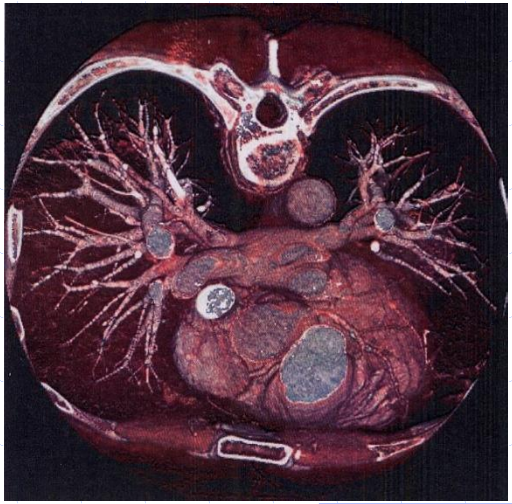
FORMATION TIC (Phymed, STIC,Télécom)

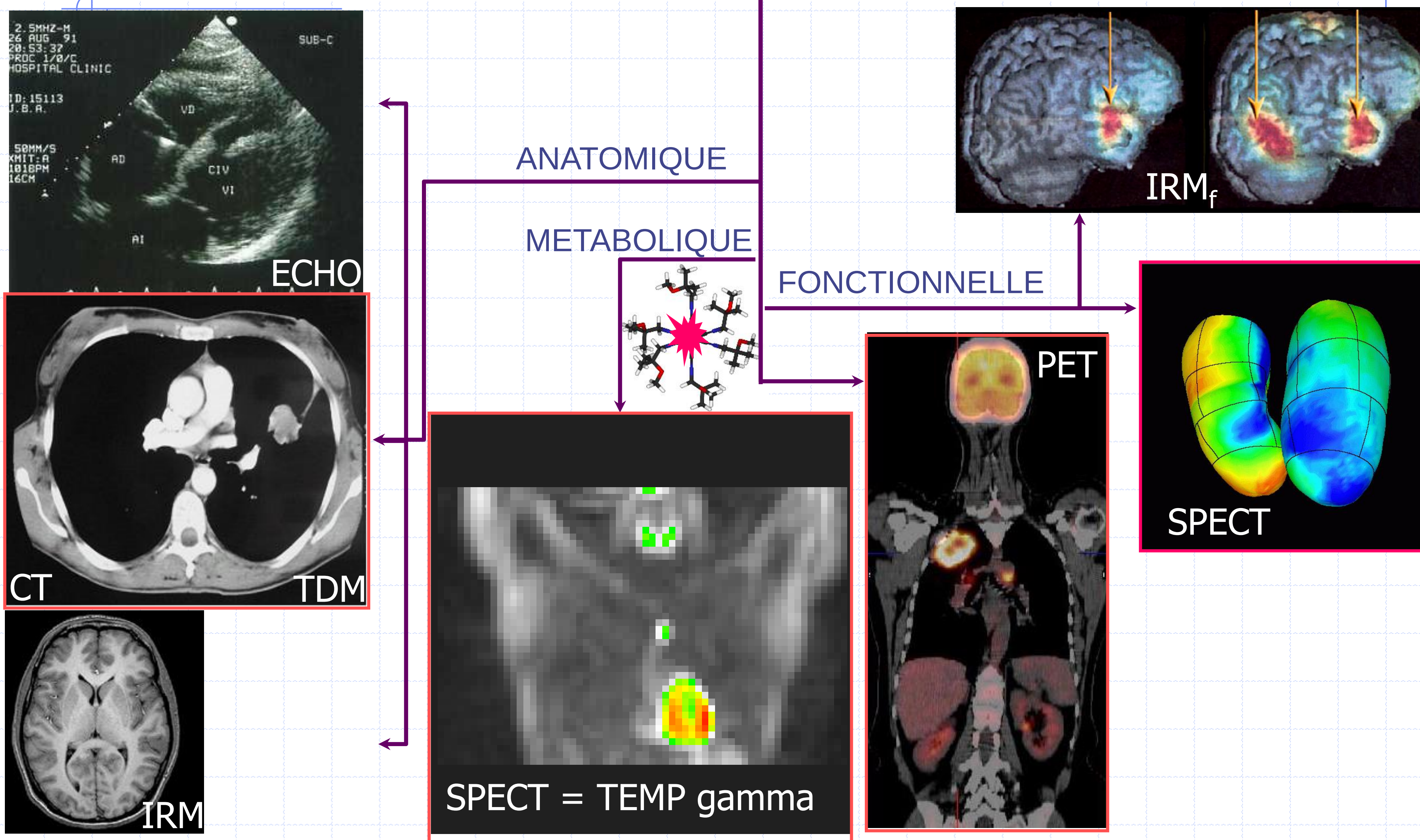
BASES DE TOMOGRAPHIE MEDICALE



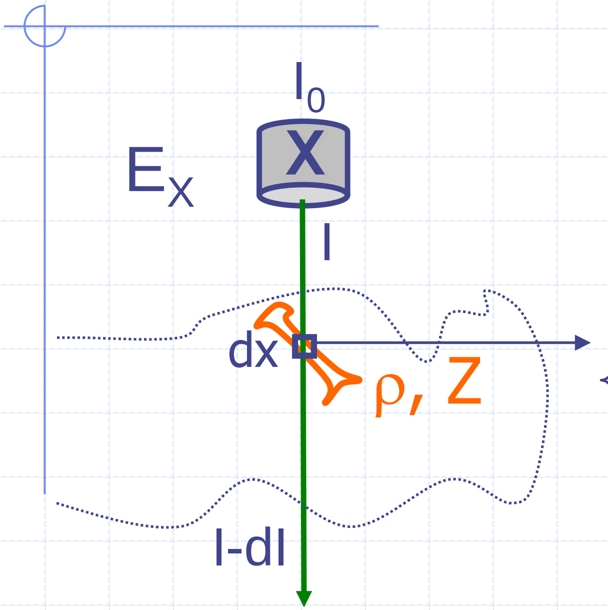
Fayçal Ben Bouallègue - faybenb@hotmail.com

