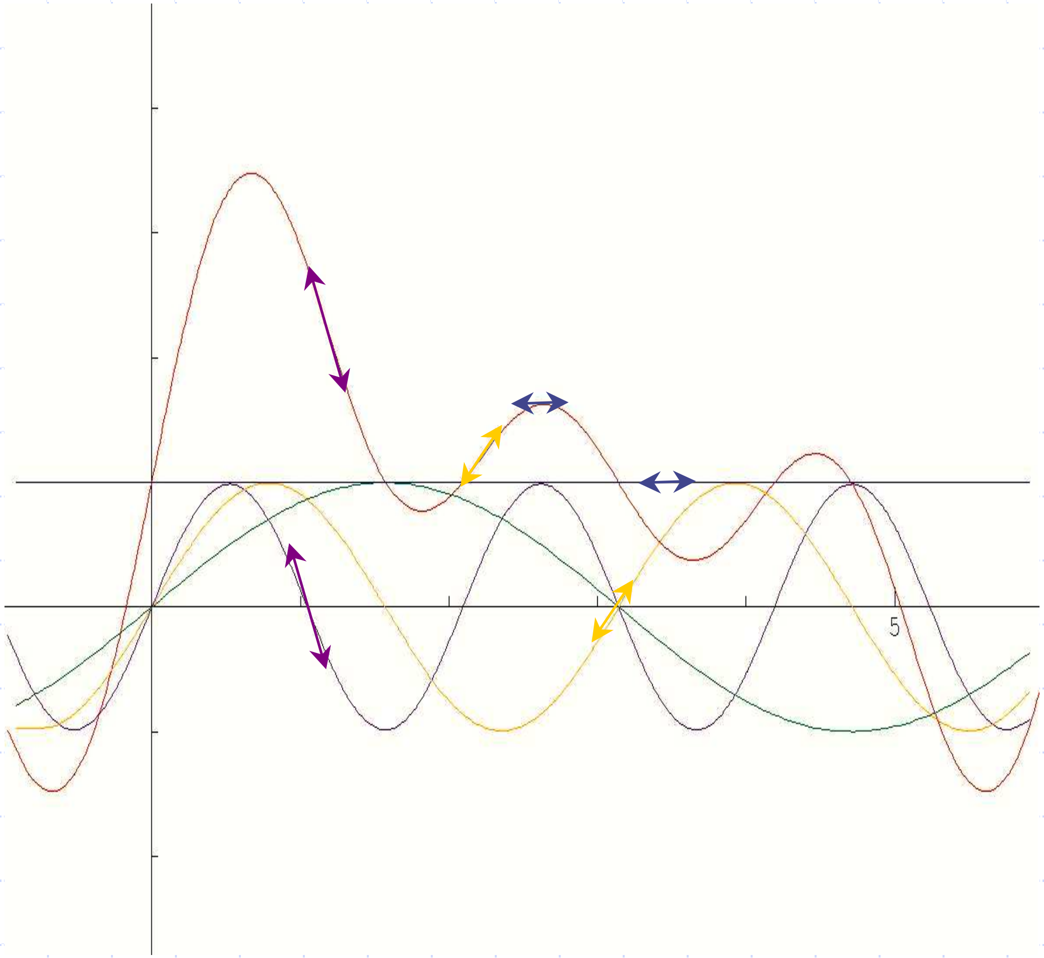
TRANSFORMATION DE FOURIER

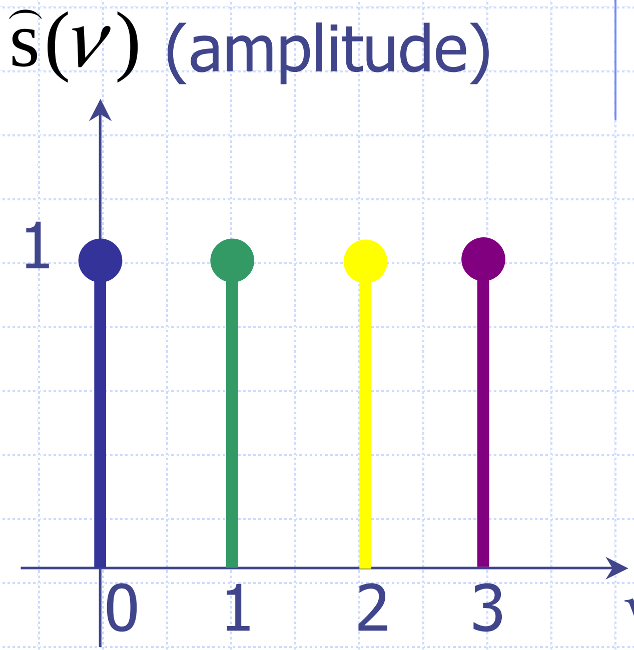
Représentation fréquentielle



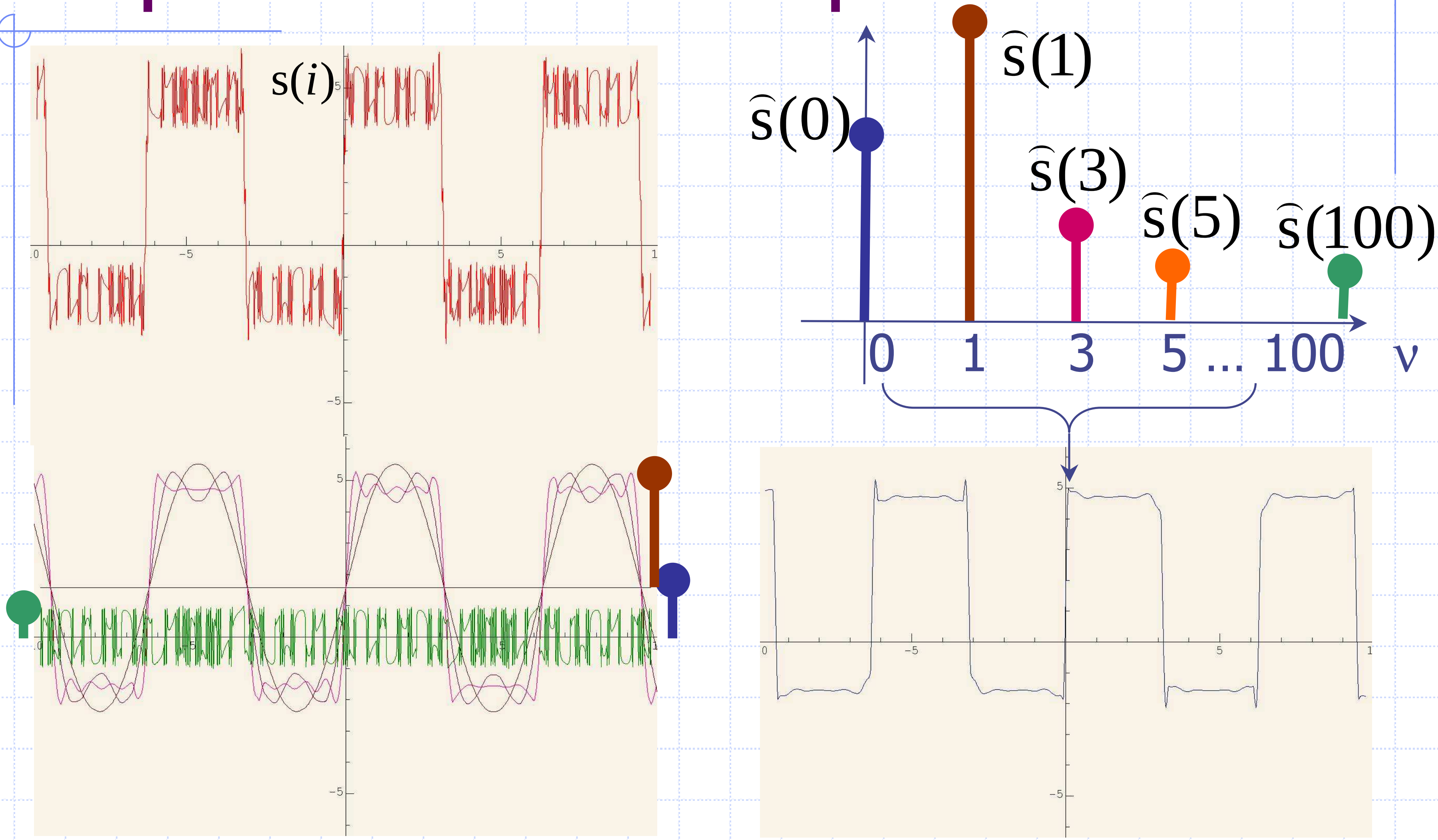


Représentation fréquentielle

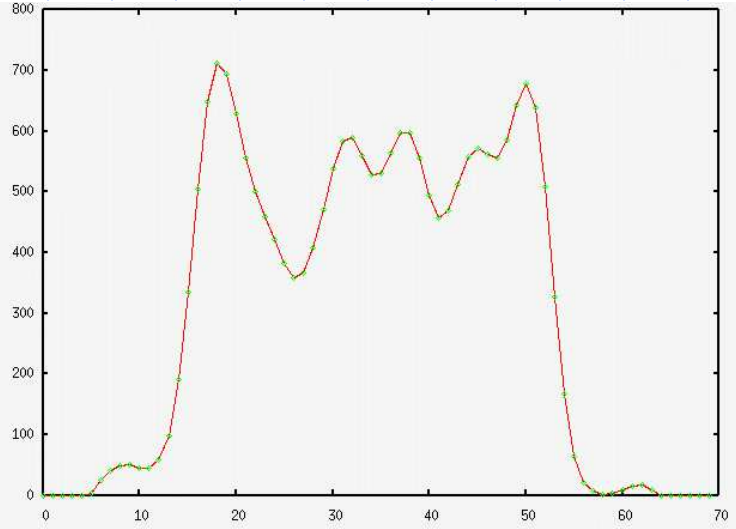


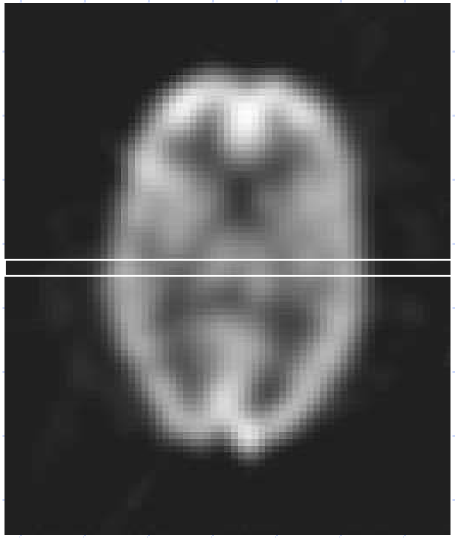


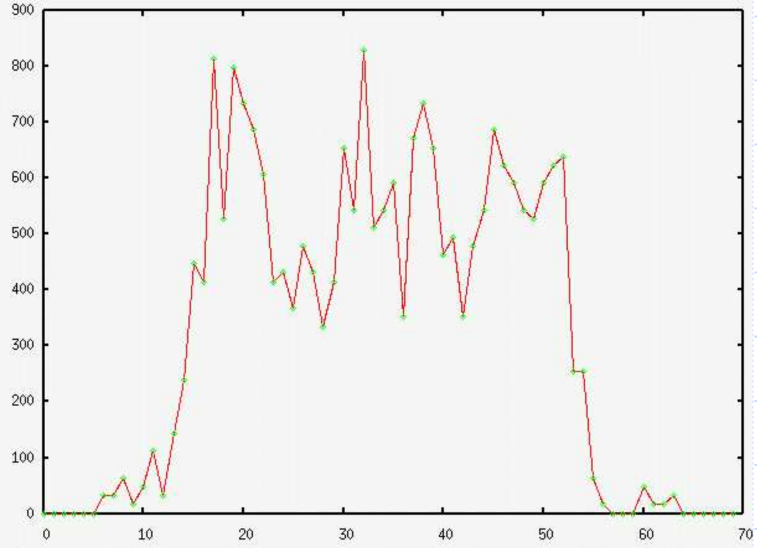
Représentation fréquentielle

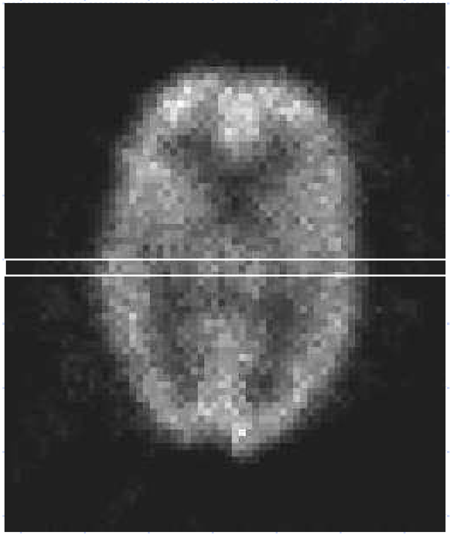


Représentation fréquentielle

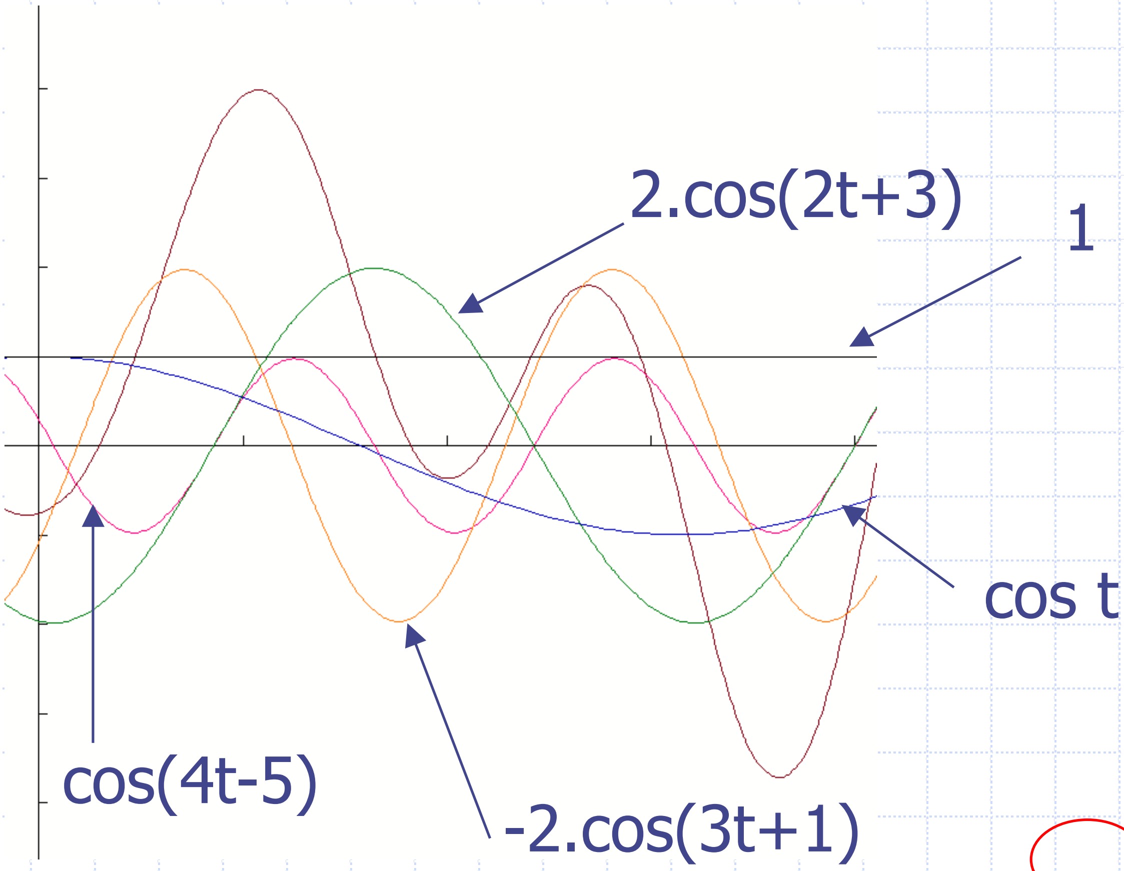






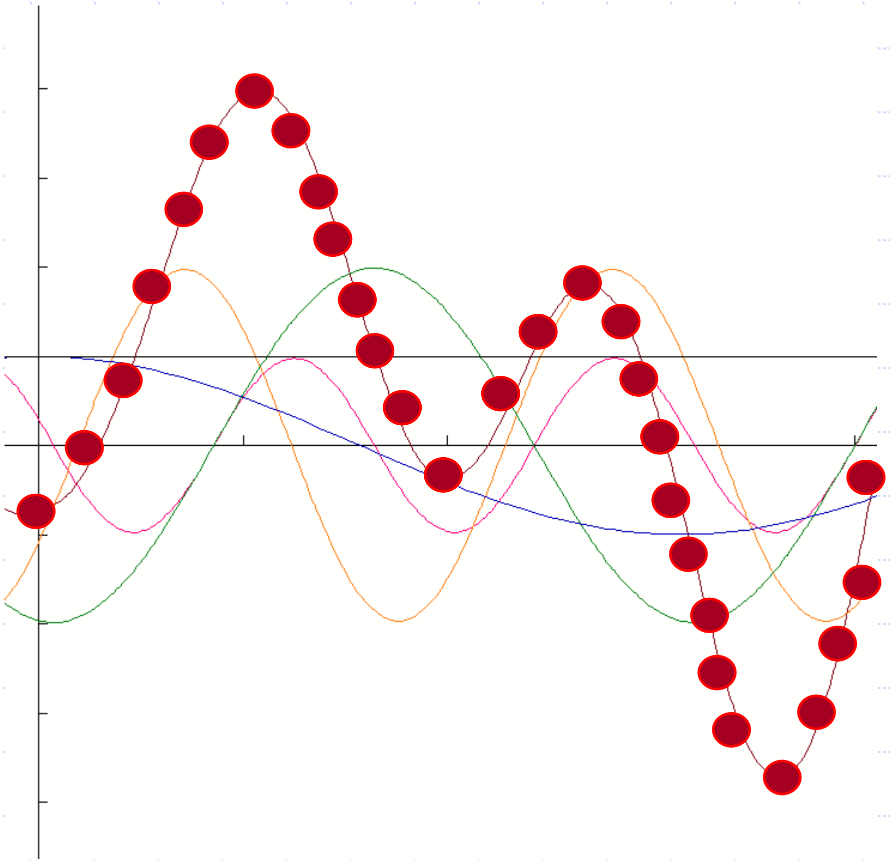


C'est un peu plus compliqué…



$$s(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos\left[(k\omega)t + \varphi_k\right]$$

C'est un peu plus compliqué…



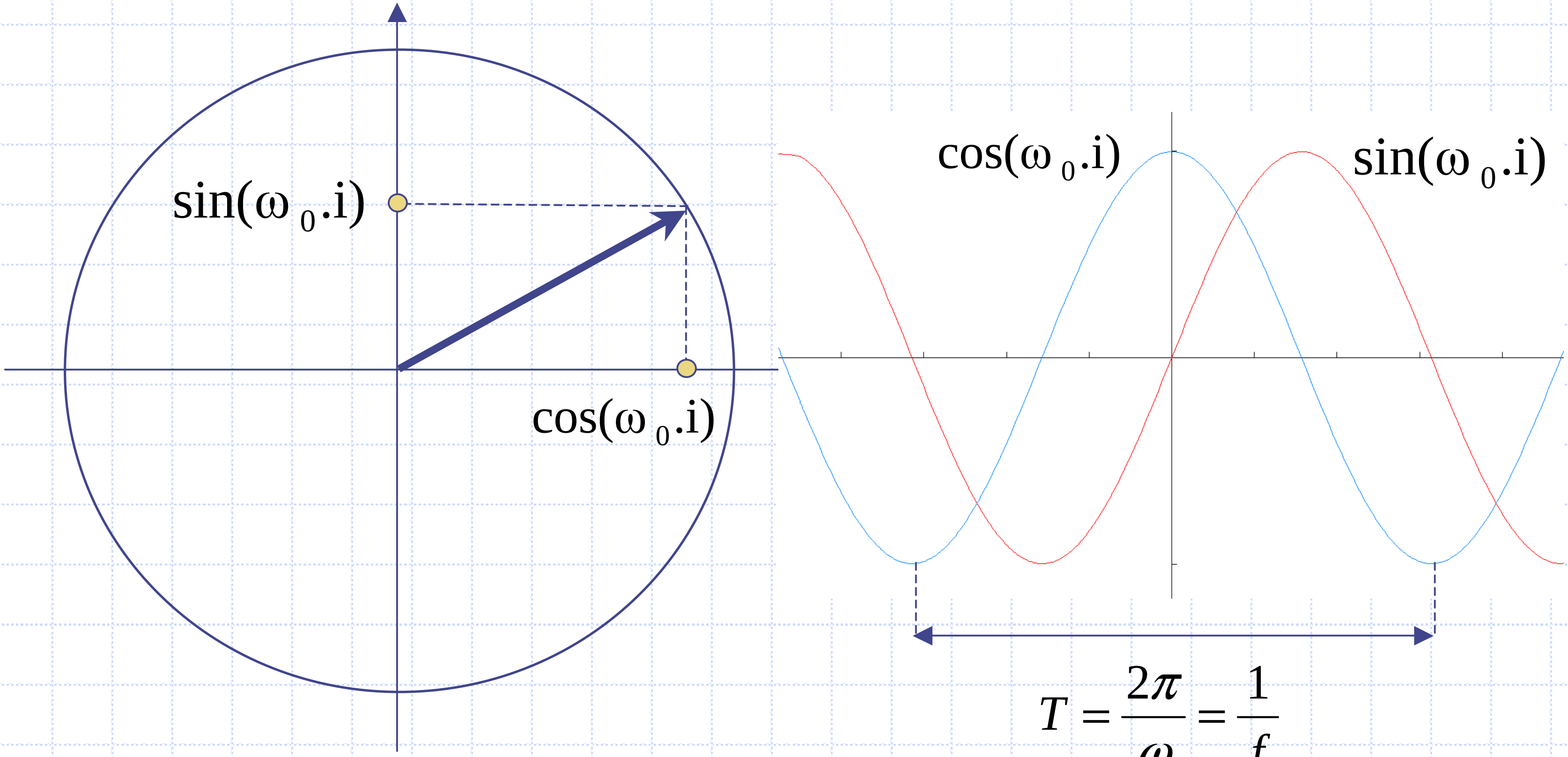
$$A_k \cos[\varphi_k]\cos[(k\omega)t]$$
$$-\,A_k \sin[\varphi_k]\sin[(k\omega)t]$$

$$=$$

$$s(t) = A_0 + \sum_{k=1}^{\cancel{\infty}} A_k \cos[(k\omega)t + \varphi_k]$$

Rappel: nombres complexes

$$f(i) = e^{j.\omega_0.i} = \cos(\omega_0.i) + j.\sin(\omega_0.i)$$



$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Transformée de Fourier discrète

$$\boxed{s(i) = \frac{1}{N}\sum_{k=0}^{N-1} \hat{s}(k).e^{j.(k.\omega_0)i}}$$

$$\omega_0 = 2\pi\frac{1}{N}$$ fondamentale

$$s(i) = \frac{1}{N}[$$

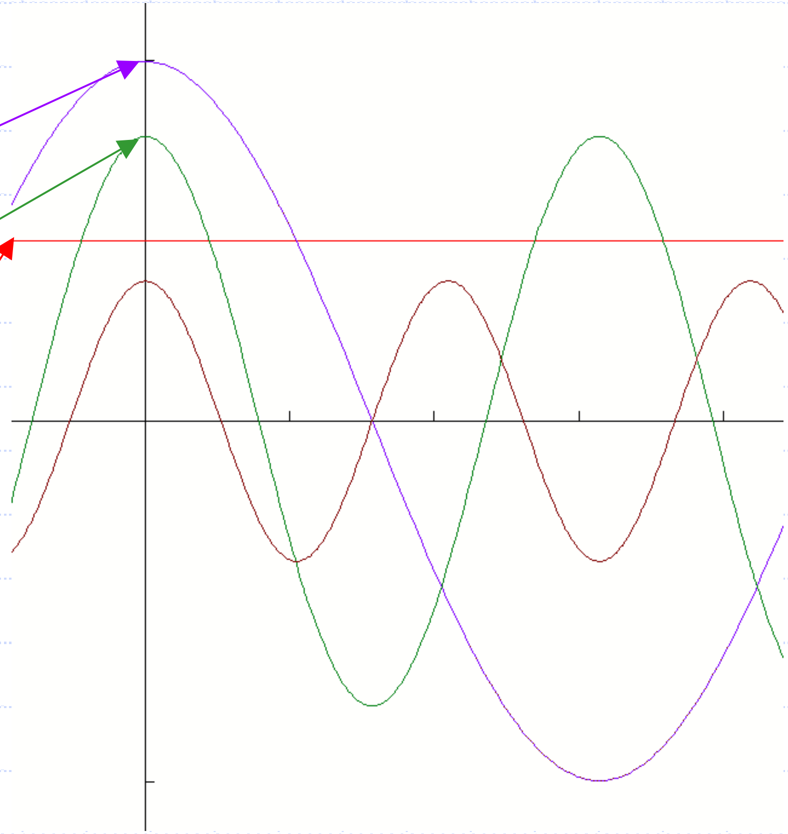
$$\hat{s}(0)$$

$$+\hat{s}(1).e^{j.(\omega_0)i}$$

$$+\hat{s}(2).e^{j.(2.\omega_0)i} + \ldots$$

$$+\hat{s}(N-1).e^{j.((N-1).\omega_0)i}$$

$$]$$



Transformée de Fourier discrète

$$s(i) = \frac{1}{N}\sum_{k=0}^{N-1} \hat{s}(k).e^{j.(k\frac{2\pi}{N})i}$$

$$\widehat{s}(\nu) = \sum_{k=0}^{N-1} s(k).e^{-j.(k\frac{2\pi}{N})\nu} = A_\nu e^{j\varphi_\nu}$$

Transformée de Fourier discrète

$$s(i) = \frac{1}{N}\sum_{k=0}^{N-1} \hat{s}(k).e^{j.(k\frac{2\pi}{N})i}$$

$$\hat{s}(\nu) = \sum_{k=0}^{N-1} s(k).e^{-j.(k\frac{2\pi}{N})\nu}$$

$$s(i) \stackrel{?}{=} \frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{k'=0}^{N-1} s(k').e^{-j.(k'\frac{2\pi}{N})k}\right].e^{j.(k\frac{2\pi}{N})i}$$

$$s(i) \stackrel{?}{=} \frac{1}{N}\sum_{k'=0}^{N-1} s(k')\left[\sum_{k=0}^{N-1}.e^{k.(j\frac{2\pi}{N})(i-k')}\right]$$

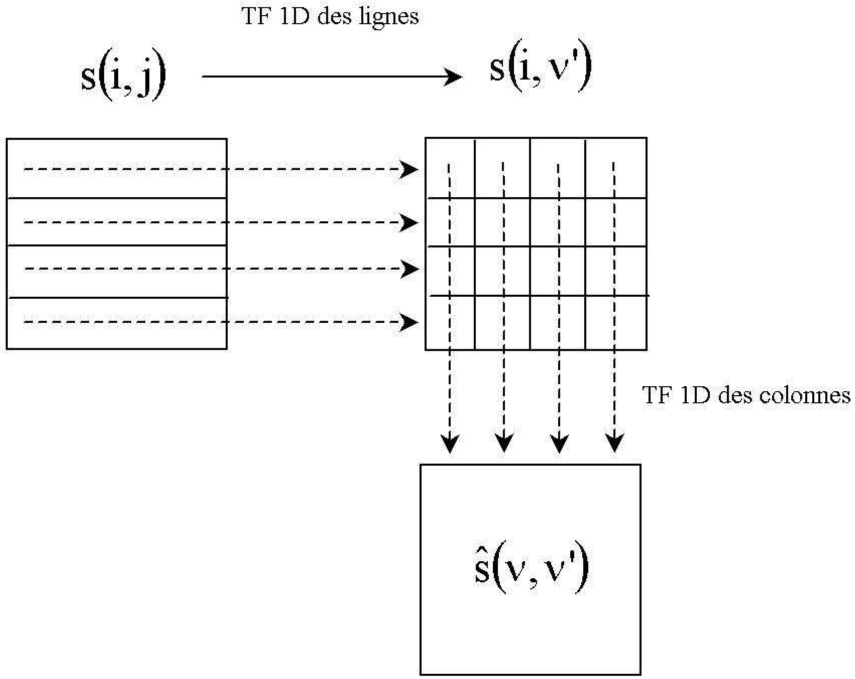
$$k' \neq i \Rightarrow \sum_{k=0}^{N-1}.e^{k.(j\frac{2\pi}{N})(i-k')} = \frac{1-e^{j.2\pi(i-k')}}{1-e^{j\frac{2\pi}{N}(i-k')}} = 0,$$

$$k' = i \Rightarrow \sum_{k=0}^{N-1}.e^{k.(j\frac{2\pi}{N})(i-k')} = N$$

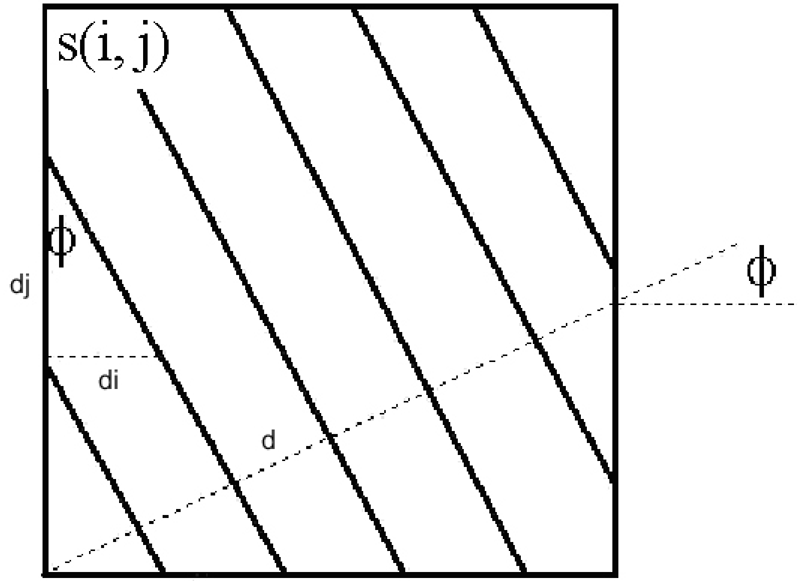
TF discrète 2D

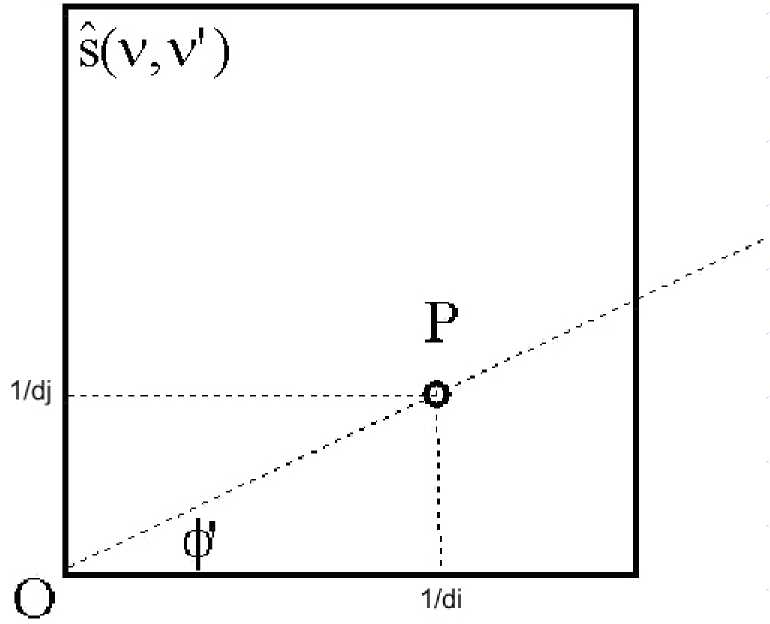
$$s(i,j)=\frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{k'=0}^{N-1}\hat{s}(k,k').e^{j.[ki+k'j]\omega_0}$$

$$\hat{s}(\nu,\nu')=\sum_{k=0}^{N-1}\sum_{k'=0}^{N-1}s(k,k').e^{-j.[k\nu+k'\nu']\omega_0}$$



