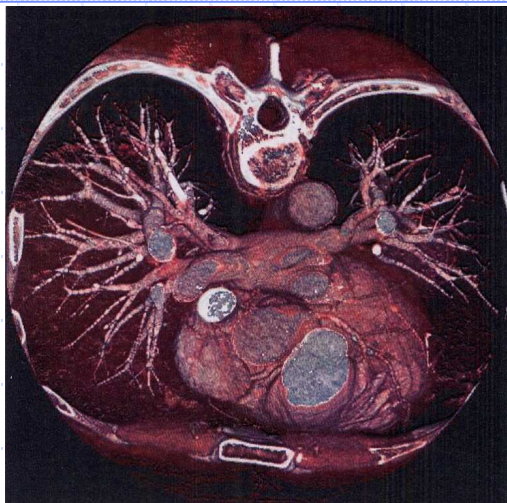


FORMATION TIC (Phymed, STIC, Télécom) BASES DE TOMOGRAPHIE MEDICALE

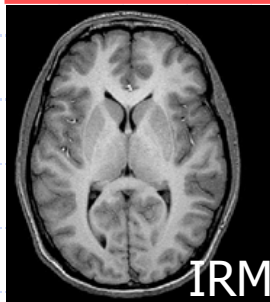
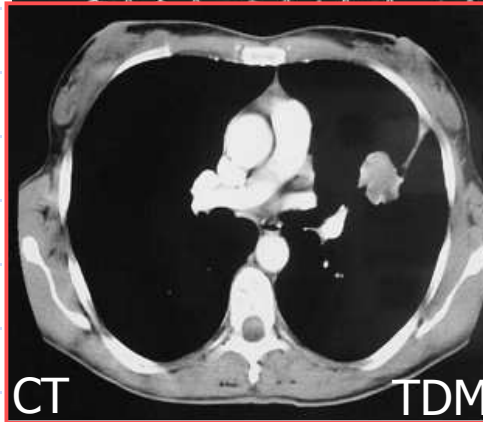


Denis Mariano-Goulart

Faculté de médecine & CHRU de Montpellier

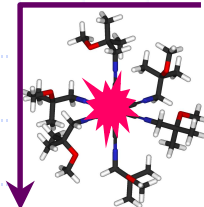
<http://scinti.etud.univ-montp1.fr>

Imagerie médicale

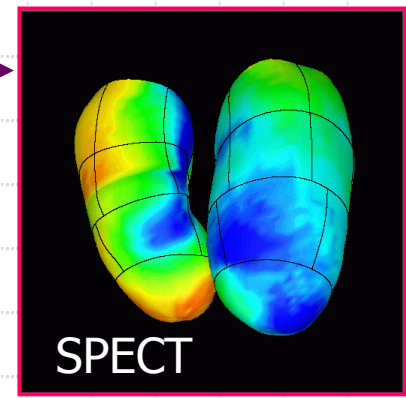
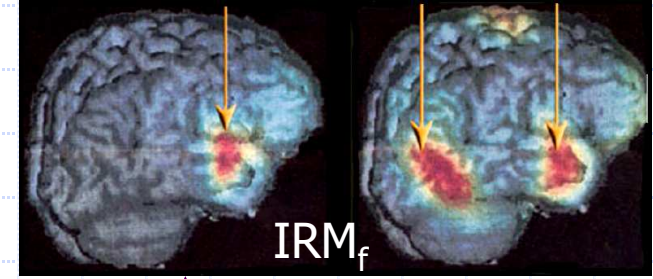
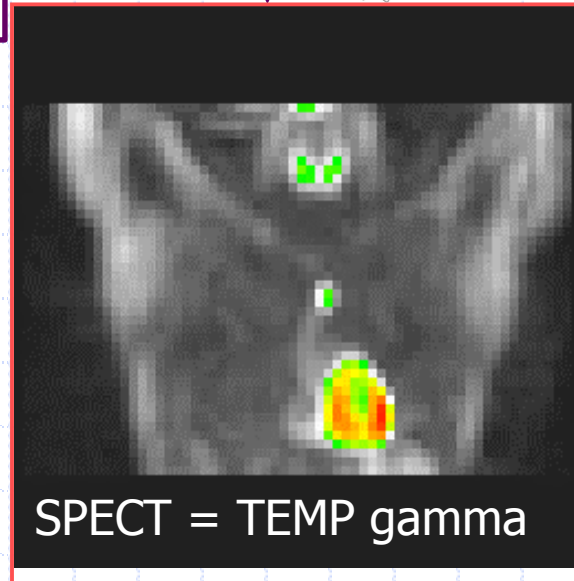


ANATOMIQUE

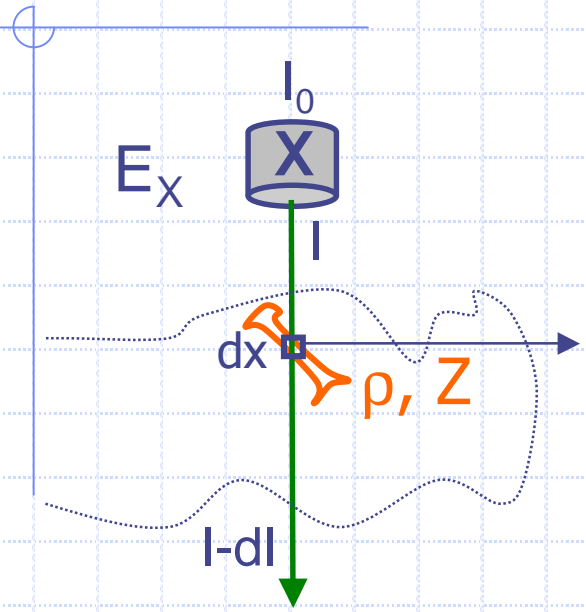
METABOLIQUE



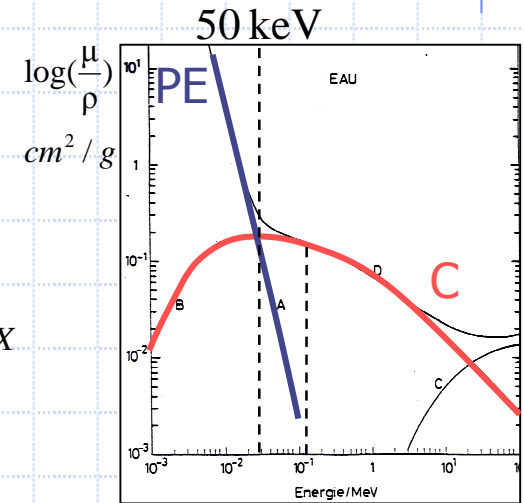
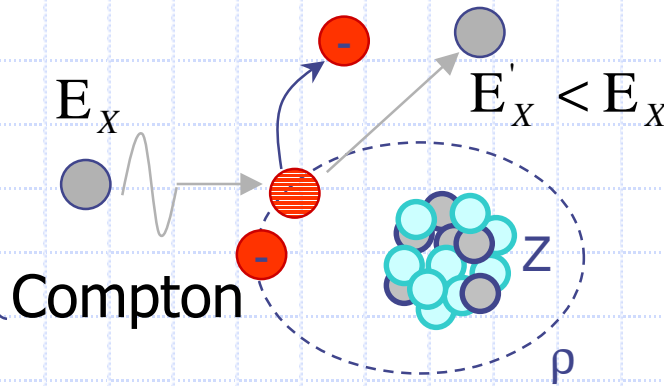
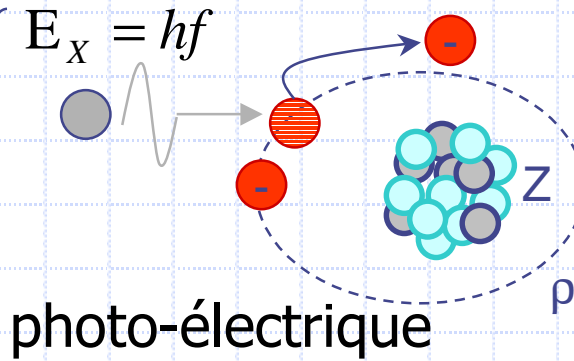
FONCTIONNELLE



IMAGERIE DE TRANSMISSION X

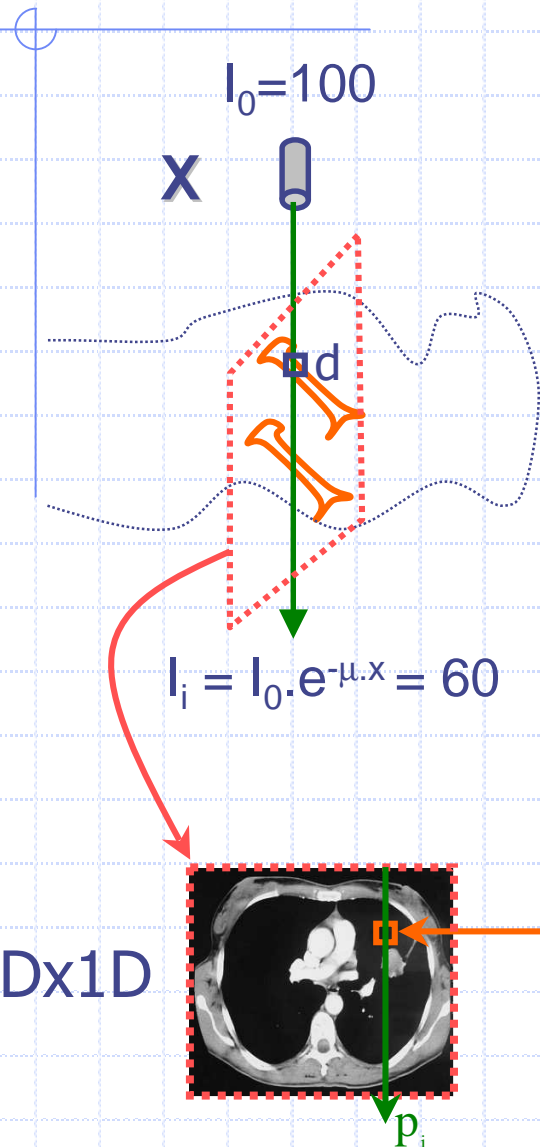


μ probabilité d'atténuation/cm
 μ est proportionnelle
 à la masse volumique ρ



$$\mu = -\frac{dI}{I dx} \text{ cm}^{-1} \propto \rho \Rightarrow I = I_0 \cdot e^{-\mu \cdot x}$$