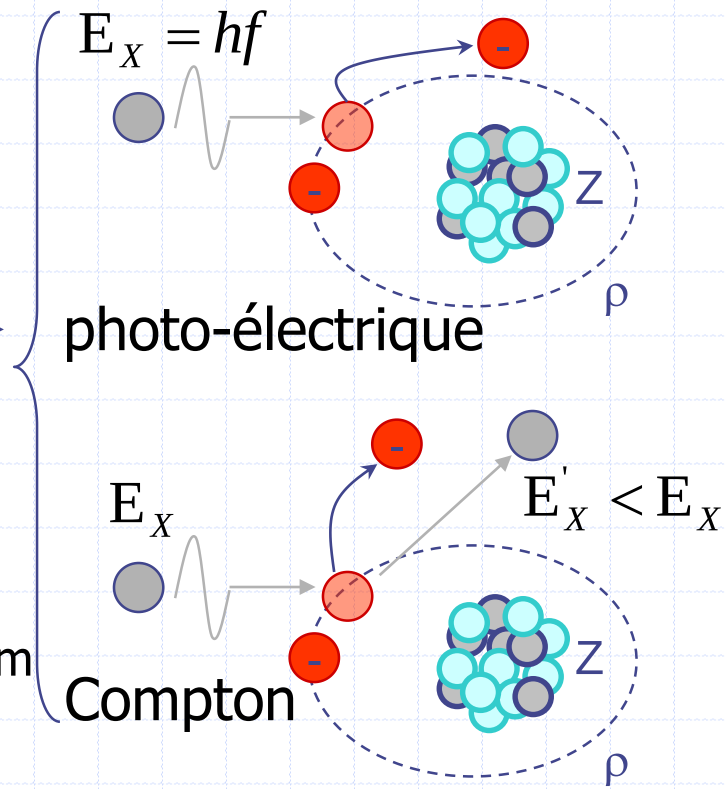
http://scinti.etud.univ-montp1.fr

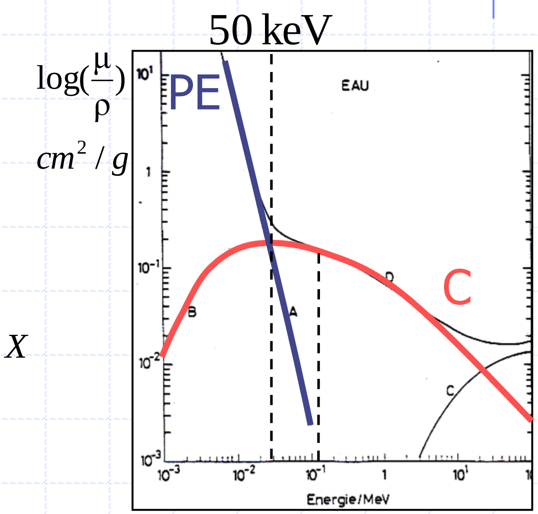
Imagerie médicale



IMAGERIE DE TRANSMISSION X



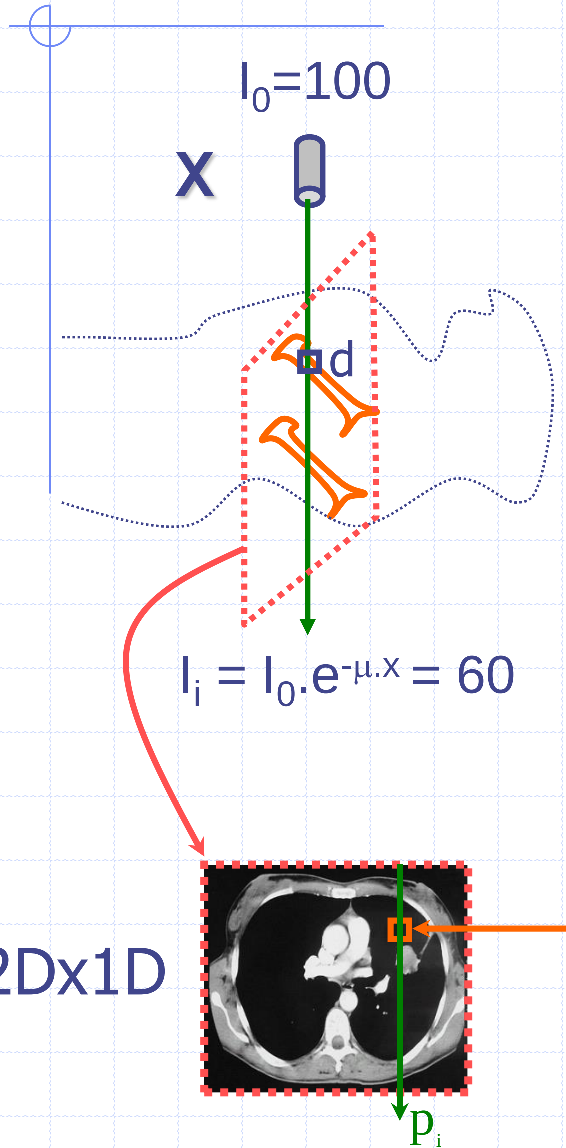




μ probabilité d'atténuation/cm
μ est proportionnelle
à la masse volumique ρ

$$\mu = -\frac{\frac{dI}{I}}{dx} \ \mathrm{cm}^{-1} \propto \rho \Rightarrow I = I_0 . e^{-\mu . x}$$

Scanner X = Computed Tomography



2Dx1D

$$I_i = I_0 e^{-\sum \mu_j . d} = I_0 e^{-d.\sum \mu_j}$$

$$\Rightarrow p_i = \sum \mu_j = -\frac{1}{d} \ln \frac{I_i}{I_0}$$

mesure

$r_{i,j}$ = contribution du pixel j à la projection i

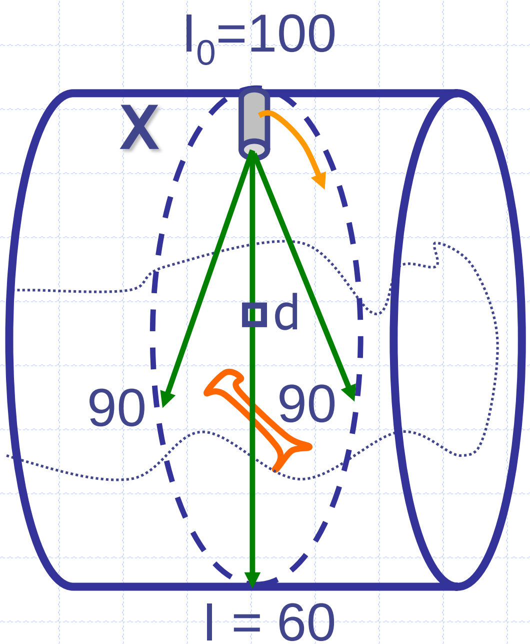
paramètres géométriques connus du scanner

$$p_i = \sum r_{i,j} . \mu_j$$

?

mesure

Scanner X = Computed Tomography



$$I_i = I_0 e^{-\sum \mu_j . d} = I_0 e^{-d . \sum \mu_j}$$

$$\Rightarrow p_i = \sum \mu_j = -\frac{1}{d} \ln \frac{I_i}{I_0}$$

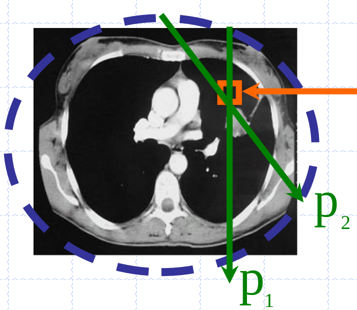
$r_{i,j}$ = contribution du pixel j à la projection i

paramètres géométriques connus du scanner

$$p_i = \sum r_{i,j} . \mu_j$$?

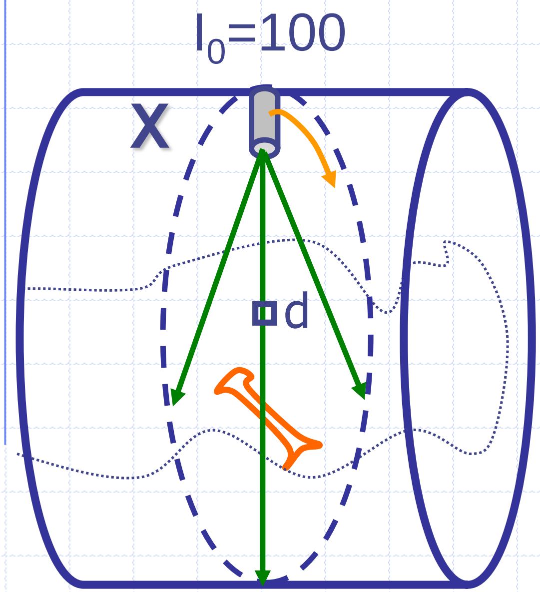
mesure

2Dx1D

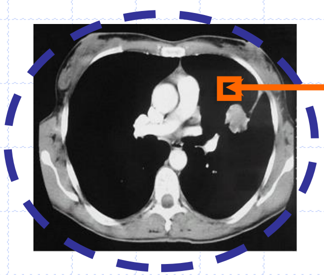


$$\begin{cases} p_1 = r_{1,1}\mu_1 + r_{1,2}\,\mu_2 + \ldots + r_{1,n}\mu_n \\ p_2 = r_{2,1}\mu_1 + r_{2,2}\,\mu_2 + \ldots + r_{2,n}\mu_n \\ \ldots \end{cases}$$

Scanner X = Computed Tomography

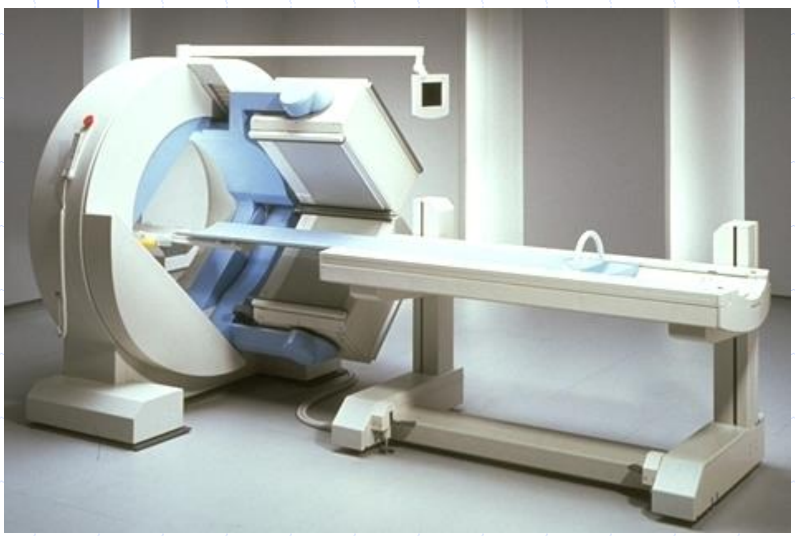


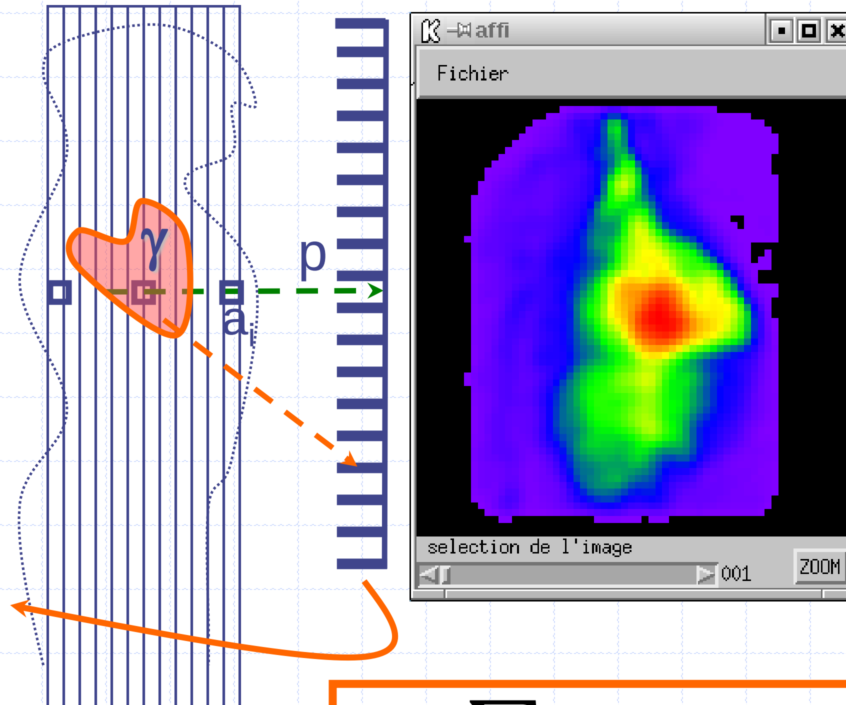
$$p_i = \sum_j r_{i,j}\,\mu_j, \quad i = 0\text{-}360°$$



