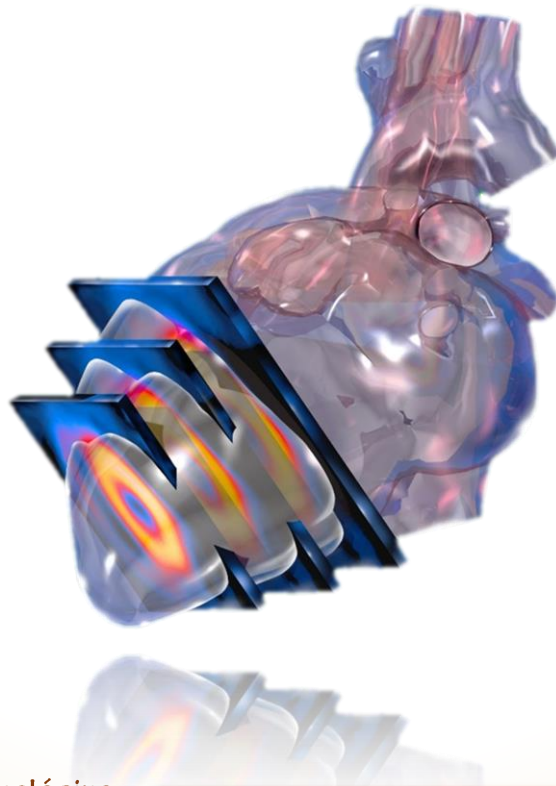


Tomographie d'émission mono-photonique

— 0 —



Imagerie médicale

Imagerie anatomique



Imagerie médicale

Imagerie anatomique

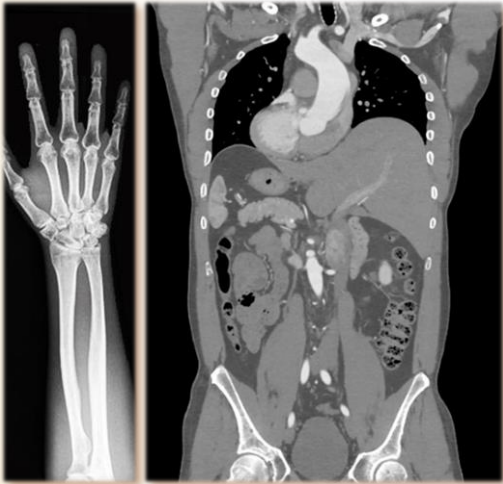
Radiographie X



Imagerie médicale

Imagerie anatomique

Radiographie X



Imagerie médicale

Imagerie anatomique

Radiographie X



IRM



Imagerie médicale

Imagerie anatomique

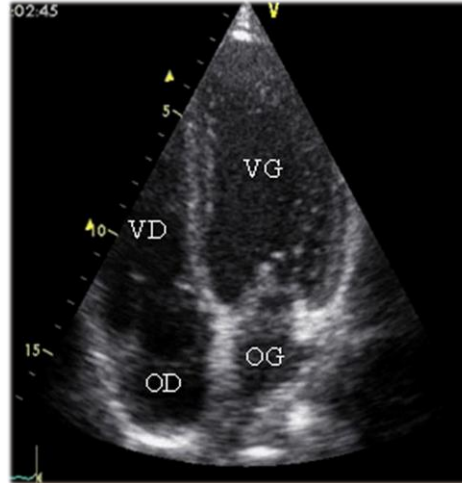
Radiographie X



IRM



Echographie



Imagerie médicale

Imagerie anatomique

Imagerie
métabolique



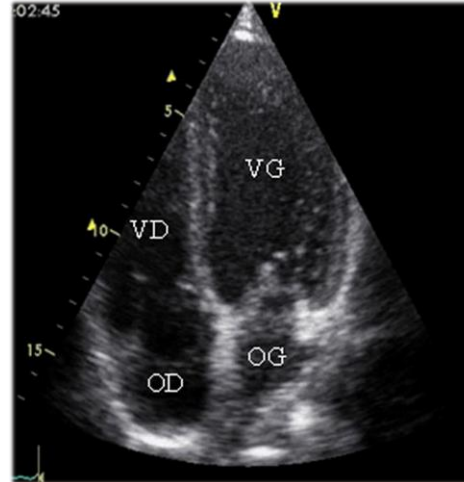
Radiographie X



IRM



Echographie



Imagerie médicale

Imagerie anatomique