TF discrète 2D: interprétation





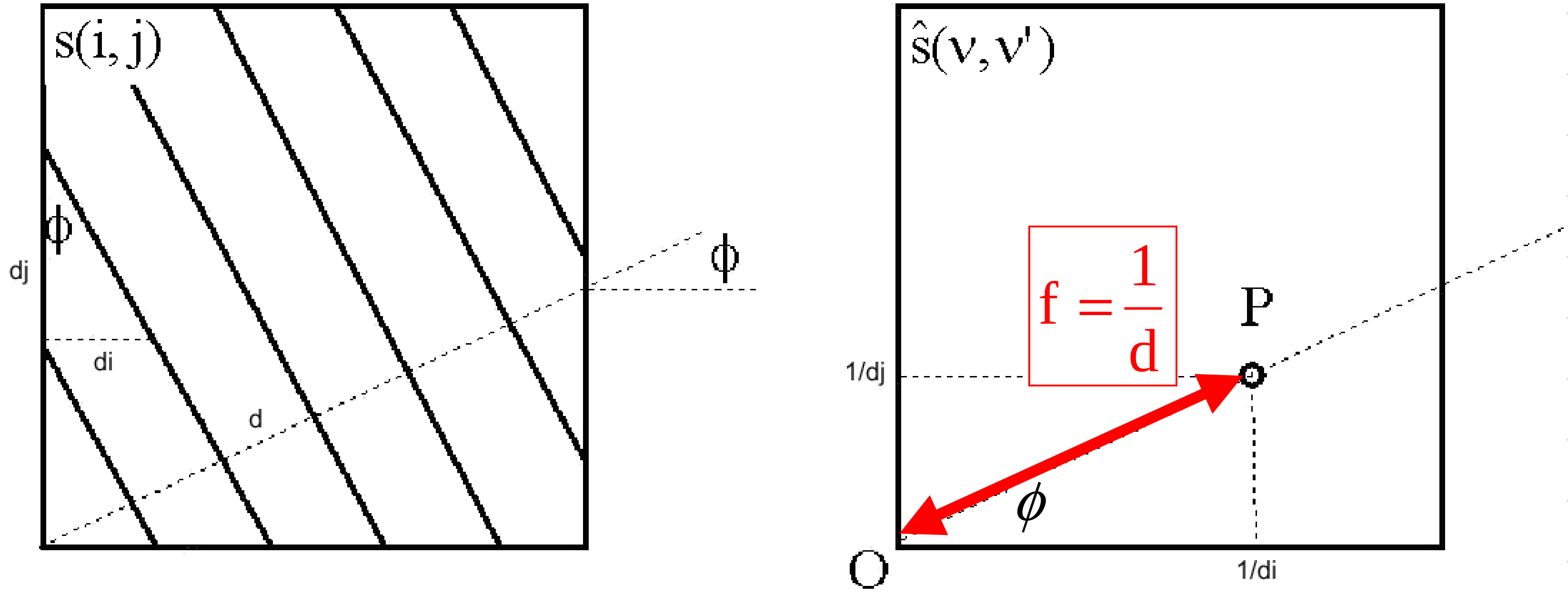
$$tg\phi = \frac{d_i}{d_j}$$

$$tg\phi' = \frac{1/d_j}{1/d_i} \Rightarrow \phi' = \phi$$

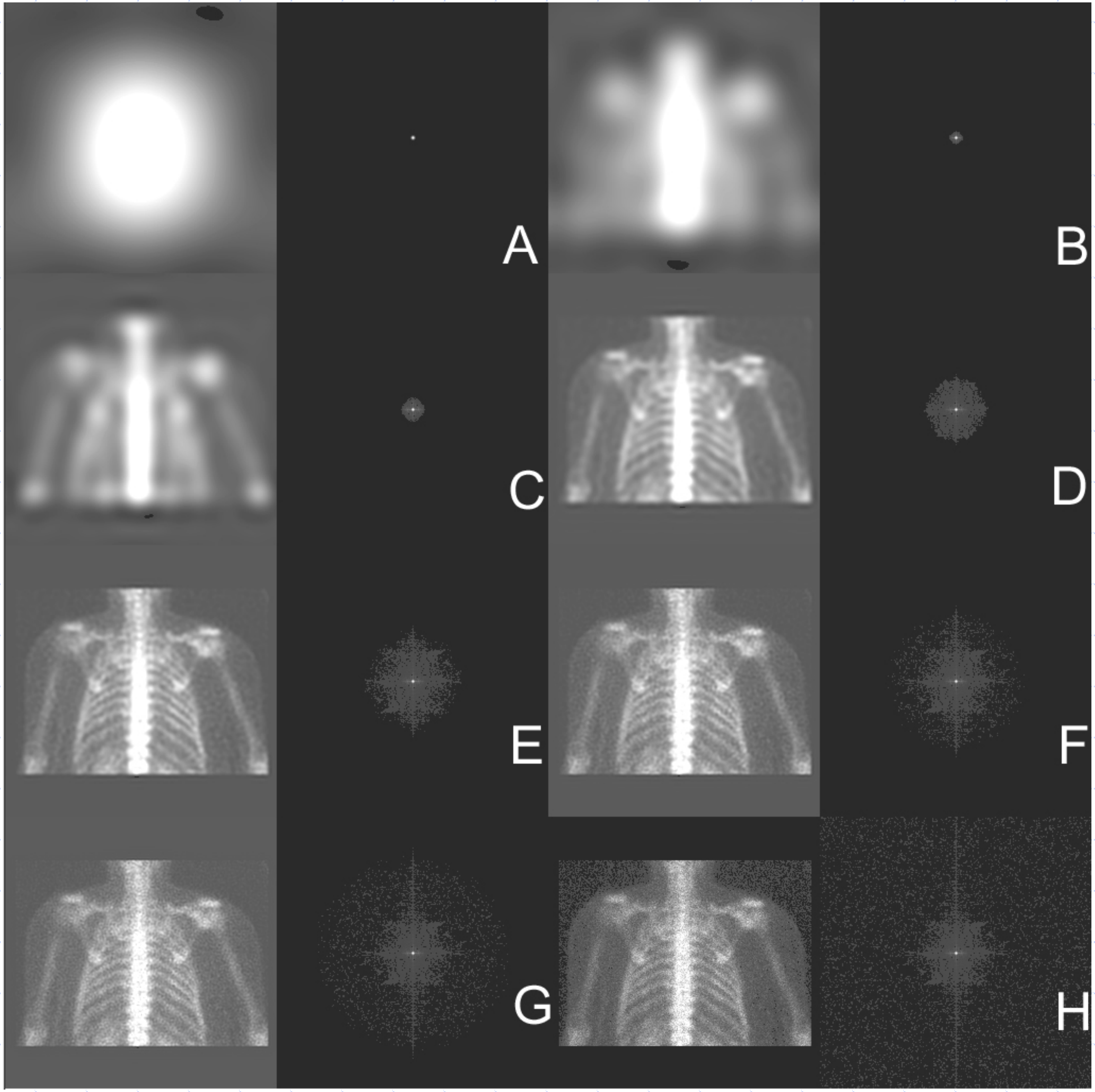
$$\cos\phi = \frac{d}{d_i}$$

$$\cos\phi = \frac{1/d_i}{OP} \Rightarrow OP = \frac{1}{d}$$

TF discrète 2D: interprétation



② TFD



H=original

Fast Fourier Transform (FFT)

$$\hat{s}(\nu) = \sum_{k=0}^{N-1} s(k).e^{-j.(k\omega_0)\nu} = \sum_{k=0}^{N-1} s(k).W_N^{k\nu}$$

$$W_N = e^{-j.\frac{2.\pi}{N}} = e^{-j.\omega_0}$$

$$W_N = \cos(\frac{2\pi}{N}) - j.\sin(\frac{2\pi}{N})$$

$$\hat{s}(\nu) = \sum_{k=0}^{\frac{N}{2}-1} s(2k).W_N^{2.k.\nu} + \sum_{k=0}^{\frac{N}{2}-1} s(2k+1).W_N^{(2.k+1).\nu}$$

$$= \sum_{k=0}^{\frac{N}{2}-1} s(2k).W_N^{2.k.\nu} + W_N^{\nu} \sum_{k=0}^{\frac{N}{2}-1} s(2k+1).W_N^{2.k.\nu}$$

$$= \sum_{k=0}^{\frac{N}{2}-1} s(2k).W_{N/2}^{k.\nu} + W_N^{\nu} \sum_{k=0}^{\frac{N}{2}-1} s(2k+1).W_{N/2}^{k.\nu}$$

$$= G(\nu) + W_N^{\nu}.H(\nu)$$

Fast Fourier Transform (FFT)

$$\cos(\frac{2\pi.\nu}{N}) - j.\sin(\frac{2\pi.\nu}{N})$$

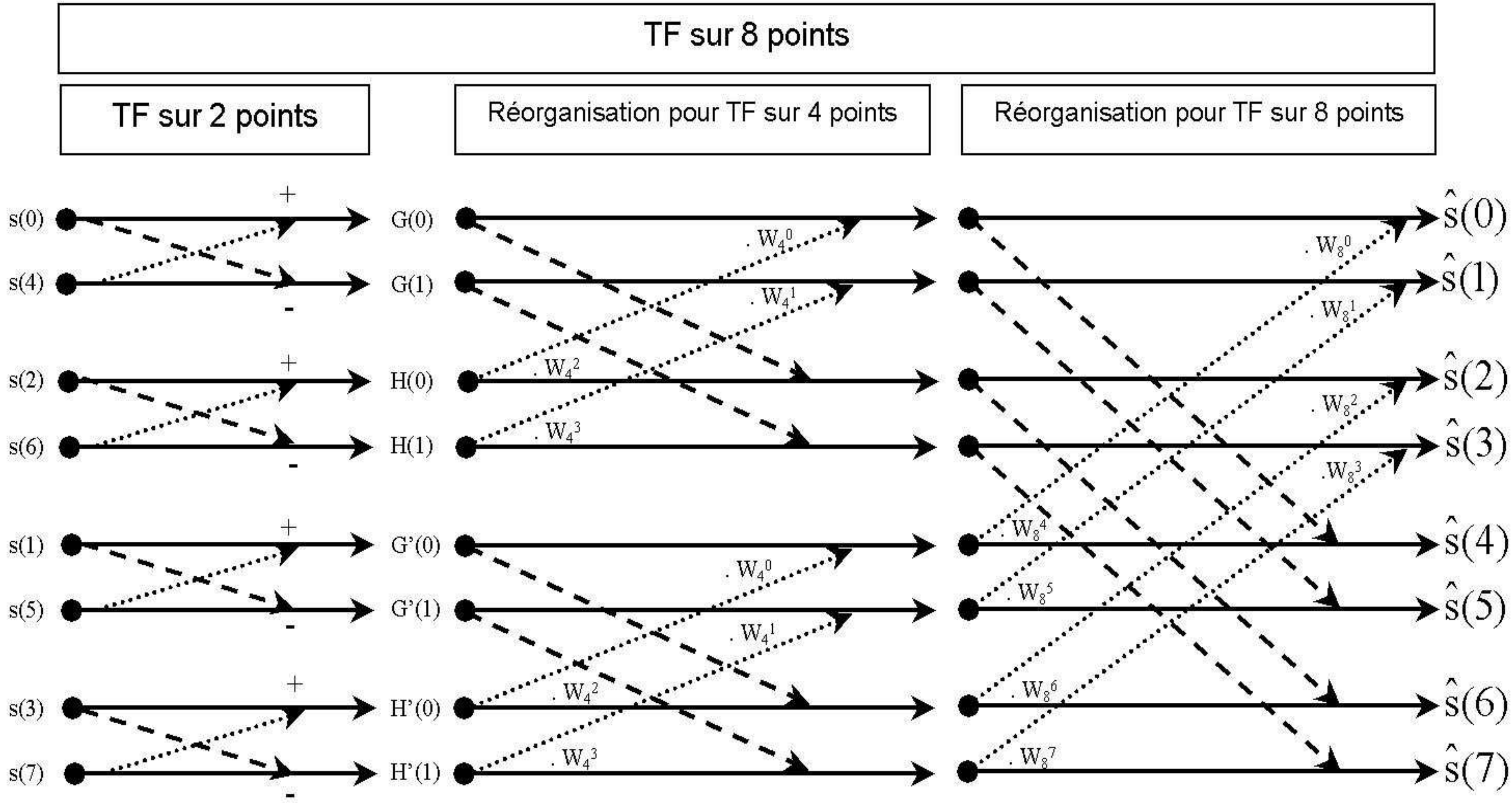
$$\boxed{\hat{s}(\nu) = G(\nu) + W_N^{\nu}.H(\nu)}$$

TF sur N points TF sur N/2 points

TF sur 2 points: $\hat{s}(\nu) = \sum_{k=0}^{1} s(k).e^{-j.(k\frac{2\pi}{2})\nu} = s(0) + (-1)^{\nu} s(1)$

Complexité N² $\rightarrow$ N.$\log_2$ N (512² $\rightarrow$ 512x9 i.e 57 fois moins)

Algorithme FFT



Autres décompositions de Fourier

Série de Fourier
f(x) périodique

$$f(x)=\frac{1}{T}\sum_{k\in Z}\hat{f}(k).e^{j.(k\frac{2\pi}{T})x}$$

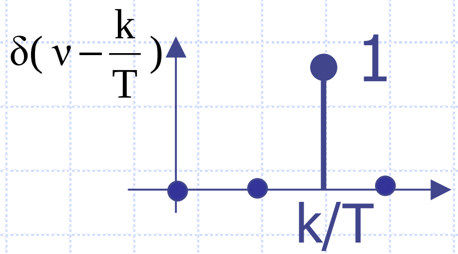
$$\hat{f}(k)=\int_0^T f(x).e^{-j.(k\frac{2\pi}{T})x}dx$$

Transformée de Fourier

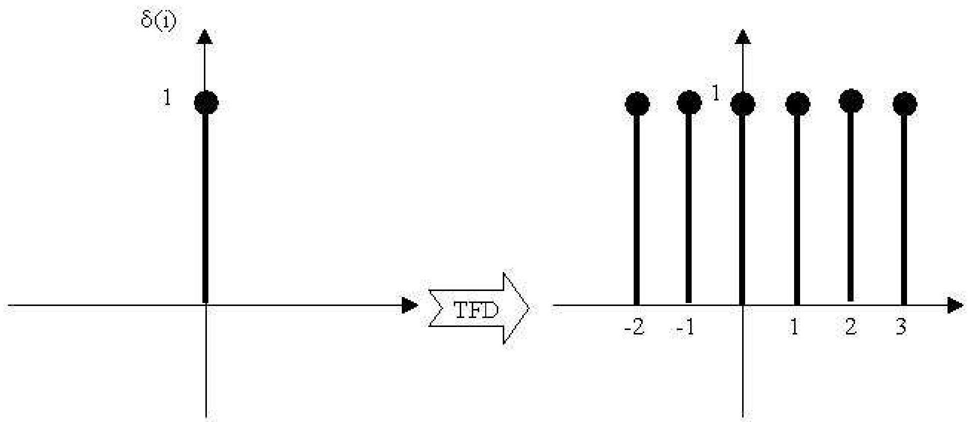
$$f(x)=\int_{-\infty}^{+\infty}\hat{f}(\nu).e^{2j\pi\nu x}d\nu$$

$$\hat{f}(\nu)=\int_{-\infty}^{+\infty}f(x).e^{-2j\pi\nu x}dx$$

$$\hat{f}(\nu)=\sum_{k\in Z}\hat{f}(k).\delta(\nu-\frac{k}{T})$$



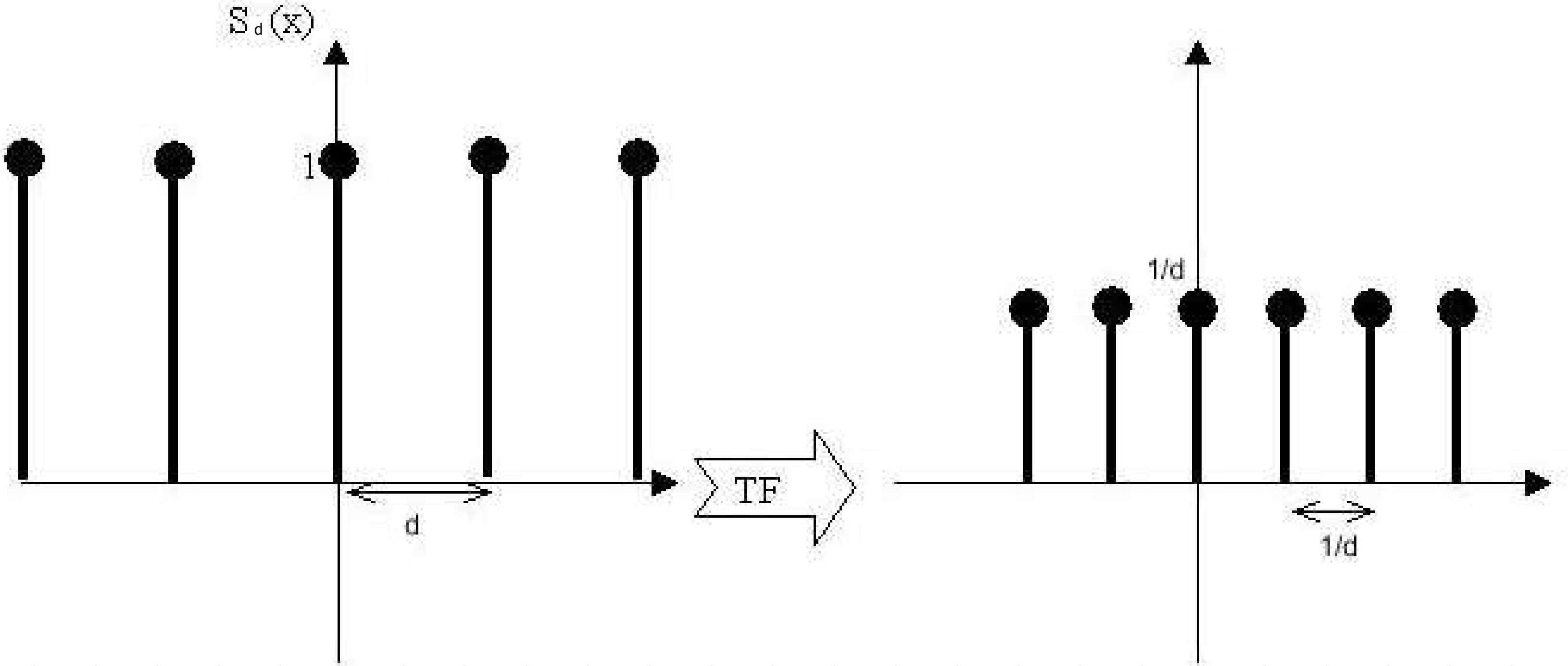
TF utiles: impulsion de Dirac



$$\hat{\delta}(\nu)=\sum_{k=0}^{N-1}\delta(k).e^{-j.(k\frac{2\pi}{N})\nu}=\delta(0).e^{0}=1$$

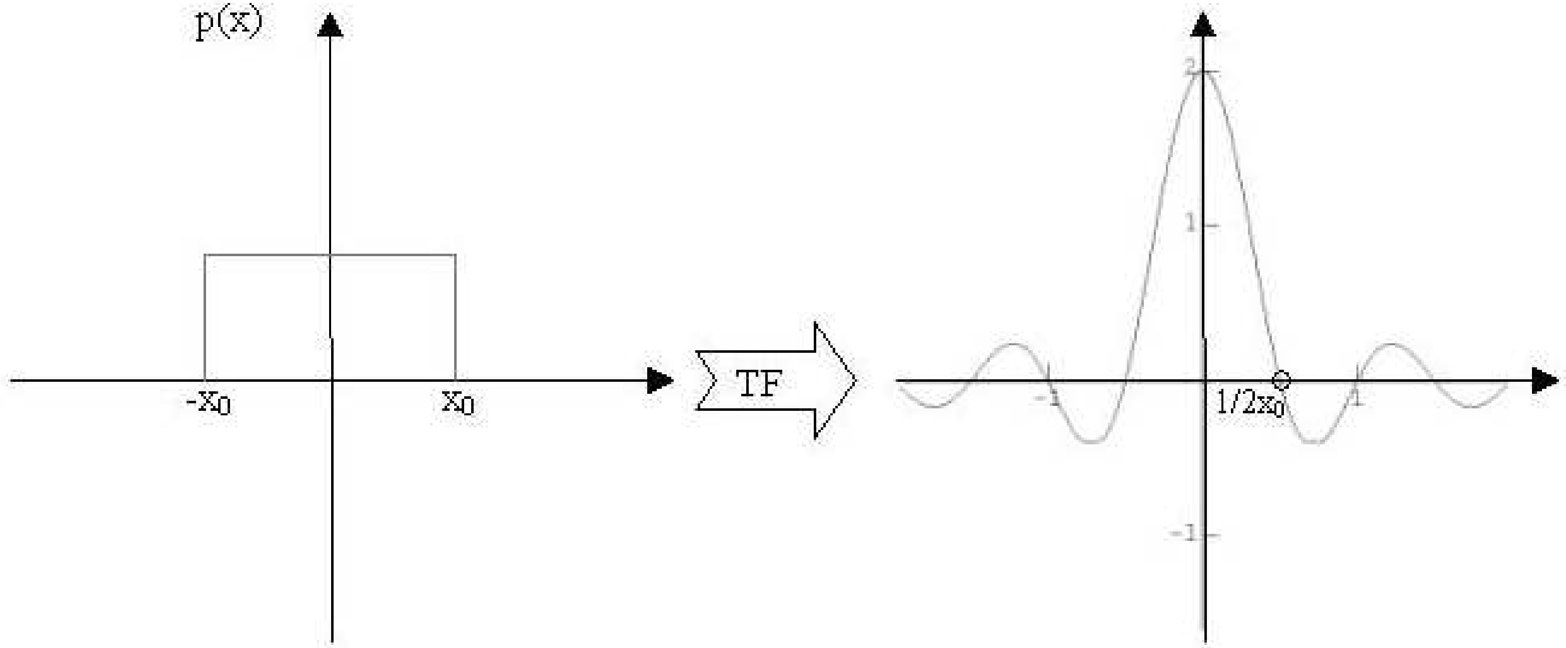
$$\hat{\delta}_R(\nu)=\sum_{k=0}^{N-1}\delta(k-R).e^{-j.(k\frac{2\pi}{N})\nu}=\delta(0).e^{-j.(R\frac{2\pi}{N})\nu}=W_N^{R.\nu}$$

TFD utiles : « peigne de Dirac »



$$S_d(x) = \sum_{k\in Z} \delta(x - kd) \qquad \hat{S}(\nu) = \frac{1}{d}\sum_{k\in Z} \delta\left(\nu - \frac{k}{d}\right) = \frac{1}{d} S_{\frac{1}{d}}(\nu)$$

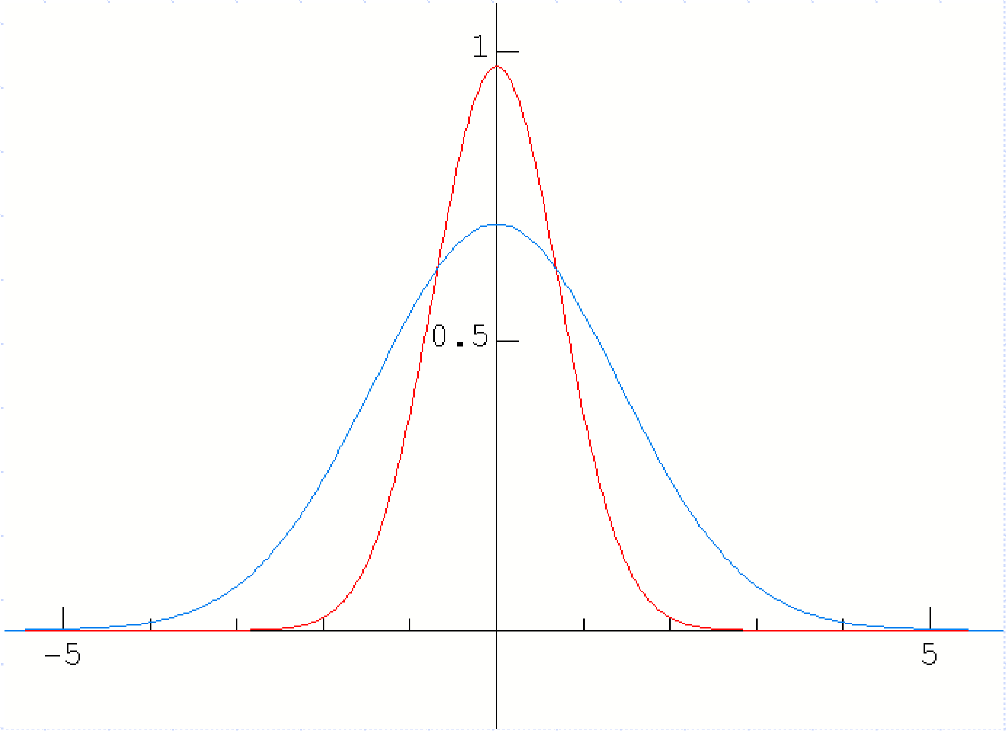
TF utiles: créneau



$$\hat{p}(\nu) = \int_{-\infty}^{+\infty} p(x).e^{-2j\pi\nu x}dx = \int_{-x_0}^{+x_0} e^{-2j\pi\nu x}dx = \frac{\sin(2x_0\pi\nu)}{\pi\nu}$$

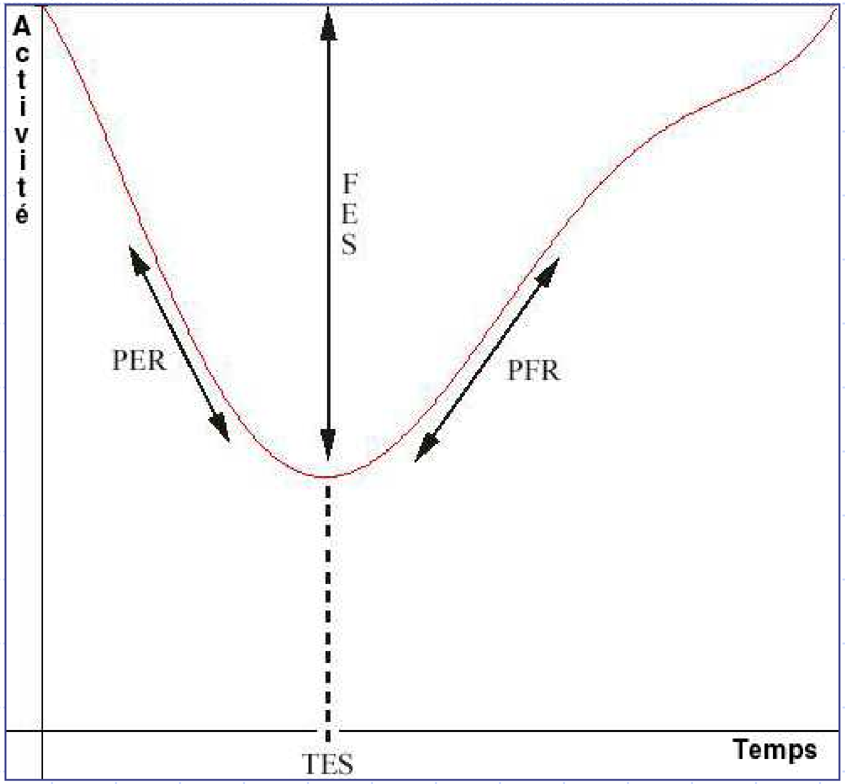
TF utiles: gaussienne

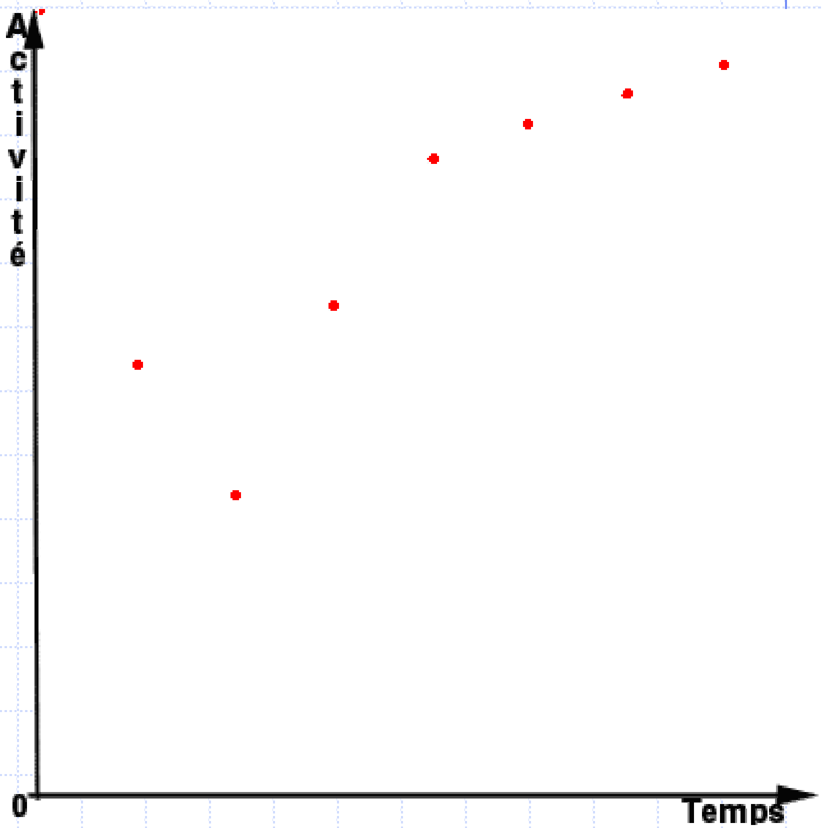
$$h(i) = e^{-\alpha i^2} \Leftrightarrow \hat{h}(\nu) = \frac{1}{\sqrt{2\alpha}} e^{-\frac{\nu^2}{4.\alpha}}$$