Scanner X = Computed Tomography



$$I_i = I_0 e^{-\sum \mu_j \cdot d} = I_0 e^{-d \cdot \sum \mu_j}$$

$$\Rightarrow p_i = \sum \mu_j = -\frac{1}{d} \ln \frac{I_i}{I_0} \text{ mesure}$$

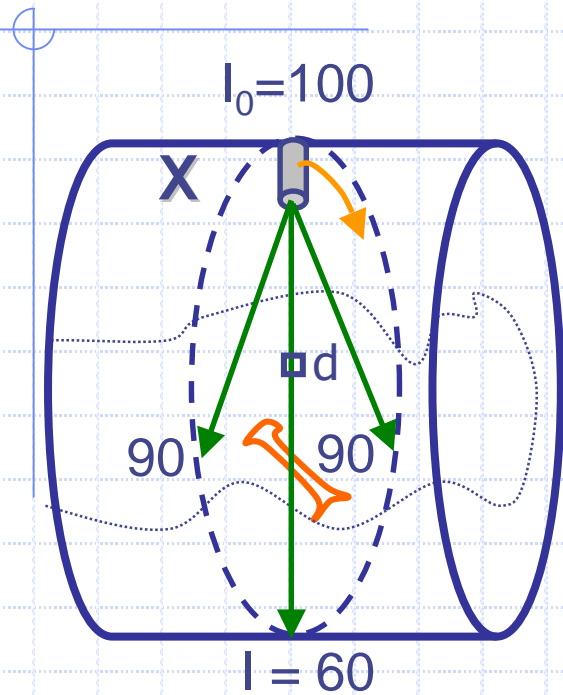
$r_{i,j}$ = contribution du pixel j à la projection i
paramètres géométriques connus du scanner

$$p_i = \sum r_{i,j} \cdot \mu_j ?$$

mesure

2Dx1D

Scanner X = Computed Tomography



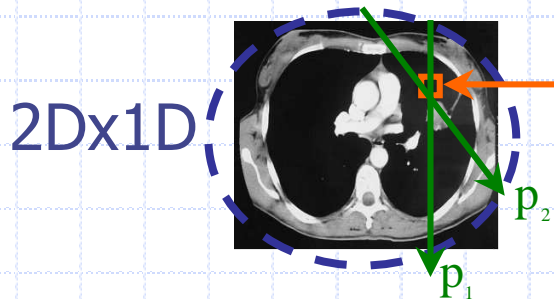
$$I_i = I_0 e^{-\sum \mu_j \cdot d} = I_0 e^{-d \cdot \sum \mu_j}$$

$$\Rightarrow p_i = \sum \mu_j = -\frac{1}{d} \ln \frac{I_i}{I_0}$$

$r_{i,j}$ = contribution du pixel j à la projection i
 paramètres géométriques connus du scanner

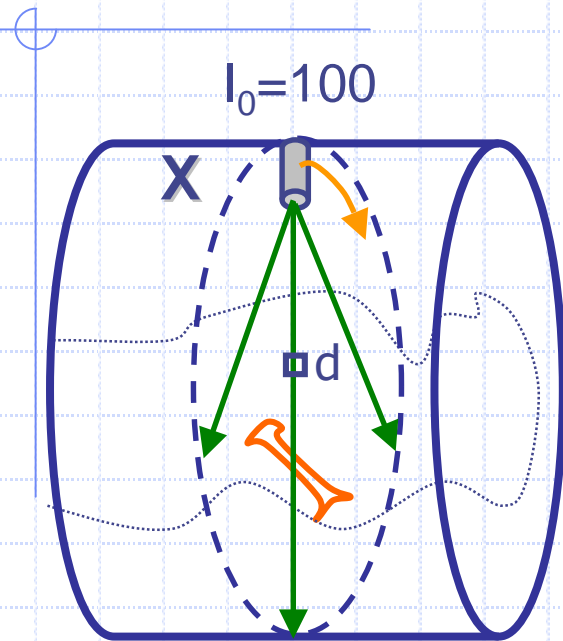
$$p_i = \sum r_{i,j} \cdot \mu_j \quad ?$$

mesure

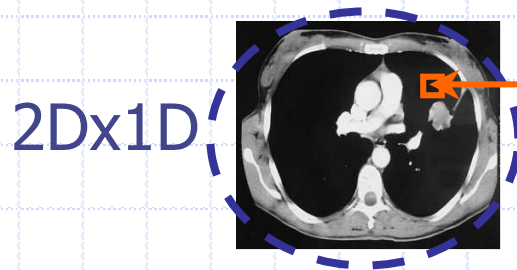


$$\begin{cases} p_1 = r_{1,1} \mu_1 + r_{1,2} \mu_2 + \dots + r_{1,n} \mu_n \\ p_2 = r_{2,1} \mu_1 + r_{2,2} \mu_2 + \dots + r_{2,n} \mu_n \\ \dots \end{cases}$$

Scanner X = Computed Tomography

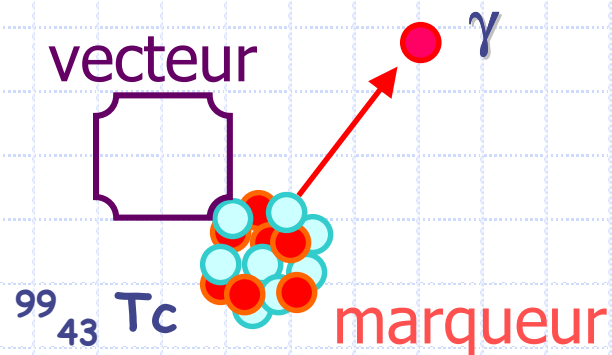
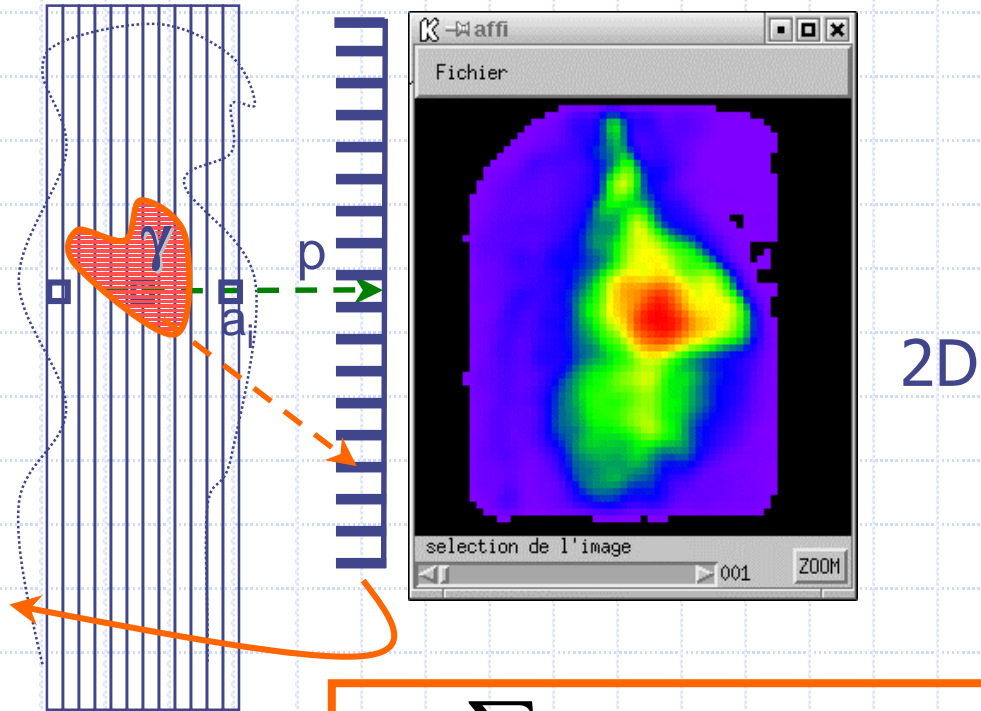


$$p_i = \sum_j r_{i,j} \mu_j, \quad i = 0 - 360^\circ$$

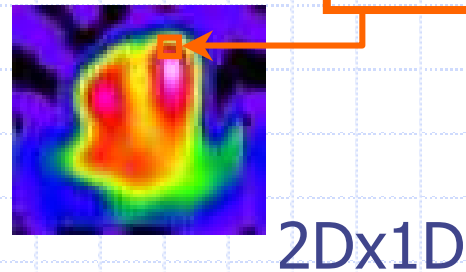


$r_{i,j}$ = contribution du pixel j à la projection i

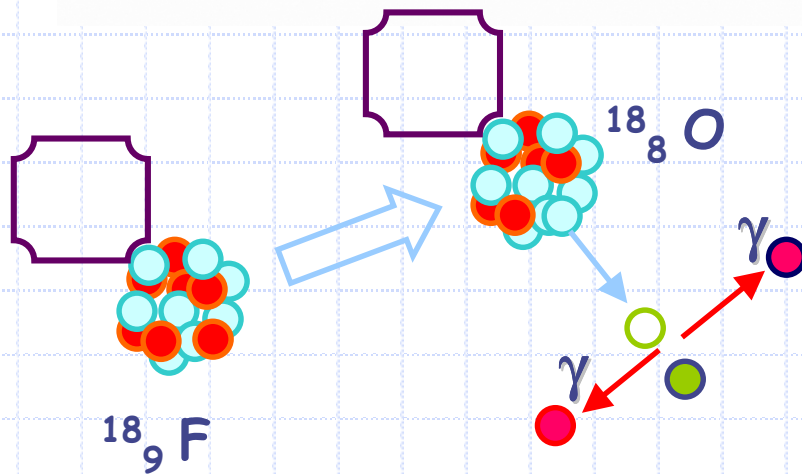
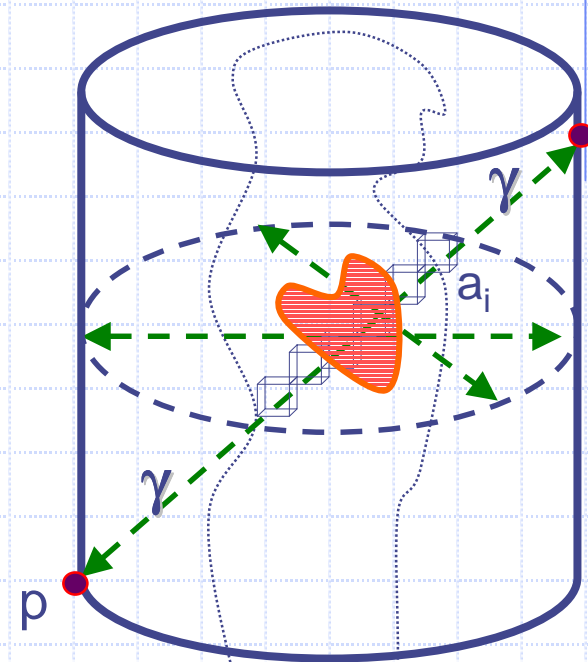
Single Photon Emission CT