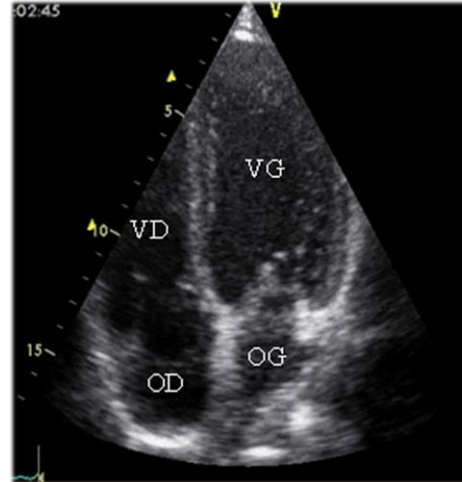
Radiographie X



IRM



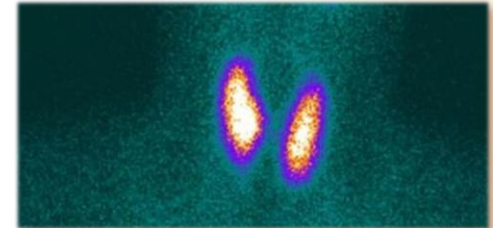
Echographie



Imagerie métabolique



Scintigraphie



Imagerie médicale

Imagerie anatomique



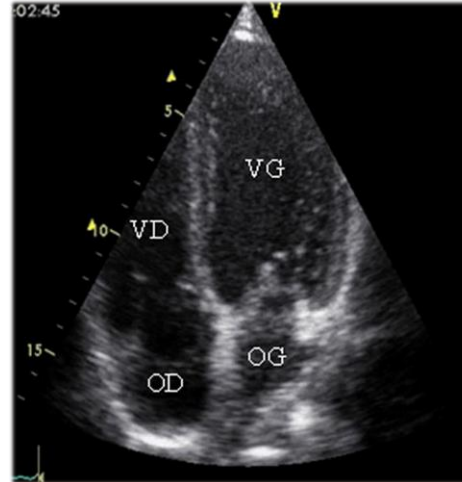
Radiographie X



IRM



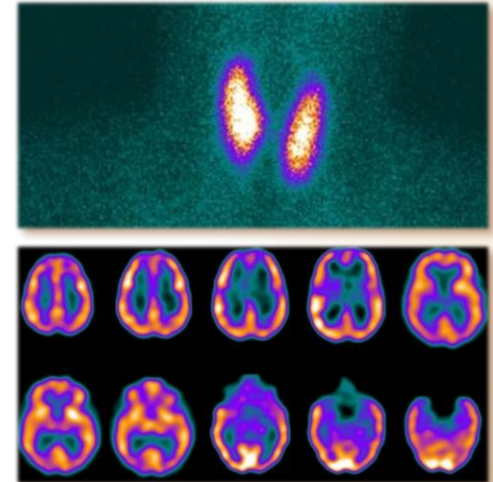
Echographie



Imagerie métabolique



Scintigraphie



Imagerie médicale

Imagerie anatomique

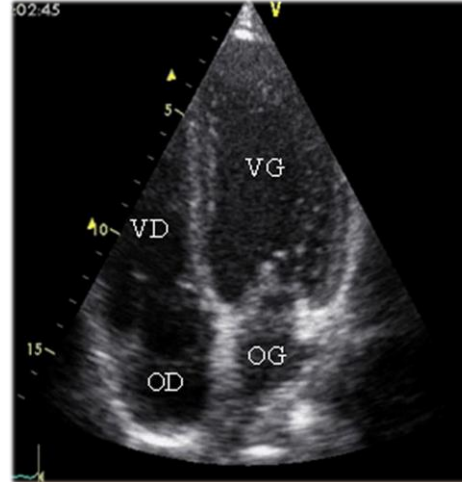
Radiographie X



IRM



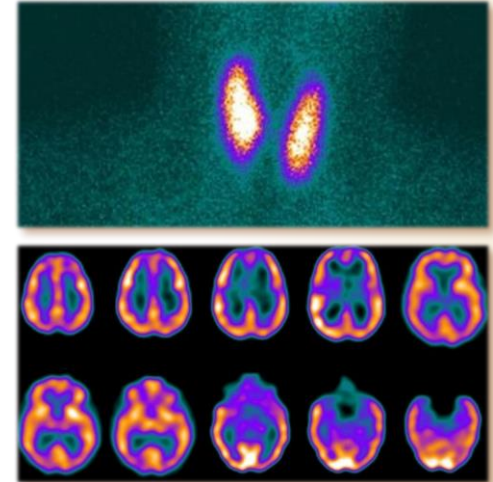
Echographie



Imagerie
métabolique



Scintigraphie



Imagerie fonctionnelle

Imagerie médicale

Ceci est un pied

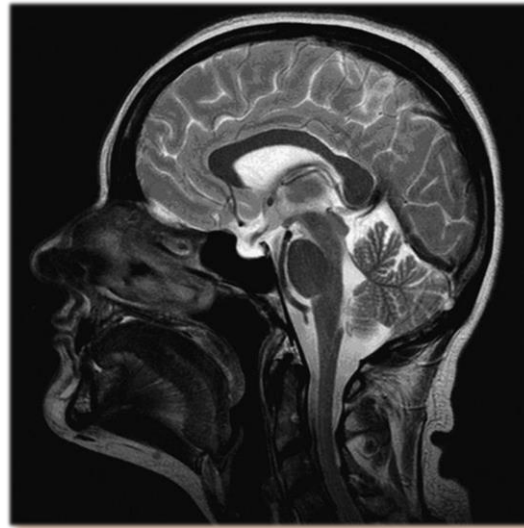