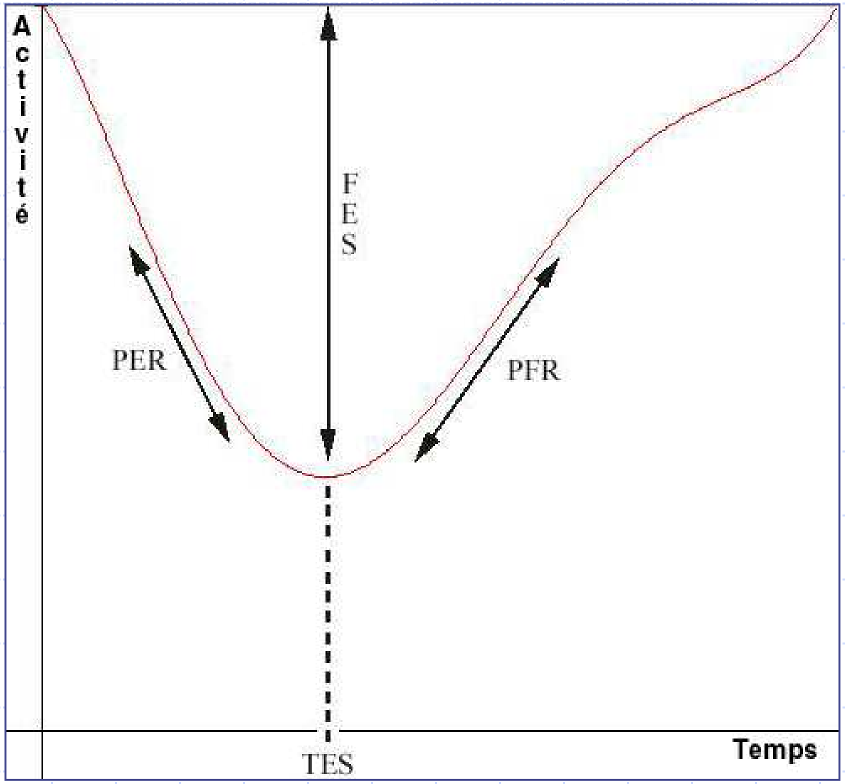
$$\mathrm{Rm} : G(i) = \frac{e^{-\frac{i^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \Rightarrow \sigma_1 = \frac{1}{\sigma_2}$$

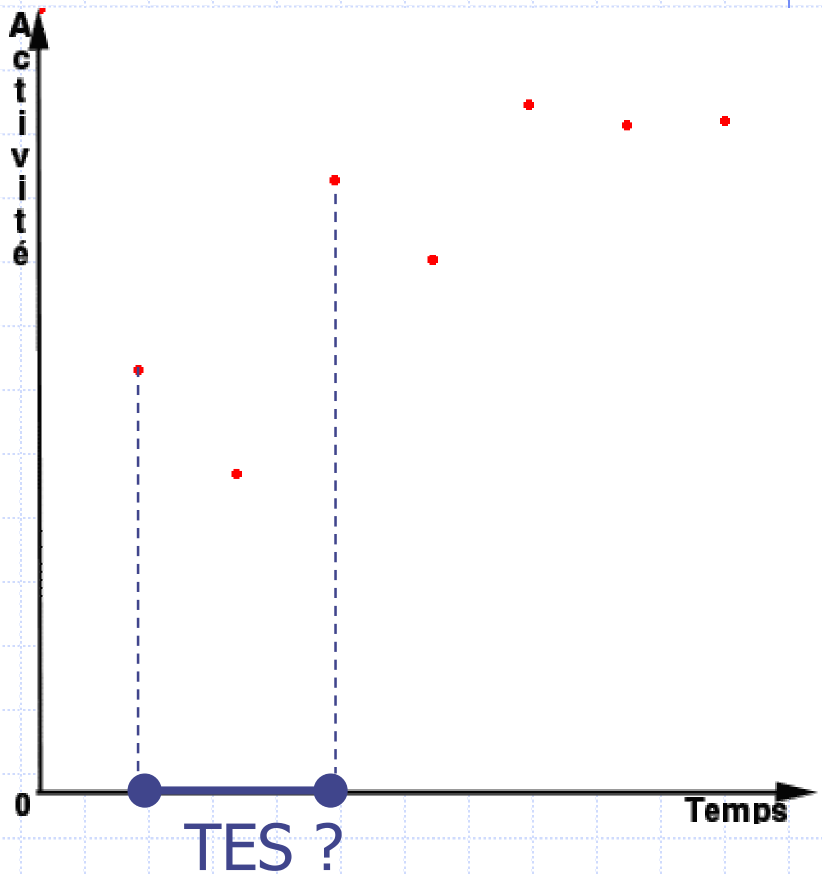
Application: détermination des TES





Application: détermination des TES

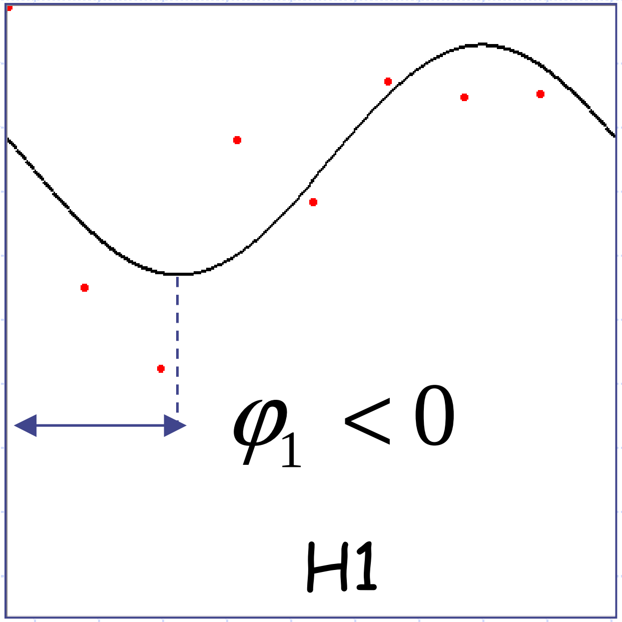


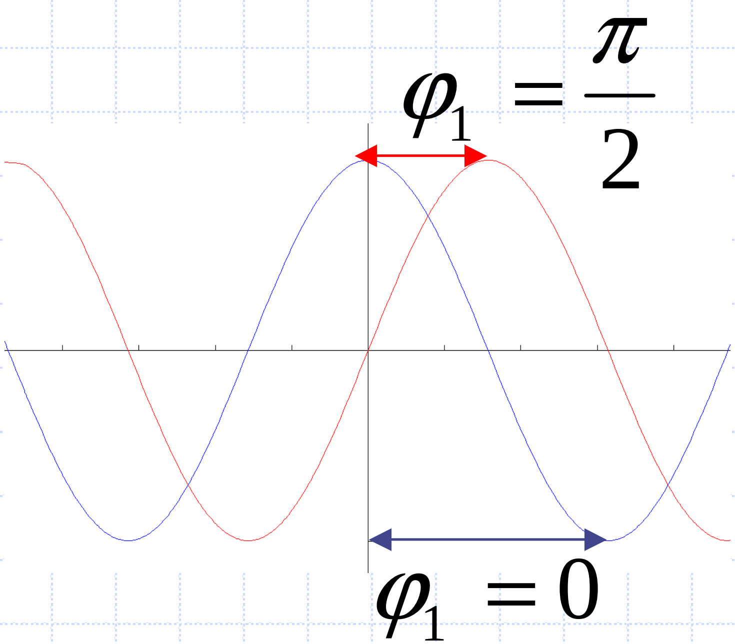


1° harmonique

$$s(t) = \frac{1}{N}\sum_{k=0}^{N-1} \hat{s}(k).e^{j.(k\omega_0)t}$$

$$s(t) = \frac{\hat{s}(0)}{N} + \frac{\hat{s}(1)}{N} e^{j.(\omega_0)t} = A_0 + A_1 e^{j.(\omega_0)t+\varphi_1}$$

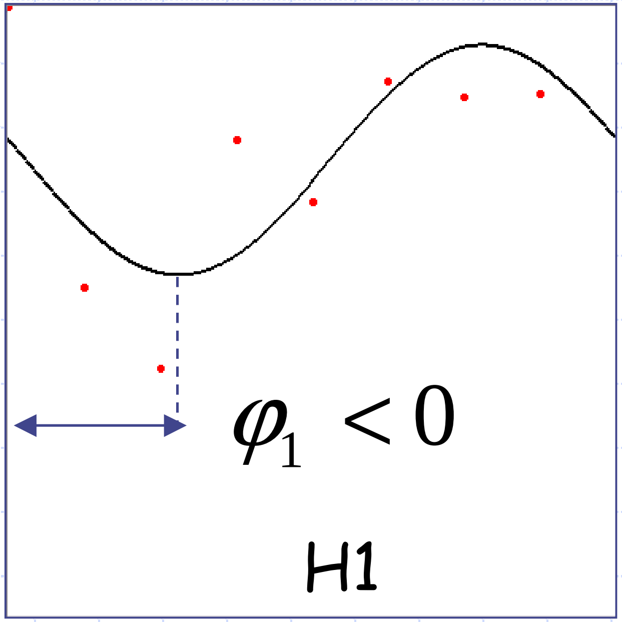


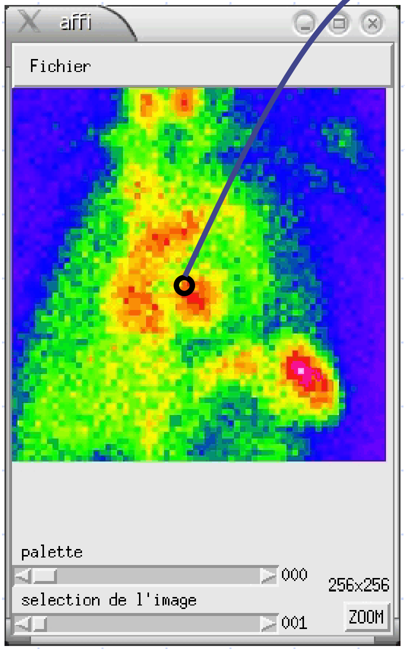


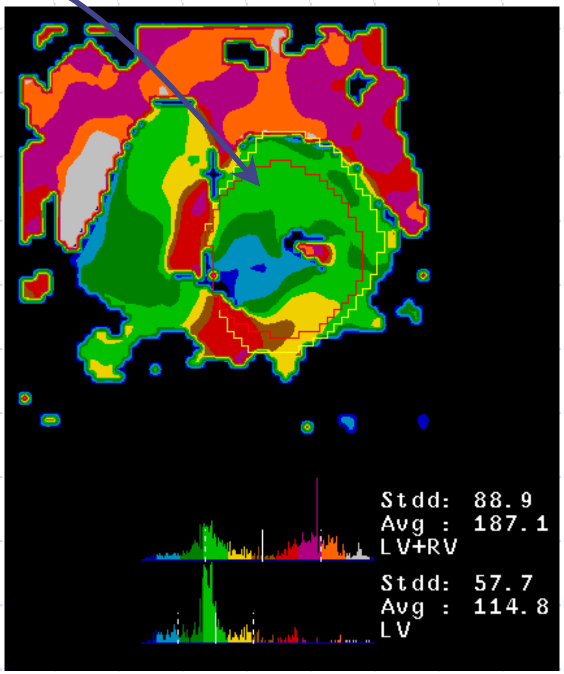
1° harmonique

$$s(t) = A_0 + A_1\, e^{j.(\omega_0)t+\varphi_1}$$

φ_1

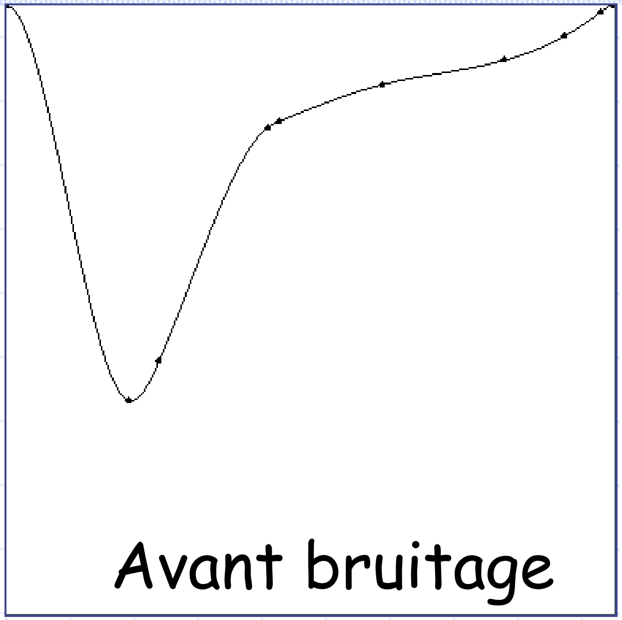


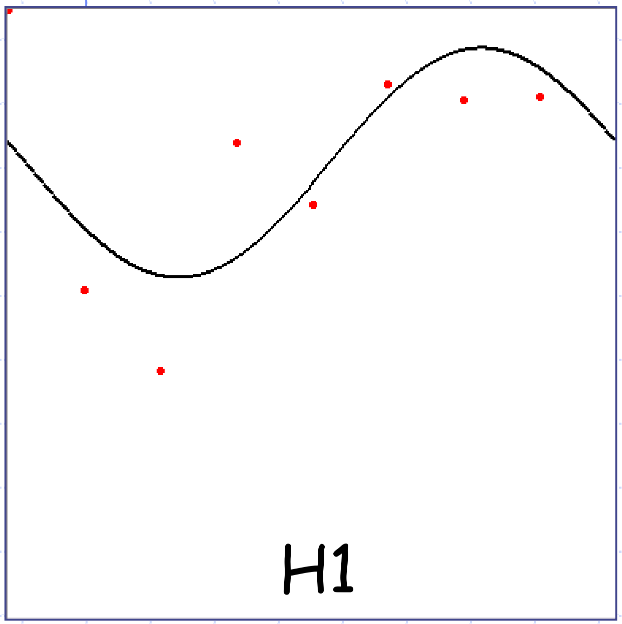


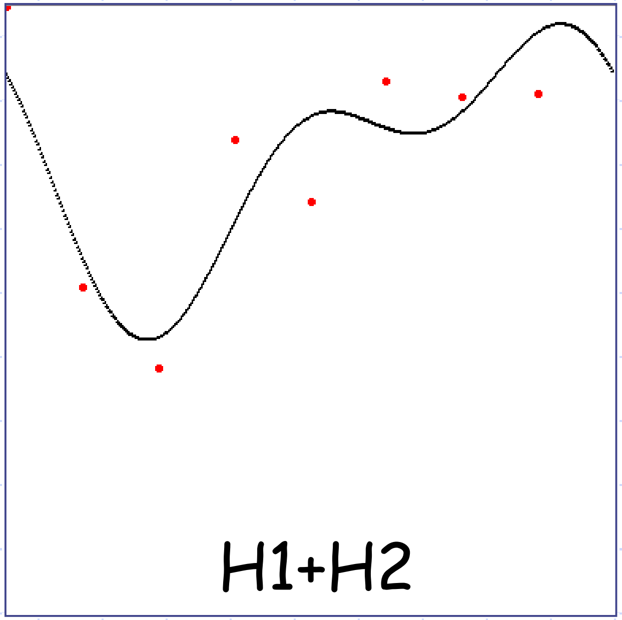


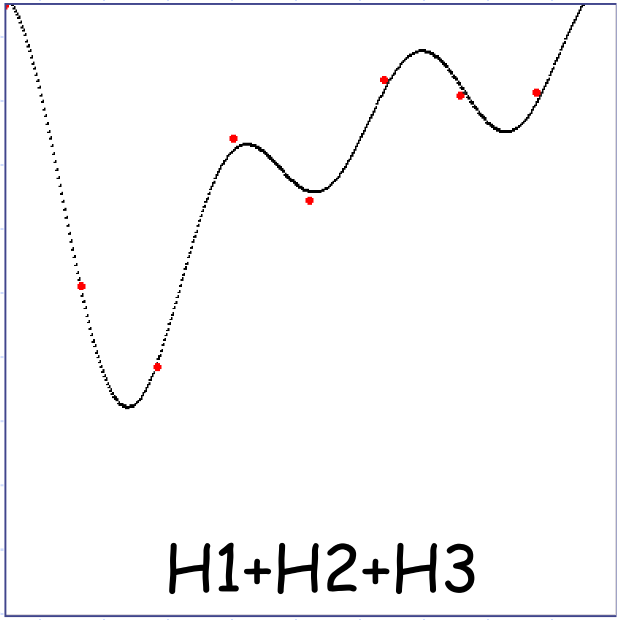
Multi-harmoniques

$$s(t)=\frac{1}{N}\sum_{k=0}^{N-1}\hat{s}(k).e^{j.(k\omega_0)t}$$

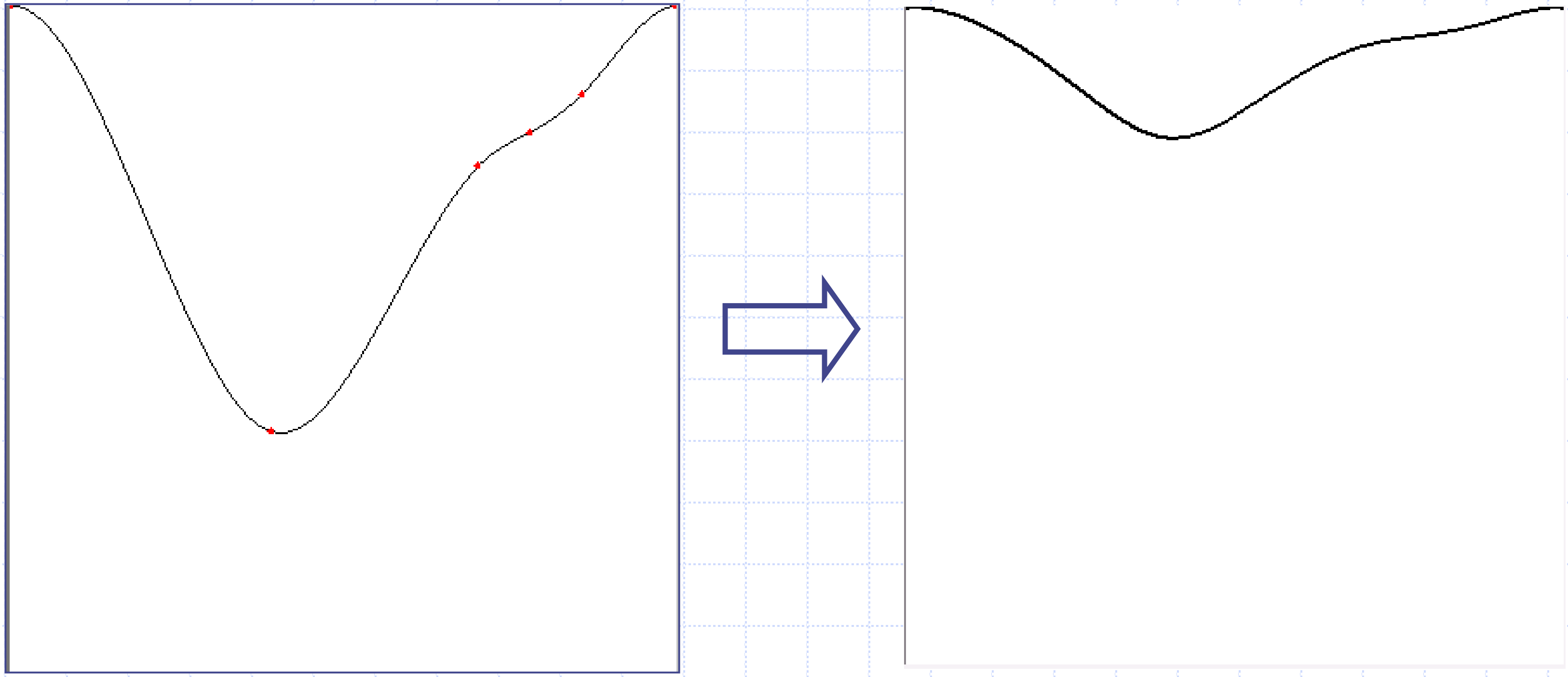








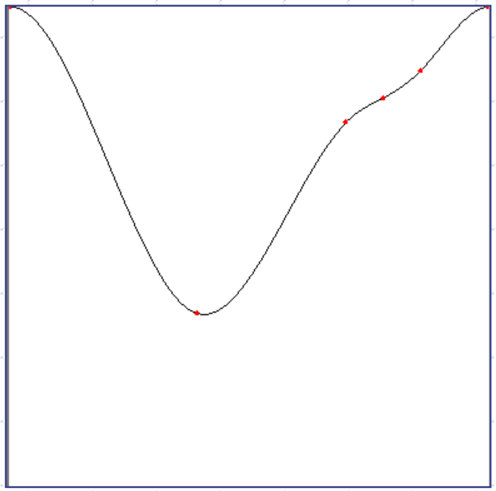
Ajustement en amplitude

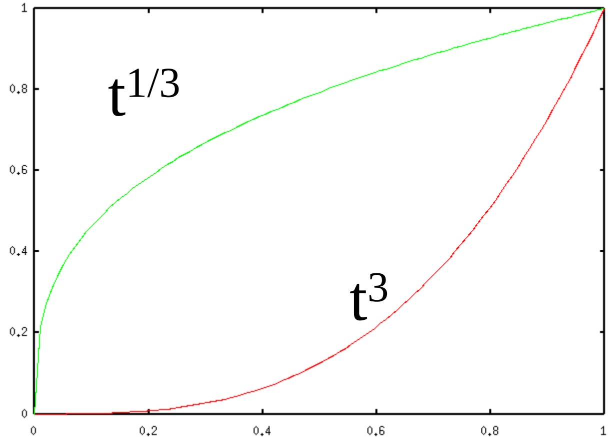


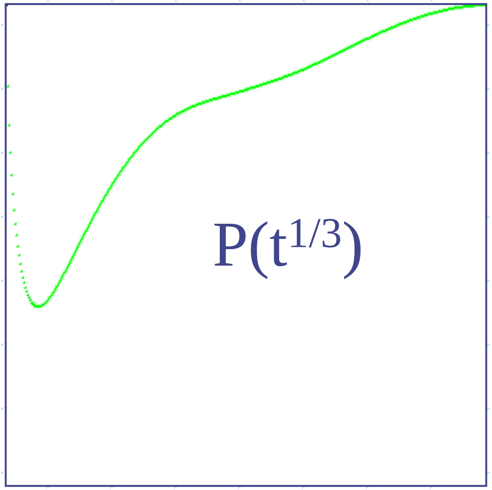
P(t) $\qquad\qquad$ $D(t) = P(t)^{\beta}$

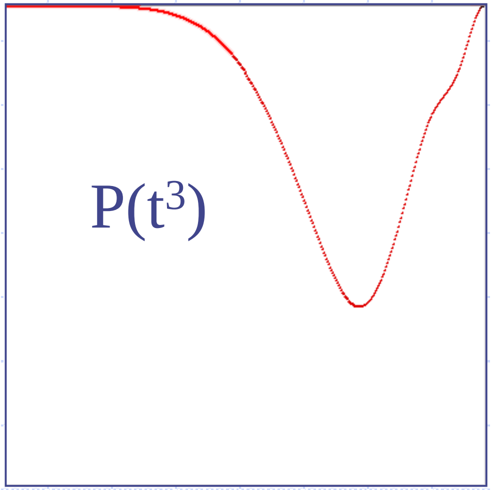
(Caderas de Kerleau et Mariano-Goulart, *IEEE TRANS MED IMAGING* 2004)

Ajustement en temps

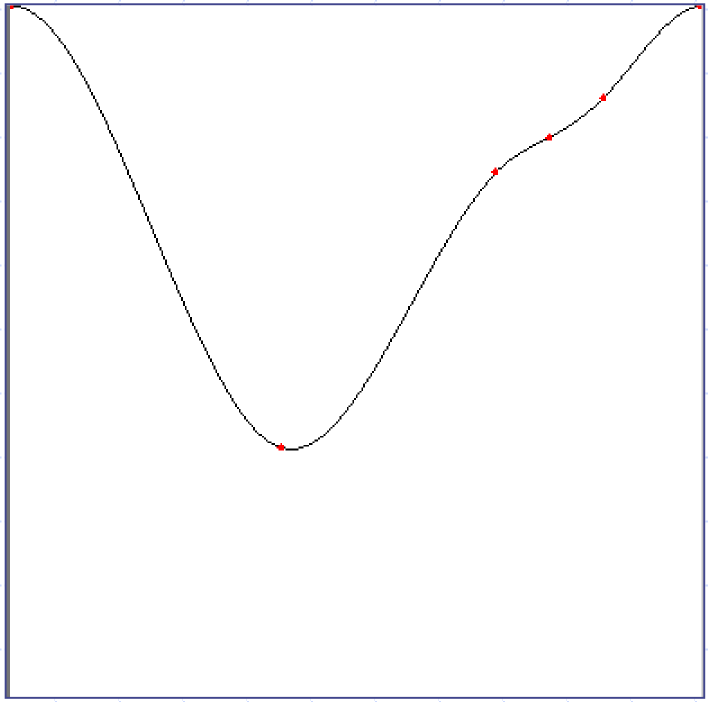




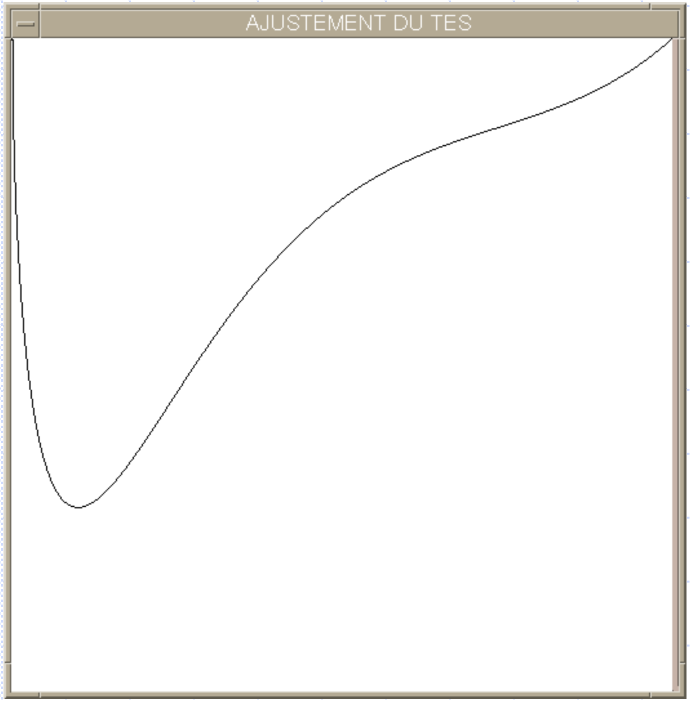




Ajustement en temps

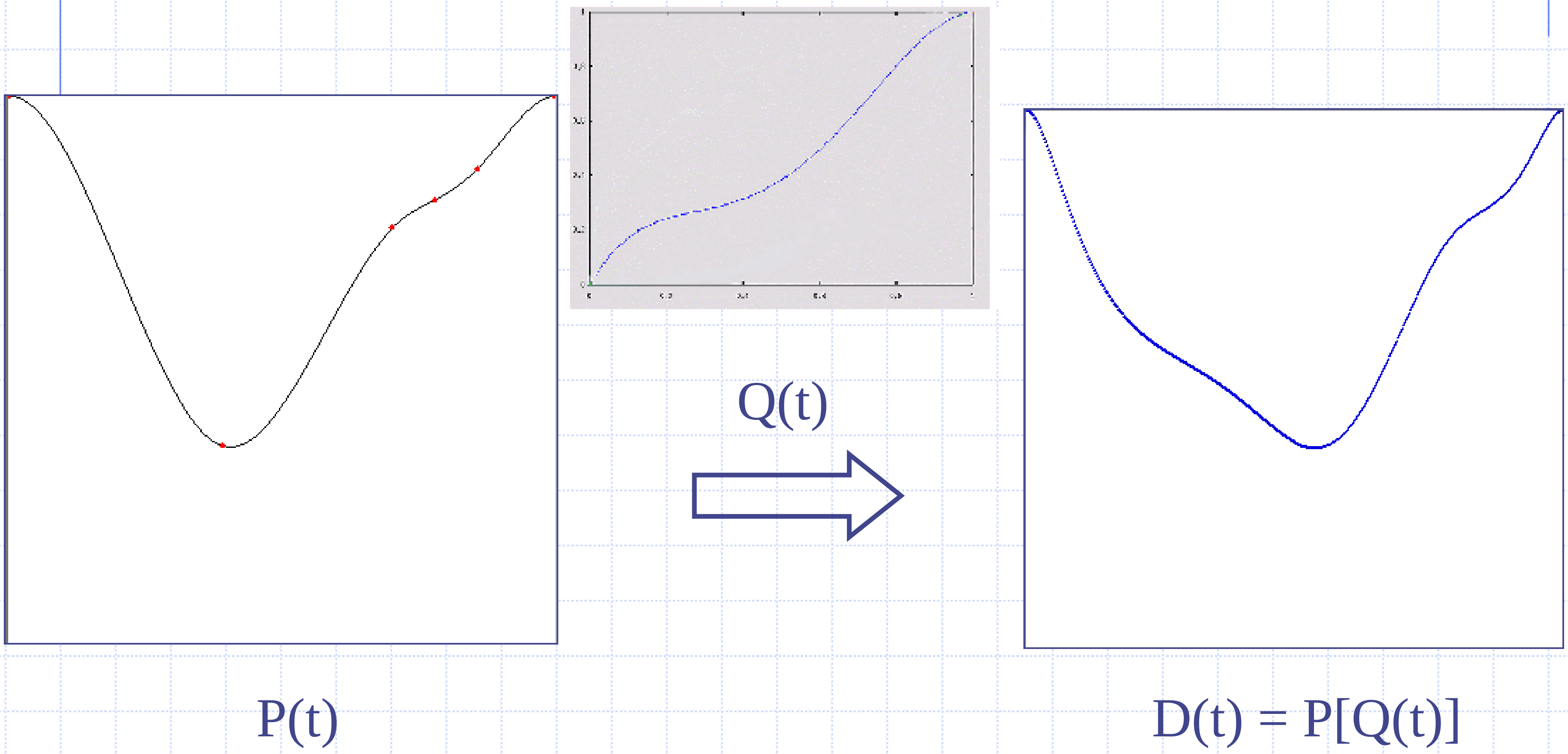


P(t)