$$r_{i,j} = \text{contribution du pixel } j \text{ à la projection } i$$

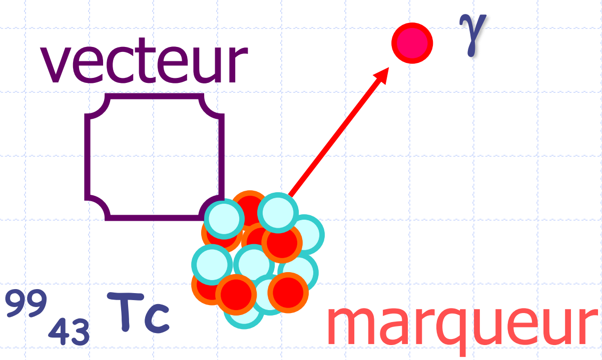
Single Photon Emission CT

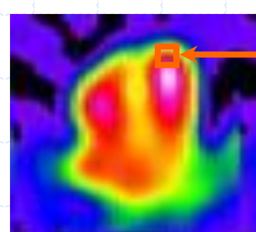




2D

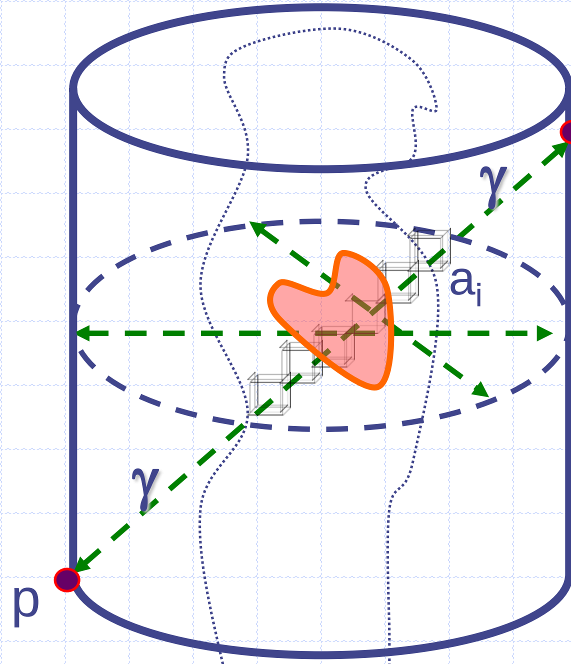
$$\boxed{p_i = \sum_j r_{i,j}\, a_j, \quad i = 0 - 360°}$$