Imagerie médicale

Ceci est un pied



Ceci est un cerveau

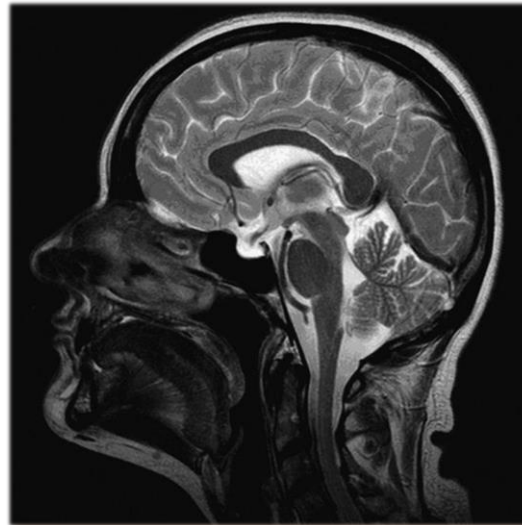


Imagerie médicale

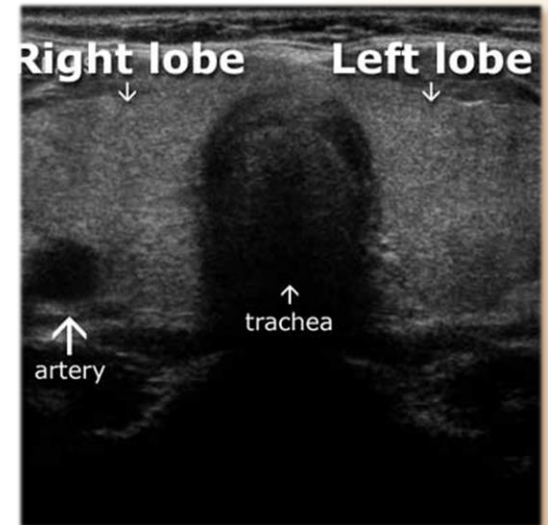
Ceci est un pied



Ceci est un cerveau

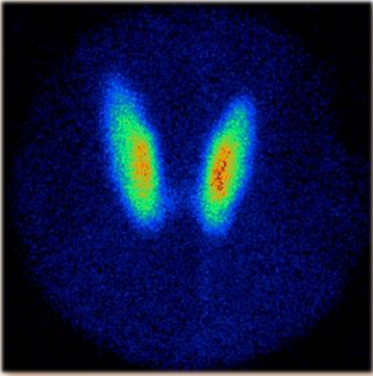


Ceci est une thyroïde



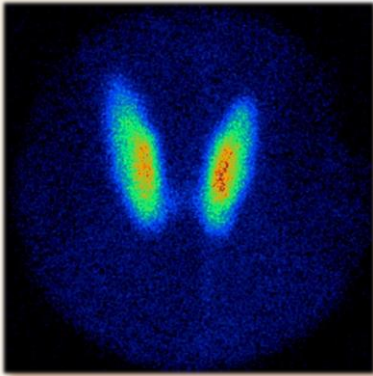
Imagerie médicale

Ceci n'est PAS une thyroïde

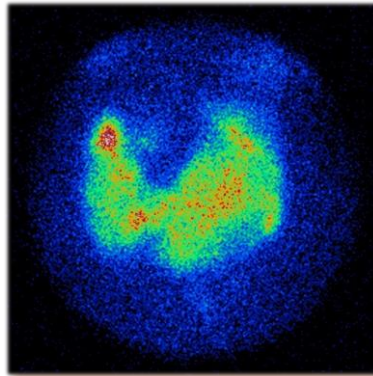


Imagerie médicale

Ceci n'est PAS une thyroïde

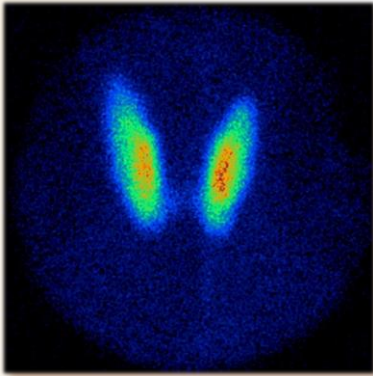


Ceci non plus

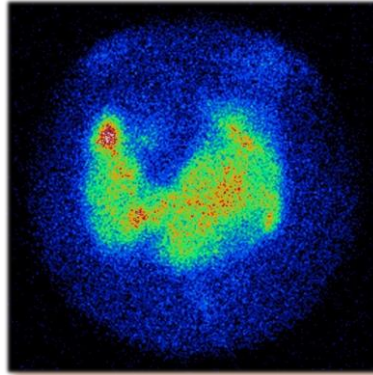


Imagerie médicale

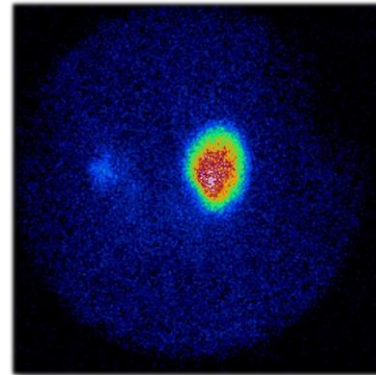
Ceci n'est PAS une thyroïde



Ceci non plus

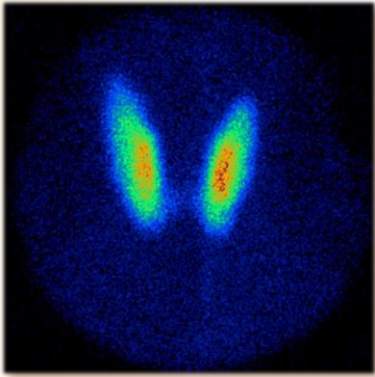