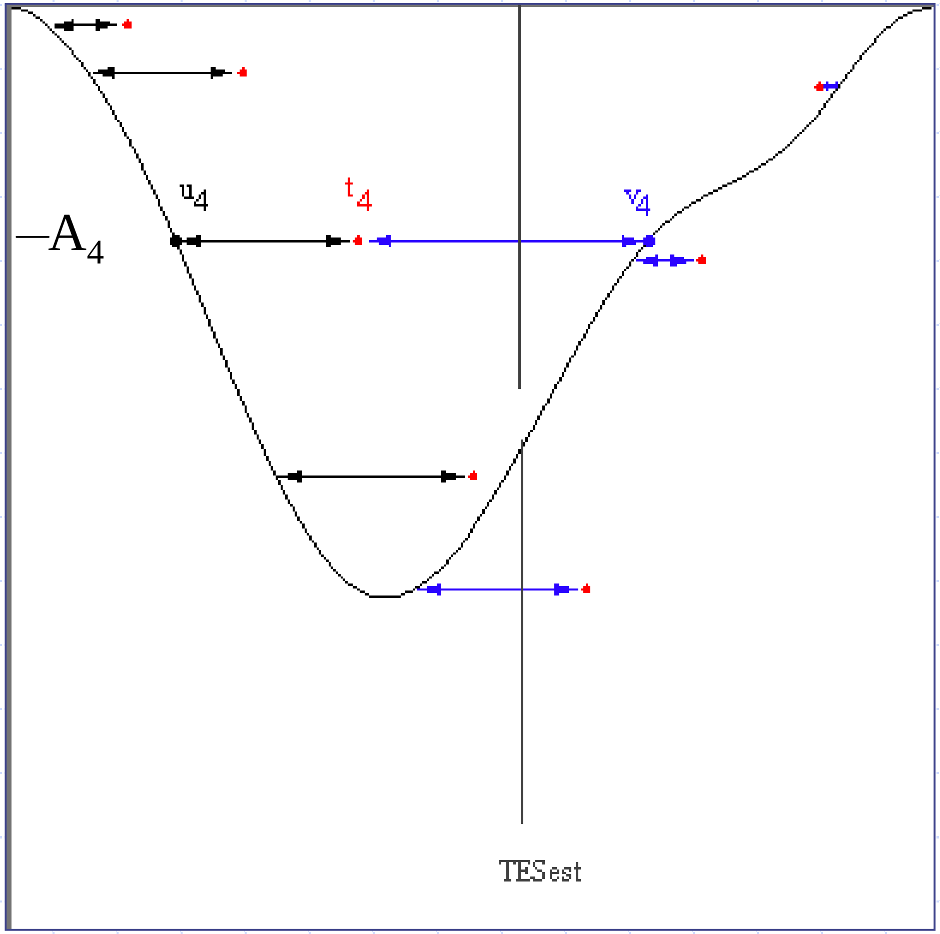


$D(t) = P(t^{\alpha})$

Ajustement en temps

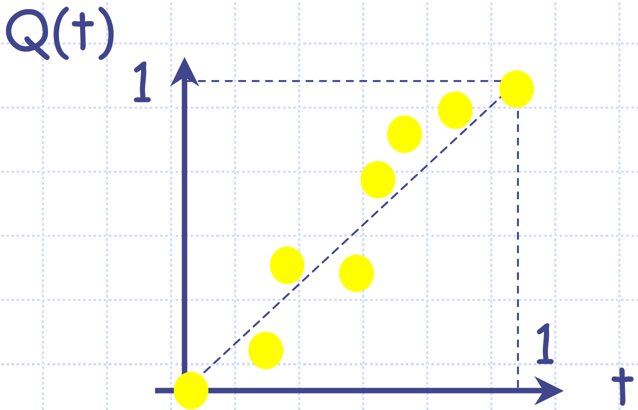


Restauration du signal



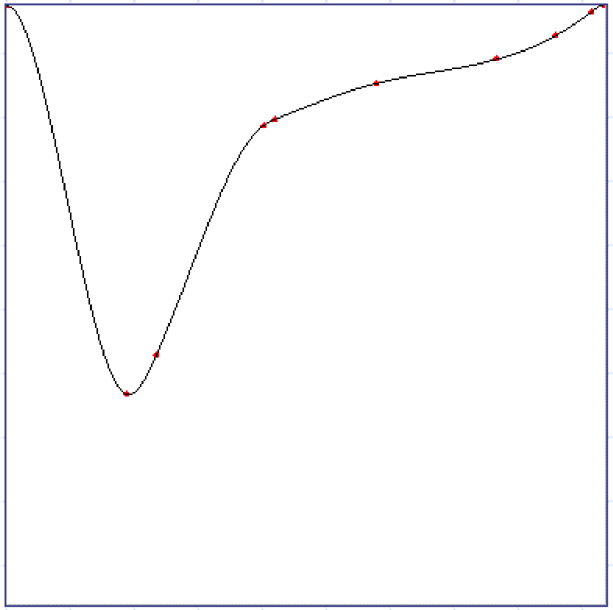
Acquisition bruitée (t_4 , A_4)

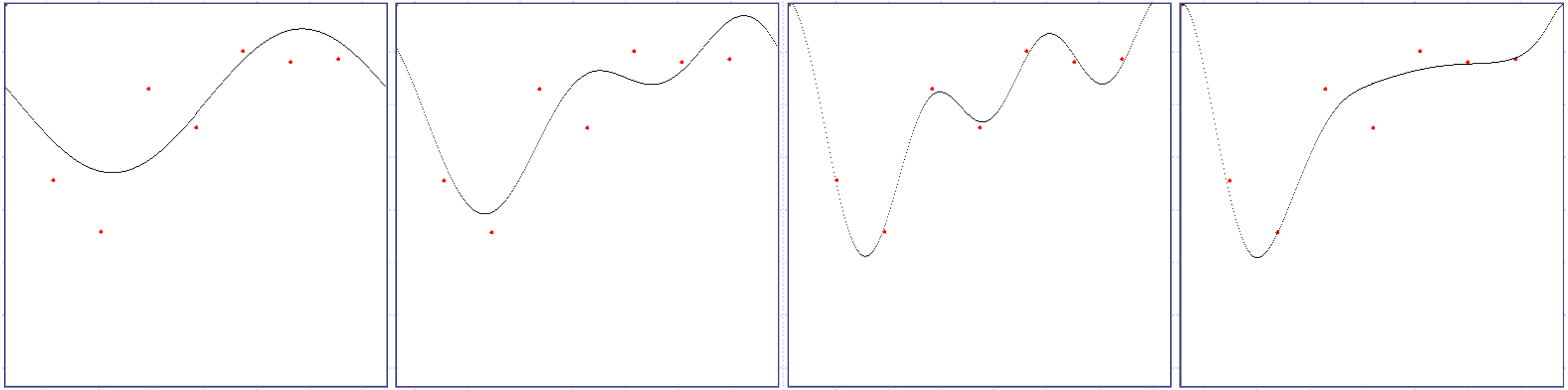
$$A_4 = P(u_4)$$
$$A_4 = D(t_4) = P[\ Q(t_4)]$$
$$\boxed{Q(t_4) = u_4}$$



(Caderas de Kerleau et Mariano-Goulart, *IEEE TRANS MED IMAGING* 2004)

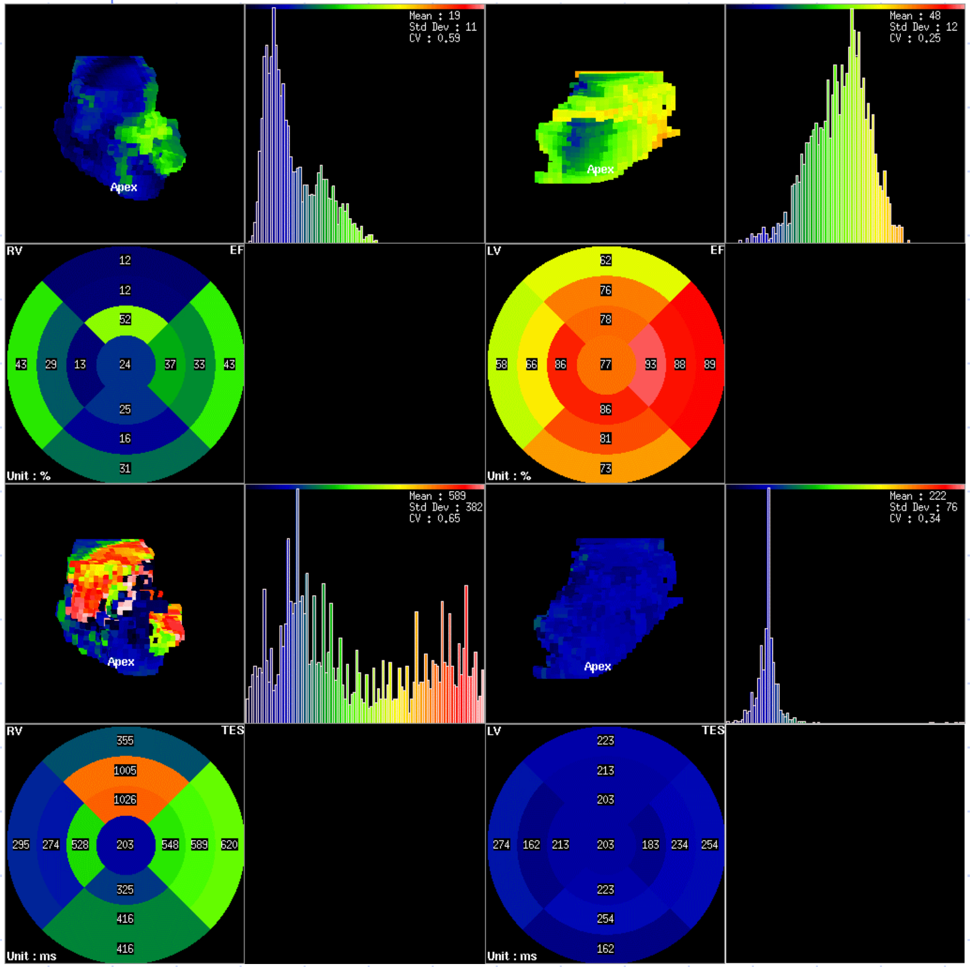
Résultats (I)

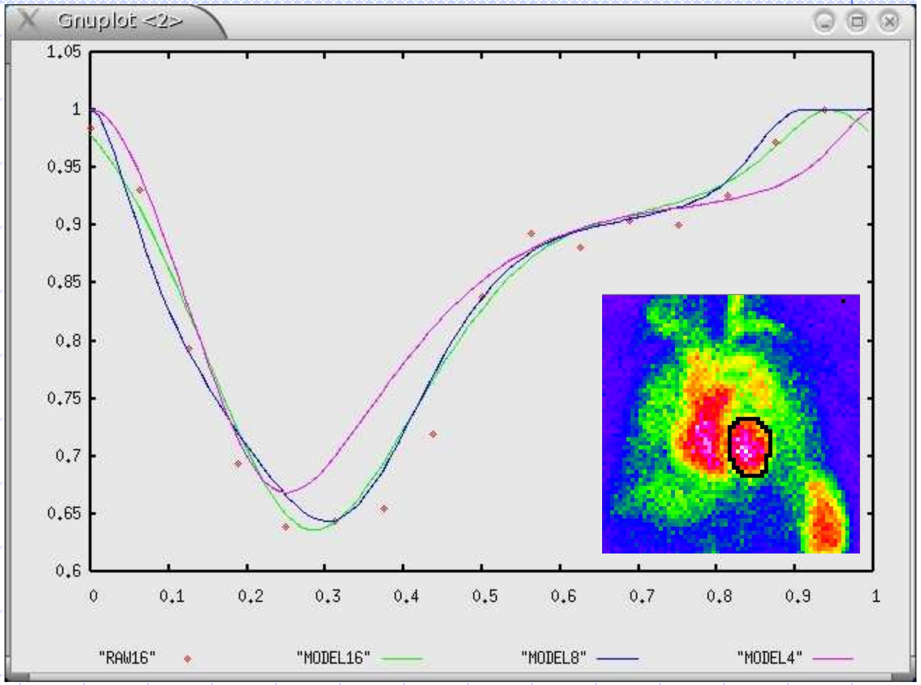




H1 H1+H2 H1+H2+H3 D(t)

Résultats (II)



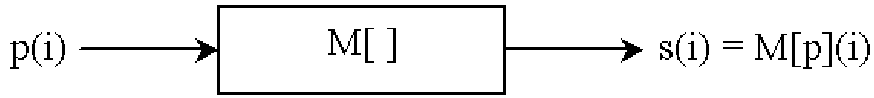


FORMATION DE L'IMAGE

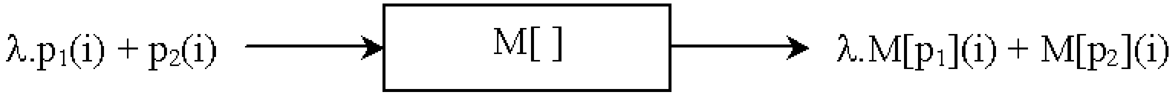
SLID

RESOLUTION

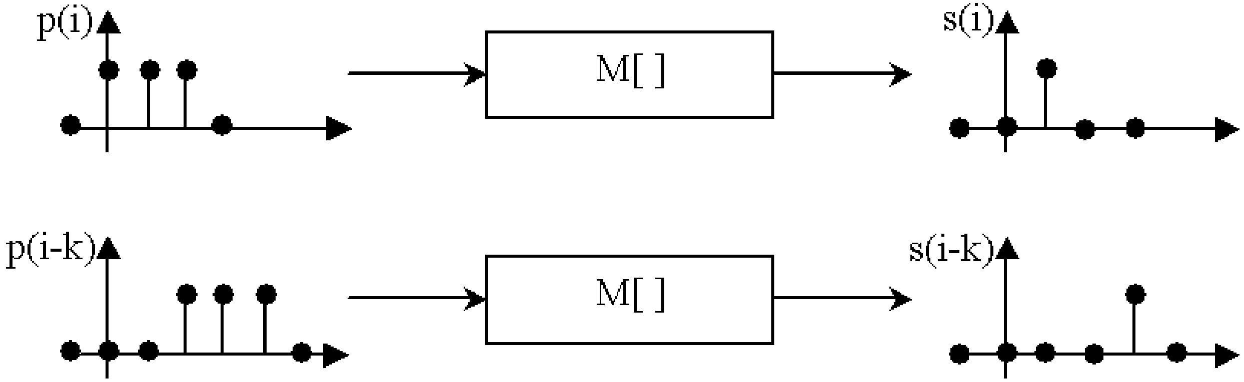
Systèmes linéaires & invariants dans le décalage



Système linéaire et invariant dans le décalage



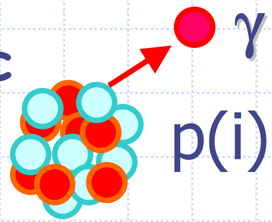
Linéarité



Invariance dans le décalage (dans cet exemple : k=2)

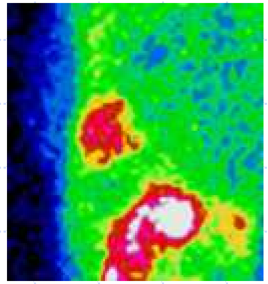
Formation de l'image

$^{99}_{43}$Tc



p(i) →

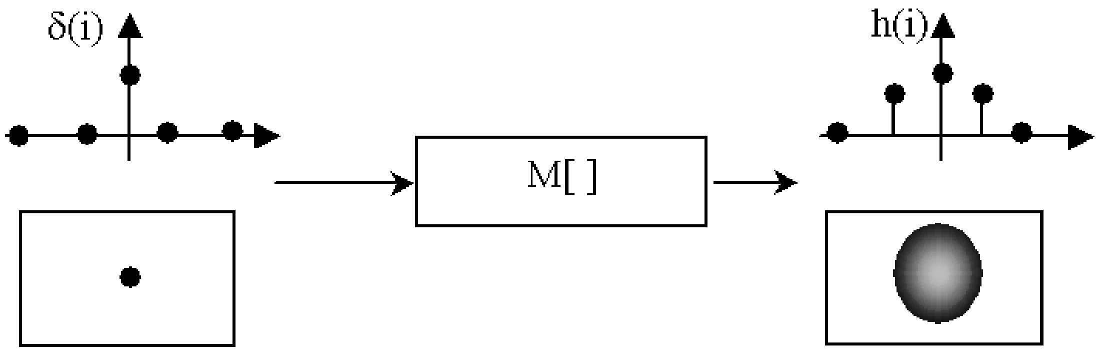
→ s(i)



$$p(i) = \sum_{k=-\infty}^{+\infty} p(k).\delta(i-k)$$

M[ ]

$$s(i) = \sum_{k=-\infty}^{+\infty} p(k).M[\delta](i-k)$$



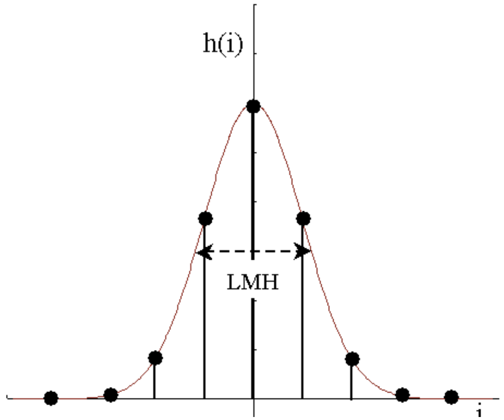
$$s(i) = \sum_{k=-\infty}^{+\infty} p(k).h(i-k)$$

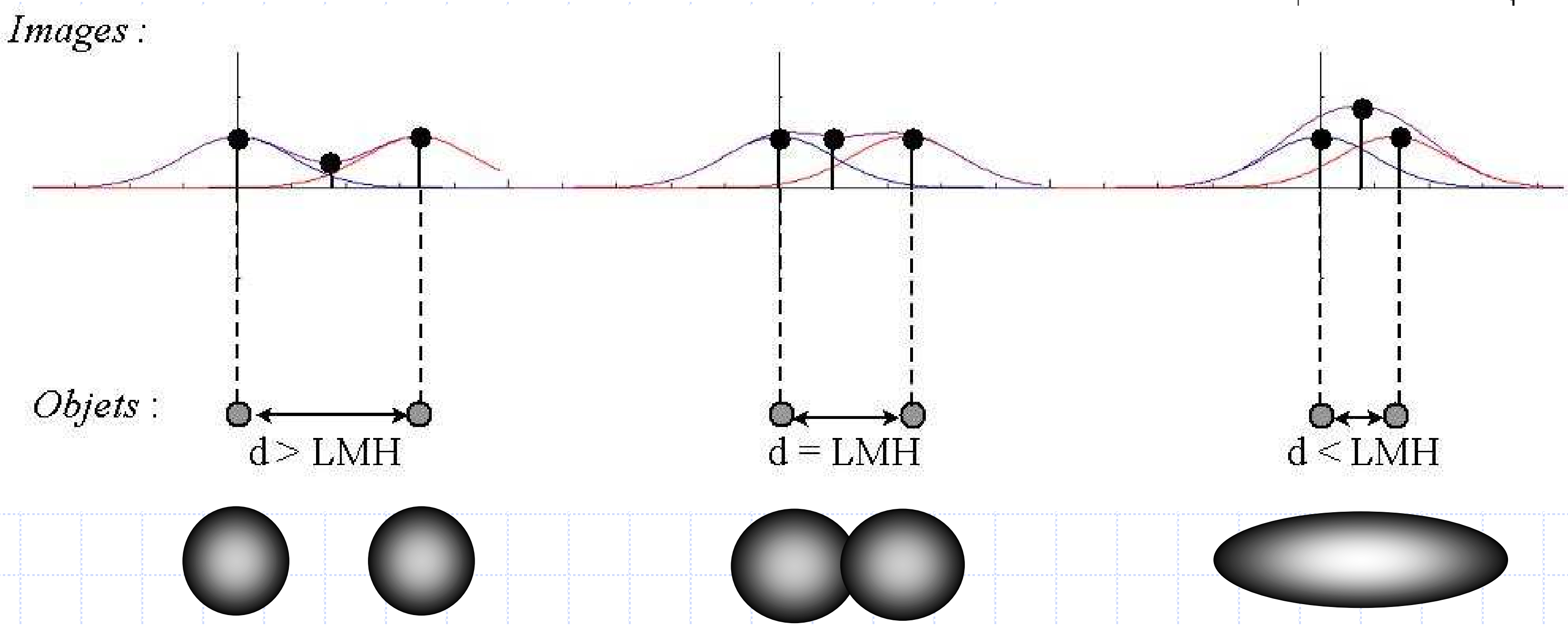
$$s = p * h = h * p$$

Produit de convolution

$$\mathrm{Rm} : p = \delta * p$$

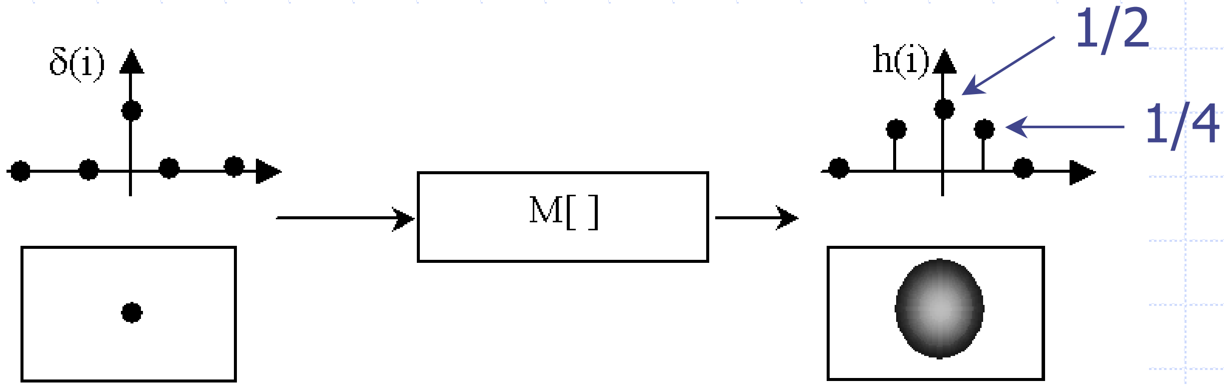
Interprétation (I)





LMH = pouvoir séparateur = résolution de l'imageur
1/LMH = fréquence spatiale maximale dans le signal

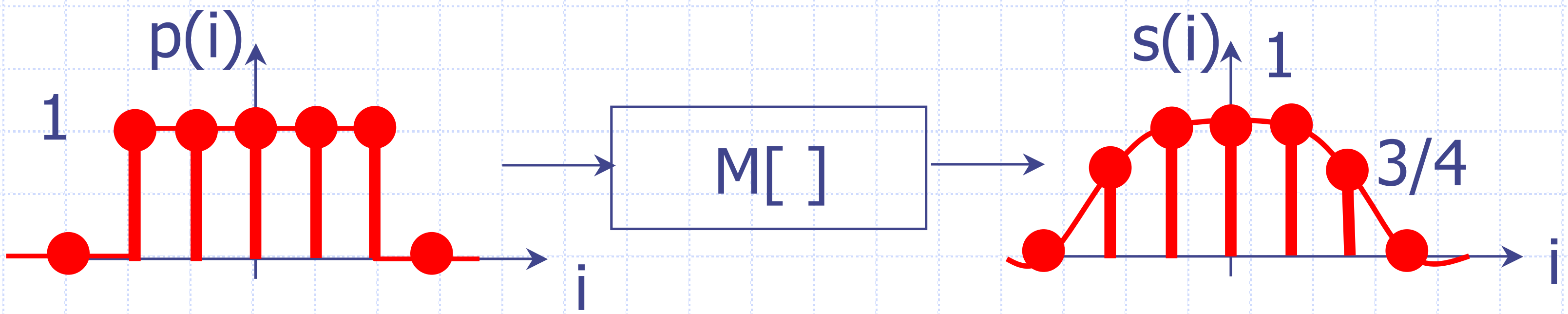
Interprétation (II)



$$s(i) = \sum_{k=-1}^{+1} h(k).p(i-k)$$

$$s(i) = \frac{1}{4}p(i+1) + \frac{1}{2}p(i) + \frac{1}{4}p(i-1) = \frac{2.p(i) + p(i+1) + p(i-1)}{4}$$

moyenne pondérée



SLID et base de Fourier

$$p(i) = e^{j.(\nu\omega_0)i} \rightarrow \boxed{M[\ ]} \longrightarrow s(i) = M[p](i)$$

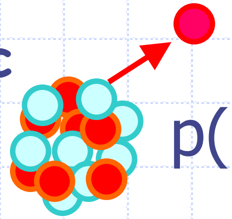
$$s(i) = \sum_{k=-\infty}^{k=+\infty} h(k).e^{j.(\nu.\omega_0).(i-k)} = e^{j.(\nu.\omega_0).i} \sum_{k=-\infty}^{k=+\infty} h(k).e^{-j.(\nu.\omega_0).k}$$

$$p(i) = e^{j.(\nu\omega_0)i} \quad \Rightarrow \quad M[p](i) = \hat{h}(\nu).p(i)$$

Un SLID agit sur l'harmonique ν
en l'amplifiant par la réponse en fréquence en ν

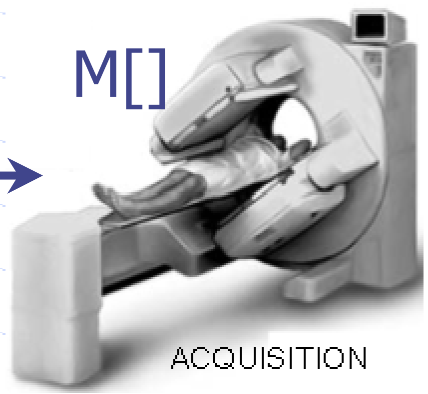
SLID et base de Fourier

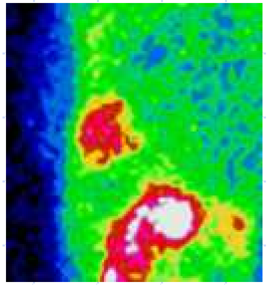
$^{99}_{43}$Tc



γ

p(i) → M[] → s(i)





$$p(i) = \frac{1}{N} \sum_{\nu=0}^{\nu=N-1} \hat{p}(\nu).e^{j.\omega_0.\nu.i} \Rightarrow M[p](i) = \frac{1}{N} \sum_{\nu=0}^{\nu=N-1} \hat{p}(\nu).\hat{h}(\nu).e^{j.\omega_0.\nu.i}$$

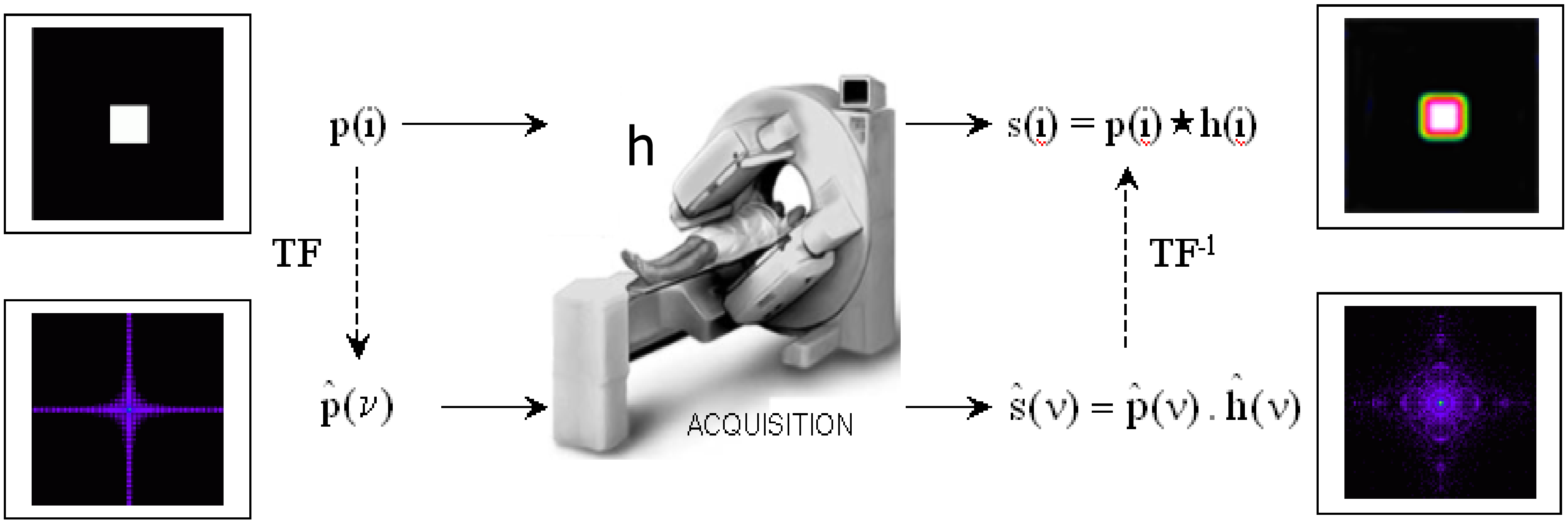
$=$

$$s(i) = \frac{1}{N} \sum_{\nu=0}^{\nu=N-1} \hat{s}(\nu).e^{j.\omega_0.\nu.i}$$

$$\boxed{s = p * h \Rightarrow \hat{s} = \hat{p}\,.\,\hat{h}}$$

$$\text{Rm}:\; s = \delta * h = h \Rightarrow \hat{s} = \hat{\delta}\,.\,\hat{h} = \hat{h},\; \text{d'où}\;\; \hat{\delta} = 1$$

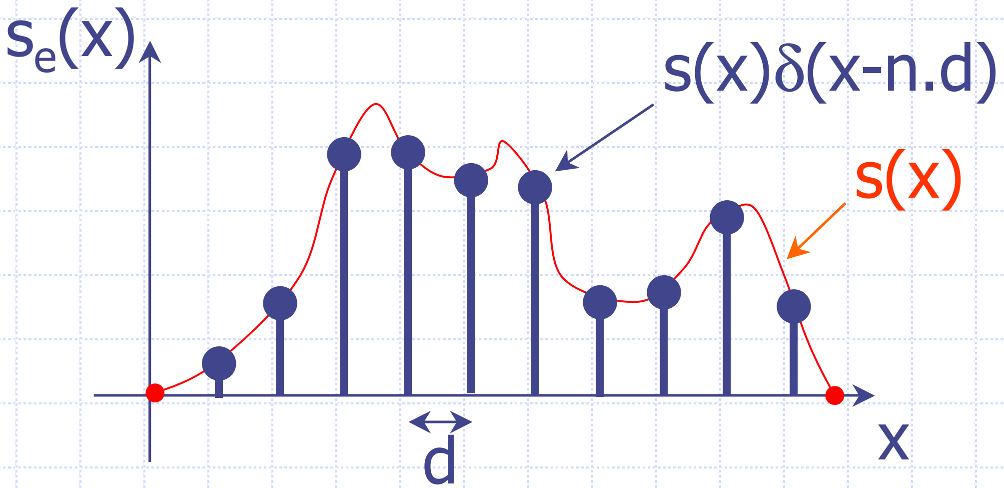
Formation de l'image: synthèse



ECHANTILLONNAGE

THEOREME DE SHANNON

Opérateur d'échantillonnage



$$s_e(x) = s(x).S_d(x) = \sum_{n=-\infty}^{+\infty} s(x).\delta(x-nd) = \sum_{n=-\infty}^{+\infty} s(n.d).\delta(x-nd)$$