$$p_i = \sum_j r_{i,j} a_j, \quad i = 0-360^\circ$$



Tomographie par Emission de Positons

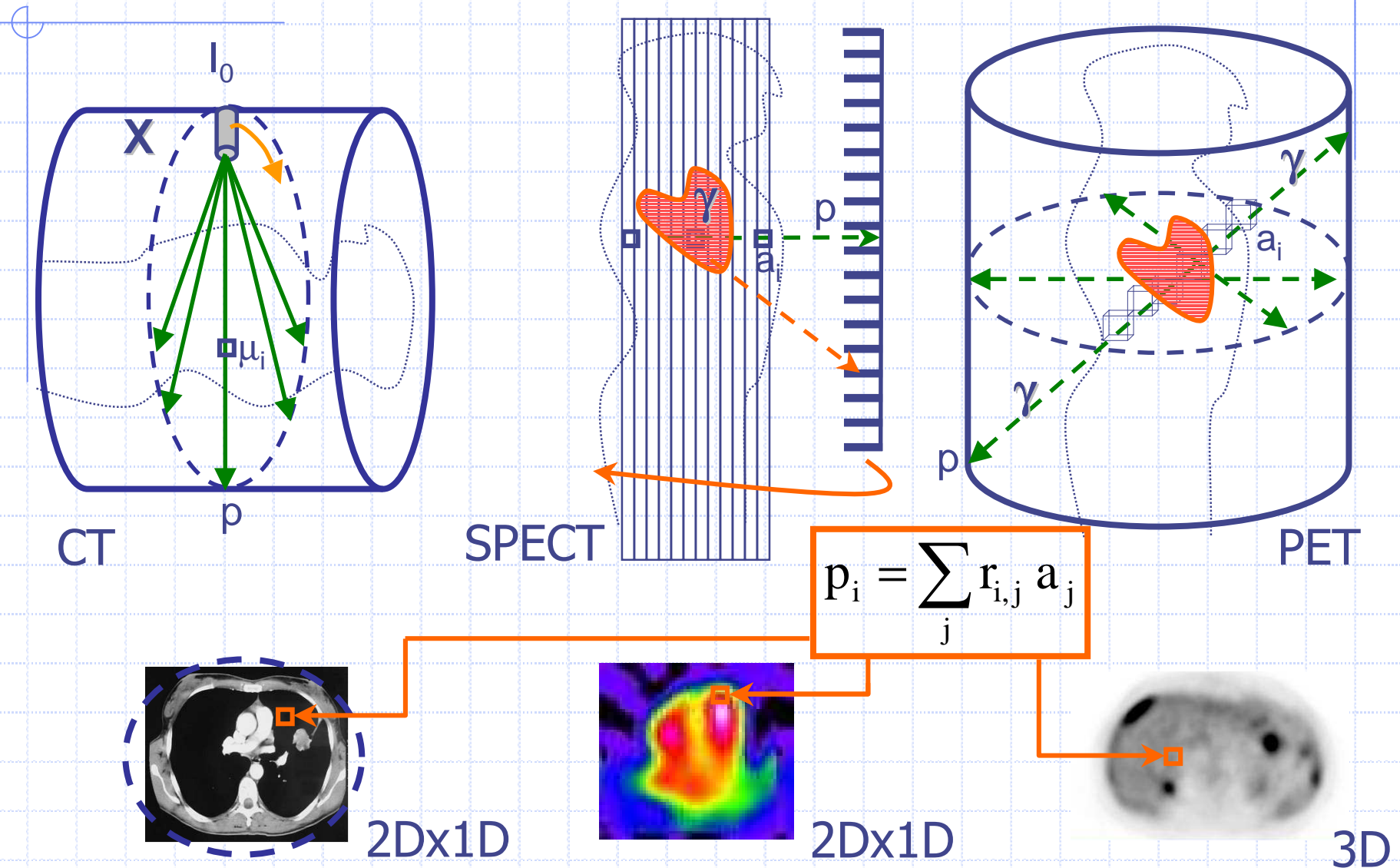


$$p_i = \sum_j r_{i,j} a_j$$

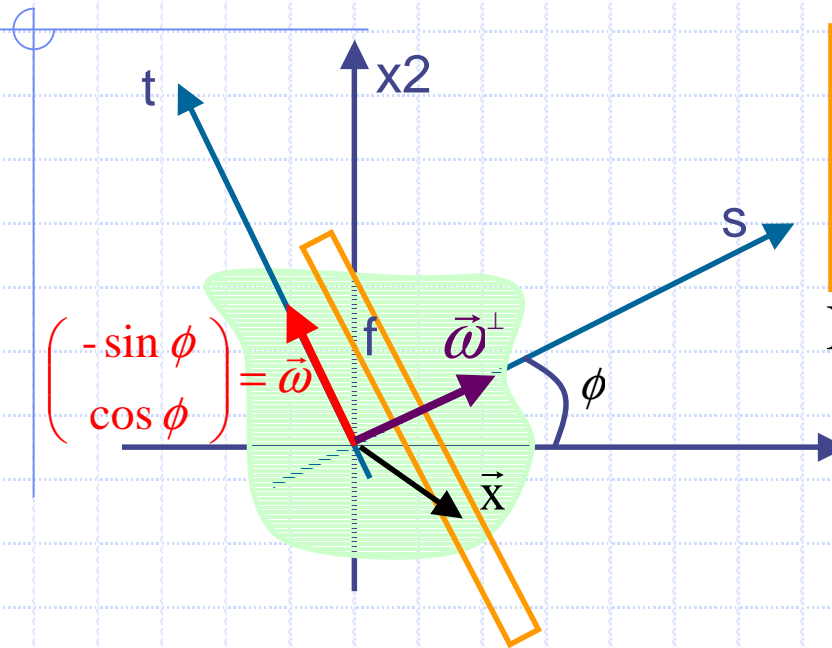


3D

Tomographie: problème inverse linéaire



Modélisation analytique



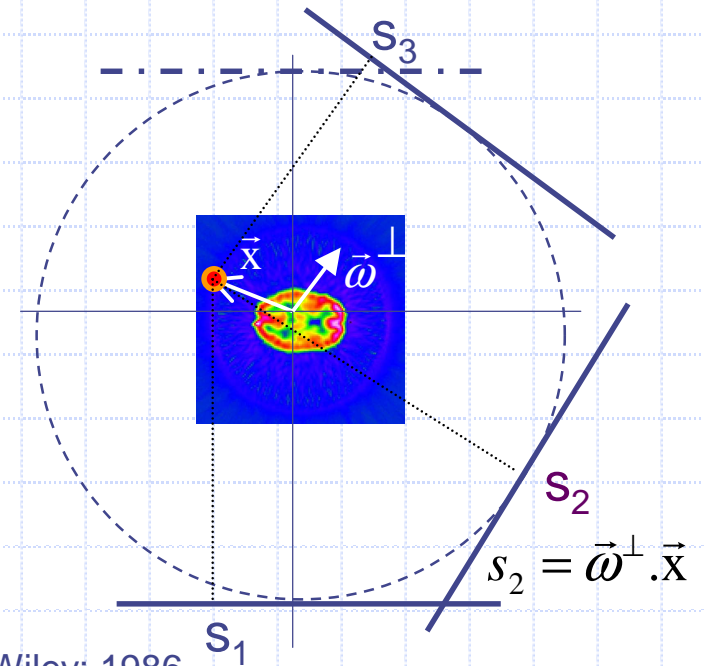
$$p(\vec{\omega}, s) = p_{\vec{\omega}}(s) = \int_t f(s \vec{\omega}^\perp + t \vec{\omega}) dt$$