Pas plus

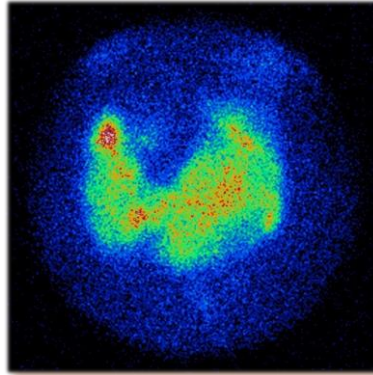


Imagerie médicale

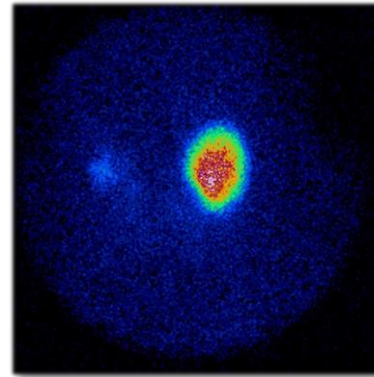
Ceci n'est PAS une thyroïde



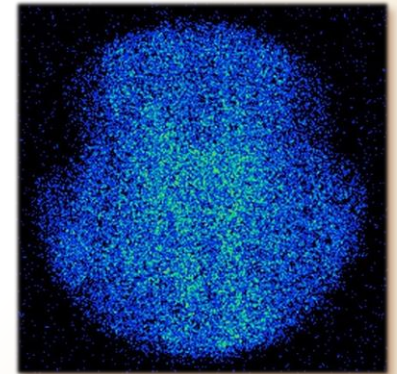
Ceci non plus



Pas plus

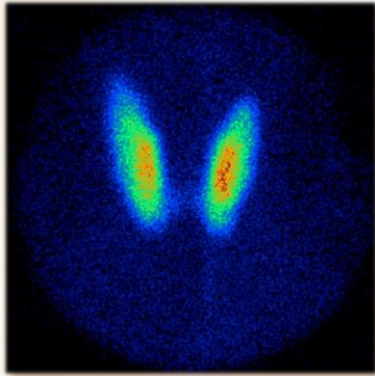


...

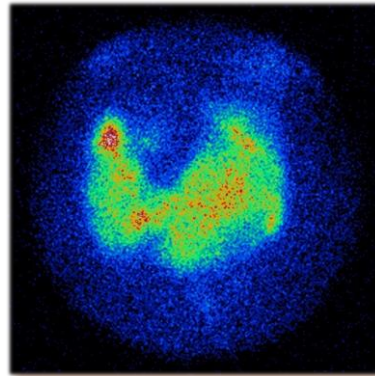


Imagerie médicale

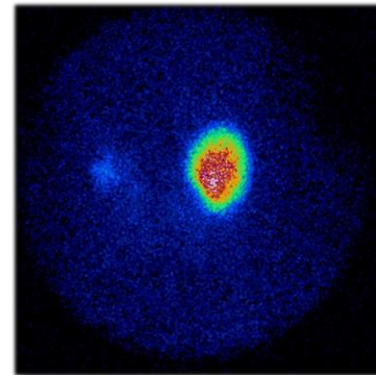
Ceci n'est PAS une thyroïde



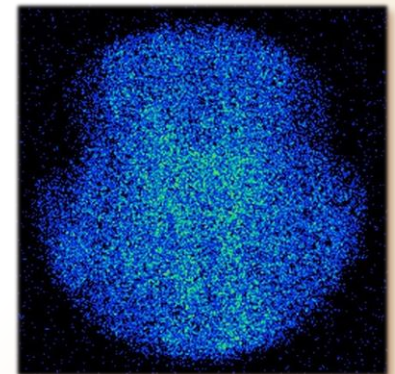
Ceci non plus



Pas plus

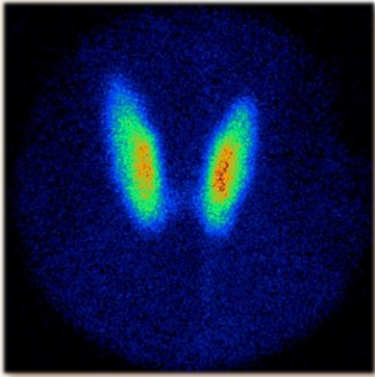


...



Scintigraphie =
Imagerie fonctionnelle

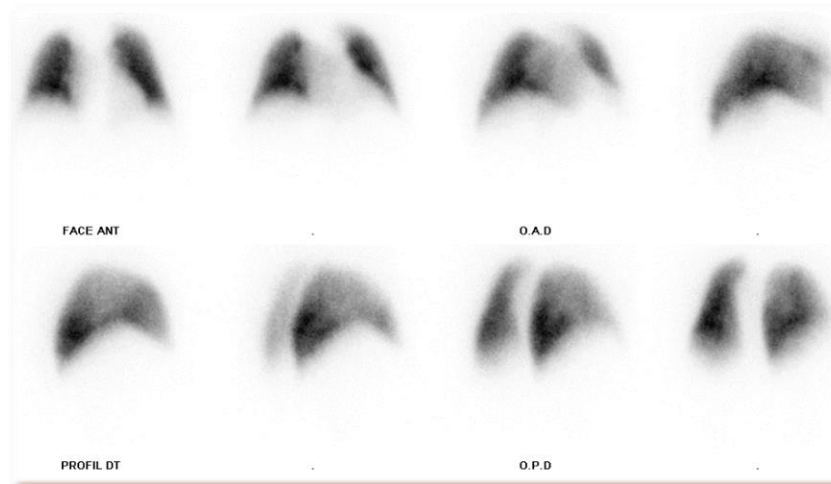
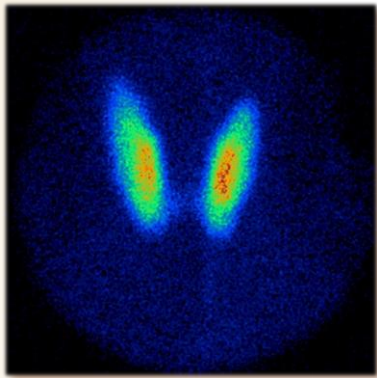
Imagerie médicale