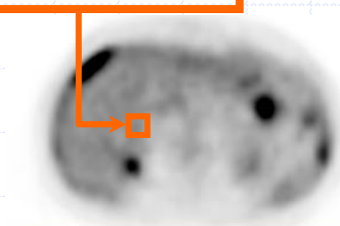


2Dx1D

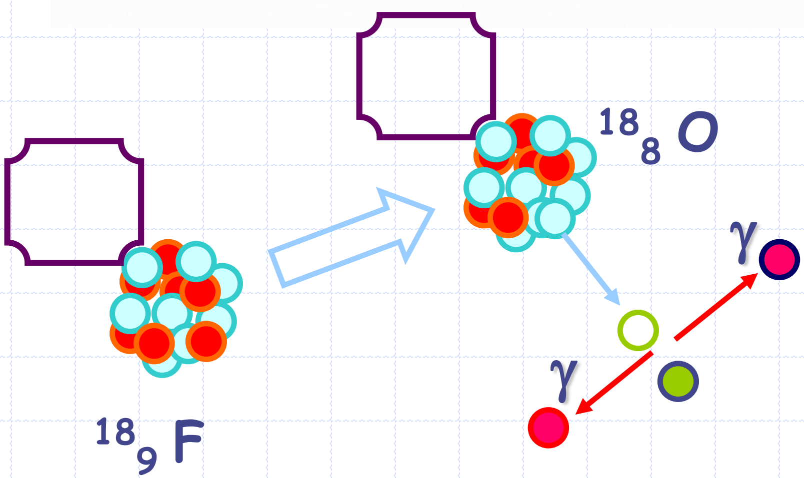
Tomographie par Emission de Positons



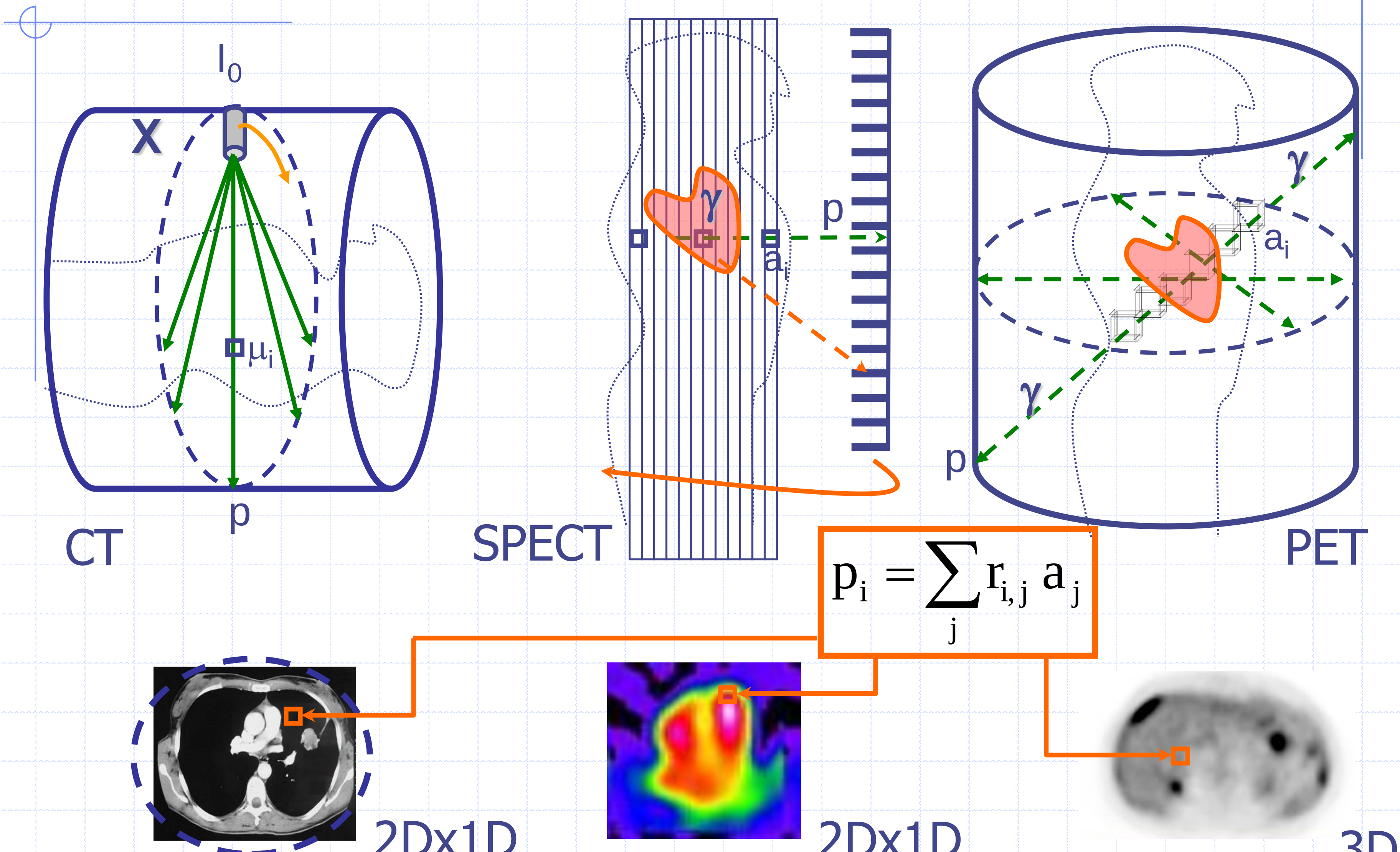
$$p_i = \sum_j r_{i,j}\, a_j$$



3D

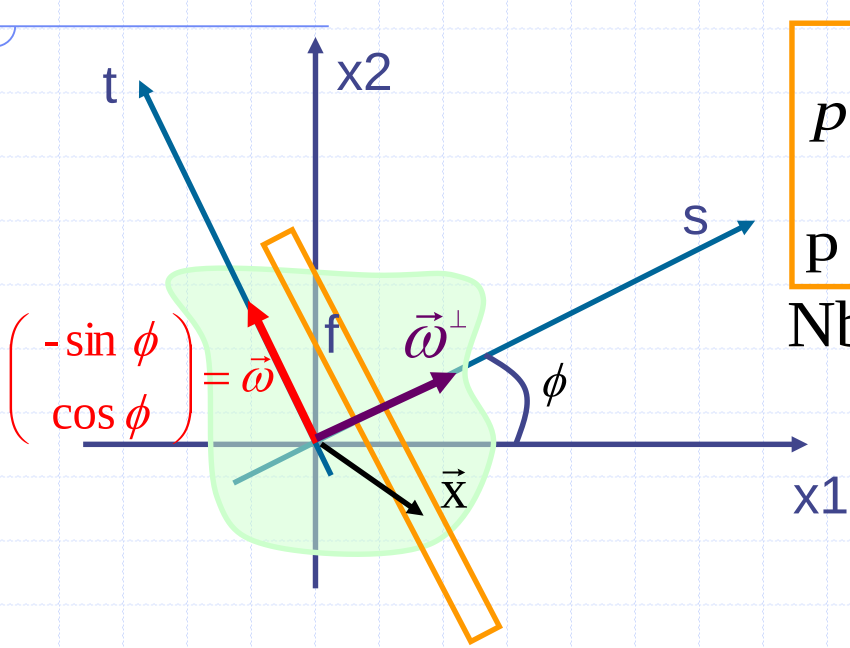


Tomographie: problème inverse linéaire



$$p_i = \sum_j r_{i,j}\, a_j$$

Modélisation analytique



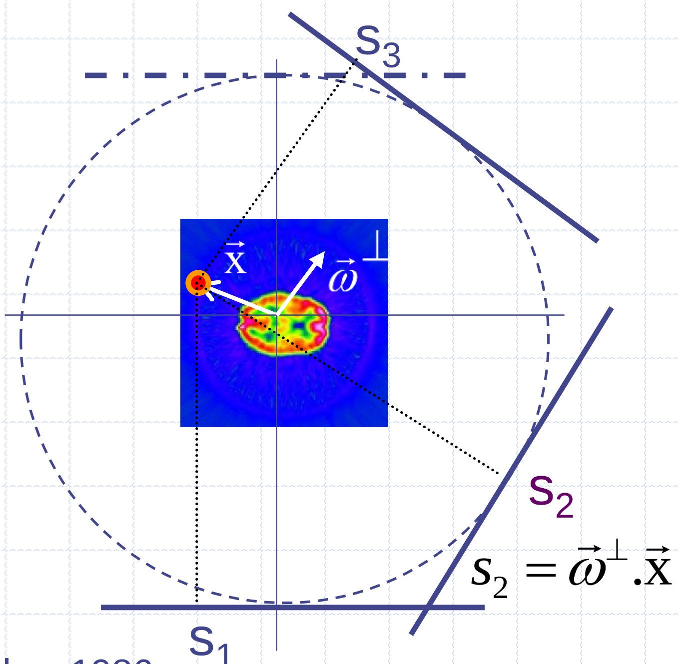
$$p(\vec{\omega},s) = p_{\vec{\omega}}(s) = \int_t f(s\vec{\omega}^{\perp} + t\vec{\omega})\,dt$$

$p = Rf$ transformée de Radon

Nb : $\vec{x}.\vec{\omega}^{\perp} = s$ où $\vec{x}$ se projette suivant $\vec{\omega}$

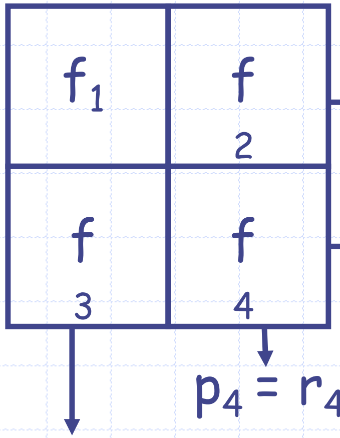
$$(R^* p)(\vec{x}) = \int_{\phi=0}^{\pi} p\left(\vec{\omega}, \vec{\omega}^{\perp}.\vec{x}\right) d\phi$$

rétroprojection = épandage



Natterer F. The mathematics of computerized tomography. New York: Wiley; 1986.

Modélisation algébrique



$p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2$

$p_2 = r_{2,3}\, f_3 + r_{2,4}\, f_4$

$p_4 = r_{4,2}\, f_2 + r_{4,4}\, f_4$

$p_3 = r_{3,1}\, f_1 + r_{3,3}\, f_3$

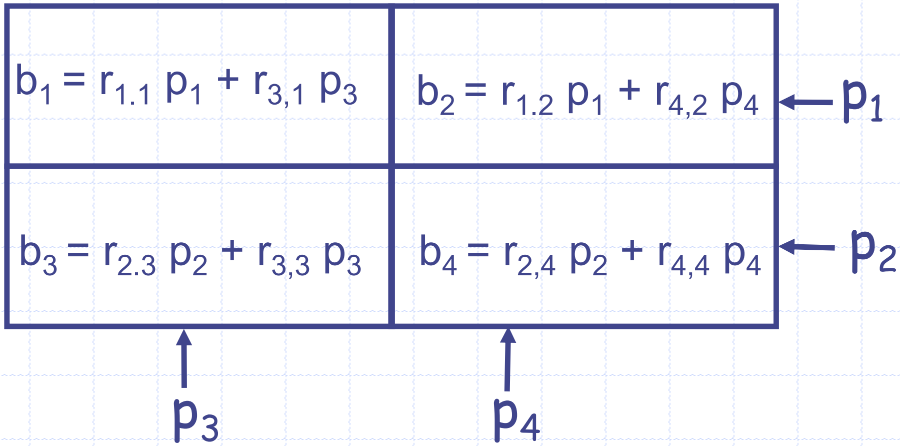
$$\begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ r_{2,1} & r_{2,2} & r_{2,3} & r_{2,4} \\ r_{3,1} & r_{3,2} & r_{3,3} & r_{3,4} \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{pmatrix} . \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

$r_{i,j}$ = % du pixel j intersecté par la projection i

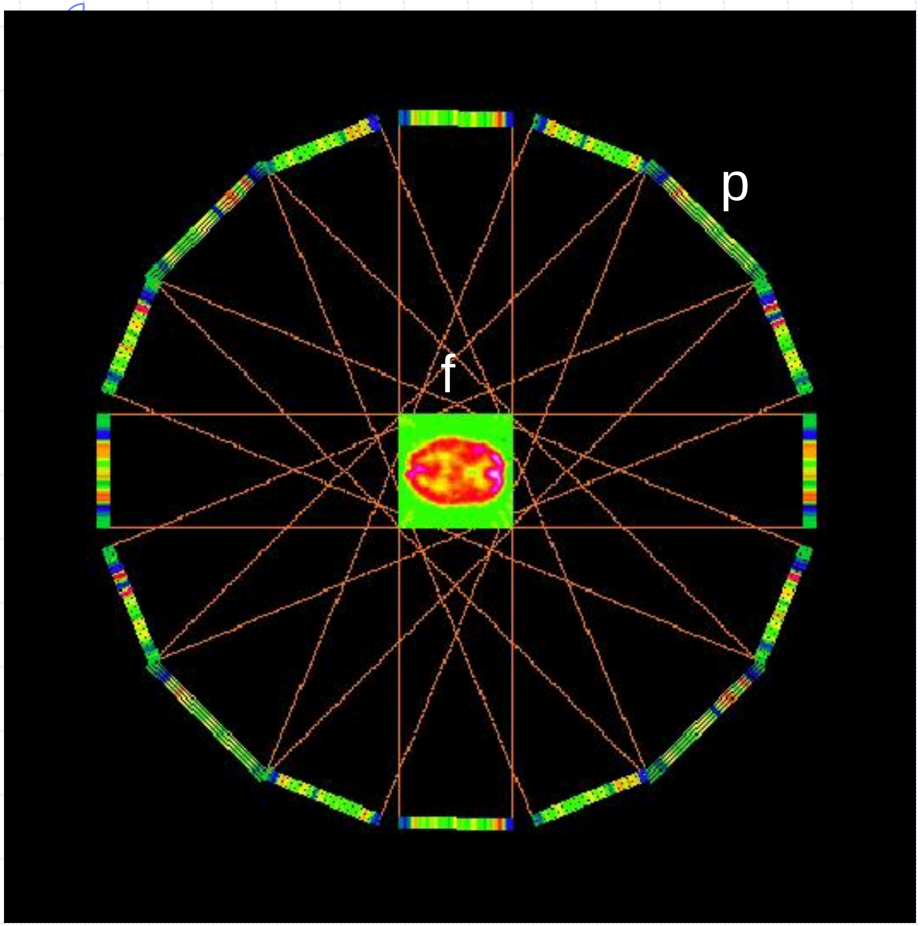
$$\boxed{R.\vec{f} = \vec{p}}$$

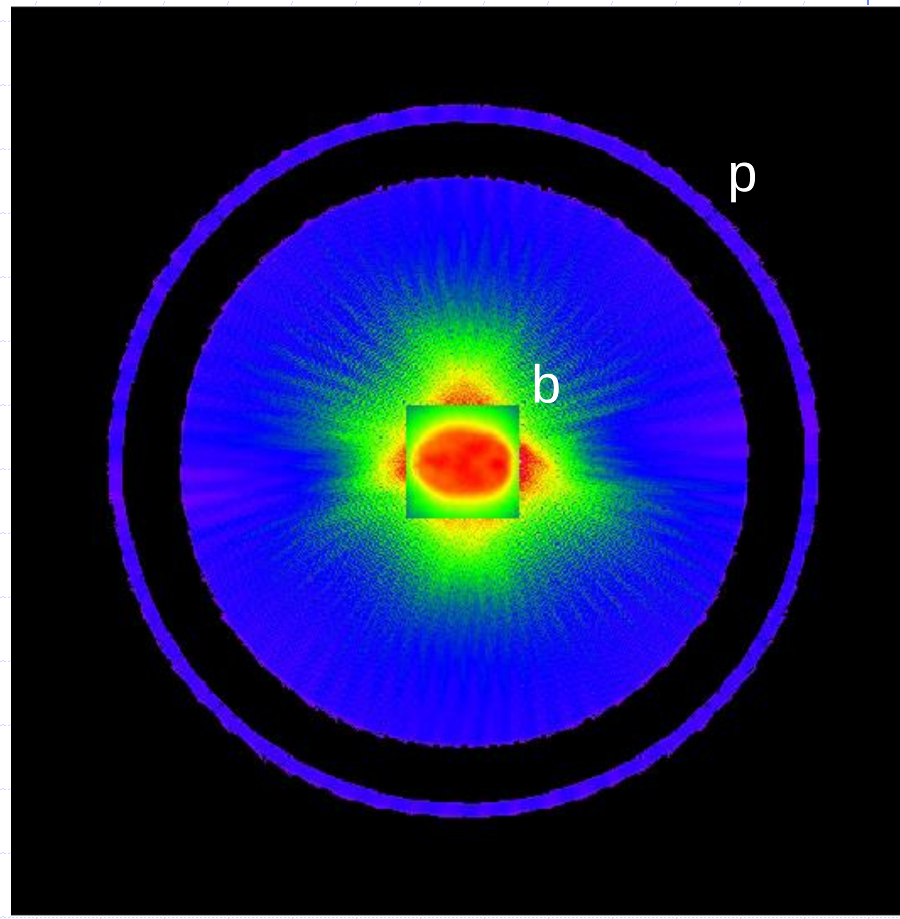
$$\begin{pmatrix} r_{1,1} & r_{2,1} & r_{3,1} & r_{4,1} \\ r_{1,2} & r_{2,2} & r_{3,2} & r_{4,2} \\ r_{1,3} & r_{2,3} & r_{3,3} & r_{4,3} \\ r_{1,4} & r_{2,4} & r_{3,4} & r_{4,4} \end{pmatrix} . \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$$\boxed{{}^{t}R.\vec{p} = \vec{b}}$$