$p = Rf$ transformée de Radon

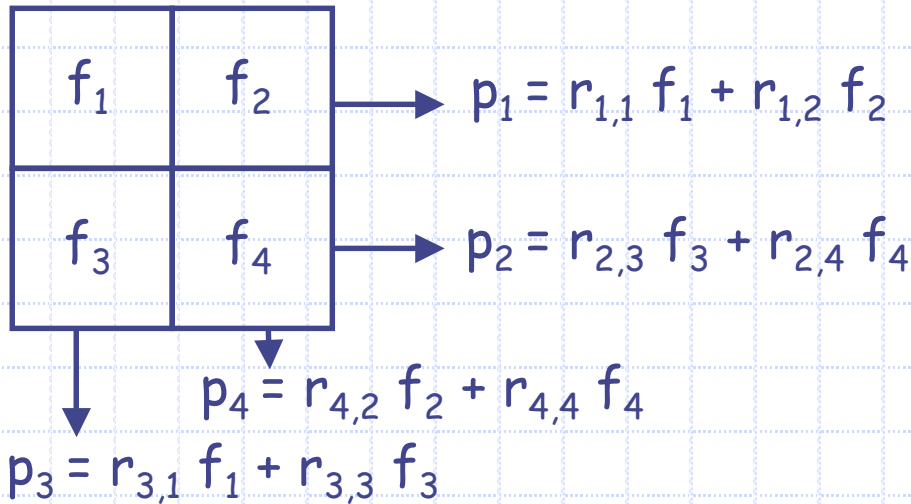
Nb : $\vec{x} \cdot \vec{\omega}^\perp = s$ où \vec{x} se projette suivant $\vec{\omega}$

$$(R^* p)(\vec{x}) = \int_{\phi=0}^{\pi} p(\vec{\omega}, \vec{\omega}^\perp \cdot \vec{x}) d\phi$$

rétroprojection = épandage



Modélisation algébrique



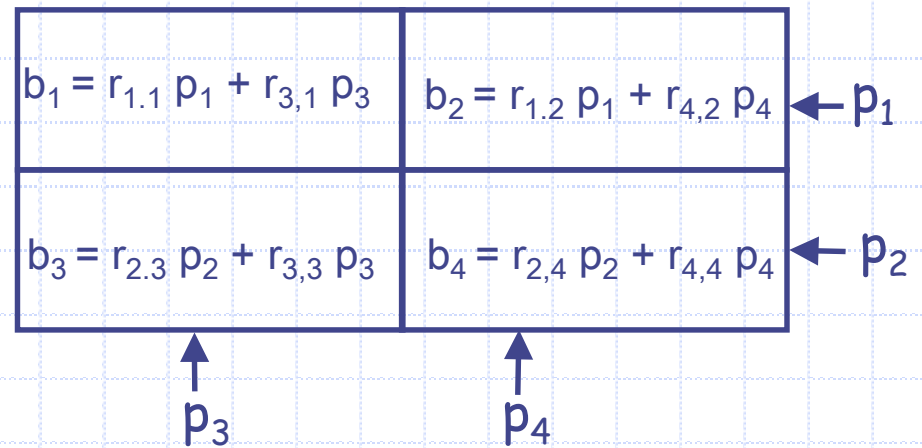
$$\begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ r_{2,1} & r_{2,2} & r_{2,3} & r_{2,4} \\ r_{3,1} & r_{3,2} & r_{3,3} & r_{3,4} \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{pmatrix} \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

$r_{i,j}$ = % du pixel j intersecté par la projection i

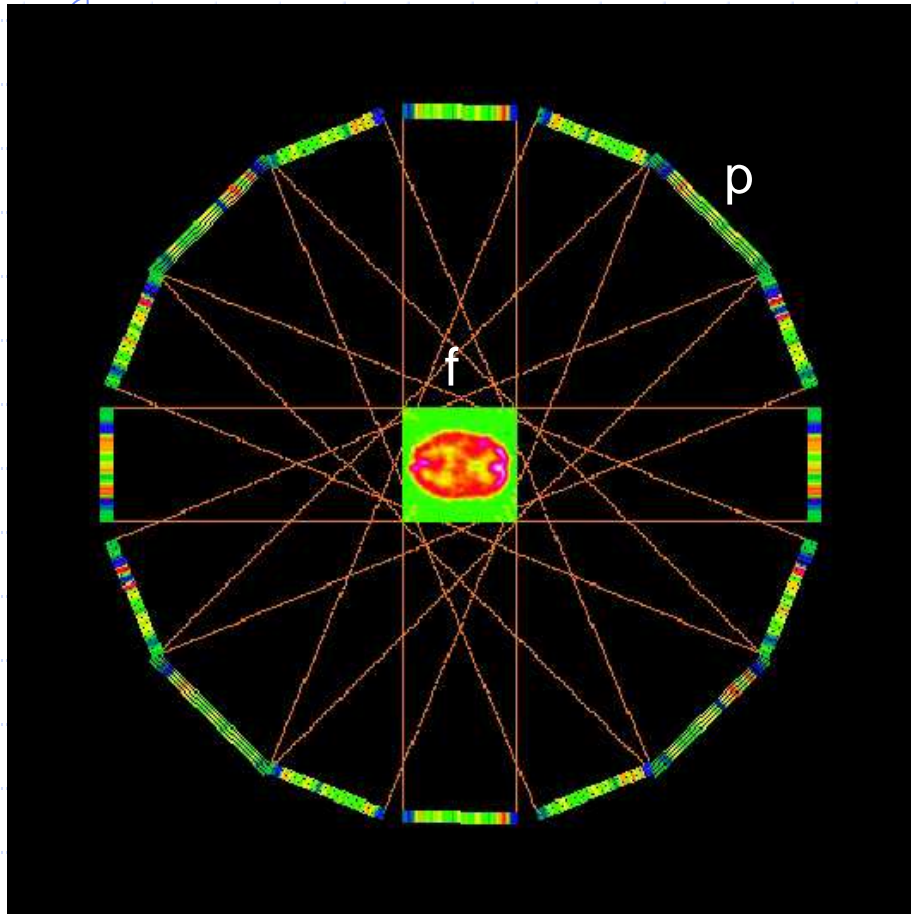
$$\mathbf{R} \cdot \vec{f} = \vec{p}$$

$$\begin{pmatrix} r_{1,1} & r_{2,1} & r_{3,1} & r_{4,1} \\ r_{1,2} & r_{2,2} & r_{3,2} & r_{4,2} \\ r_{1,3} & r_{2,3} & r_{3,3} & r_{4,3} \\ r_{1,4} & r_{2,4} & r_{3,4} & r_{4,4} \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

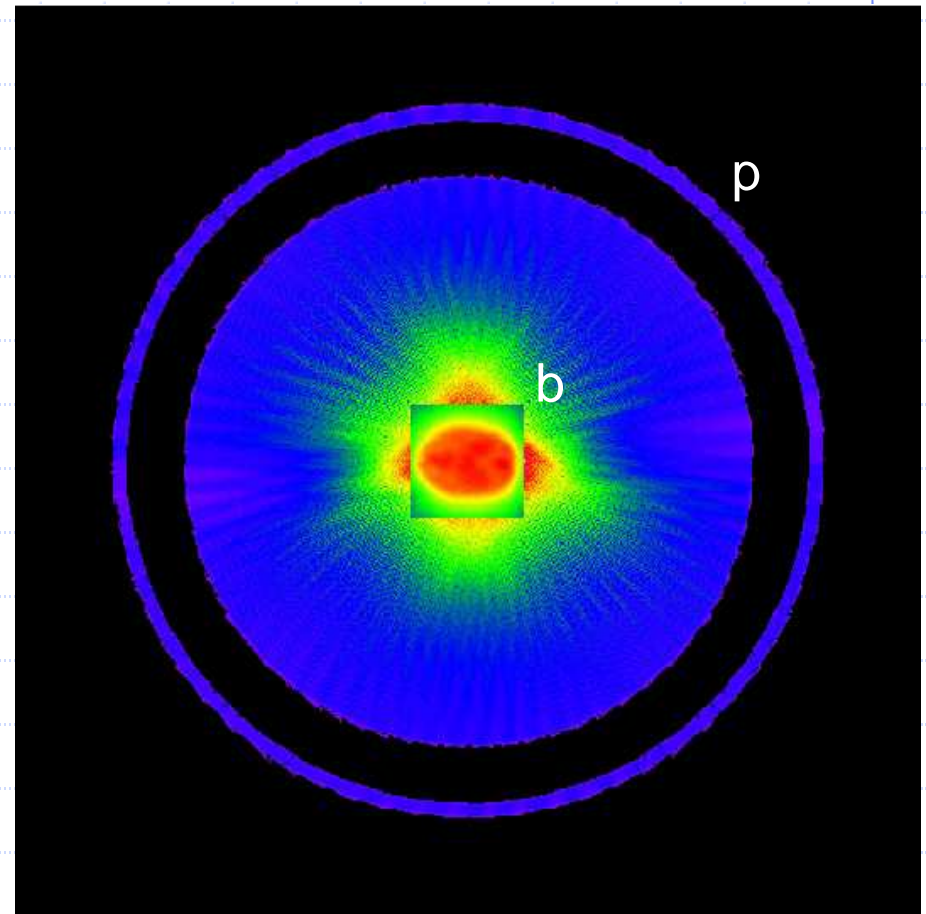
$${}^t\mathbf{R} \cdot \vec{p} = \vec{b}$$



Projection / Rétroprojection

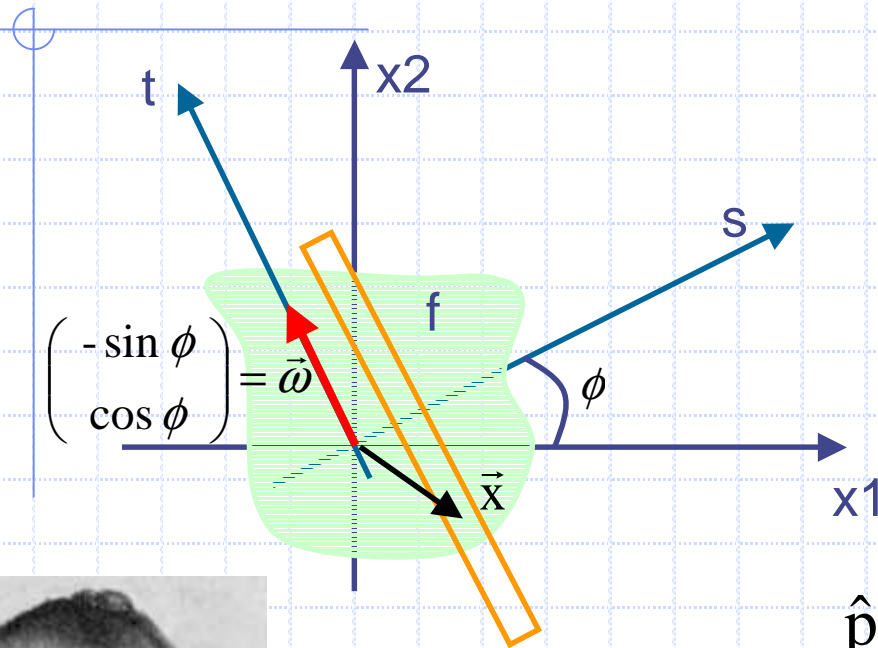


$$\mathbf{R} \cdot \vec{f} = \vec{p}$$



$${}^t\mathbf{R} \cdot \vec{p} = \vec{b}$$

Théorème de coupe centrale



$$\begin{cases} p_{\vec{\omega}}(s) = \int_t f(s\vec{\omega}^\perp + t\vec{\omega}) dt \\ \hat{p}_{\vec{\omega}}(\sigma) = \int_s p_{\vec{\omega}}(s) \cdot e^{-i \cdot s \cdot \sigma} ds \end{cases}$$

$$\hat{p}_{\vec{\omega}}(\sigma) = \int_s \int_t f(s\vec{\omega}^\perp + t\vec{\omega}) e^{-i \cdot s \cdot \sigma} dt ds$$