>>> Imagerie planaire

**Scintigraphie =
Imagerie fonctionnelle**

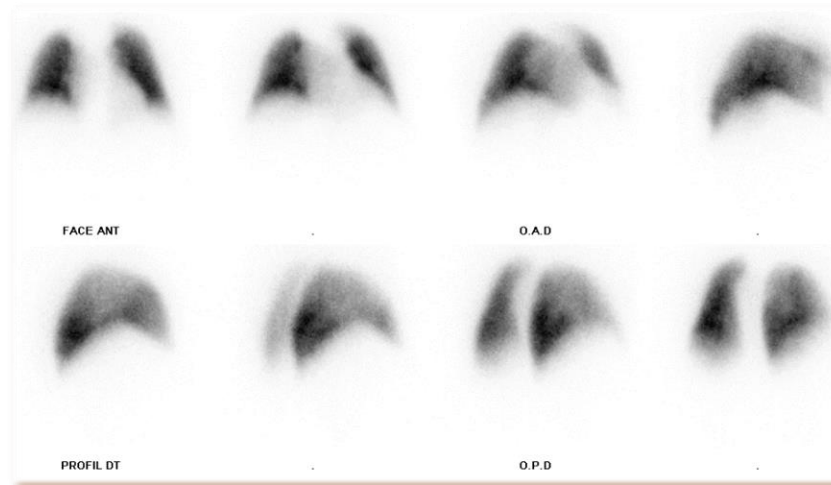
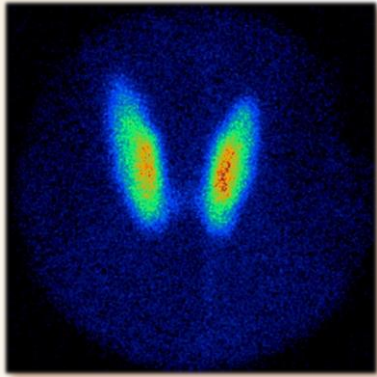
Imagerie médicale