Projection / Rétroprojection

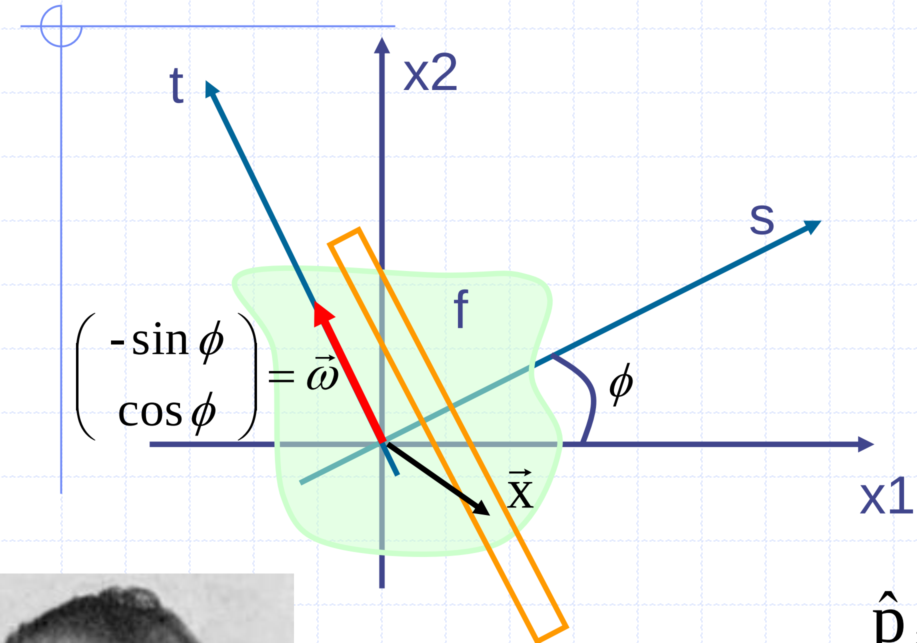




$$\boxed{R.\vec{f} = \vec{p}}$$

$$\boxed{{}^{t}R.\vec{p} = \vec{b}}$$

Théorème de coupe centrale



$$\begin{cases} p_{\vec{\omega}}(s) = \int_t f(s\vec{\omega}^{\perp} + t\vec{\omega})\,dt \\ \hat{p}_{\vec{\omega}}(\sigma) = \int_s p_{\vec{\omega}}(s).e^{-i.s.\sigma}\,ds \end{cases}$$

$$\hat{p}_{\vec{\omega}}(\sigma) = \int_s \int_t f(s\vec{\omega}^{\perp} + t\vec{\omega})\, e^{-i.s.\sigma}\, dt\, ds$$

$$\hat{p}_{\vec{\omega}}(\sigma) = \iint f(\vec{x})\, e^{-i.\sigma\, \vec{x}.\vec{\omega}^{\perp}}\, d\vec{x}$$

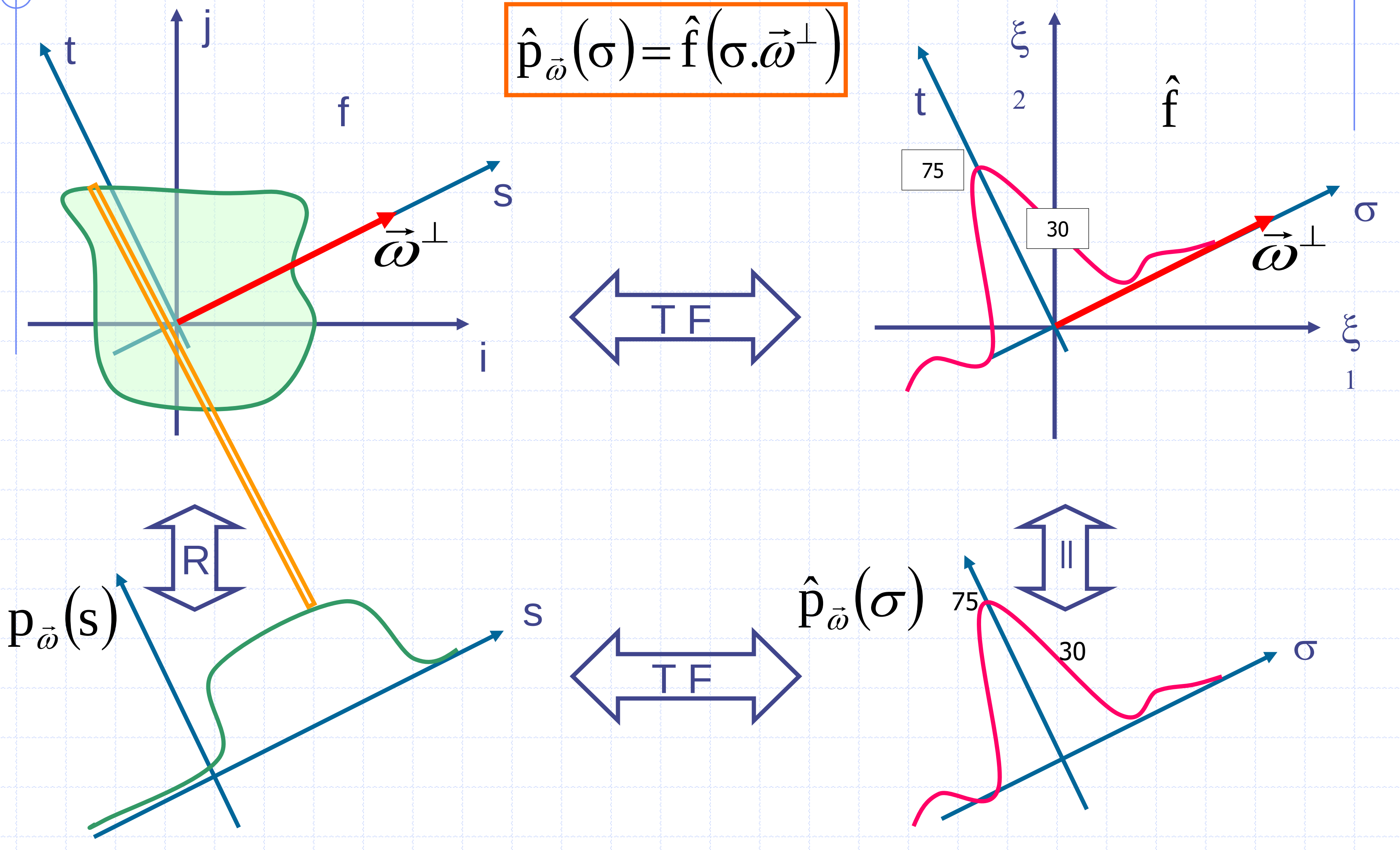
$$\boxed{\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}(\sigma.\cos\phi\,,\,\sigma.\sin\phi) = \hat{f}(\sigma.\vec{\omega}^{\perp})}$$

J. Radon
1887-1956

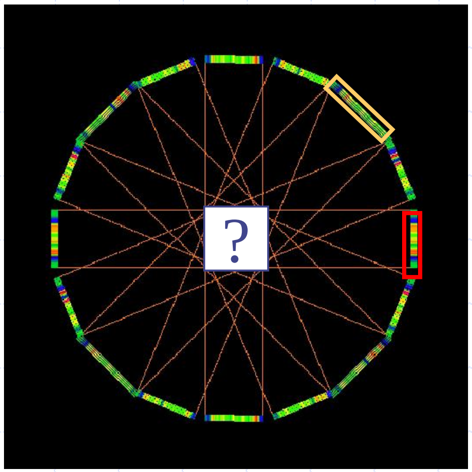
Johann Radon, ¨ Uber die Bestimmung von Funktionen durch ihre Integralwerte langs gewisser Mannigfaltigkeiten, Ber. Verh. Sach. Akad. 69 (1917), 262–77

Interprétation (I)