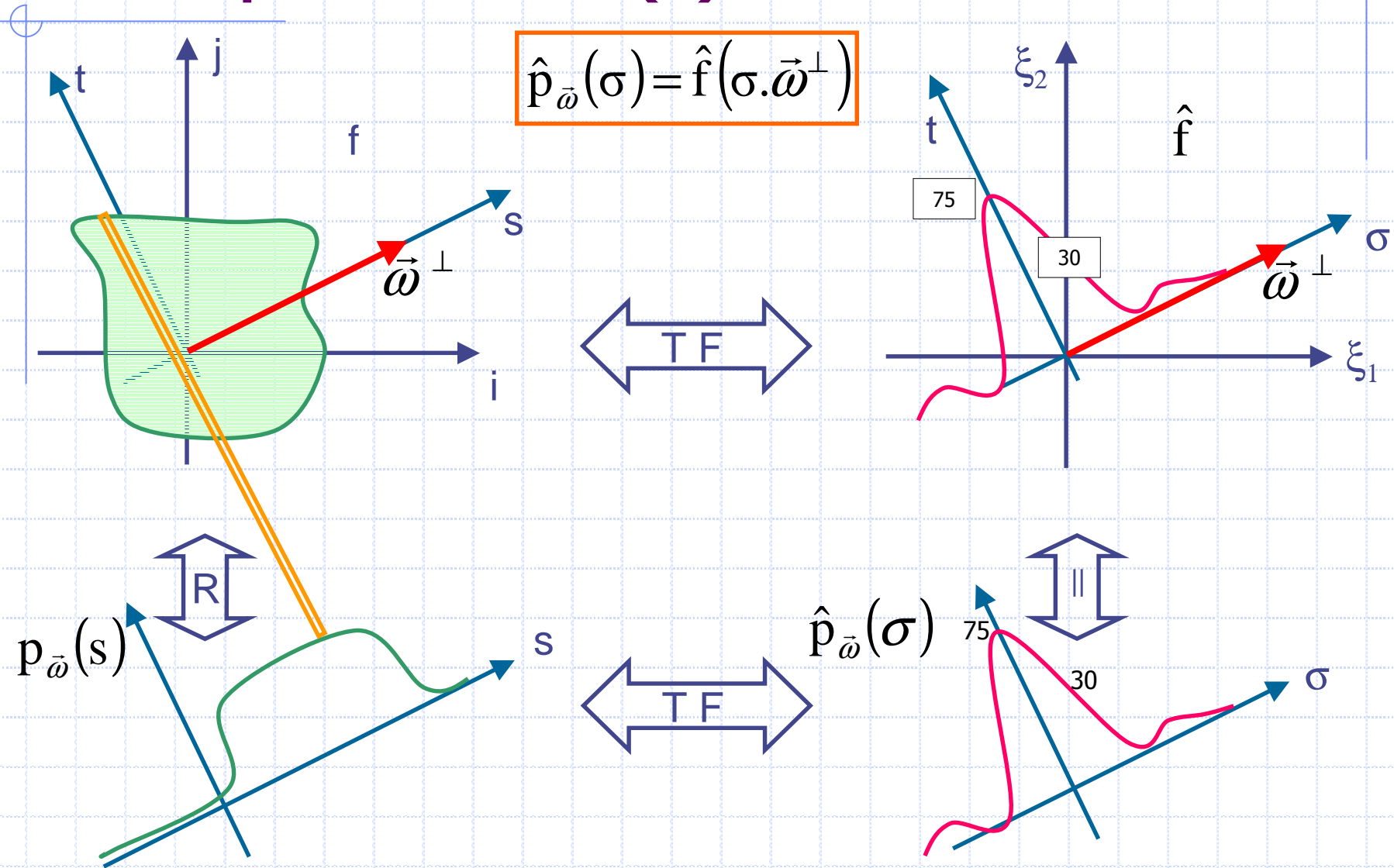
$$\hat{p}_{\vec{\omega}}(\sigma) = \iint f(\vec{x}) e^{-i \cdot \sigma \vec{x} \cdot \vec{\omega}^\perp} d\vec{x}$$

$$\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}(\sigma \cdot \cos \phi, \sigma \cdot \sin \phi) = \hat{f}(\sigma \cdot \vec{\omega}^\perp)$$

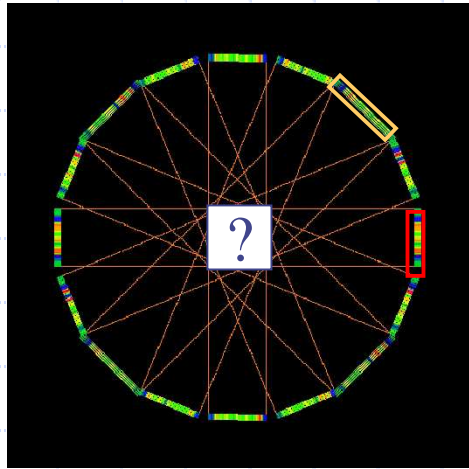


J. Radon
1887-1956

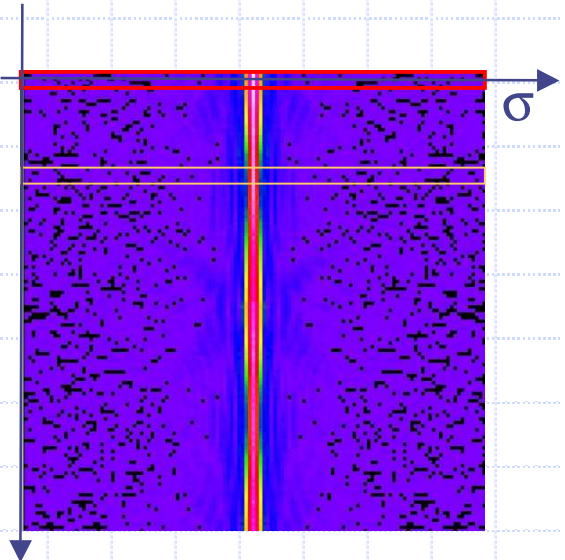
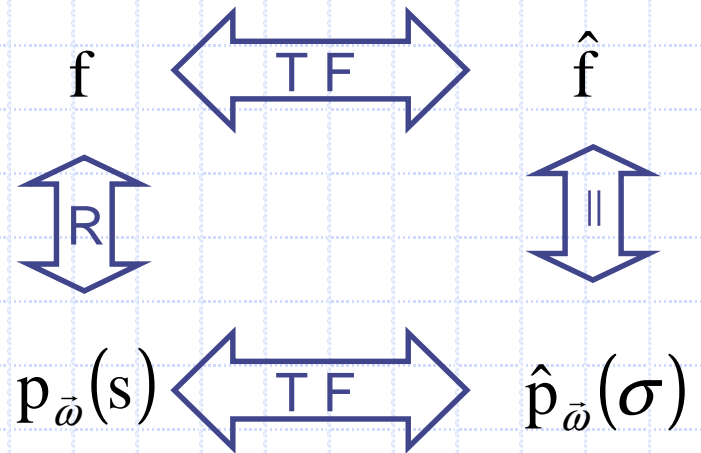
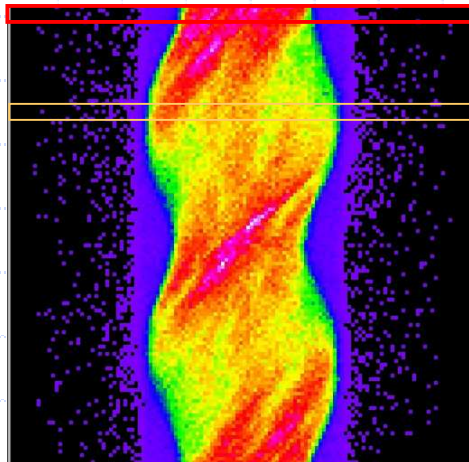
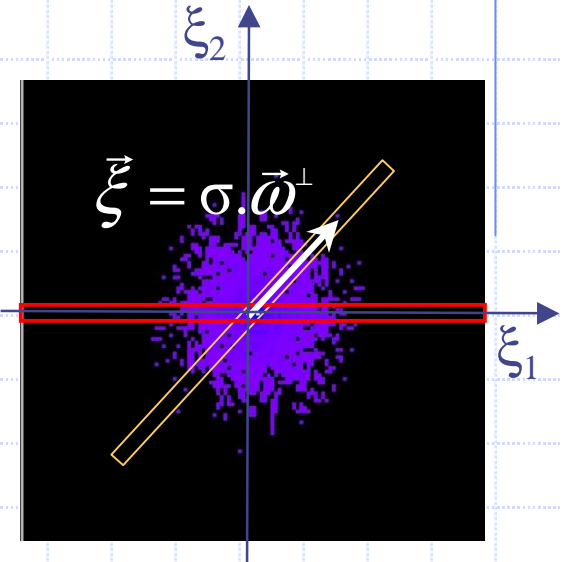
Interprétation (I)