>>> Imagerie planaire

**Scintigraphie =
Imagerie fonctionnelle**

Imagerie médicale

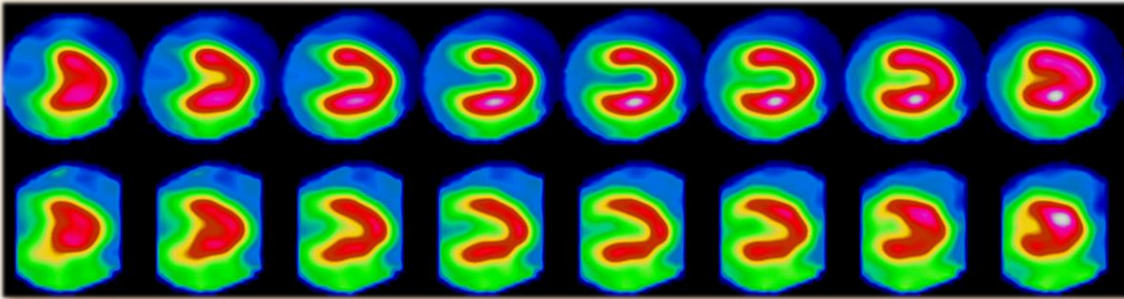


>>> Imagerie planaire

Scintigraphie =
Imagerie fonctionnelle

Imagerie médicale

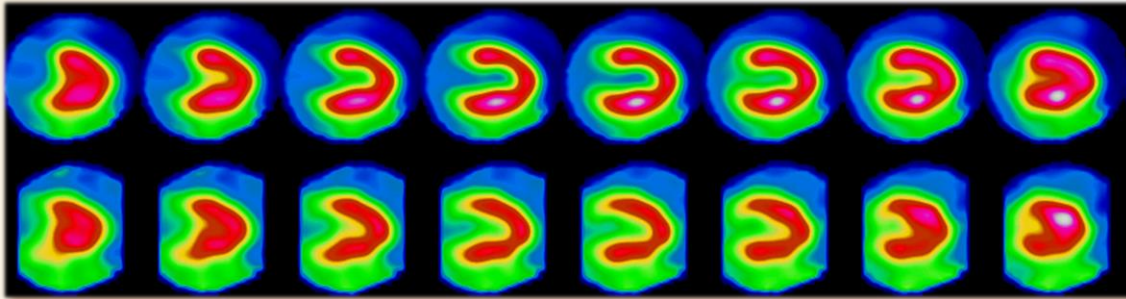
>>> Imagerie en coupes = TEMP / SPECT



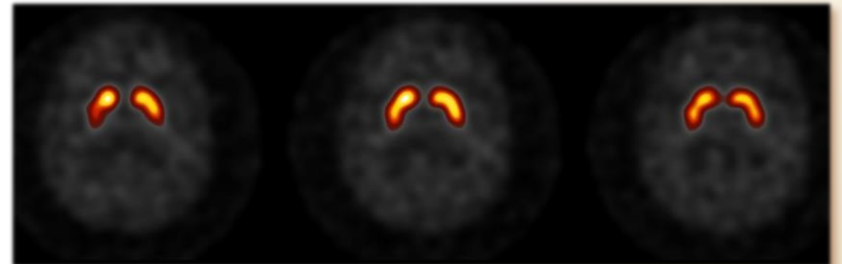
Scintigraphie =
Imagerie fonctionnelle

Imagerie médicale

>>> Imagerie en coupes = TEMP / SPECT

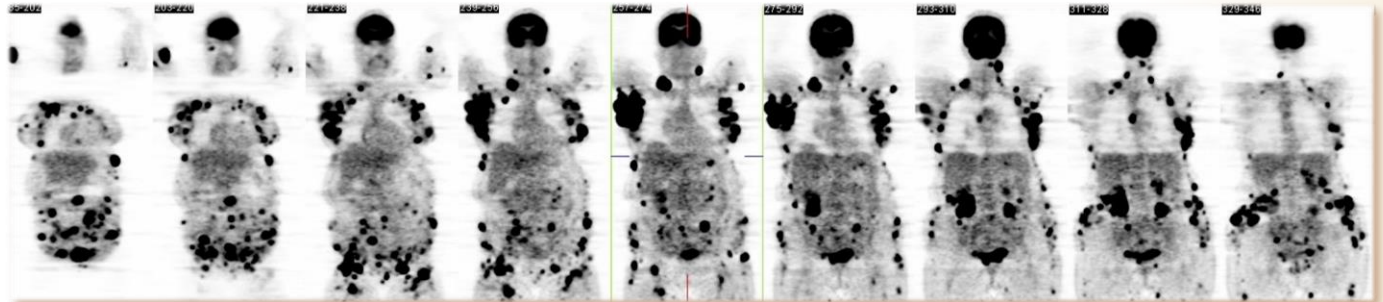
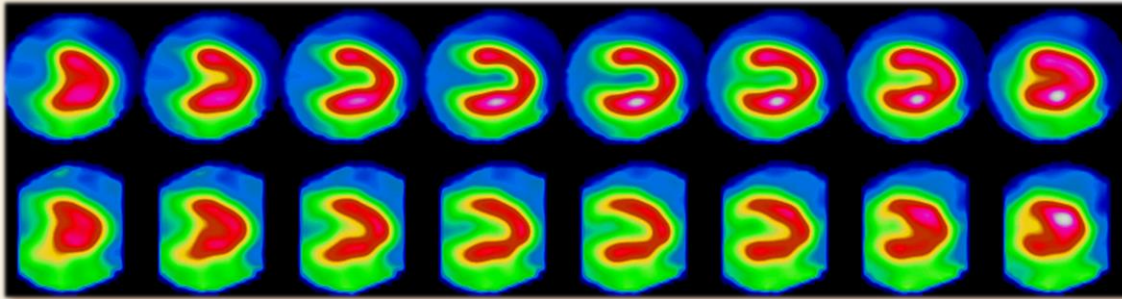


Scintigraphie =
Imagerie fonctionnelle

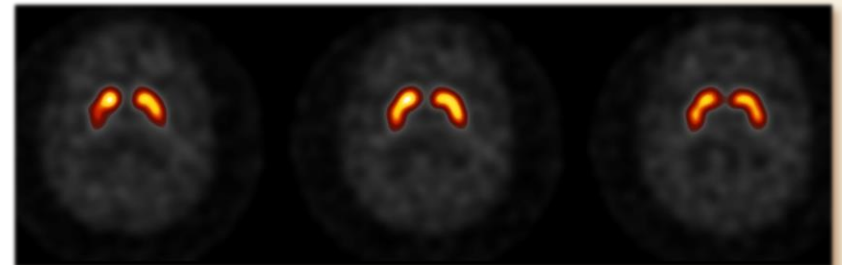


Imagerie médicale

>>> Imagerie en coupes = TEMP / SPECT

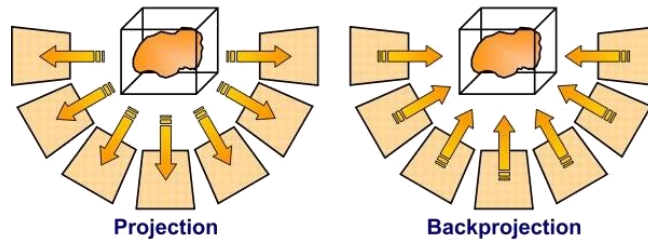


Scintigraphie =
Imagerie fonctionnelle



TEMP (SPECT)