Théorème de Shannon (I)

$$s_e(x) = s(x) . \sum_{n=-\infty}^{+\infty} \delta(x - nd)$$

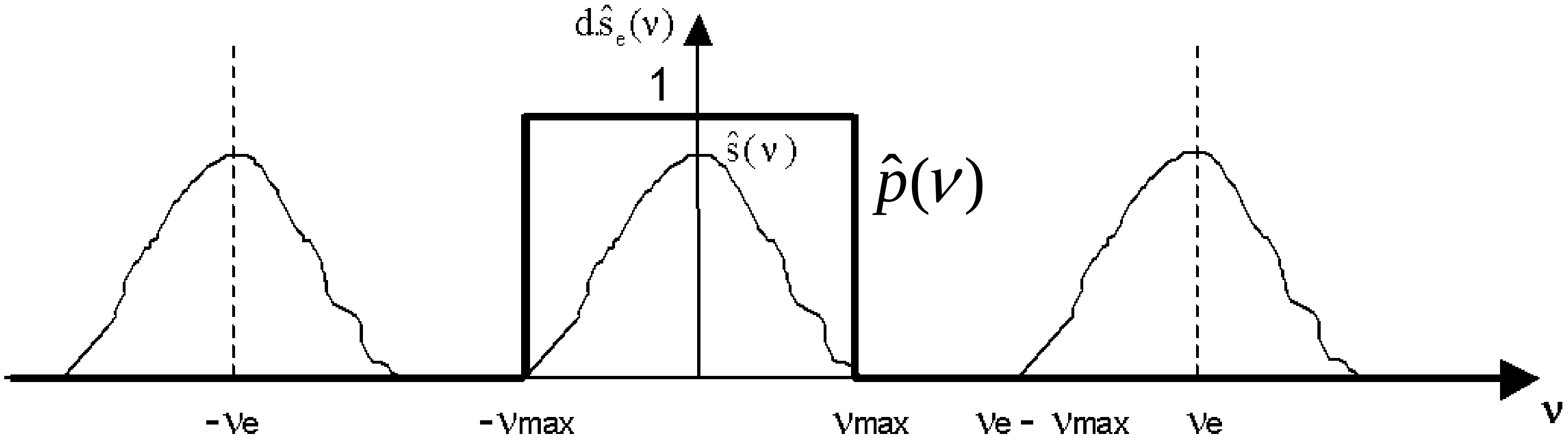
$$\hat{s}_e(\nu) = \frac{1}{d}\hat{s}(\nu) * \sum_{n=-\infty}^{+\infty} \delta\left(\nu - \frac{n}{d}\right)$$

$$\text{mais } \hat{s}(\nu) * \delta\left(\nu - \frac{n}{d}\right) = \sum_{k=-\infty}^{+\infty} \hat{s}(\text{k}) . \delta\left(\nu - \frac{n}{d} - k\right) = \hat{s}\left(\nu - \frac{n}{d}\right)$$

$$\text{donc} \quad \hat{s}_e(\nu) = \frac{1}{d}\sum_{n=-\infty}^{+\infty} \hat{s}\left(\nu - \frac{n}{d}\right)$$

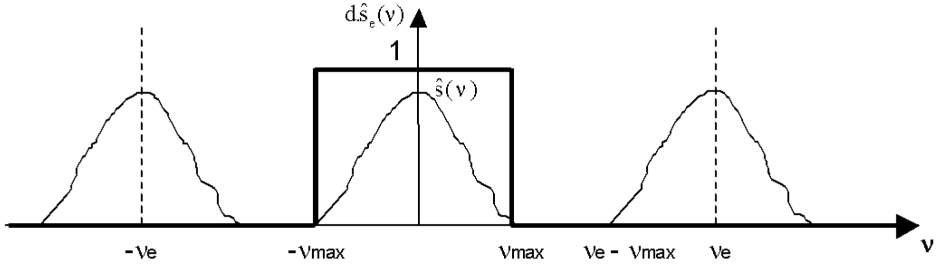
Théorème de Shannon (II)

$$d\,.\,\hat{s}_e(\nu) \;=\; \sum_{n=-\infty}^{+\infty} \hat{s}\left(\nu - \frac{n}{d}\right)$$



$$\nu_e > 2.\nu_{\max} \Rightarrow d.\hat{s}_e(\nu).\hat{p}(\nu) = \hat{s}(\nu)$$

Théorème de Shannon (III)



$$\nu_e > 2.\nu_{\max} \Rightarrow d.\hat{s}_e(\nu).\hat{p}(\nu) = \hat{s}(\nu)$$

$$\mathrm{s(x) = d.p(x)} * \sum_{\mathrm{n}=-\infty}^{\infty} \mathrm{s(n.d).\delta(x - n.d) = d.} \sum_{\mathrm{n}=-\infty}^{\infty} \mathrm{s(n.d).p(x - n.d)}$$

$$\boxed{s(x) = d.\sum_{n=-\infty}^{\infty} s(n.d).\frac{\sin[2\pi\nu_{\max}(x - n.d)]}{\pi(x - n.d)}}$$

Théorème de Shannon (IV)

$$\boxed{\nu_e > 2.\nu_{\max} \Rightarrow s(x) = d.\sum_{n=-\infty}^{\infty} s(n.d).\frac{\sin[2\pi\nu_{\max}(x-n.d)]}{\pi(x-n.d)}}$$

donc échantillonnage sans perte d'information

$2.\nu_{max}$ = 2/LMH est appelé fréquence de Nyquist

Exemple : Nombre de pixels en scintigraphie myocardique
champ 50x50 cm
LMH = 16 mm

Théorème de Shannon (IV)

$$\boxed{\nu_e > 2.\nu_{\max} \Rightarrow s(x) = d.\sum_{n=-\infty}^{\infty} s(n.d).\frac{\sin[2\pi\nu_{\max}(x-n.d)]}{\pi(x-n.d)}}$$

donc échantillonnage sans perte d'information

$2.\nu_{max}$ = 2/LMH est appelé fréquence de Nyquist

Exemple : Nombre de pixels en scintigraphie myocardique
champ 50x50 cm
LMH = 16 mm

Réponse:

ν_{max} =1/LMH =1/16 = 0.0625 pixel/mm

ν_e =2.ν_{max} =2/16 = 0.125 pixel/mm

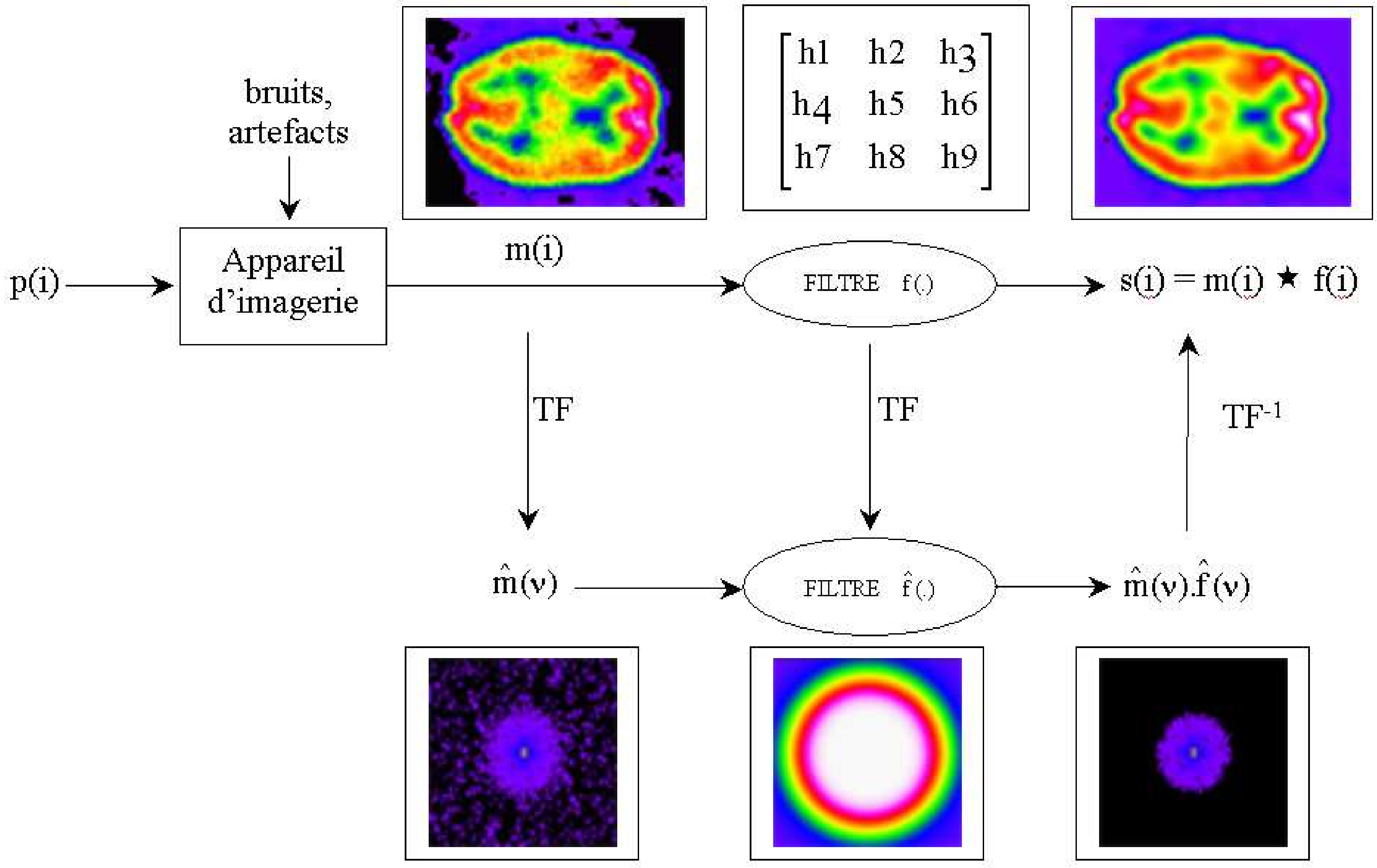
Donc 0.125 x 500 =62.5 i.e 64 pixels/côté

FILTRAGE

Filtrage linéaire (et invariant dans le décalage)

Filtrage non linéaire

Filtrage linéaire d'image



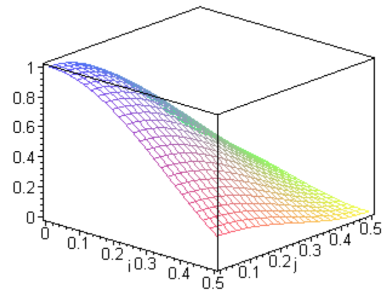
Filtres passe-bas

- Convolution: $s(i)=\sum_{k=-\infty}^{+\infty} f(k).m(i-k)$

- Remplace chaque NG par une moyenne pondérée des NG voisins $\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

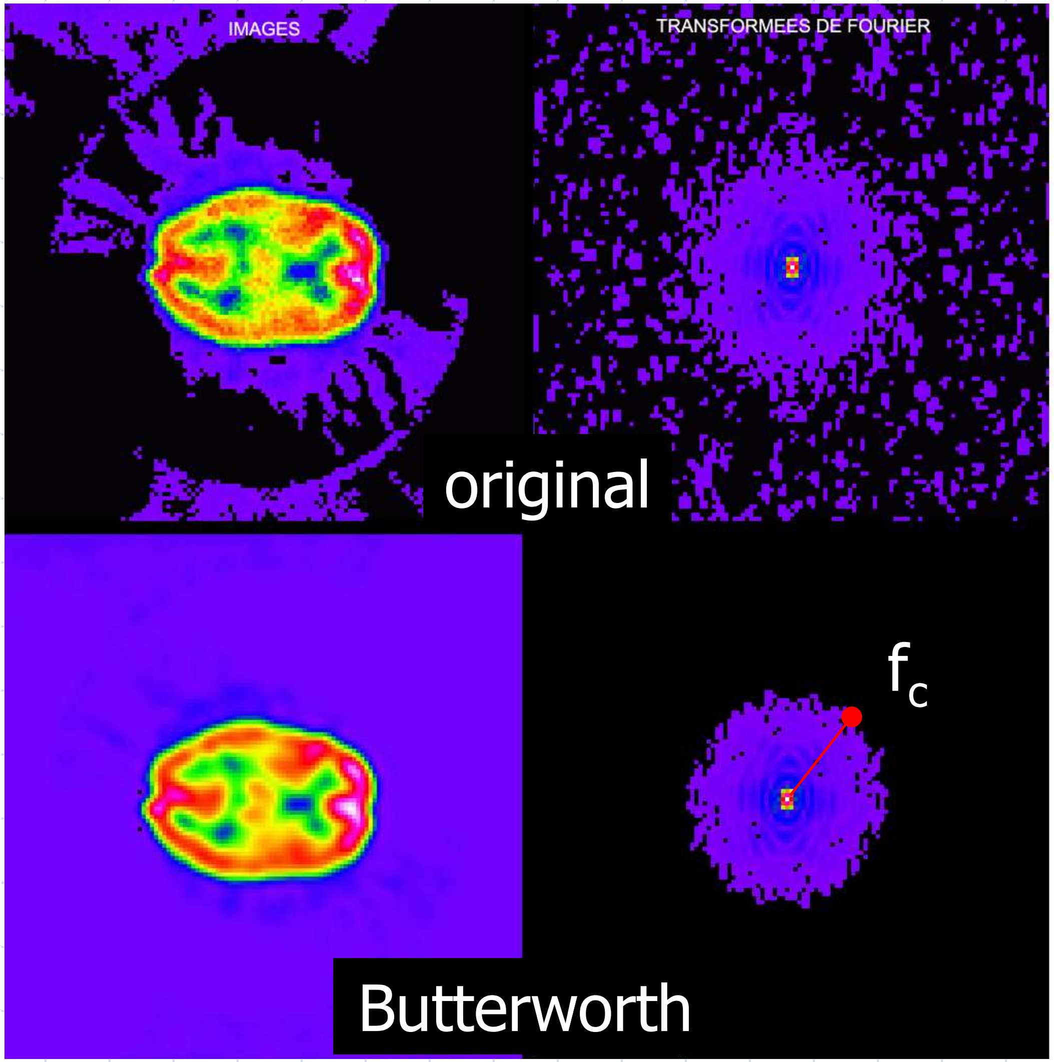
- Atténuation sélective de certaines fréquences

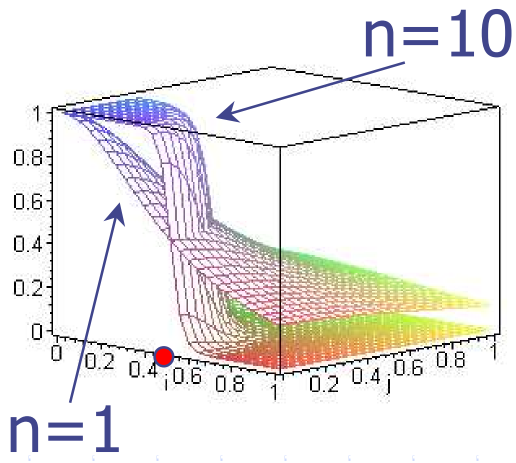
$$\hat{f}(\nu)=0{,}5.[1+\cos(\pi\frac{\nu}{\nu_e})]$$



$$\sqrt{\nu^2+\nu'^2}$$

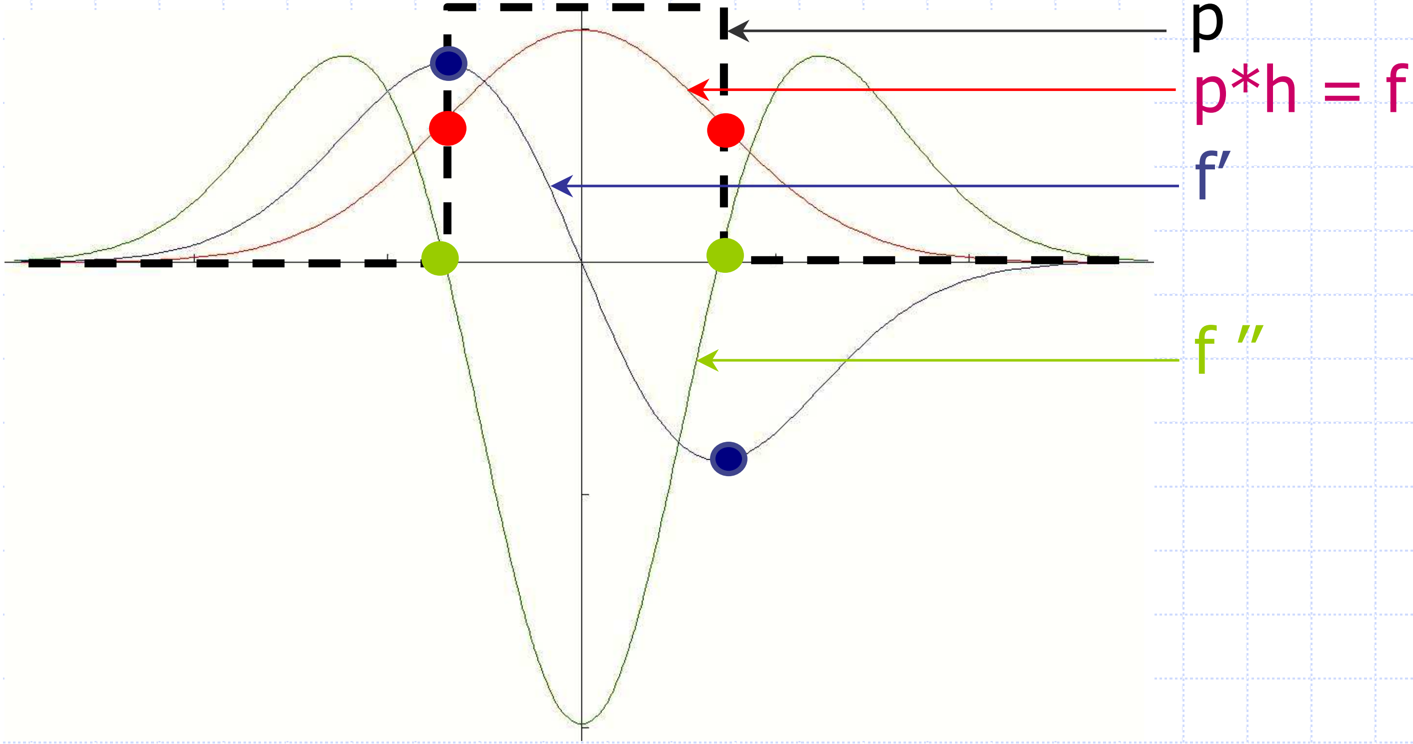
Exemple: filtres de Butterworth





$$\hat{m}(\nu,\nu') = \frac{1}{1+\left(\frac{\sqrt{\nu^2+\nu'^2}}{f_c}\right)^{2.n}}$$

Exemple: segmentation

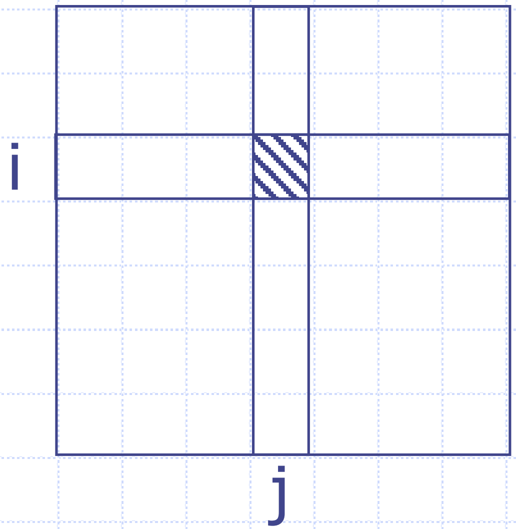


- Frontières :
  - Extrema du gradient (f')
  - Passages par zéro du Laplacien (f'')

Filtres passe-haut: Gradients

$$g_h(i,j) = \frac{1}{2}[f(i+1,j) - f(i-1,j)] \longrightarrow \begin{bmatrix} -1/2 & 0 & 1/2 \end{bmatrix}$$

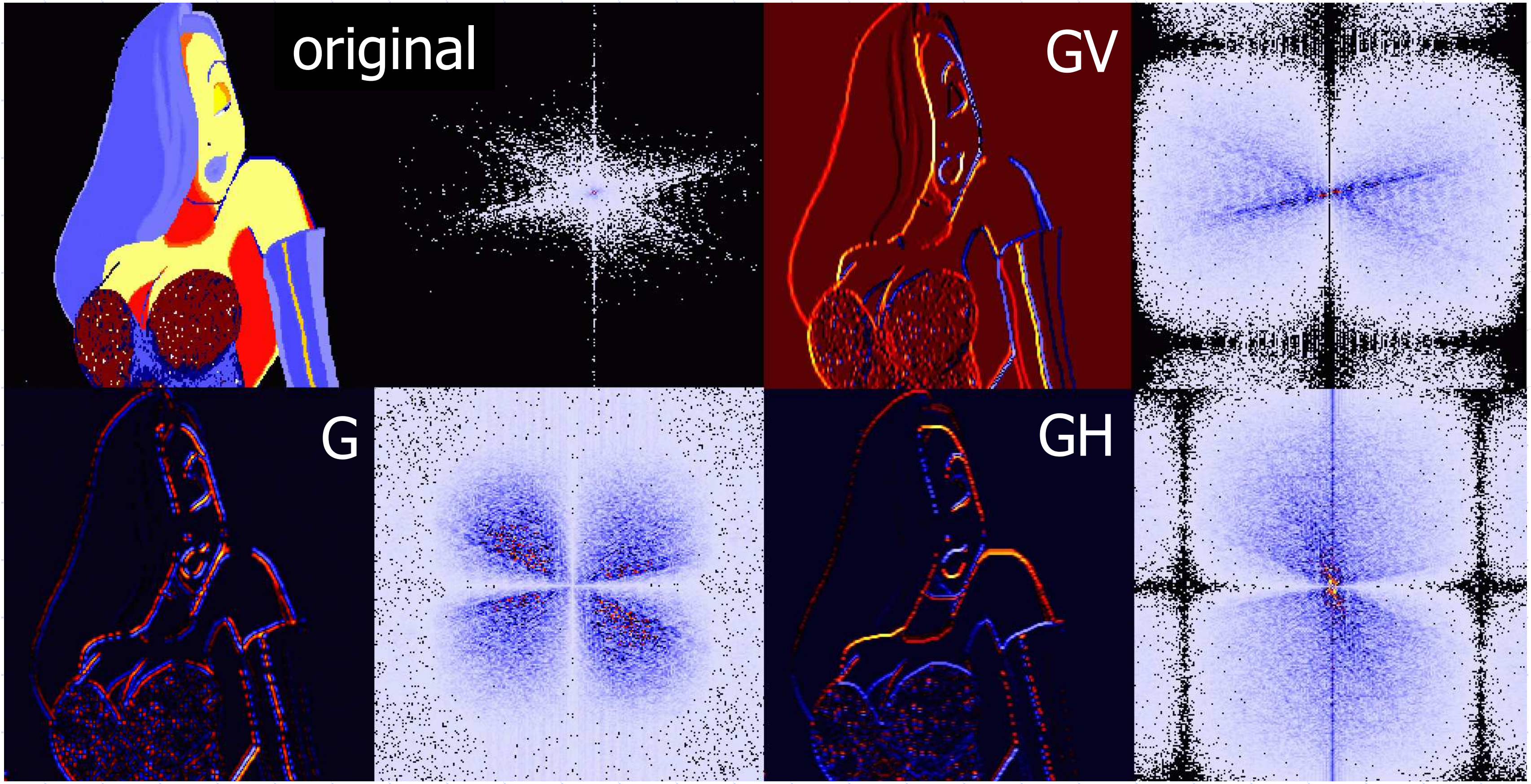
$$g_v(i,j) = \frac{1}{2}[f(i,j+1) - f(i,j-1)] \longrightarrow \begin{bmatrix} -1/2 \\ 0 \\ 1/2 \end{bmatrix}$$



Généralisation 2d:

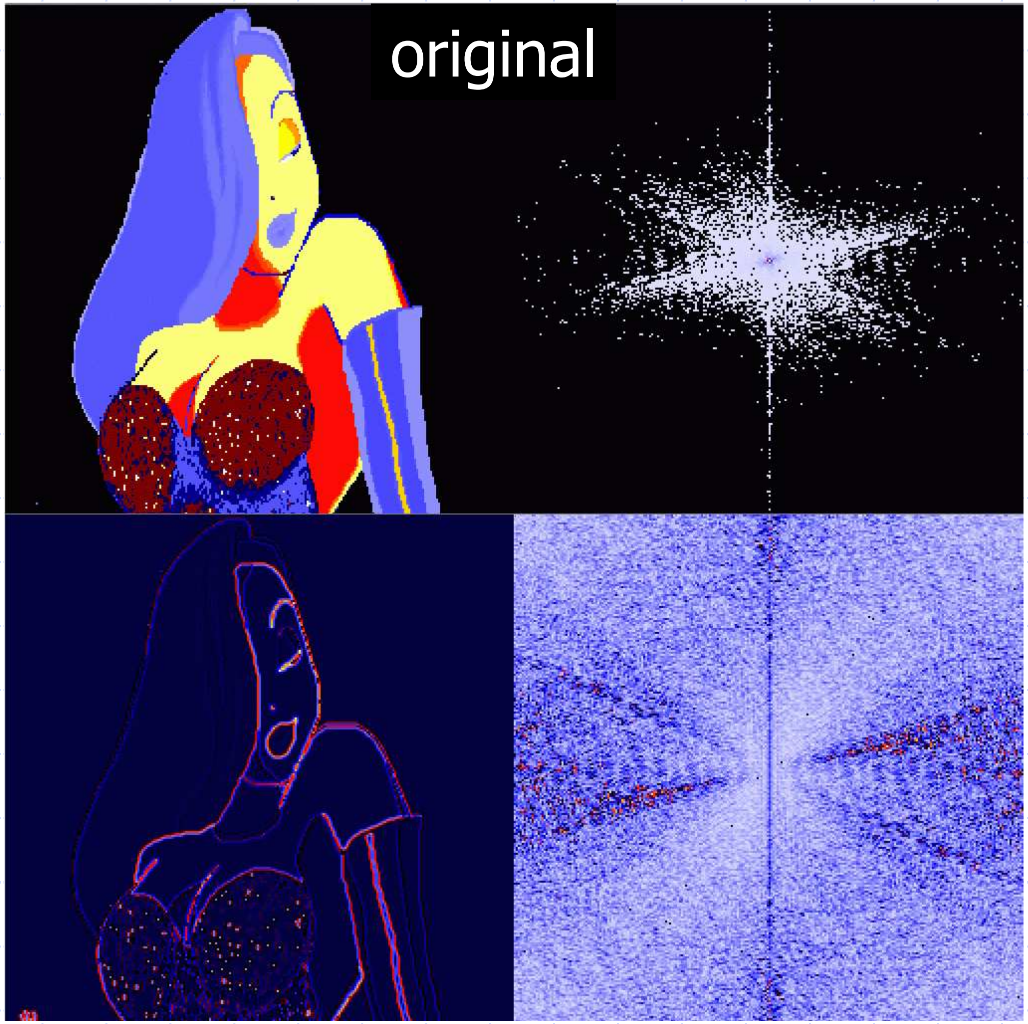
$$G_h = \begin{bmatrix} -1 & 0 & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \\ -1 & 0 & 1 \end{bmatrix} \qquad G_v = \begin{bmatrix} -1 & -\sqrt{2} & -1 \\ 0 & 0 & 0 \\ 1 & \sqrt{2} & 1 \end{bmatrix}$$

Filtres passe-haut: Gradients



GH (GV) efface les frontières verticales (horizontales)

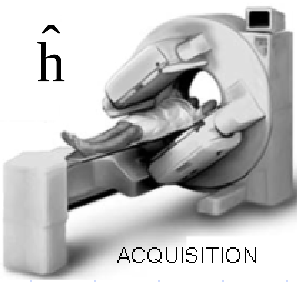
Filtres passe-haut: Laplacien



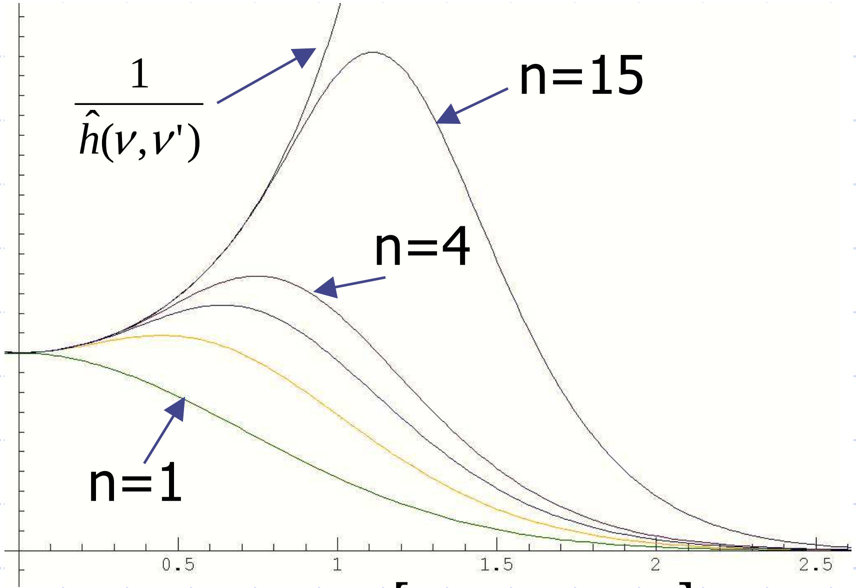
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Filtre de déconvolution de Metz