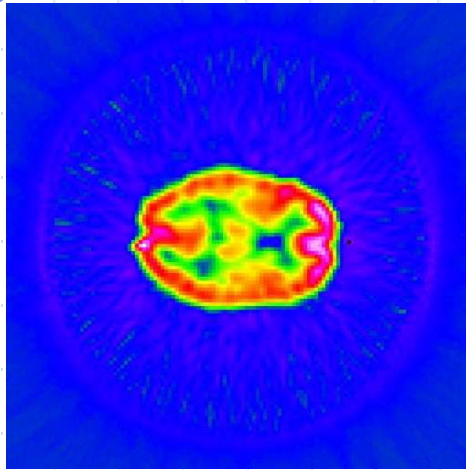
Interprétation (II)



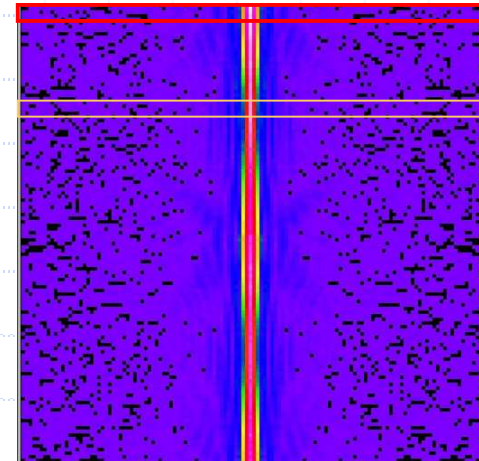
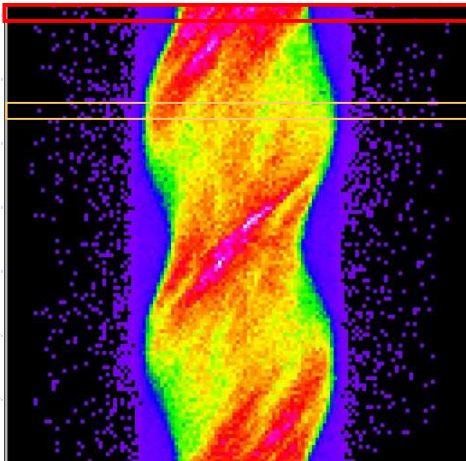
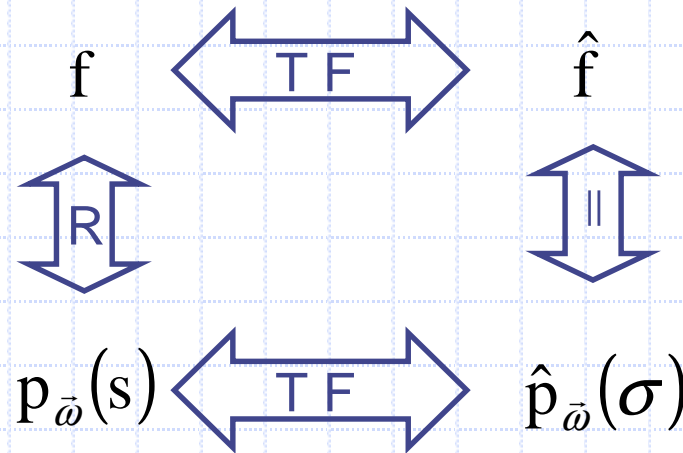
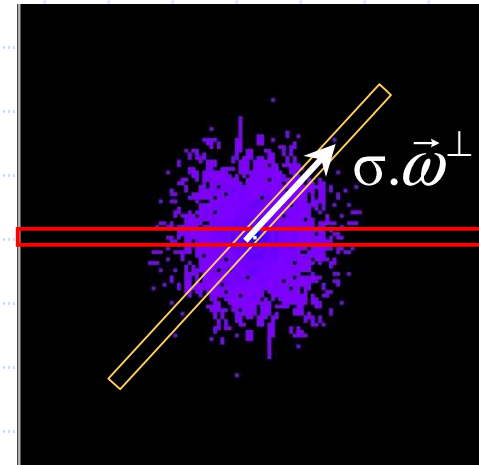
$$\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}(\sigma \cdot \vec{\omega}^\perp)$$



Interprétation (II)



$$\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}(\sigma \cdot \vec{\omega}^\perp)$$



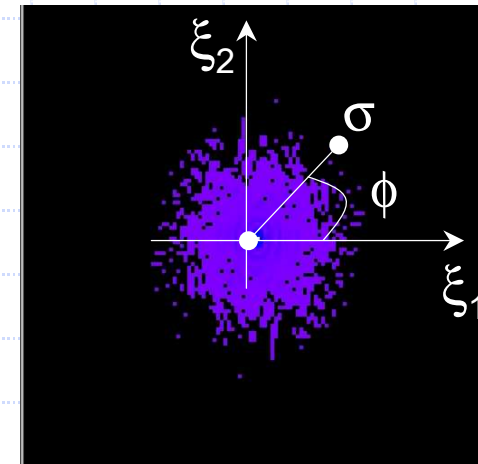
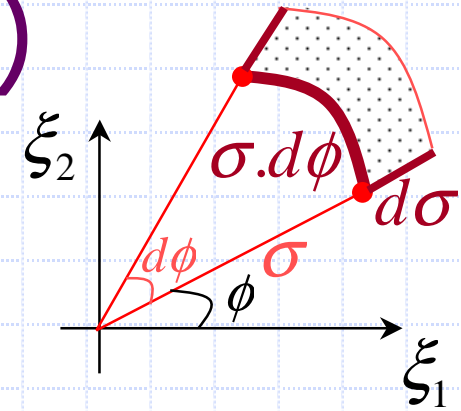
Rétroprojection filtrée (I)

$$f(\vec{x}) = \iint \widehat{f}(\vec{\xi}) e^{i\vec{x} \cdot \vec{\xi}} d\vec{\xi}$$

$$f(\vec{x}) = \int_{\phi=0}^{\pi} \int_{\sigma=-\infty}^{\sigma=+\infty} \widehat{f}(\sigma \vec{\omega}^{\perp}) e^{i\sigma \vec{\omega}^{\perp} \cdot \vec{x}} |\sigma| d\sigma d\phi$$

$$f(\vec{x}) = \int_{\phi=0}^{\pi} \int_{\sigma=-\infty}^{\sigma=+\infty} \widehat{p}_{\vec{\omega}}(\sigma) |\sigma| e^{i\sigma \vec{\omega}^{\perp} \cdot \vec{x}} d\sigma d\phi$$

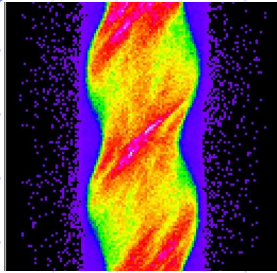
$$\underbrace{\text{TF}_s^{-1}[\widehat{p}_{\vec{\omega}} \cdot \text{abs}]}_{p_{\vec{\omega}}^f}(\vec{\omega}^{\perp} \cdot \vec{x})$$



J. Radon
1887-1956

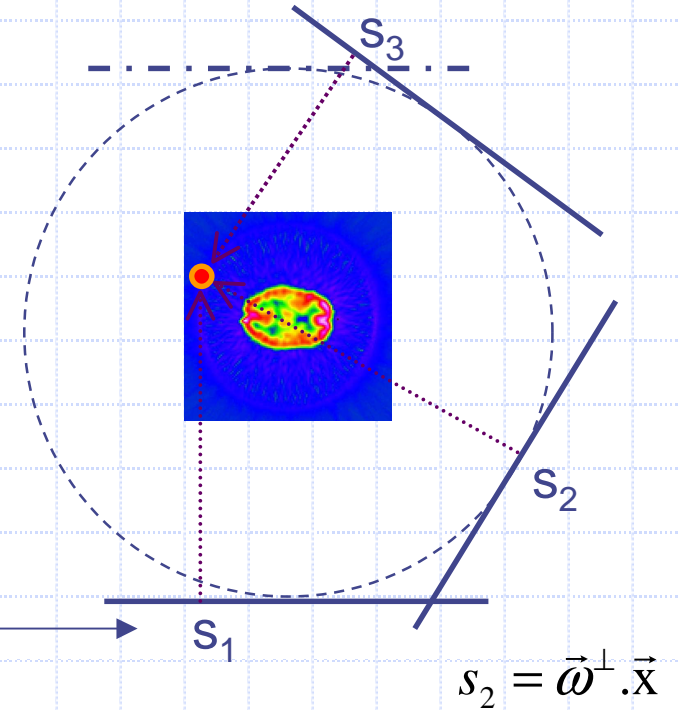
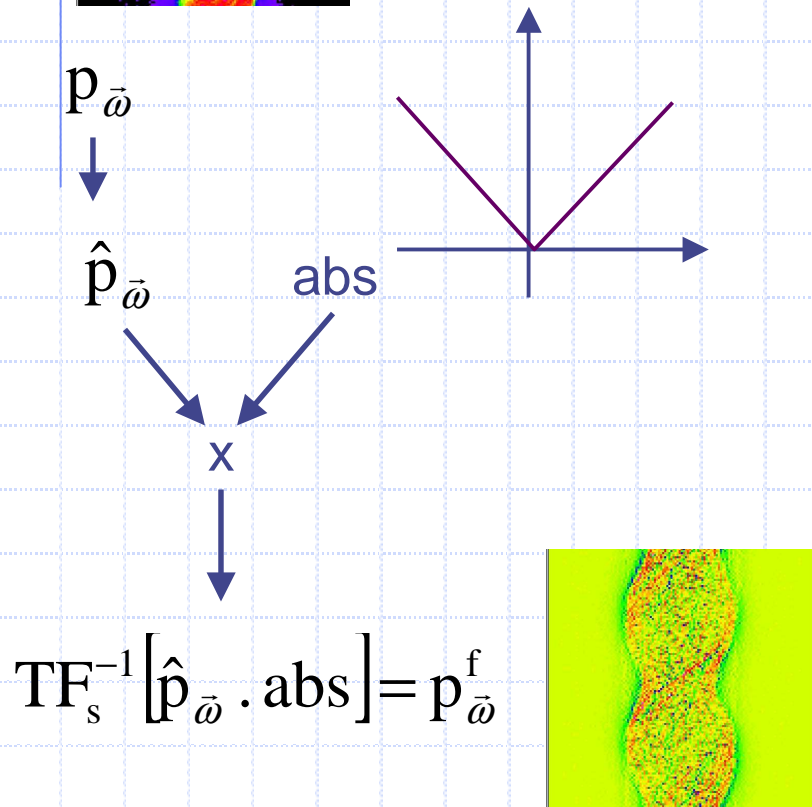
$$f(\vec{x}) = (\mathbf{R}^* p^f)(\vec{x})$$