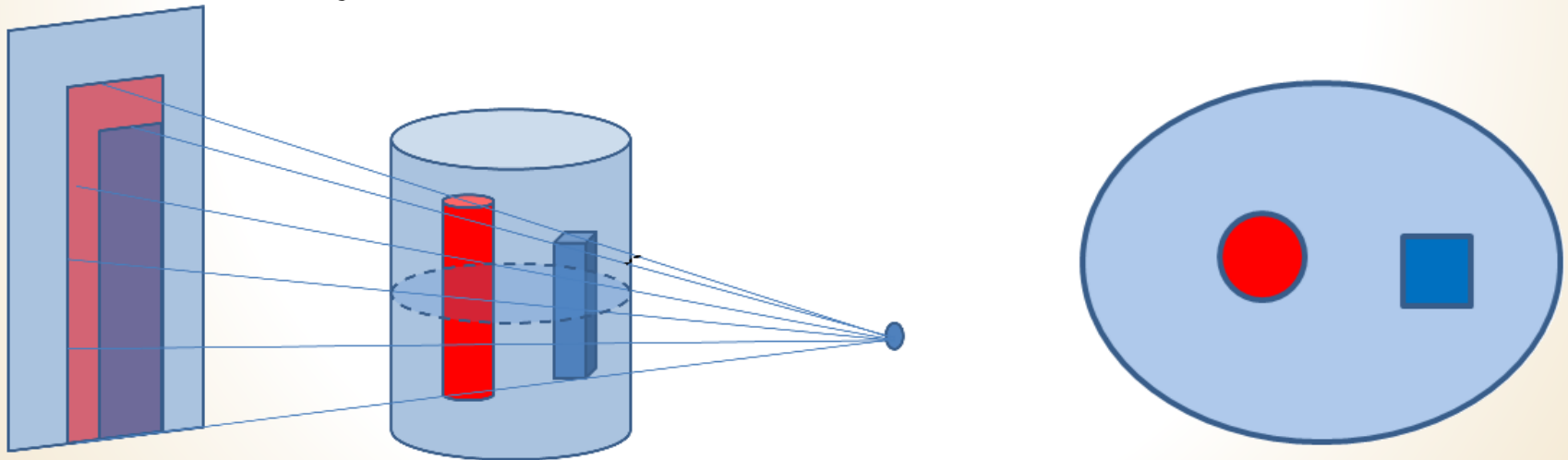
Tomographie d'émission mono-photonique



Projections



Coupes

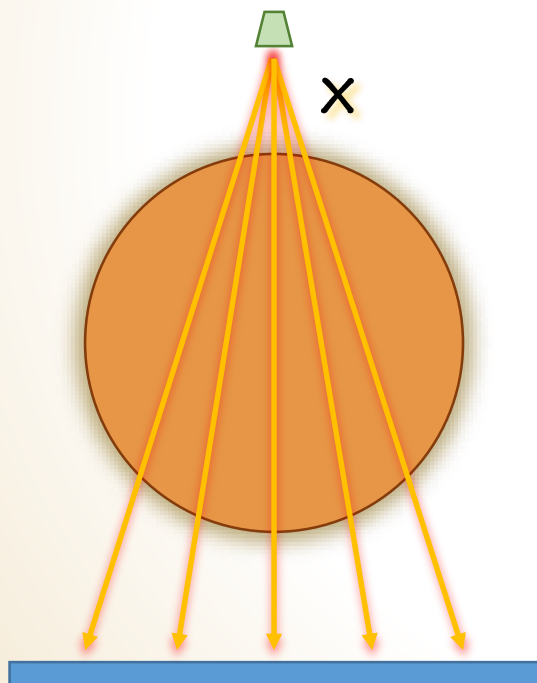


TEMP (SPECT)

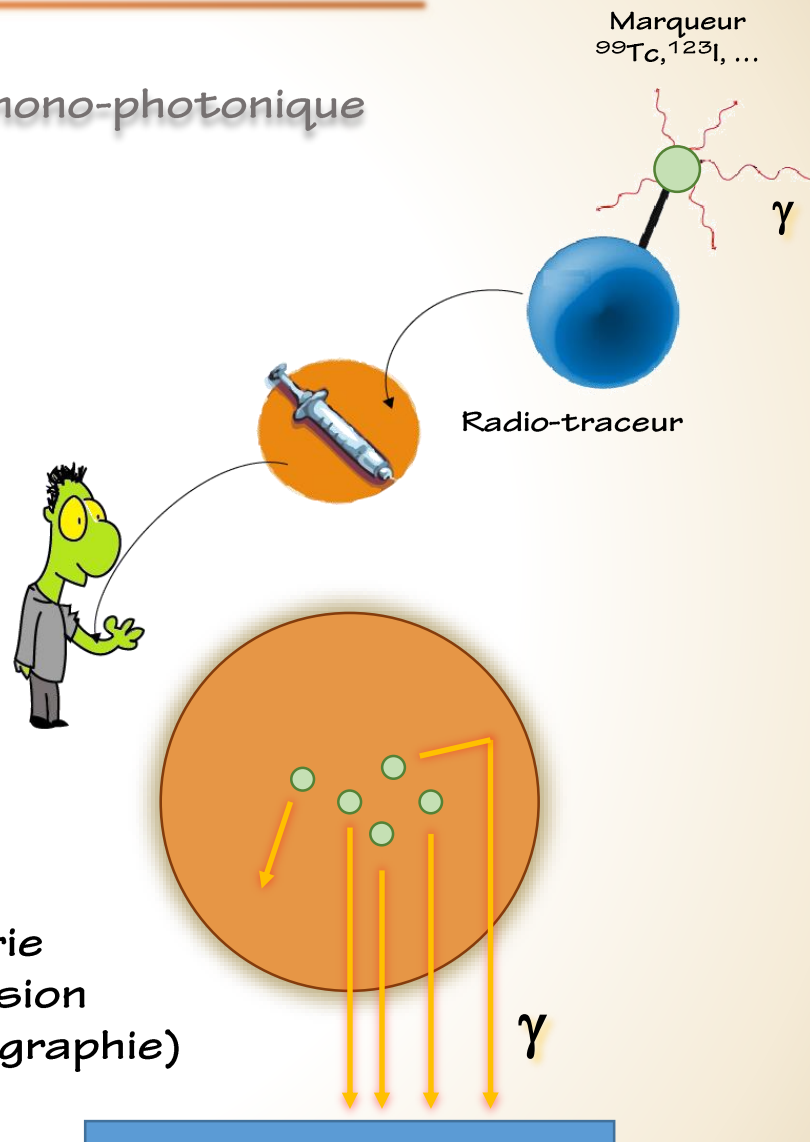
Tomographie d'émission mono-photonique

= tomo-scintigraphie

Imagerie de transmission



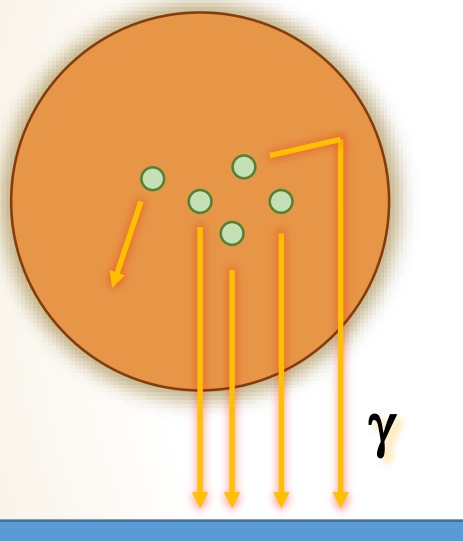
Imagerie d'émission (scintigraphie)