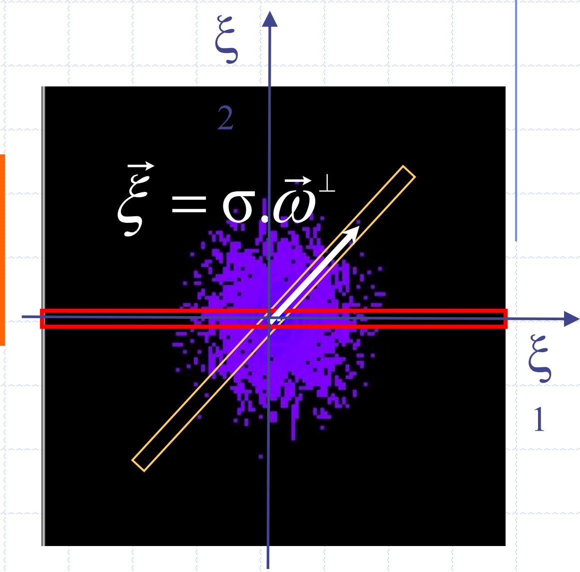
$$\boxed{\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}\left(\sigma.\vec{\omega}^{\perp}\right)}$$



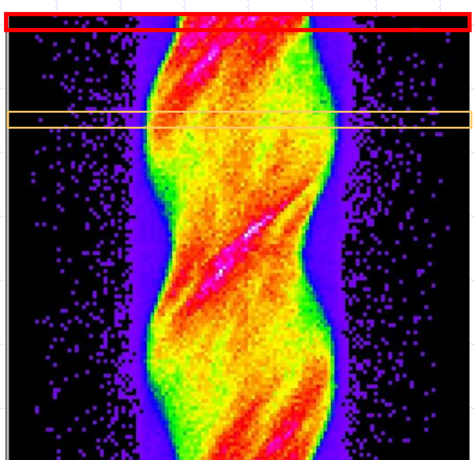
Interprétation (II)

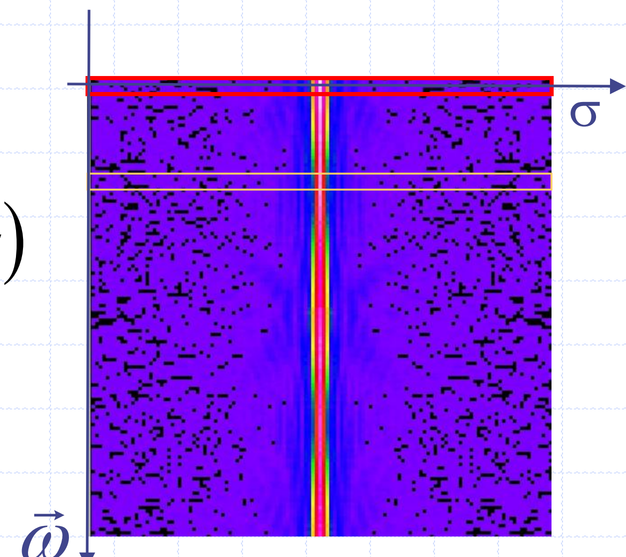


$$\boxed{\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}\left(\sigma.\vec{\omega}^{\perp}\right)}$$

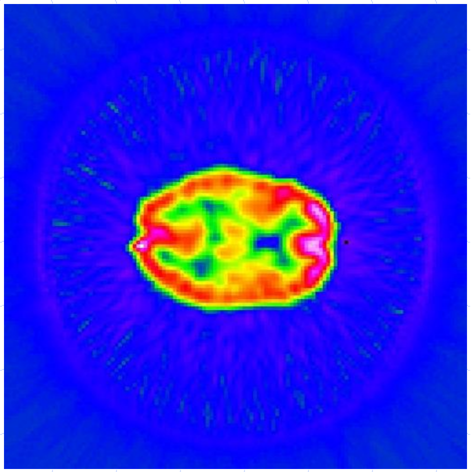


$$\begin{tikzcd}
f \arrow[r, leftrightarrow, "TF"] \arrow[d, leftrightarrow, "R"'] & \hat{f} \arrow[d, leftrightarrow, "="] \\
p_{\vec{\omega}}(s) \arrow[r, leftrightarrow, "TF"] & \hat{p}_{\vec{\omega}}(\sigma)
\end{tikzcd}$$

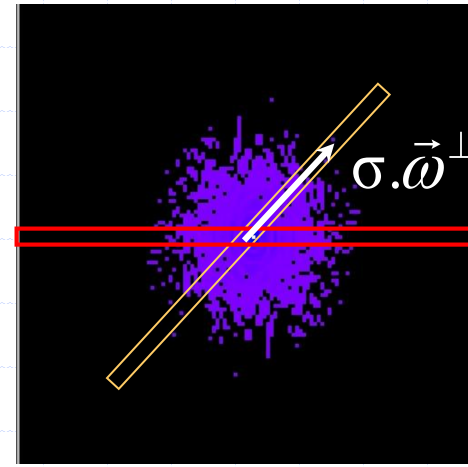




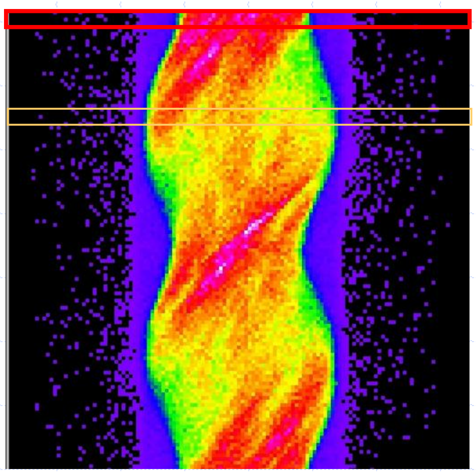
Interprétation (II)

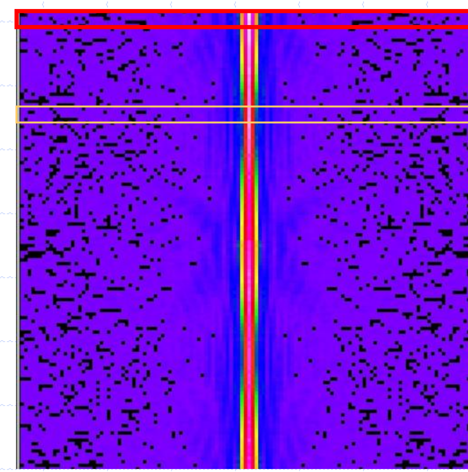


$$\boxed{\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}\left(\sigma.\vec{\omega}^{\perp}\right)}$$

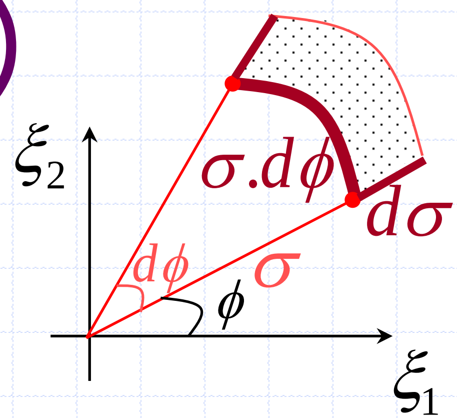


$$\begin{array}{ccc} f & \overset{TF}{\longleftrightarrow} & \hat{f} \\ {\scriptstyle R}\big\updownarrow & & \big\updownarrow{\scriptstyle \Vert} \\ p_{\vec{\omega}}(s) & \overset{TF}{\longleftrightarrow} & \hat{p}_{\vec{\omega}}(\sigma) \end{array}$$



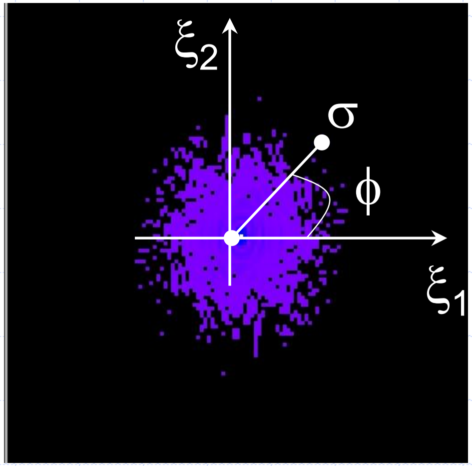


Rétroprojection filtrée (I)



$$f(\vec{x}) = \iint \hat{f}(\vec{\xi})\, e^{i\,\vec{x}.\vec{\xi}}\, d\vec{\xi}$$

$$f(\vec{x}) = \int_{\phi=0}^{\pi} \int_{\sigma=-\infty}^{\sigma=+\infty} \hat{f}\left(\sigma\, \vec{\omega}^{\perp}\right) e^{i\,\sigma\, \vec{\omega}^{\perp}.\vec{x}}\, |\sigma|\, d\sigma\, d\phi$$



$$f(\vec{x}) = \int_{\phi=0}^{\pi} \boxed{\int_{\sigma=-\infty}^{\sigma=+\infty} \hat{p}_{\vec{\omega}}(\sigma)\, |\sigma|\, e^{i\,\sigma\, \vec{\omega}^{\perp}.\vec{x}}\, d\sigma}\, d\phi$$

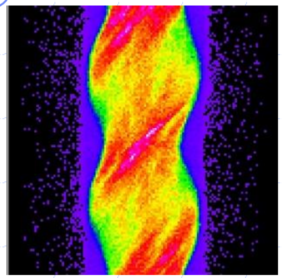
$$\| $$

$$\underbrace{TF_s^{-1}\left[\hat{p}_{\vec{\omega}}\, .\, \mathrm{abs}\right]\left(\vec{\omega}^{\perp}.\vec{x}\right)}_{p^{f}_{\vec{\omega}}}$$