Rétroprojection filtrée (II)

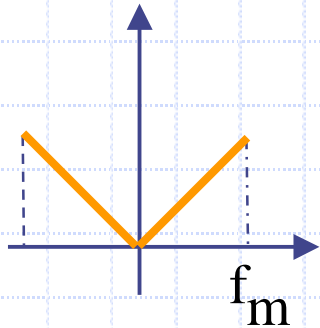


$$f(\vec{x}) = (\mathbf{R}^* p^f)(\vec{x})$$

Projections sur 180°



Rétroprojection filtrée (III)



$$RL(x) = \frac{f_{\max} \sin(2\pi \cdot f_{\max} \cdot x)}{\pi \cdot x} - \frac{1 - \cos(2\pi \cdot f_{\max} \cdot x)}{2\pi^2 \cdot x^2}$$

$$f_{\max} = \frac{1}{2 \cdot d} = \frac{f_{\text{éch.}}}{2}$$

d = taille des pixel

$$p_{\bar{\omega}}^f = \text{TF}_s^{-1} [\hat{p}_{\bar{\omega}} \cdot \text{abs}]$$

$$p_{\bar{\omega}}^f = p_{\bar{\omega}} * \text{RL}$$

$$RL(k \cdot d) = \begin{cases} \frac{1}{4d^2} & k = 0 \\ 0 & k \neq 0 \text{ pair} \\ -\frac{1}{(k\pi \cdot d)^2} & k \text{ impair} \end{cases}$$

Exemple pour $d = \frac{1}{2}$, on obtient le filtre : $\left(0 \quad -\frac{1}{2,5} \quad 1 \quad -\frac{1}{2,5} \quad 0 \right)$

Rétroprojection filtrée (IV)

0	0	0	← 15 ← 45
0	0	0	← 60 ← 90
0	0	0	← 15 ← 45

45 90 45

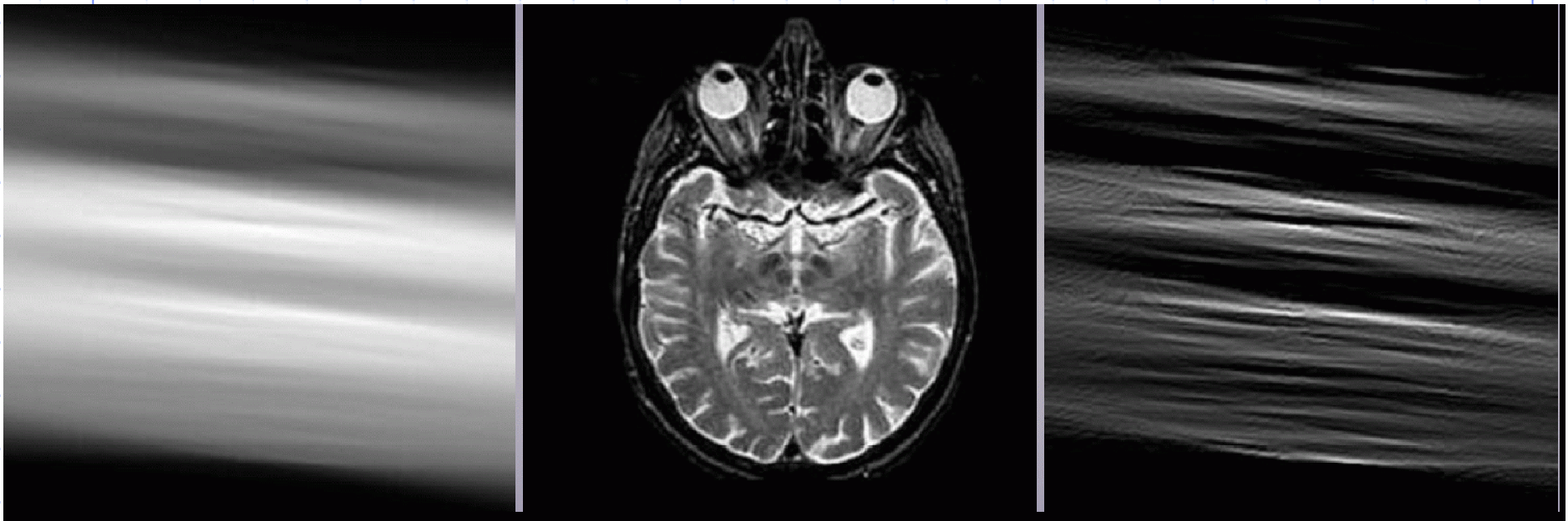
$$\text{Filtre} = \begin{pmatrix} -\frac{1}{3} & 1 & -\frac{1}{3} \end{pmatrix}$$

10	25	10
25	40	25
10	25	10

5	5	5
20	20	20
5	5	5
↑ 15	↑ 60	↑ 15

« illustration non contractuelle !... »

Rétroprojection filtrée (V)



$$R^* p$$

$$R^* TF_s^{-1} [\hat{p} . abs]$$

Limites des techniques analytiques

◆ Nécessité de données sur 180°

- ◆ Problème important en TEP 3D.
- ◆ L'inversion directe ou la RPF ne fonctionnent pas sur des données tronquées.

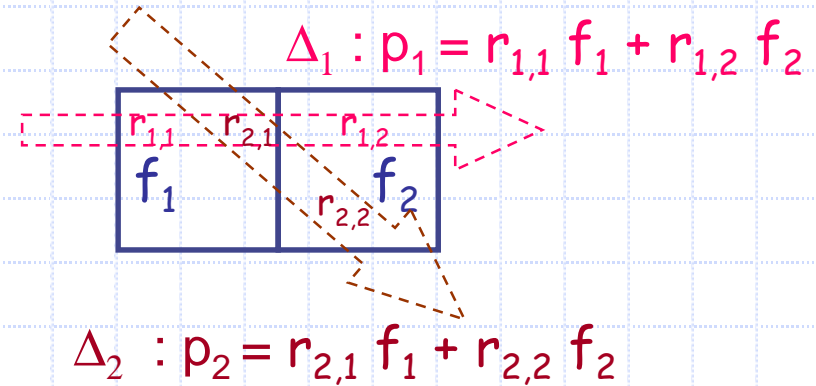
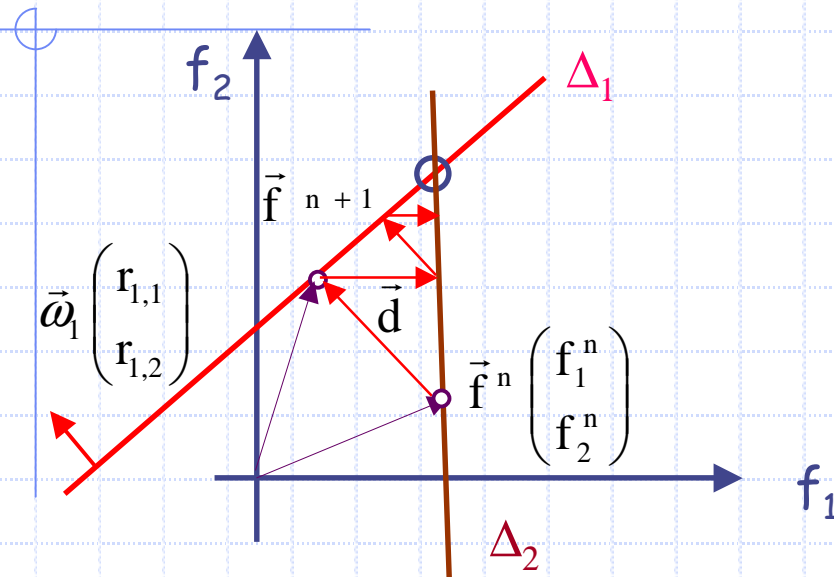
◆ Prise en compte des artefacts en SPECT et PET :

- ◆ Dans le théorème de la coupe centrale, $f(x)$ et non pas $f(x,s,\phi)$
- ◆ Difficulté majeure d'introduire des facteurs du type $\exp(-\mu L_{x,s,\phi})$
- ◆ D'où un problème pour corriger les artefacts d'atténuation (photoélectrique, Compton).
- ◆ En revanche, une déconvolution de la réponse impulsionnelle est faisable.