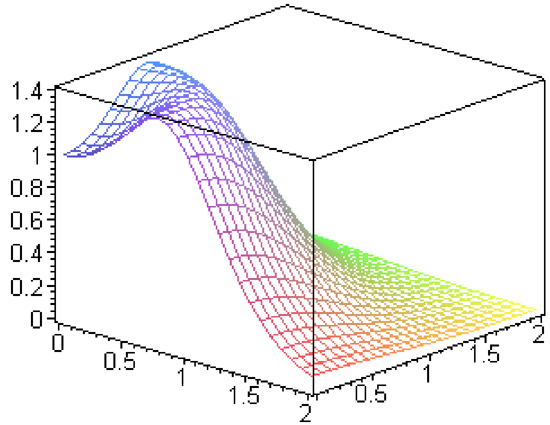
$\hat{p}$ → $\hat{h}$



→ $\hat{s} = \hat{h}.\hat{p}$ → $\boxed{\dfrac{1}{\hat{h}}}$ → $\hat{p}$



$$\hat{m}(\nu,\nu') = \frac{1-\left[1-\hat{h}(\nu,\nu')^2\right]^n}{\hat{h}(\nu,\nu')}$$

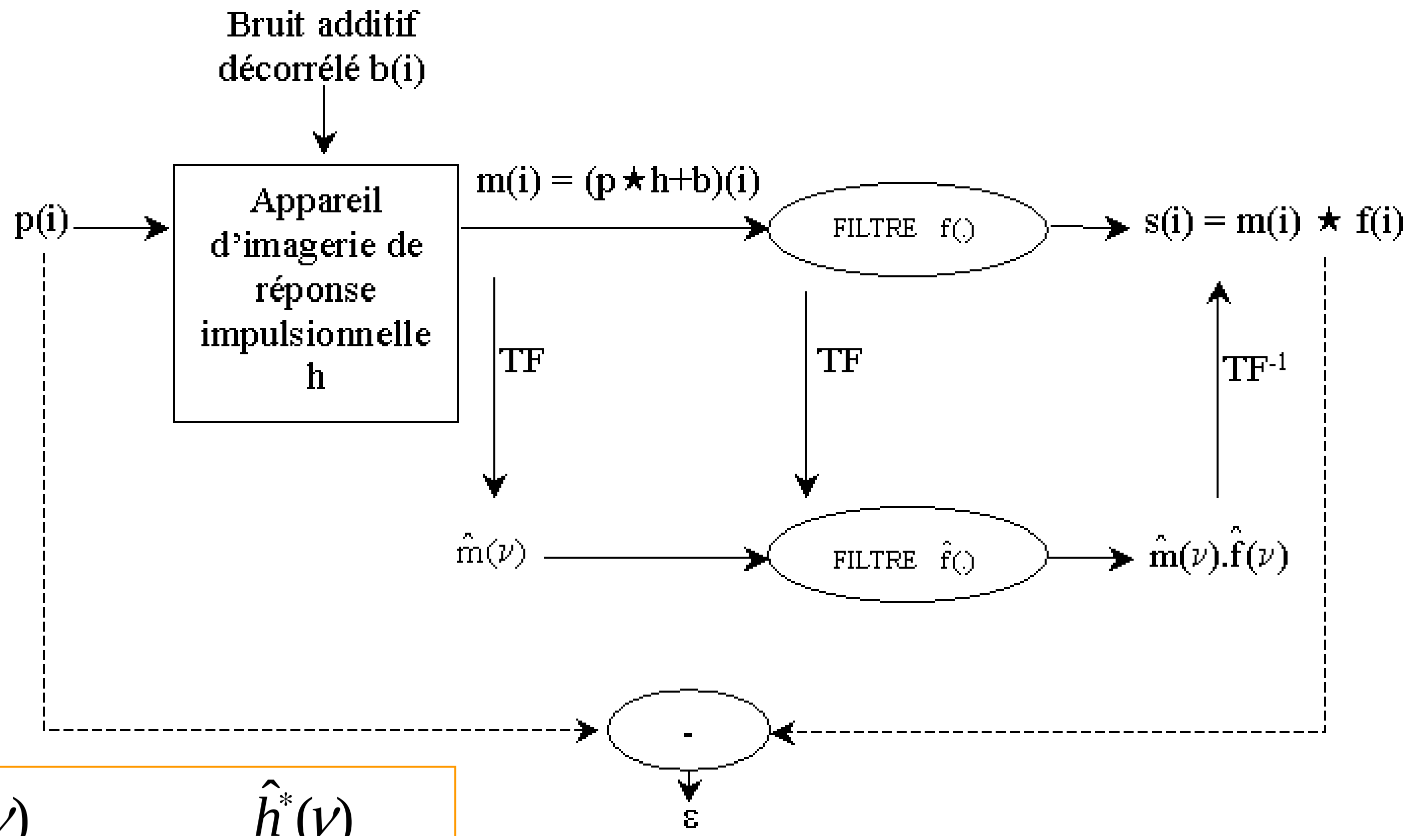


n=0,834.ln(C)-7,774

King et al. *JNM* 83;24

Metz Et al. *JNM* 73; 15

Filtre de déconvolution de Wiener



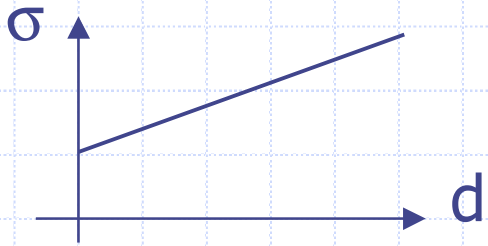
$$\hat{f}(\nu)=\frac{\hat{h}^*(\nu)}{\left|\hat{h}(\nu)\right|^2+\frac{S_b(\nu)}{S_p(\nu)}}\approx\frac{\hat{h}^*(\nu)}{\left|\hat{h}(\nu)\right|^2+\frac{\left|\hat{b}(\nu)\right|^2}{\left|\hat{p}(\nu)\right|^2}}$$

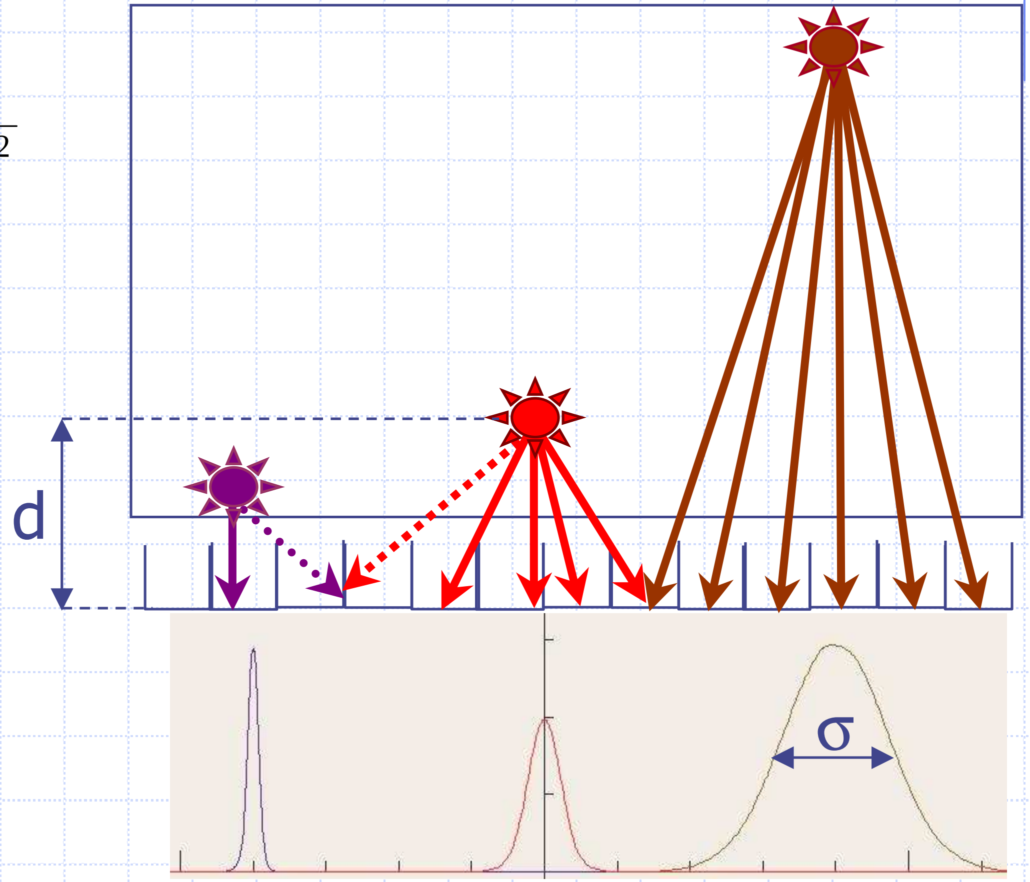
$$\hat{h}=1\Rightarrow\hat{f}(\nu)=\frac{S_p(\nu)}{S_p(\nu)+S_b(\nu)}=\frac{S_m(\nu)-S_b(\nu)}{S_m(\nu)}=1-\frac{S_b(\nu)}{S_m(\nu)}\rightarrow 0$$

Réponse impulsionnelle en MN γ

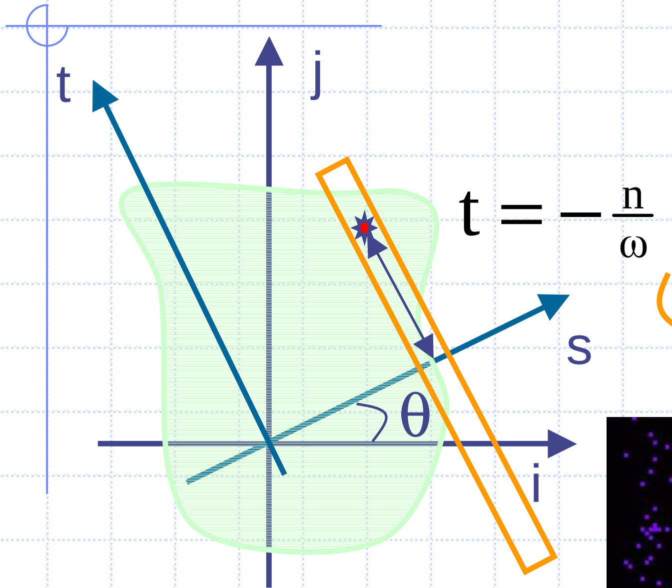
- $$h(i) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{i^2}{2\sigma^2}}$$

- $$\sigma = k.\mathrm{d} + k'$$





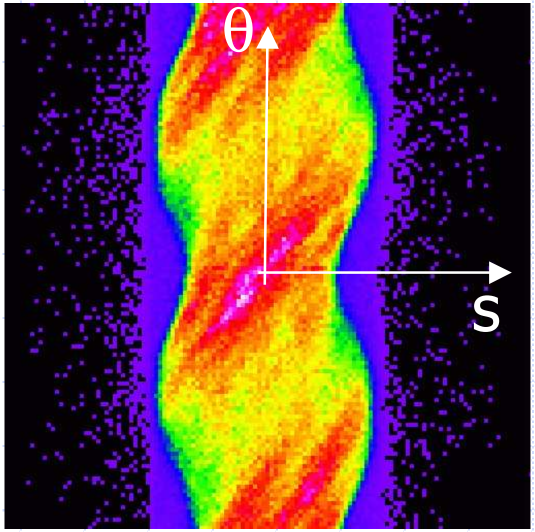
Relation fréquence-distance



Une désintégration contribue à

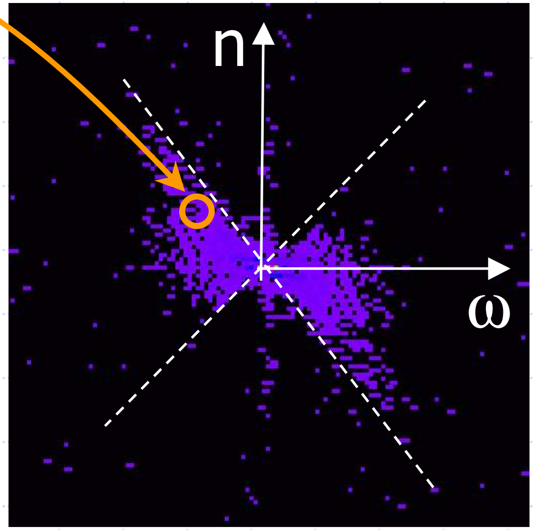
$$\hat{p}_c(\omega, n)$$

quand θ permet que $t = -\frac{n}{\omega}$



$$p_c(s,\theta) = \int f(i,j).dt$$

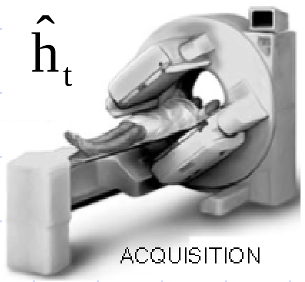
TF$_2$



$$\hat{p}_c(\omega, n)$$

Déconvolution en TEMP

$\hat{\mathrm{p}}$ $\longrightarrow$ $\hat{\mathrm{h}}_t$

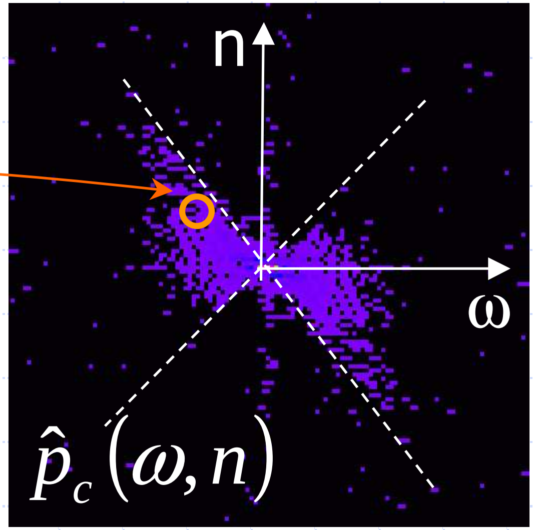


$\longrightarrow \hat{\mathrm{p}}_c = \hat{\mathrm{h}}_t \,.\, \hat{\mathrm{p}} \longrightarrow \boxed{\dfrac{1}{\hat{\mathrm{h}}_t}} \longrightarrow \hat{\mathrm{p}}$

$$h_t(i) = \frac{1}{\sqrt{2\pi}\sigma_t} e^{-\frac{i^2}{2\sigma_t^2}}$$

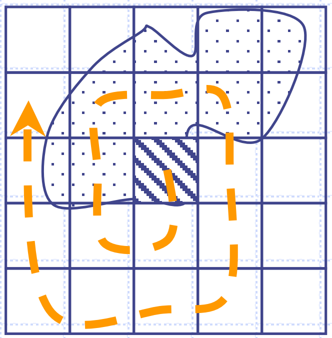
$$\hat{p}_c(\omega,n) = \hat{h}_{-\frac{n}{\omega}}(\omega).\hat{p}(\omega,n)$$

$$\boxed{\hat{p}(\omega,n) = \frac{1}{\hat{h}_{-\frac{n}{\omega}}(\omega)}.\hat{p}_c(\omega,n)}$$



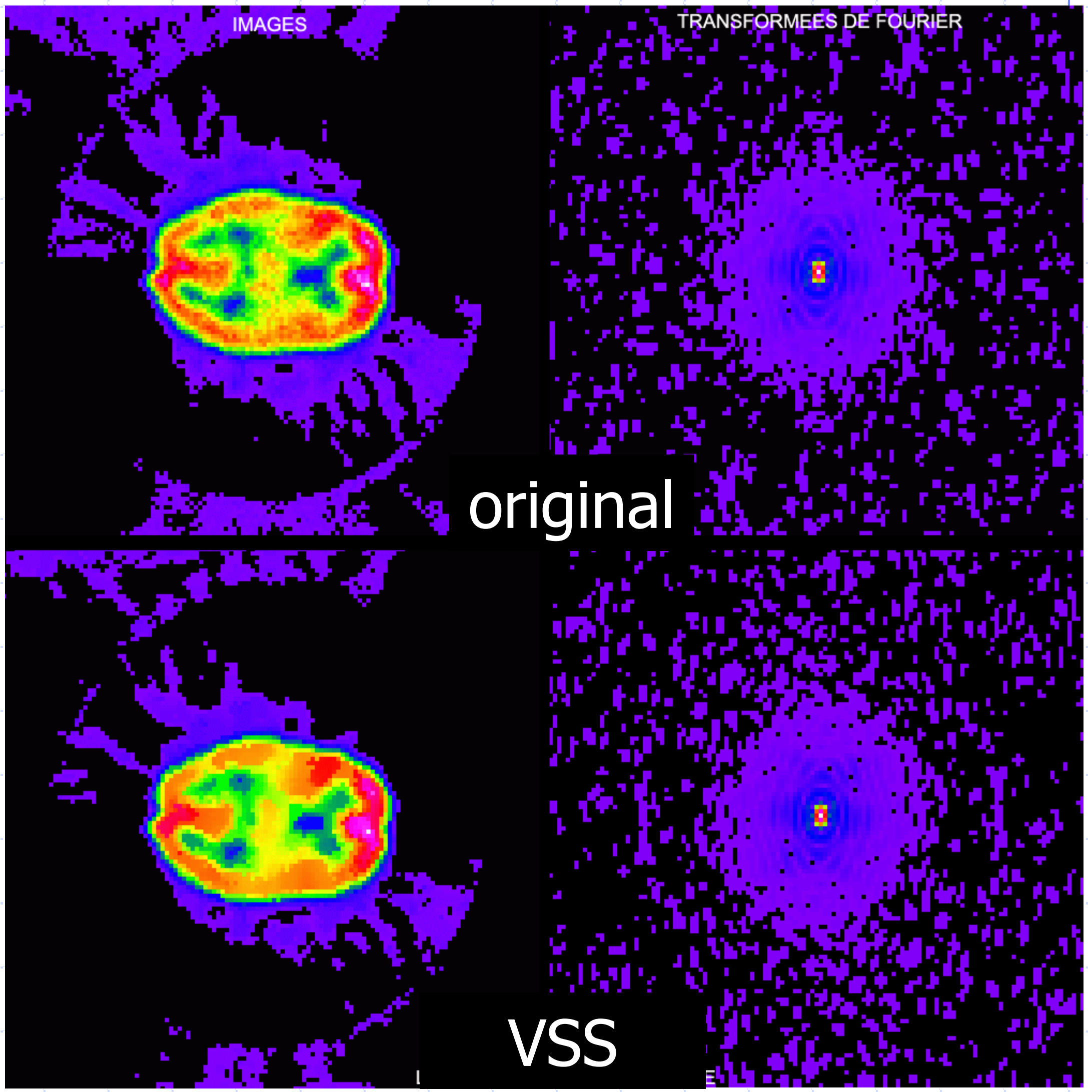
Edholm, Lewitt et al. Proc SPIE 1986, et IEEE-TMI, 1989

Lissage sur masque adapté (VSS)

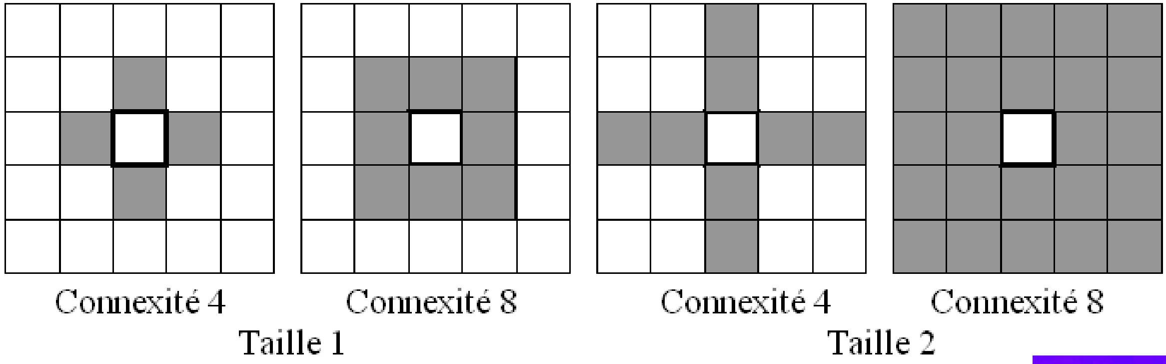


Accumulation pour moyenne des

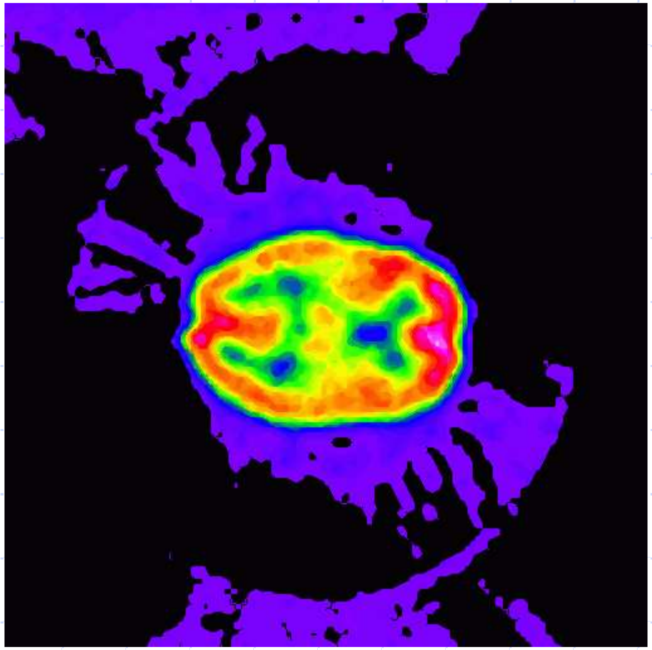
$$s(i',j') \in s(i,j) \pm 2\sqrt{s(i,j)}$$



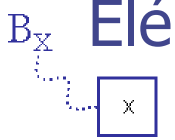
Filtre médian



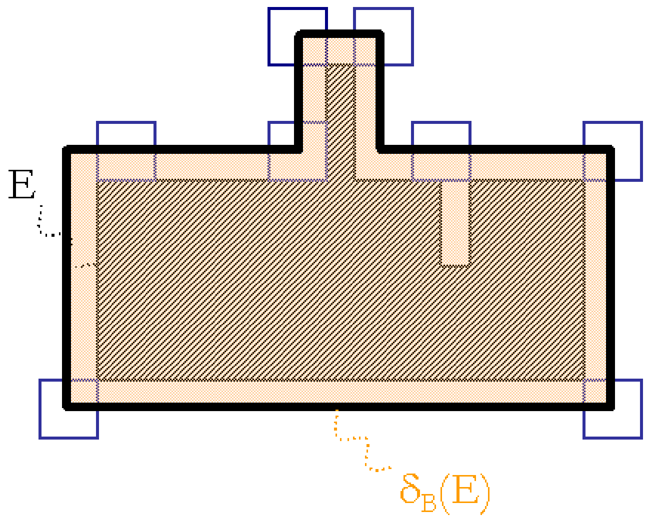
On remplace s(i,j) par le NG médian dans un voisinage fixe de (i,j)

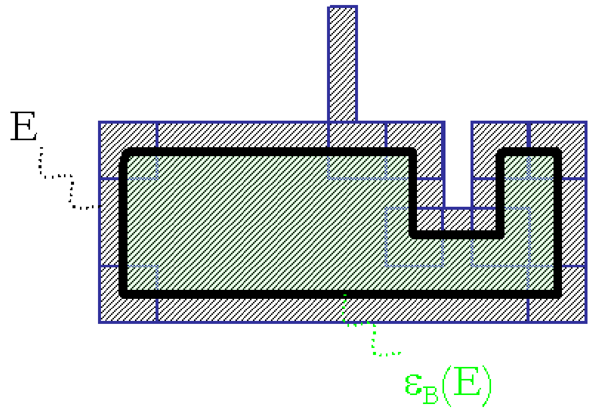


Opérateurs de Minkowski



Élément structurant centré en x





$$\delta_B(E) = \{\text{centres } X \,/\, B_X \cap E \neq \emptyset\}$$
$$= \cup\, \{B_X\,,\, X \in E\}$$

$$\varepsilon_B(E) = \{\text{centres } X \,/\, B_X \subset E\}$$

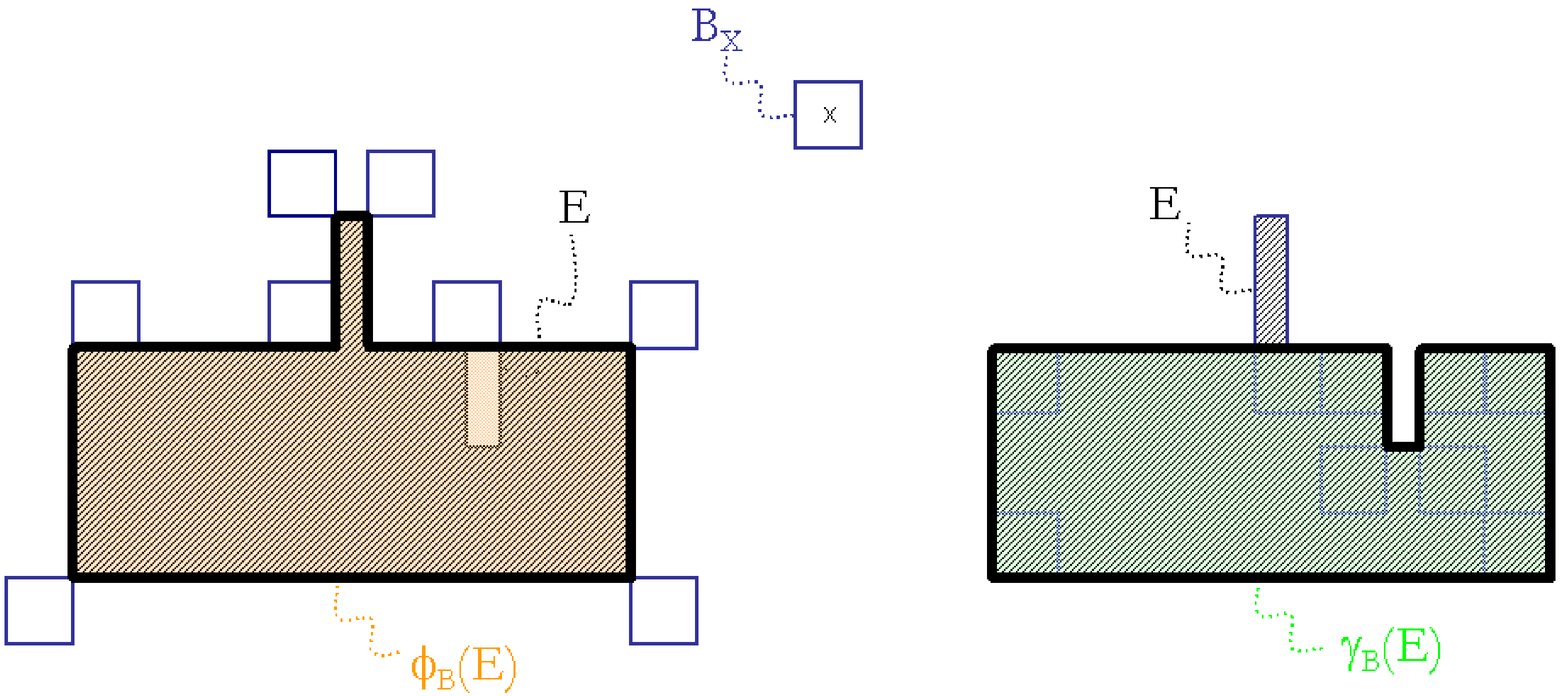
$$\boxed{\varepsilon_B(E) = [\delta_B(E^c)]^c}$$

Filtres morphologiques

ψ est un filtre morphologique si et seulement si:

- $\psi \circ \psi = \psi$
  - Modifie une fois pour toute certaines caractéristiques
- pour tout ensembles A et B, $A \subset B \Rightarrow \psi(A) \subset \psi(B)$
  - Respecte les relations contenant/contenu

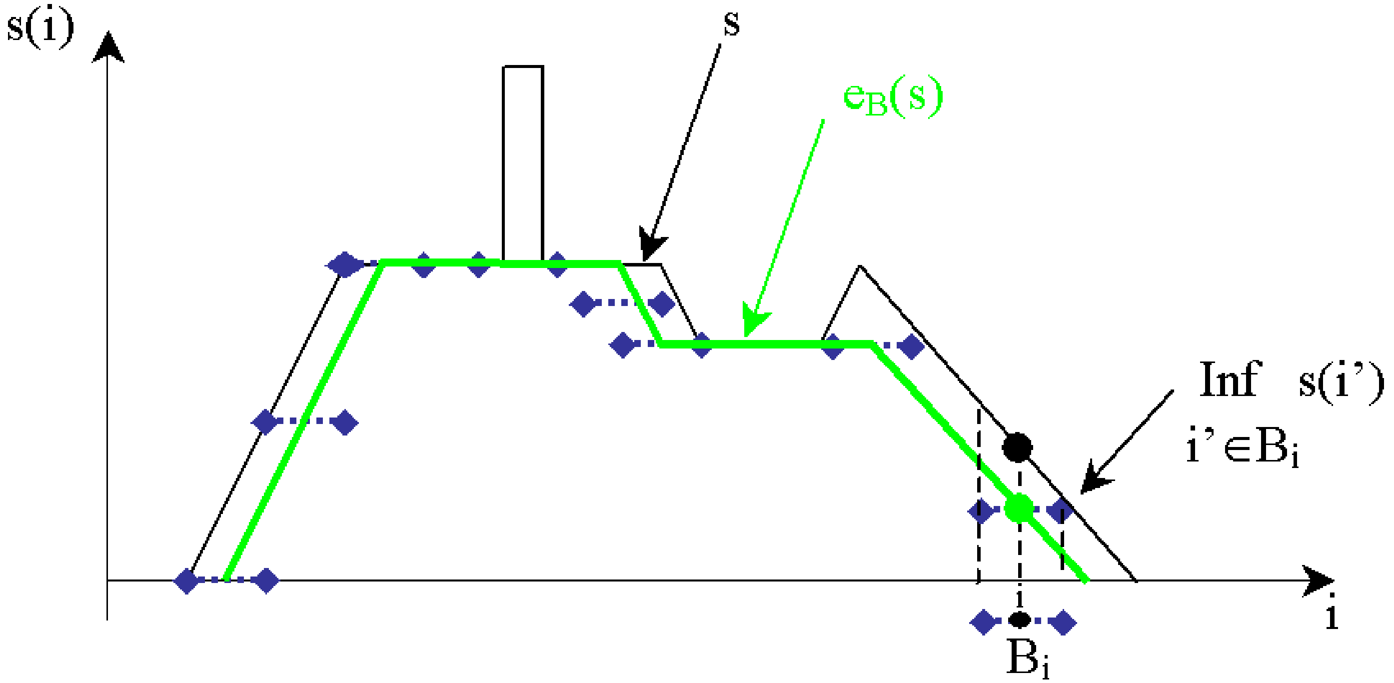
Ouvertures et fermetures binaires



$$\phi_B(E) = (\varepsilon \circ \delta)(E) = \varepsilon[\delta(E)]$$
$$= \cup \{ B_X / B_X \cap X = \varnothing \}^c$$

$$\gamma_B(E) = (\delta \circ \varepsilon)(E) = \delta[\varepsilon(E)]$$
$$= \cup \{ B_X / B_X \subset X \}$$