J. Radon
1887-1956

$$\boxed{f(\vec{x}) = (R^{*}\, p^{f})(\vec{x})}$$

Rétroprojection filtrée (II)



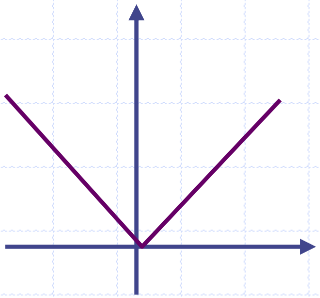
$$f(\vec{x}) = (R^* p^f)(\vec{x})$$

Projections sur 180°

$p_{\vec{\omega}}$

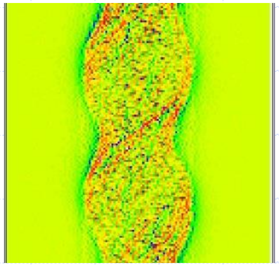
$\hat{p}_{\vec{\omega}}$

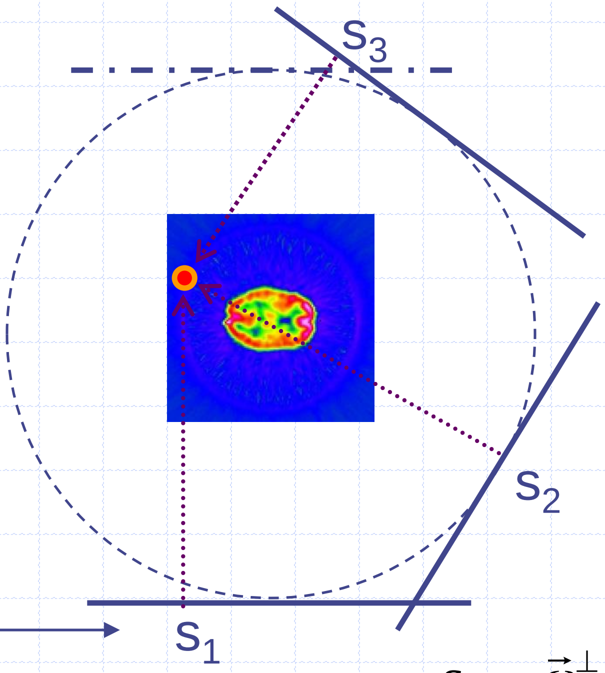
abs



x

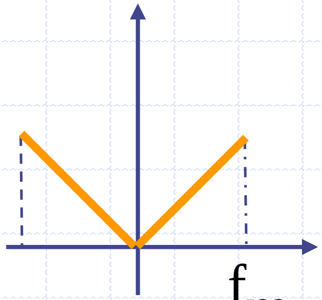
$$TF_s^{-1}\left[\hat{p}_{\vec{\omega}} \,.\, abs\right] = p^f_{\vec{\omega}}$$





$$s_2 = \vec{\omega}^{\perp}.\vec{x}$$

Rétroprojection filtrée (III)



$$\mathrm{RL}(x) = \frac{f_{max}\sin(2\pi.f_{max}x)}{\pi.x} - \frac{1-\cos(2\pi.f_{max}x)}{2\pi^2.x^2}$$

$$f_{max} = \frac{1}{2.d} = \frac{f_{éch.}}{2}$$

$$d = \text{taille des pixel}$$

$$p^f_{\vec{\omega}} = TF_s^{-1}\left[\hat{p}_{\vec{\omega}}.abs\right]$$

$$p^f_{\vec{\omega}} = p_{\vec{\omega}} * RL$$

$$\mathrm{RL}(k.d) = \begin{cases} \dfrac{1}{4d^2} & k = 0 \\ 0 & k \neq 0 \text{ pair} \\ -\dfrac{1}{(k\pi.d)^2} & k \text{ impair} \end{cases}$$

$$\text{Exemple pour } d = \frac{1}{2}\text{, on obtient le filtre : }\begin{pmatrix} 0 & -\frac{1}{2,5} & 1 & -\frac{1}{2,5} & 0 \end{pmatrix}$$

Rétroprojection filtrée (IV)

0	0	0	← 15 ← 45
0	0	0	← 60 ← 90
0	0	0	← 15 ← 45
45	90	45	

⇨

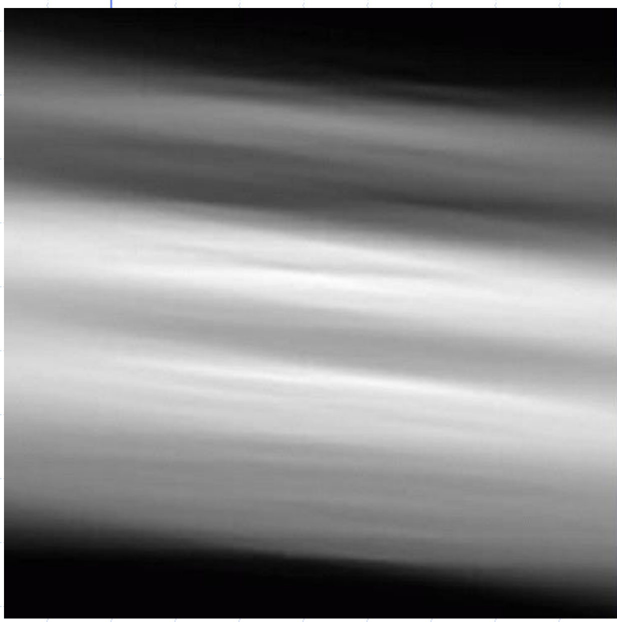
5	5	5
20	20	20
5	5	5
↑ 15	↑ 60	↑ 15

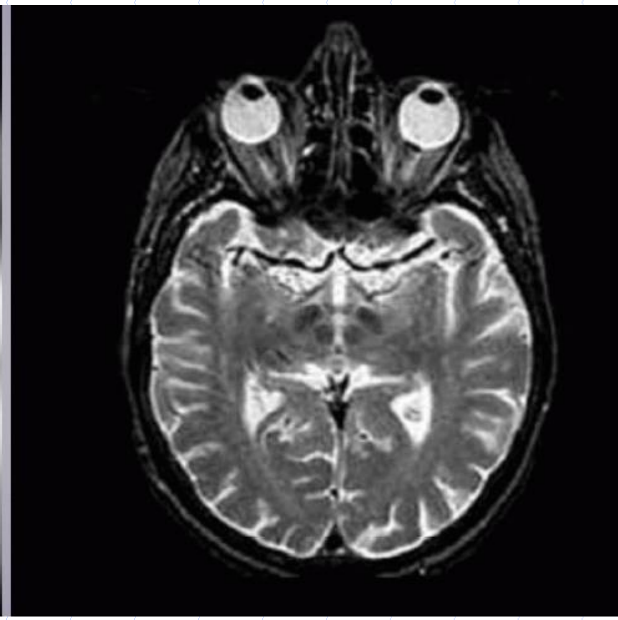
10	25	10
25	40	25
10	25	10

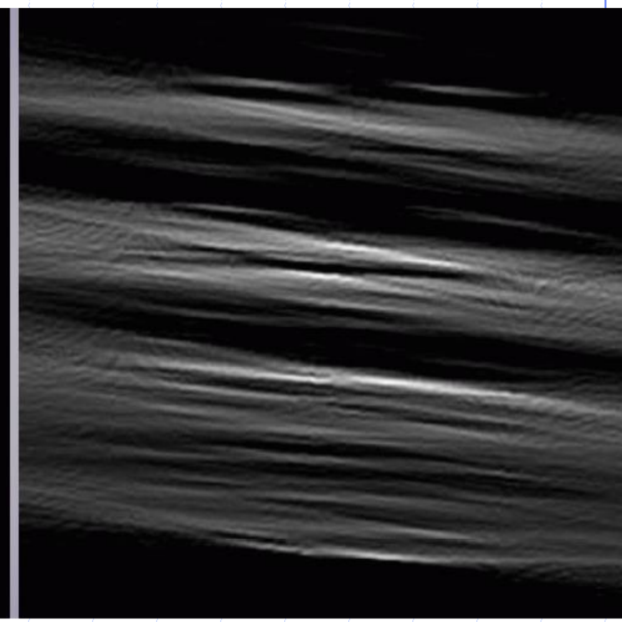
$$\text{Filtre} = \begin{pmatrix} -\frac{1}{3} & 1 & -\frac{1}{3} \end{pmatrix}$$

« illustration non contractuelle !… »

Rétroprojection filtrée (V)