◆ Ajustement de la fréquence de coupure délicate

- ◆ Nécessité d'un filtre passe-bas associé au filtre valeur absolue

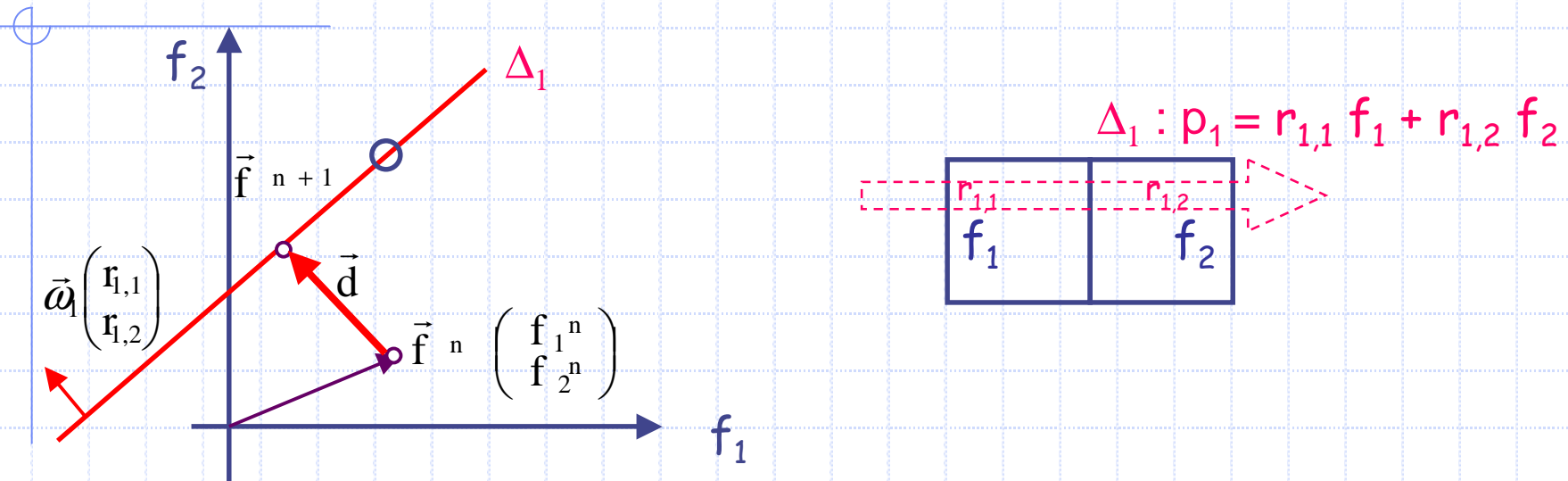
Algebraic Reconstruction Technique (I)



S. Kaczmarz
1895-1940

On construit une suite de coupes \vec{f}^n en projetant chaque itéré sur l'un puis l'autre hyperplan.

Algebraic Reconstruction Technique (II)



La distance d d'un point f^n à une droite Δ_1 est :

$$d = \frac{p_1 - \vec{f}^n \cdot \vec{\omega}_1}{\|\vec{\omega}_1\|}$$

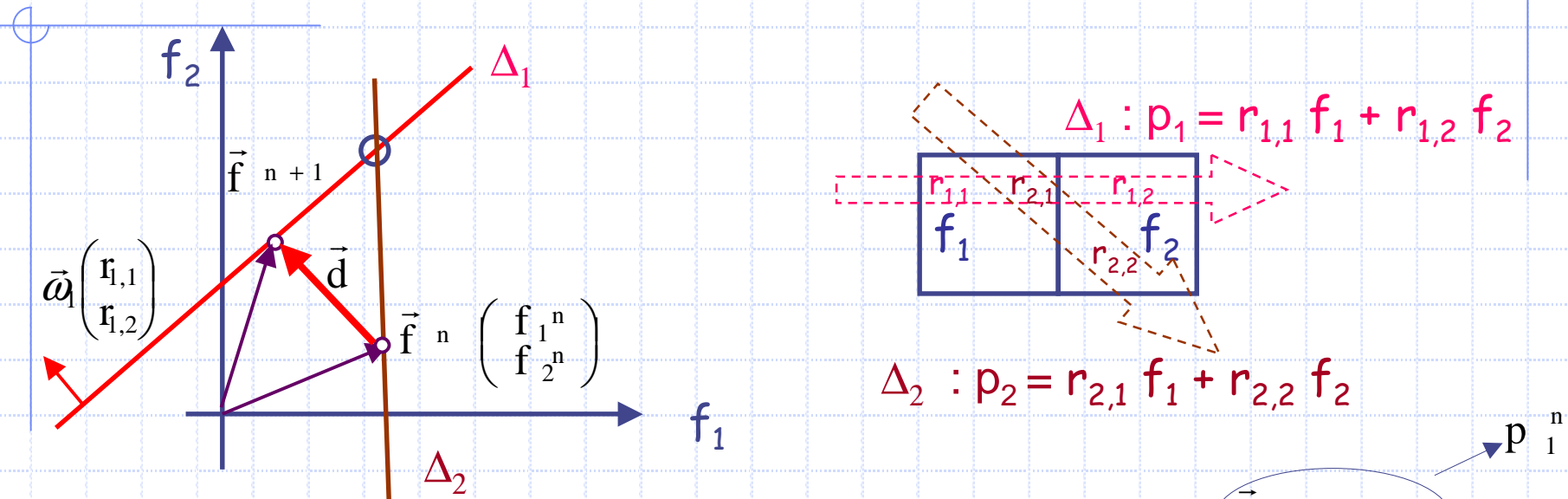
$$d = \frac{p_1 - \vec{f}^n \cdot \vec{\omega}_1}{\|\vec{\omega}_1\|} = \frac{p_1 - p_1^n}{\|\vec{\omega}_1\|}$$

$$p_1^n = r_{1,1} f_1^n + r_{1,2} f_2^n, \text{ projection qui serait mesurée si } f^n \text{ était la solution}$$



S. Kaczmarz
1895-1940

Algebraic Reconstruction Technique (II)



$$\vec{f}^{n+1} = \vec{f}^n + d \frac{\vec{\omega}_1}{\|\vec{\omega}_1\|} \quad d = \frac{p_1 - \vec{f}^n \cdot \vec{\omega}_1}{\|\vec{\omega}_1\|}$$

$$\vec{f}^{n+1} = \vec{f}^n + \frac{p_1 - p_1^n}{\|\vec{\omega}_1\|^2} \vec{\omega}_1$$

$$\vec{f}^{n+1} = \vec{f}^n + R^* (p_1 - p_1^n)$$



S. Kaczmarz
1895-1940

Algebraic Reconstruction Technique (III)

0	0	0
0	0	0
0	0	0

← 45 - 0 = 15 + 15 + 15

← 90 - 0 = 30 + 30 + 30

← 45 - 0

45 90 45

⇒

15	15	15
30	30	30
15	15	15

↓ ↓ ↓
45 90 45

- 60 60 60

-15 30 -15

10	25	10
25	40	25
10	25	10

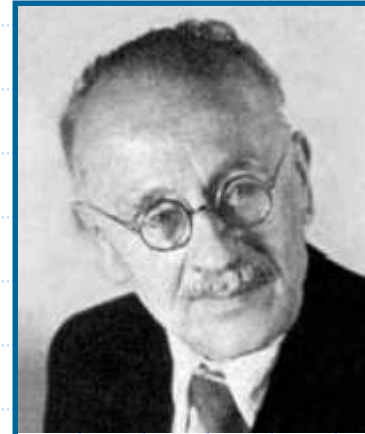
$$\vec{f}^{n+1} = \vec{f}^n + R * (p_1 - p_1^n)$$

« illustration non contractuelle !... »

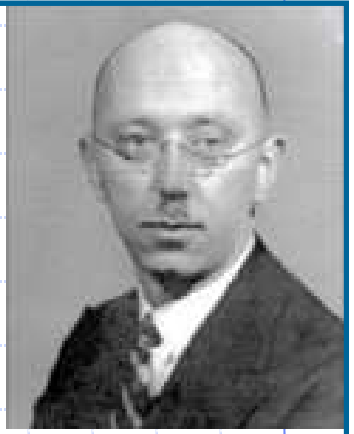
Conclusion



G. Hounsfield 1919-2004



J. Radon
1887-1956



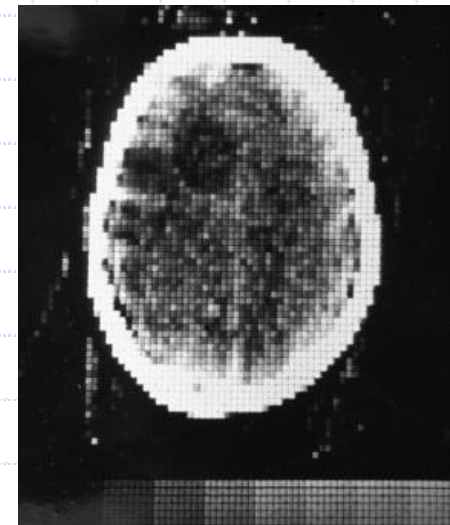
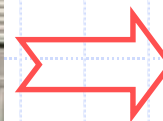
S. Kaczmarz
1895-1940

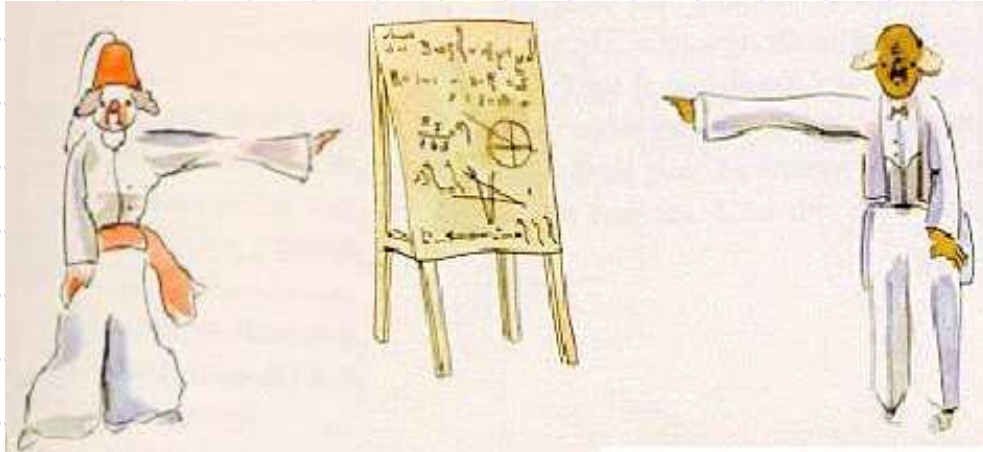


$$\hat{p}_{\vec{\theta}}(\sigma) = \hat{f}(\sigma \cdot \vec{\theta})$$

$$f(\vec{x}) = (R^* p^f)(\vec{x})$$

$$\vec{f}^{n+1} = \vec{f}^n + R^*(p_1 - p_1^n)$$





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Merci de votre attention...