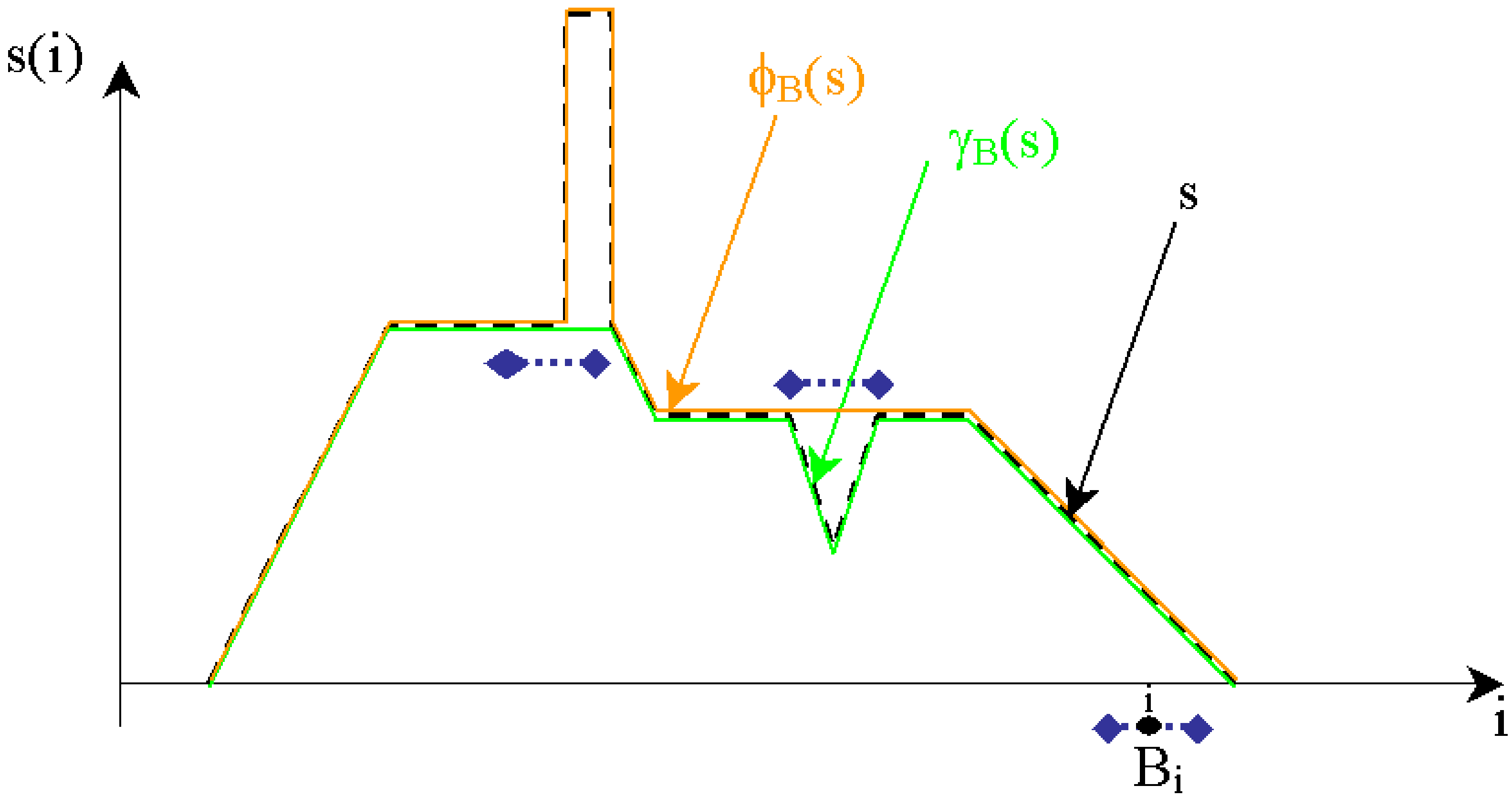
Opérateurs en niveaux de gris



$$\varepsilon_B(s)(i,j) = \underset{(i',j')\in B_{i,j}}{Inf} s(i',j')$$

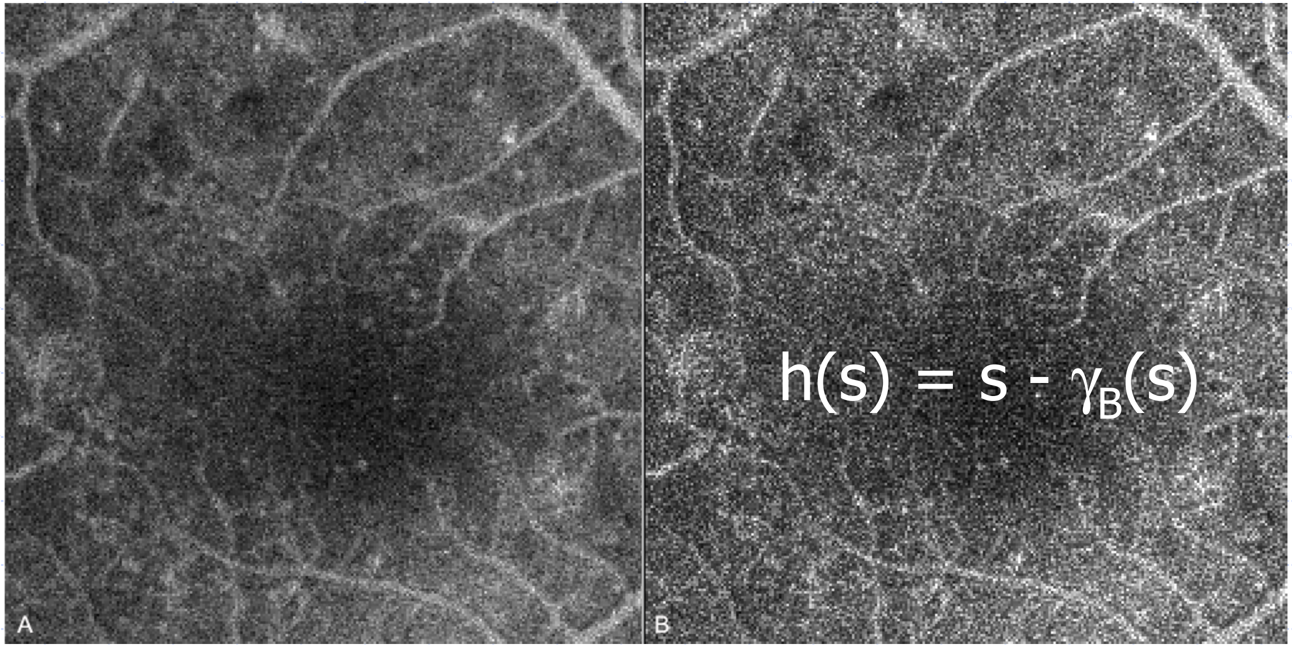
$$\delta_B(s)(i,j) = \underset{(i',j')\in B_{i,j}}{Sup} s(i',j')$$

Filtres en niveaux de gris

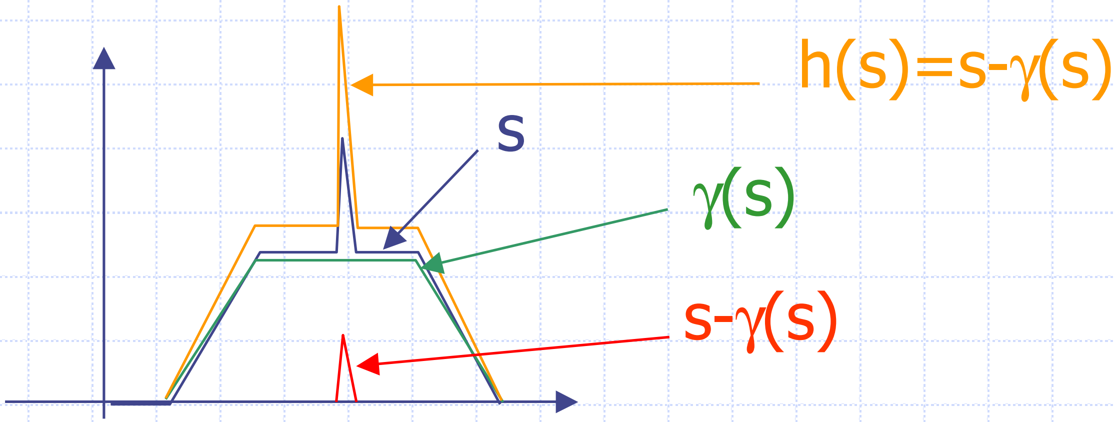


$\phi_B(s) = \varepsilon[\delta(s)]$ $\gamma_B(s) = \delta\,[\varepsilon(s)]$

Top hat transform



original



Opérateurs géodésiques

$$\Delta_s(m) = \wedge\left[\delta_B(m), s\right], \qquad \text{où } m \leq s$$

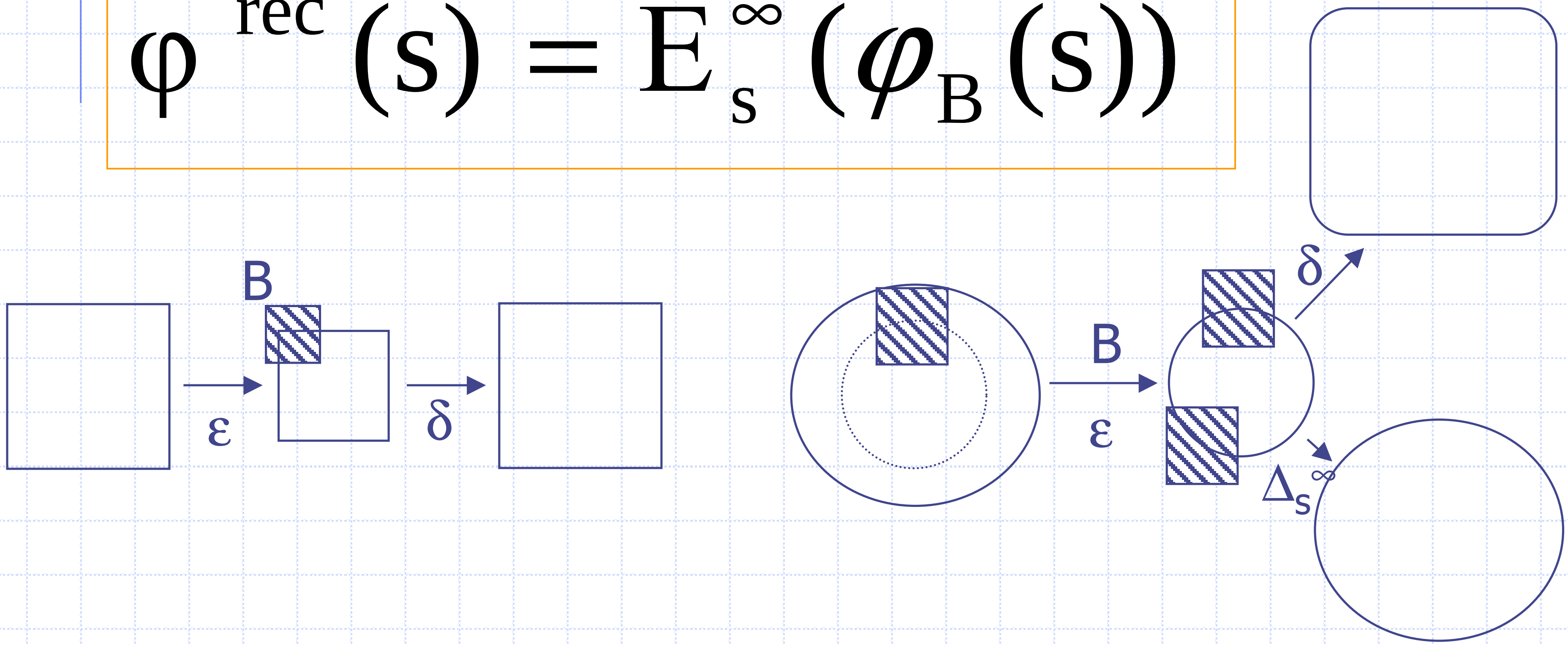
$$E_s(m) = \vee\left[\varepsilon_B(m), s\right], \qquad \text{où } m \geq s$$

$$\boxed{\begin{array}{l} \gamma^{rec}(s) = \Delta_s^{\infty}(m) \\ \varphi^{rec}(s) = E_s^{\infty}(m) \end{array}}$$

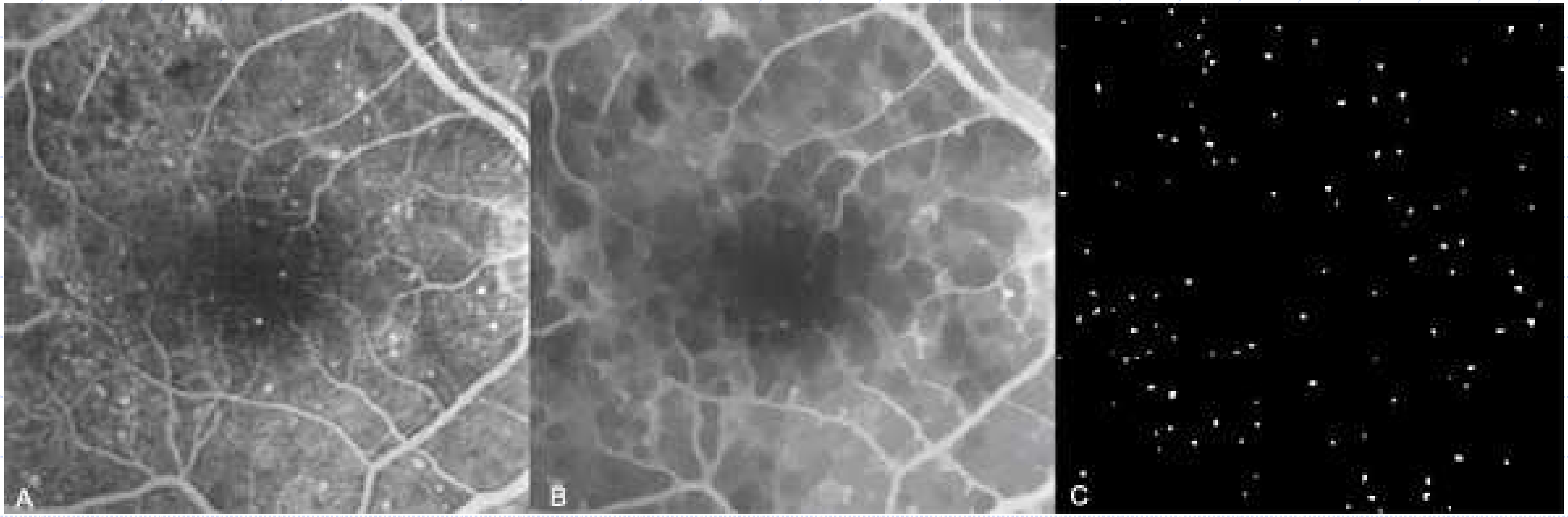
Érosion (dilatation)-reconstruction

$$\gamma^{rec}(s) = \Delta_s^{\infty}(\varepsilon_B(s))$$

$$\varphi^{rec}(s) = E_s^{\infty}(\varphi_B(s))$$



Érosion (dilatation)-reconstruction



original

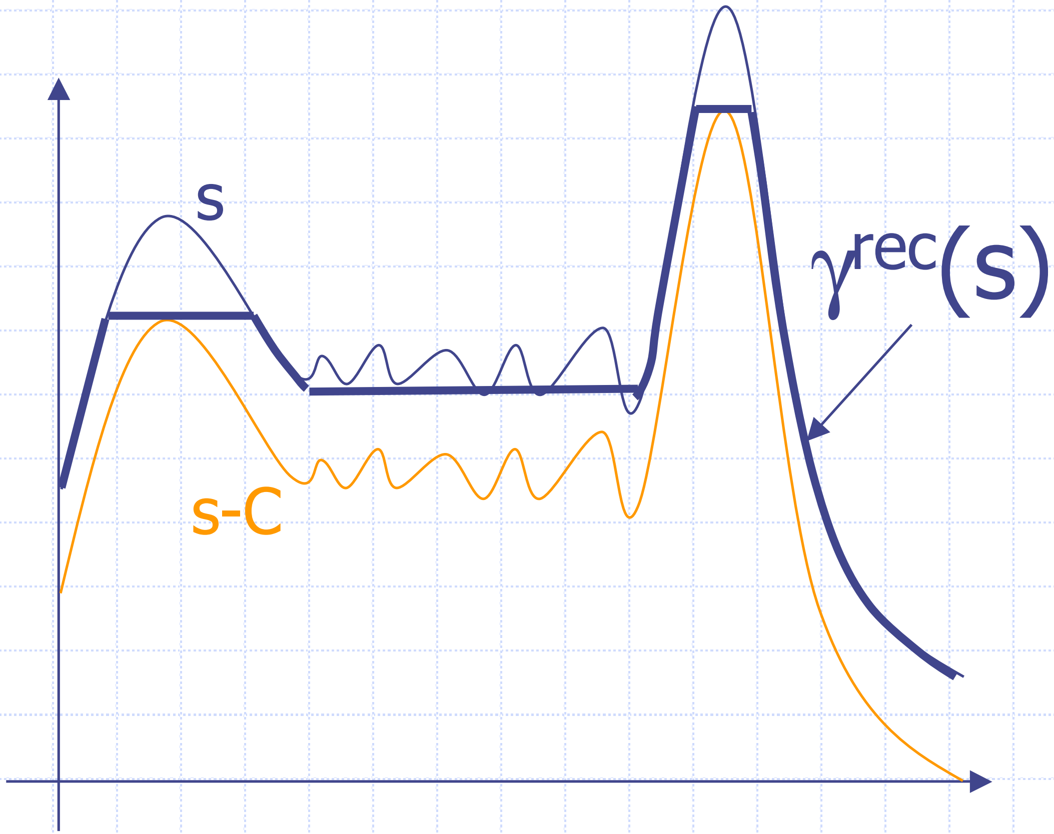
$$\gamma^{rec}(s) = \Delta_S^{\infty}(\varepsilon_B(s))$$

$$\phi^{rec}(s) = E_S^{\infty}(\delta_B(s))$$

seuil

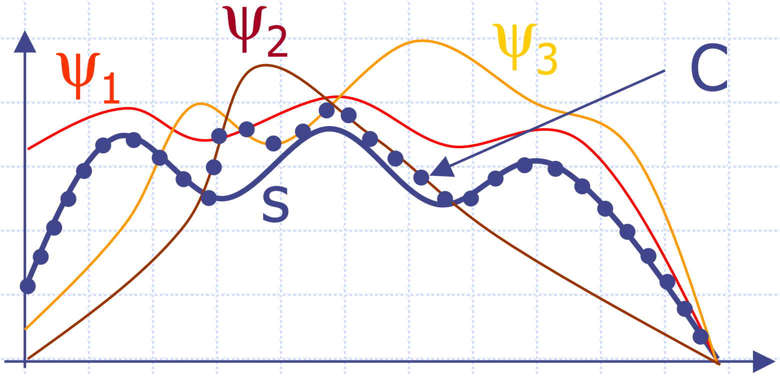
Ouverture de contraste

$$\gamma^{rec}(s) = \Delta^{\infty}_{s}(s-C)$$



- élimine les composantes de faible contraste
- s- $\gamma^{rec}(s)$ isole les maxima relatifs

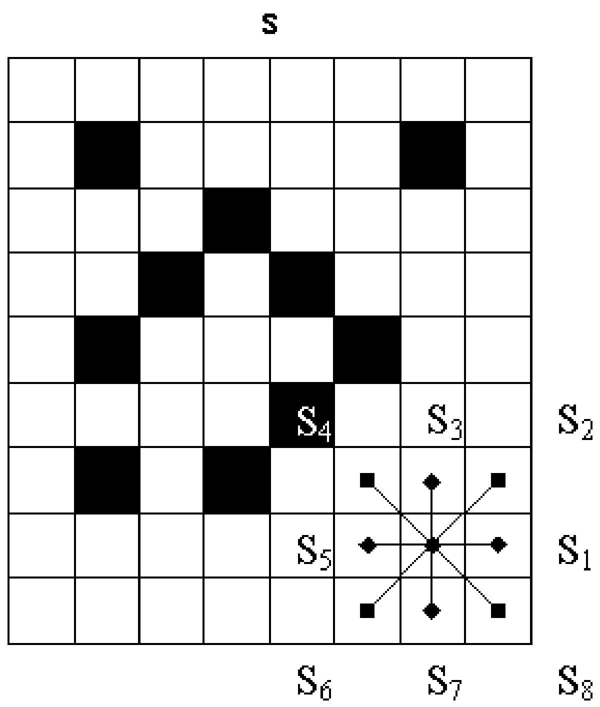
Centres morphologiques

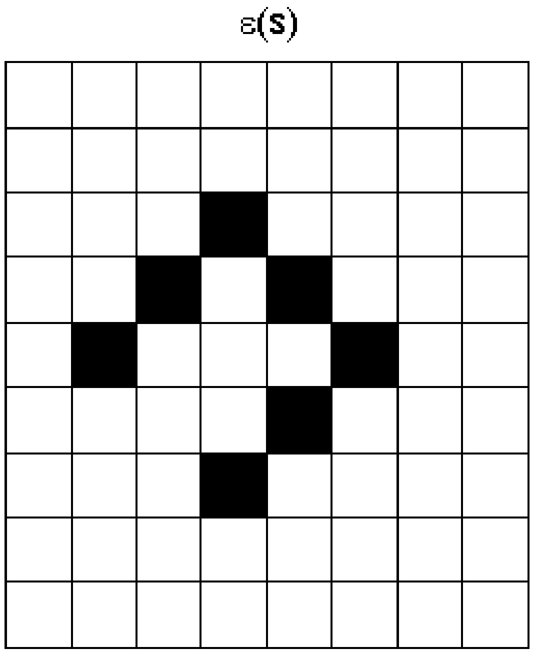


- Famille de filtres $\{\psi_k\}$
- $s(i,j) \leftarrow \wedge_k\ \psi_k(i,j)$ si $\forall k,\ \psi_k(i,j) \geq s(i,j)$
- $s(i,j) \leftarrow \vee_k\ \psi_k(i,j)$ si $\forall k,\ \psi_k(i,j) \leq s(i,j)$
- sinon s(i,j) est inchangée

$$\boxed{C = (\vee\psi_k) \wedge [\ I \vee (\wedge\psi_k)\ ] = (\wedge\psi_k) \vee [\ I \wedge (\vee\psi_k)\ ]}$$

Sup et Inf d'opérateurs

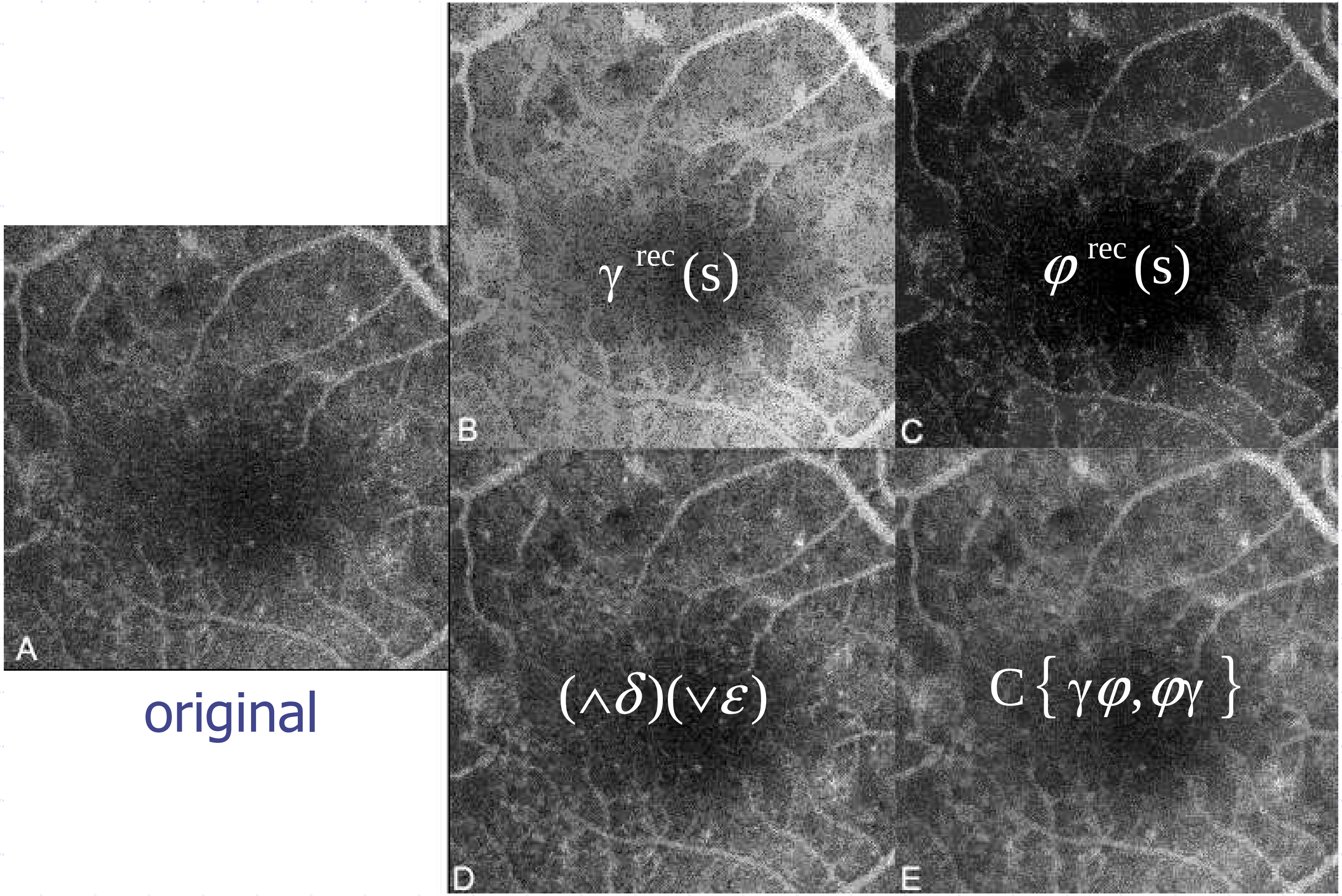




$$\varepsilon(s)(i,j) = \vee_k (\varepsilon_{Sk})(s)(i,j)$$

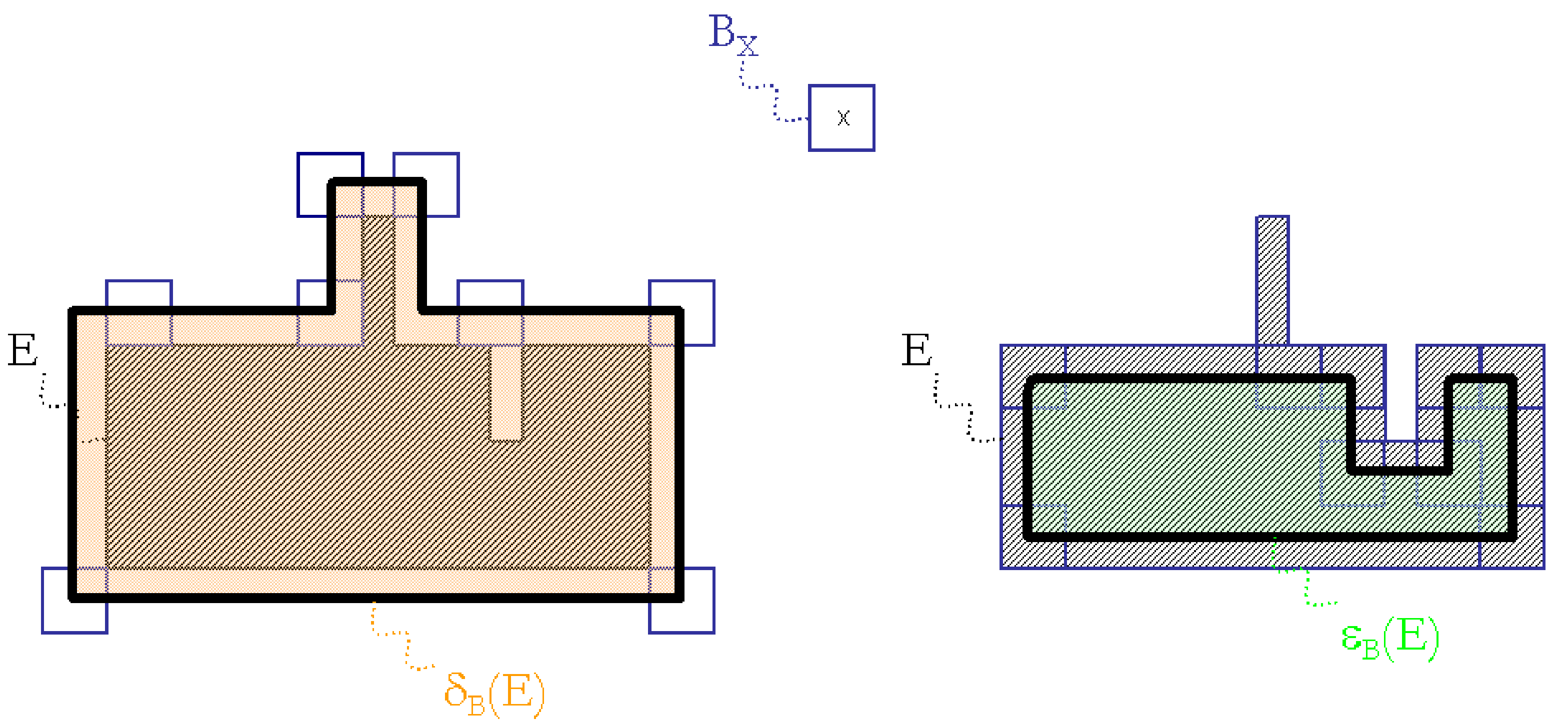
$$\delta(s)(i,j) = \wedge_k (\delta_{Sk})(s)(i,j)$$