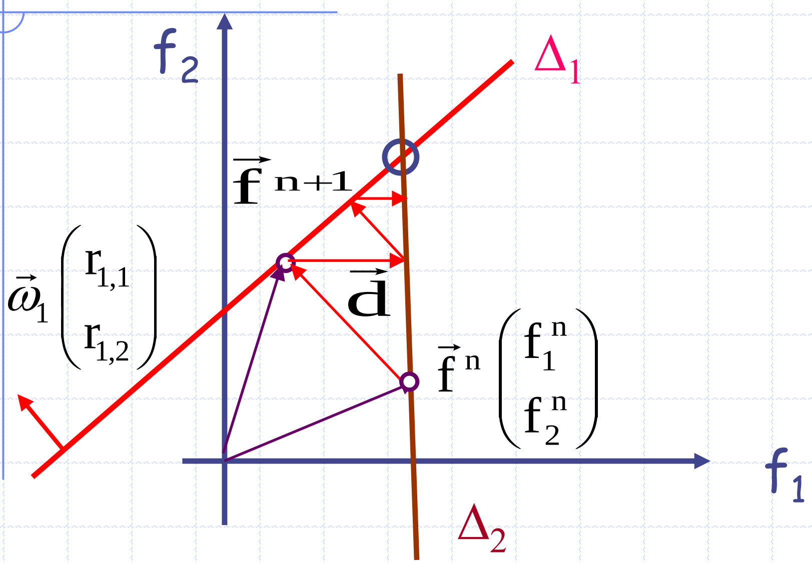
$\boxed{R^* \, p}$

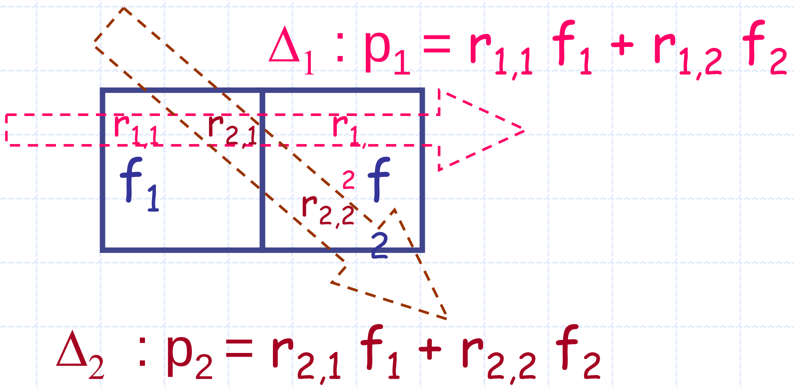
$\boxed{R^* \, TF_s^{-1}\left[\hat{p} \, . \, abs\right]}$

Limites des techniques analytiques

- Nécessité de données sur 180°
  - Problème important en TEP 3D.
  - L'inversion directe ou la RPF ne fonctionnent pas sur des données tronquées.
- Prise en compte des artefacts en SPECT et PET :
  - Dans le théorème de la coupe centrale, $f(x)$ et non pas $f(x,s,\phi)$
  - Difficulté majeure d'introduire des facteurs du type $\exp(-\mu L_{x,s,\phi})$
  - D'où un problème pour corriger les artefacts d'atténuation (photoélectrique, Compton).
  - En revanche, une déconvolution de la réponse impulsionnelle est faisable.
- Ajustement de la fréquence de coupure délicate
  - Nécessité d'un filtre passe-bas associé au filtre valeur absolue

Algebraic Reconstruction Technique (I)



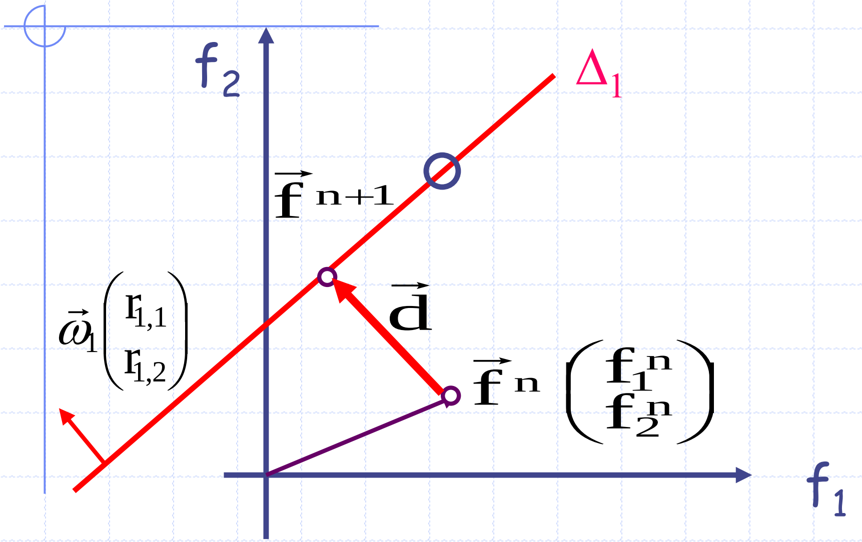


S. Kaczmarz
1895-1940

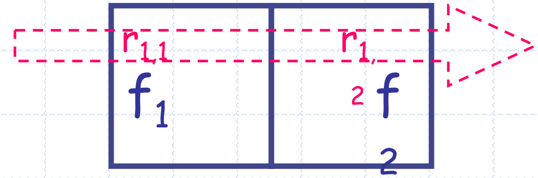
On construit une suite de coupes $\vec{f}^{\,n}$ en projetant chaque itéré sur l'un puis l'autre hyperplan.

Kaczmarz S. Angenährte Auflösung von Systemen linearer Gleichungen. *Bull Int Acad Pol Sci Lett A* 1937;35:355-7.

Algebraic Reconstruction Technique (II)



$\Delta_1 : p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2$



S. Kaczmarz
1895-1940

La distance d d'un point f^n à une droite Δ_1 est :

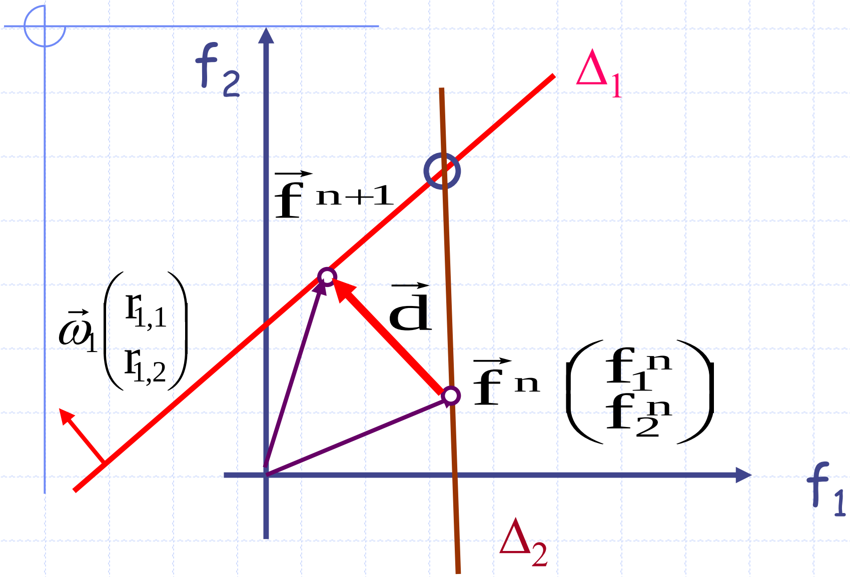
$$d = \frac{p_1 - \vec{f}^{\,n} \cdot \vec{\omega}_1}{\|\vec{\omega}_1\|}$$

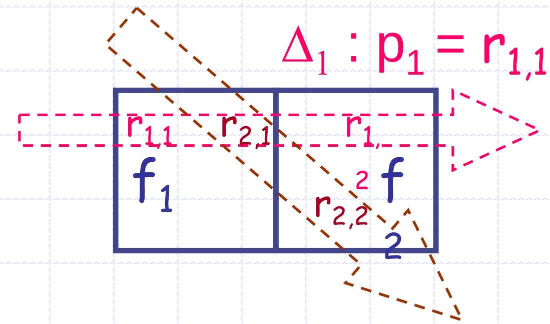
$$d = \frac{p_1 - \vec{f}^{\,n} \cdot \vec{\omega}_1}{\|\vec{\omega}_1\|} = \frac{p_1 - p_1^n}{\|\vec{\omega}_1\|}$$

$p_1^n = r_{1,1} f_1^n + r_{1,2} f_2^n$, projection qui serait mesurée si f^n était la solution

Kaczmarz S. Angenährte Auflösung von Systemen linearer Gleichungen. *Bull Int Acad Pol Sci Lett A* 1937;35:355-7.

Algebraic Reconstruction Technique (II)





$$\vec{f}^{\,n+1} = \vec{f}^{\,n} + d\,\frac{\vec{\omega}_1}{\|\vec{\omega}_1\|}$$

$$d = \frac{p_1 - \vec{f}^{\,n}\cdot\vec{\omega}_1}{\|\vec{\omega}_1\|}$$

p_1^n

$$\vec{f}^{\,n+1} = \vec{f}^{\,n} + \frac{p_1 - p_1^n}{\|\vec{\omega}_1\|^2}\,\vec{\omega}_1$$

$$\boxed{\vec{f}^{\,n+1} = \vec{f}^{\,n} + R^* \,(p_1 - p_1^n)}$$

S. Kaczmarz
1895-1940

Kaczmarz S. Angenährte Auflösung von Systemen linearer Gleichungen. *Bull Int Acad Pol Sci Lett A* 1937;35:355-7.

Algebraic Reconstruction Technique (III)

0	0	0	← 45 - 0 = 15 + 15 + 15
0	0	0	← 90 - 0 = 30 + 30 + 30
0	0	0	← 45 - 0
45	90	45	

⇨

15	15	15
30	30	30
15	15	15

	45	90	45
-	60	60	60
	-15	30	-15

10	25	10
25	40	25
10	25	10

$$\boxed{\vec{\mathbf{f}}^{\,n+1} = \vec{\mathbf{f}}^{\,n} + R^{*}\ (p_1 - p_1^n)}$$

« illustration non contractuelle !... »

Conclusion

$

G. Hounsfield 1919-2004

J. Radon
1887-1956

S. Kaczmarz
1895-1940

$$\hat{p}_{\vec{\theta}}(\sigma)=\hat{f}(\sigma.\vec{\theta})$$

$$f(\vec{x})=(R^{*}\,p^{f})(\vec{x})$$

$$\vec{f}^{\,n+1}=\vec{f}^{\,n}+R^{*}(p_{1}-p_{1}^{n})$$