Exemples



original

Gradients morphologiques



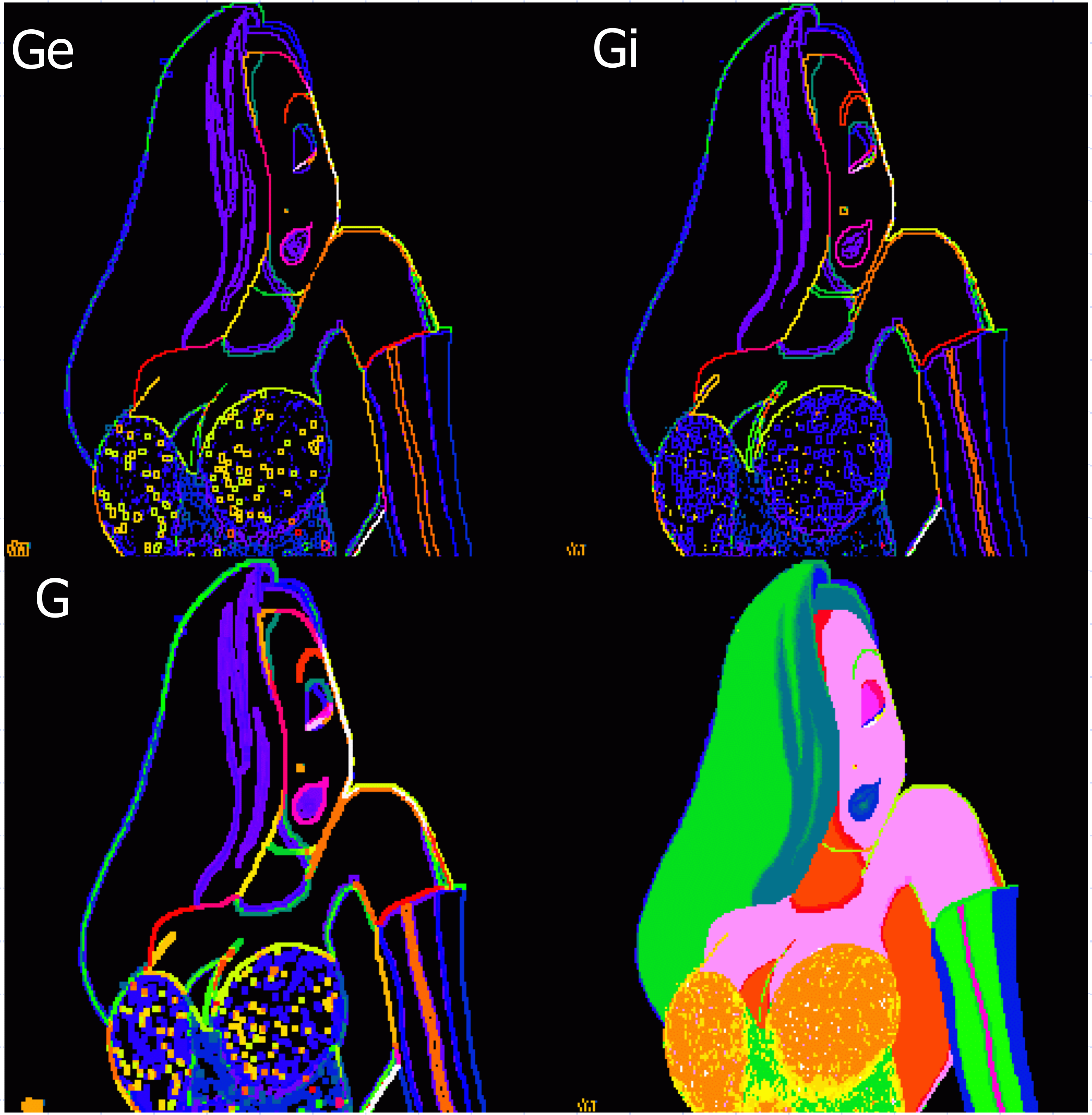
$$G_B^e(f) = \delta_B(f) - f$$

$$G_B^i(f) = f - \varepsilon_B(f)$$

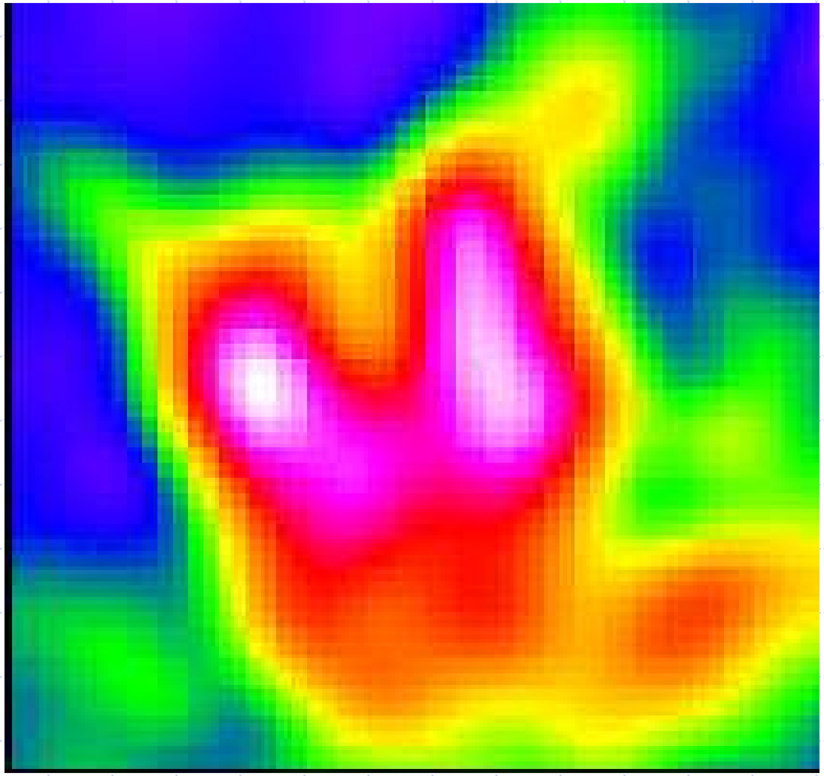
$$G_B^e(f) = \delta_B(f) - \varepsilon_B(f)$$

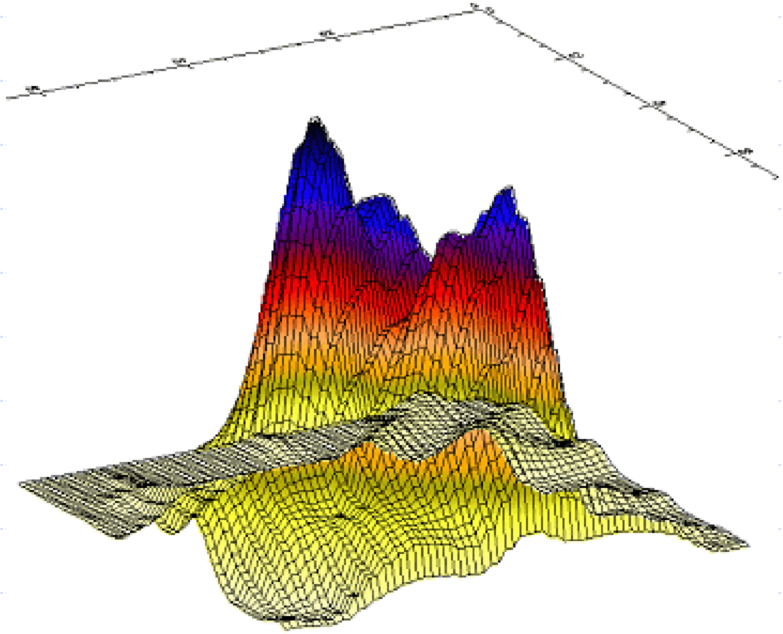
⑦ Segmentation morphologique

Gradients morpho-logiques

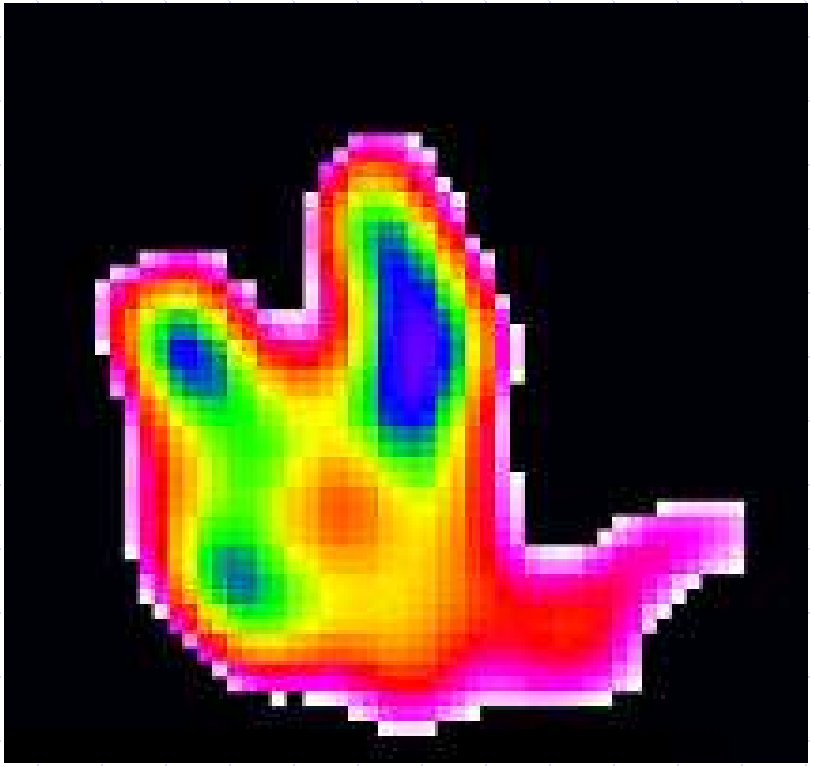


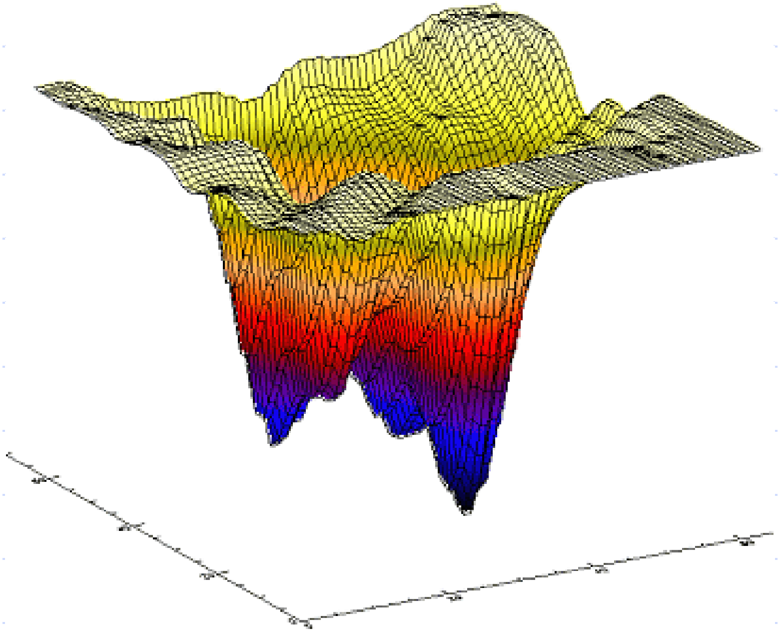
Ligne de partage des eaux





Ligne de partage des eaux

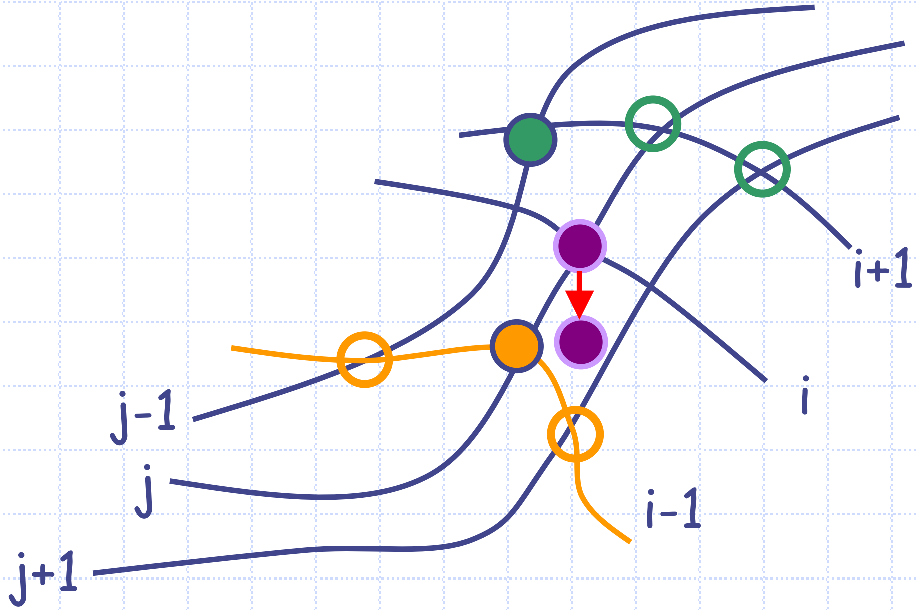




Amincissement homotopique

$$\left(f \circ \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \right)(i,j) = f_{\max} \quad \text{si} \quad f_{\max} < f(i,j) \le f_{\min}$$

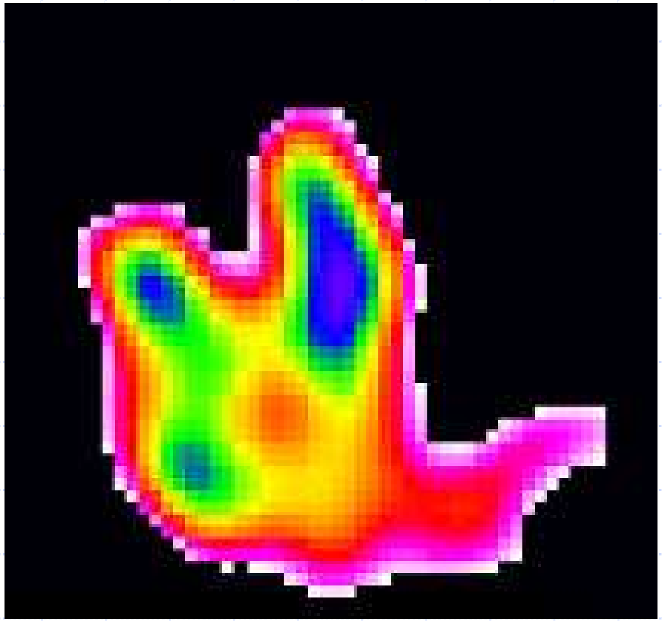
$$L = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

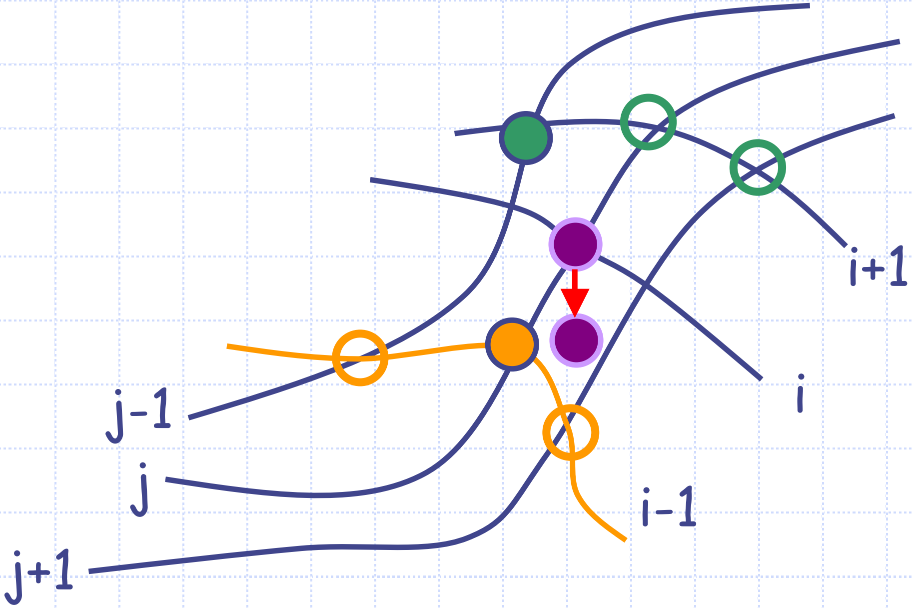


(Mariano-Goulart et al.EJNM 1998; 22:1300-07 et Revue Acomen 2000;6:69-77)

Amincissement homotopique

$$\left(f \circ \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}\right)(i,j) = f_{max} \quad \text{si} \quad f_{max} < f(i,j) \leq f_{min}$$

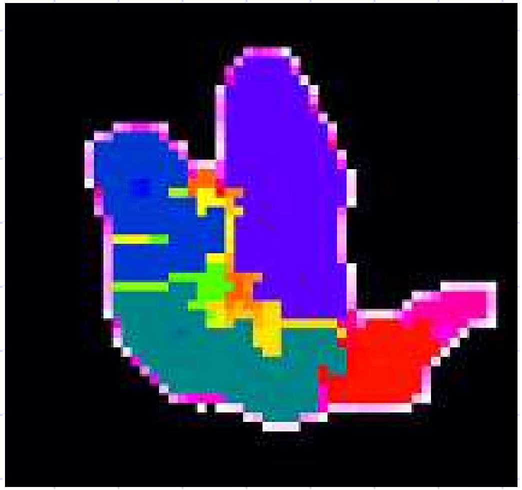




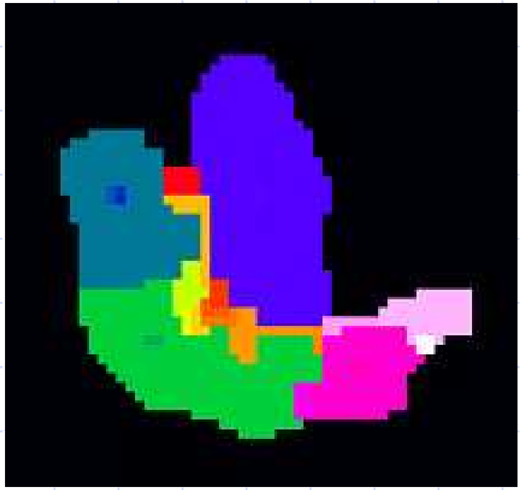
(Mariano-Goulart et al.EJNM 1998; 22:1300-07 et Revue Acomen 2000;6:69-77)

Ebarbulage par amincissement

$$Sq = (f \circ L_i)^{\infty}$$

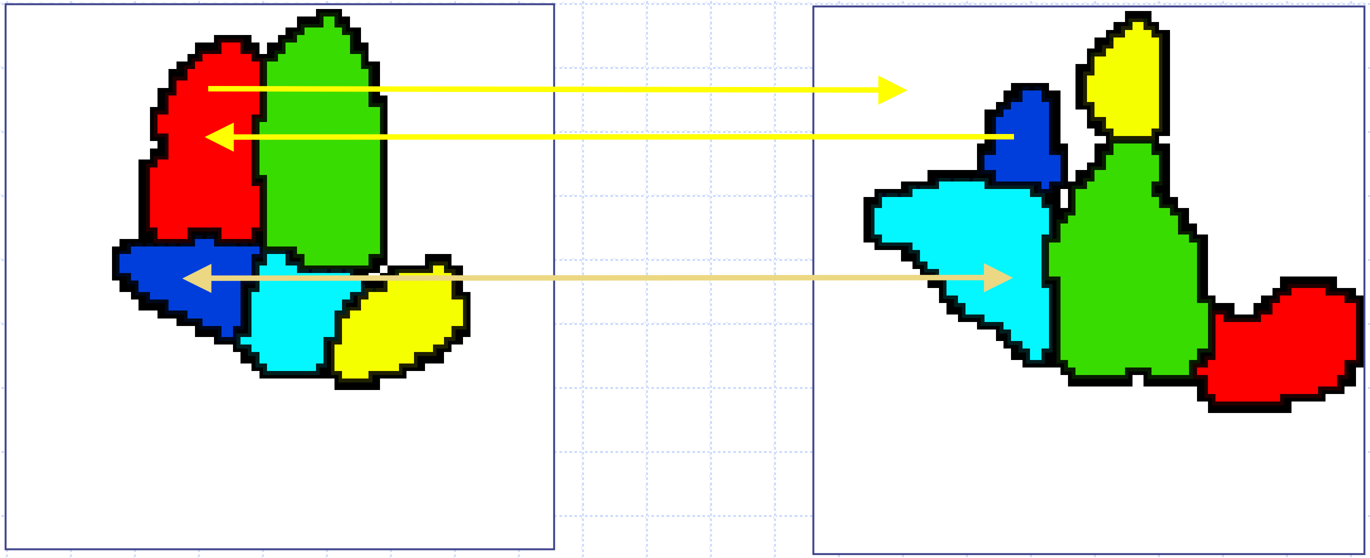


$$LPE = \left(f \circ \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_i \right)^{\infty}$$



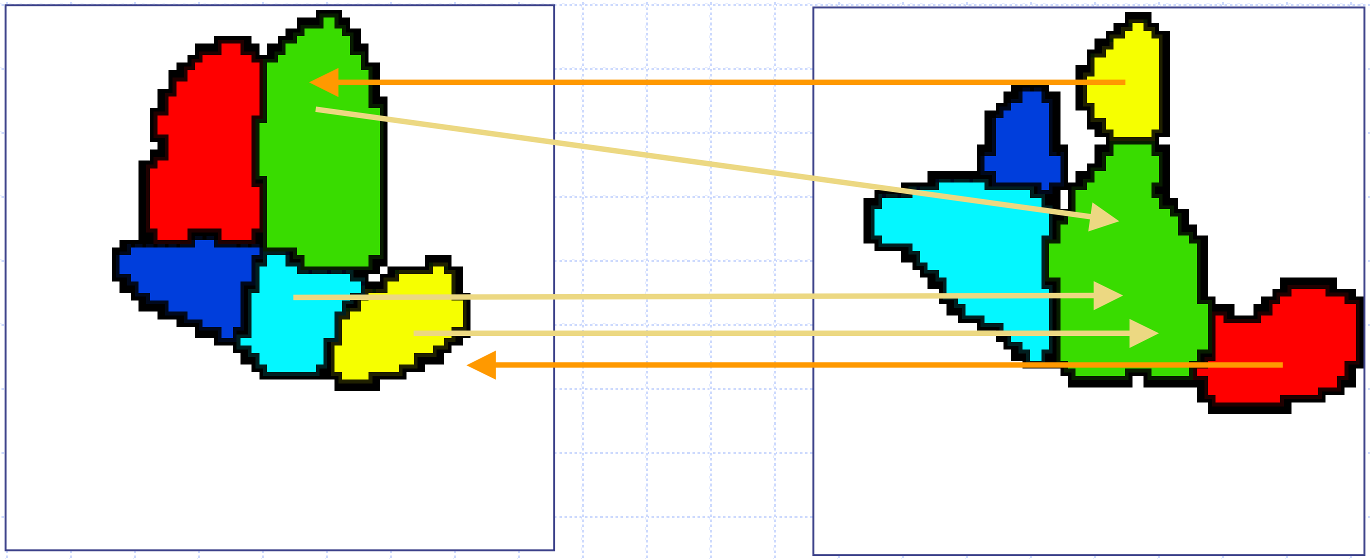
(Mariano-Goulart et al.EJNM 1998; 22:1300-07 et Revue Acomen 2000;6:69-77)

Appariements des ROIs



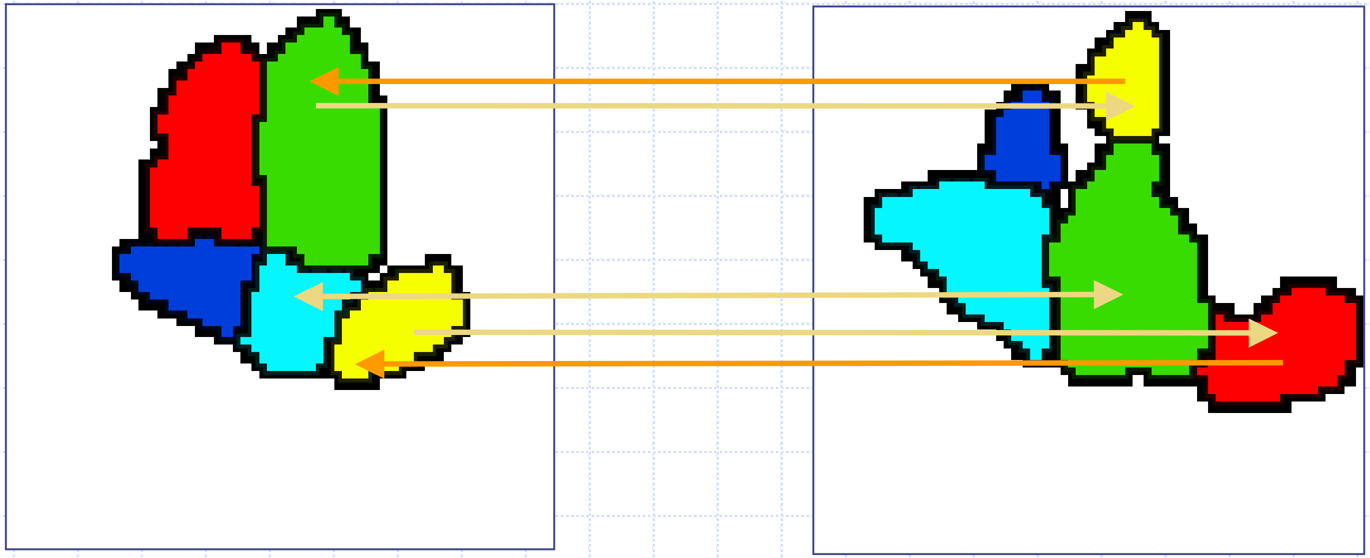
(Mariano-Goulart et al.EJNM 1998; 22:1300-07 et Revue Acomen 2000;6:69-77)

Appariements des ROIs



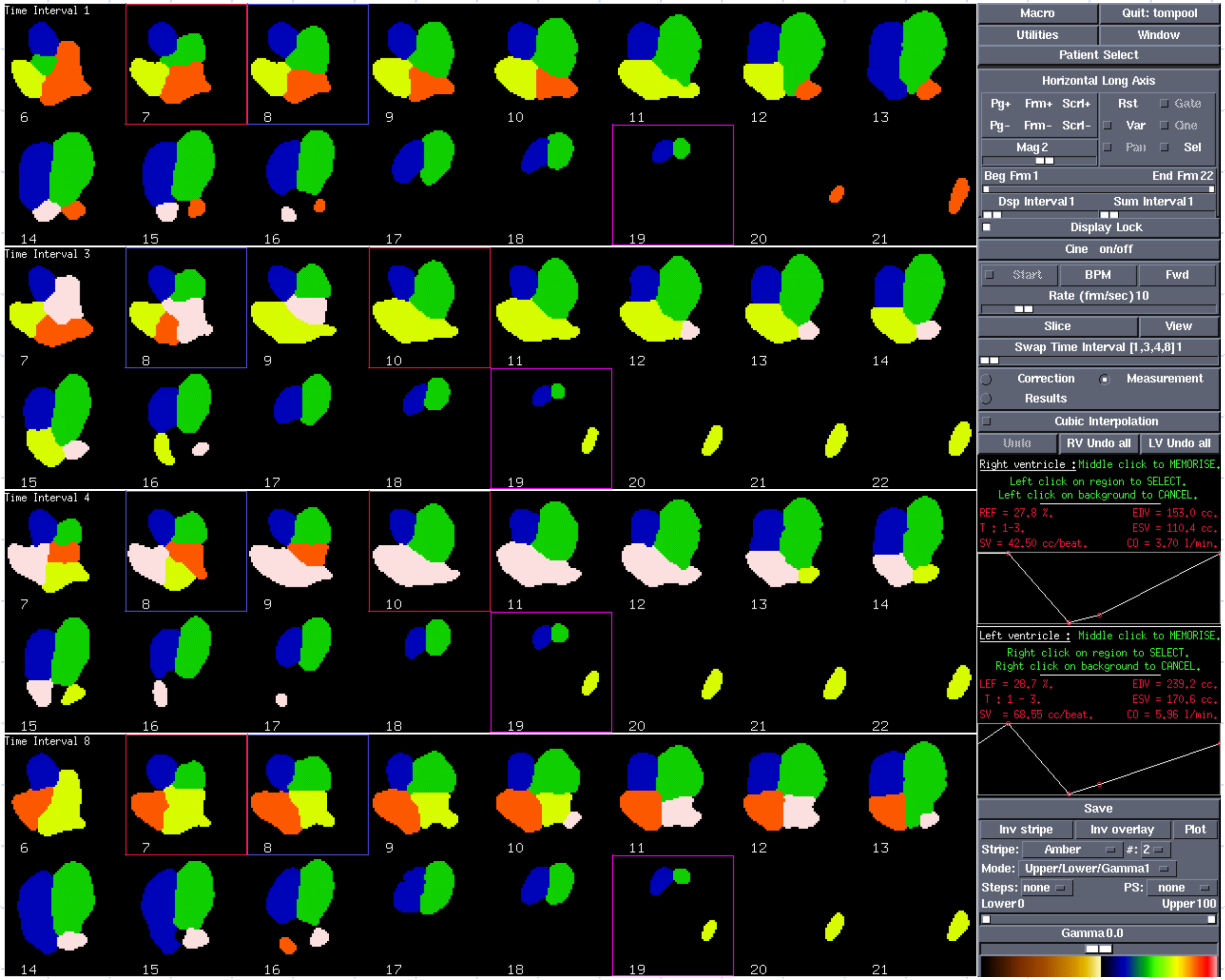
(Mariano-Goulart et al.EJNM 1998; 22:1300-07 et Revue Acomen 2000;6:69-77)

Appariements des ROIs



(Mariano-Goulart et al.EJNM 1998; 22:1300-07 et Revue Acomen 2000;6:69-77)

Résultats



(Mariano-Goulart et al.EJNM 1998;22 et EJNM 2001;28- Daou et al. JNM 2001;42)

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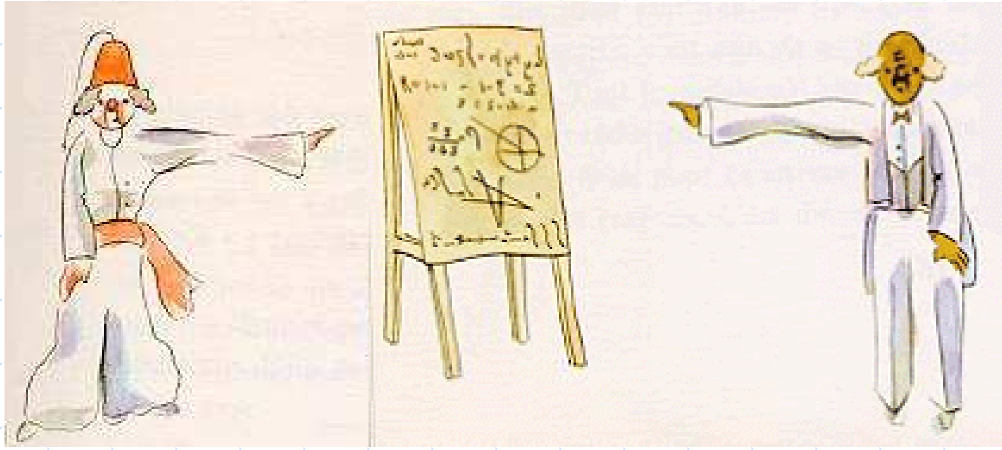

Merci de votre attention…

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