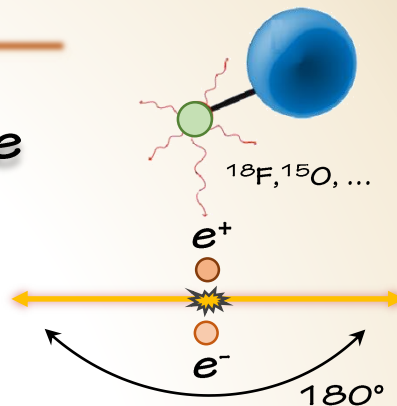
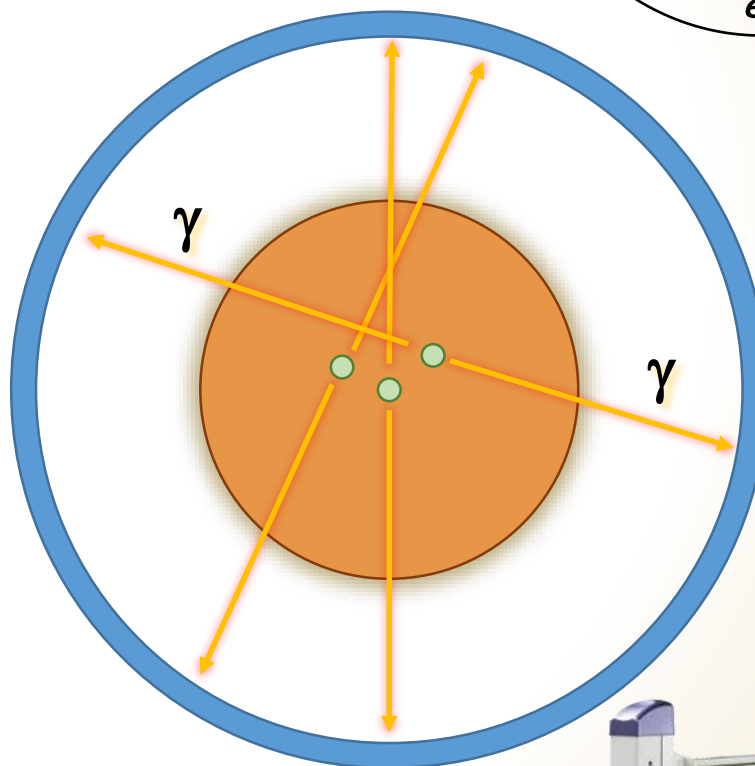
TEMP (SPECT)

Tomographie d'émission mono-photonique

SPECT

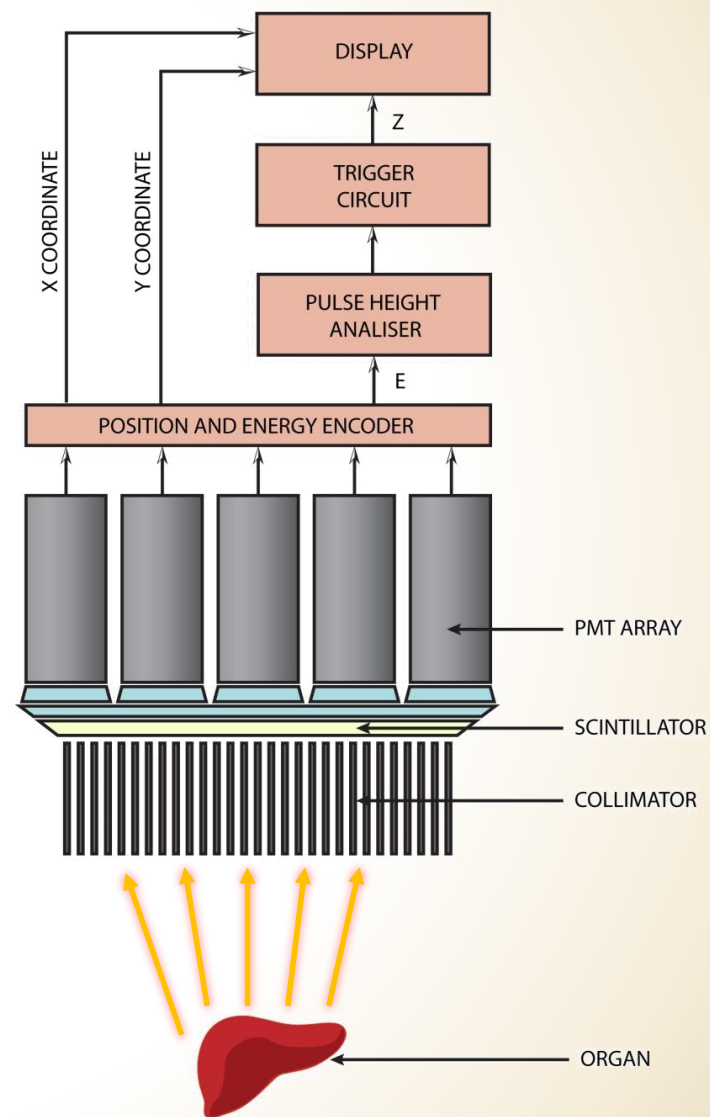
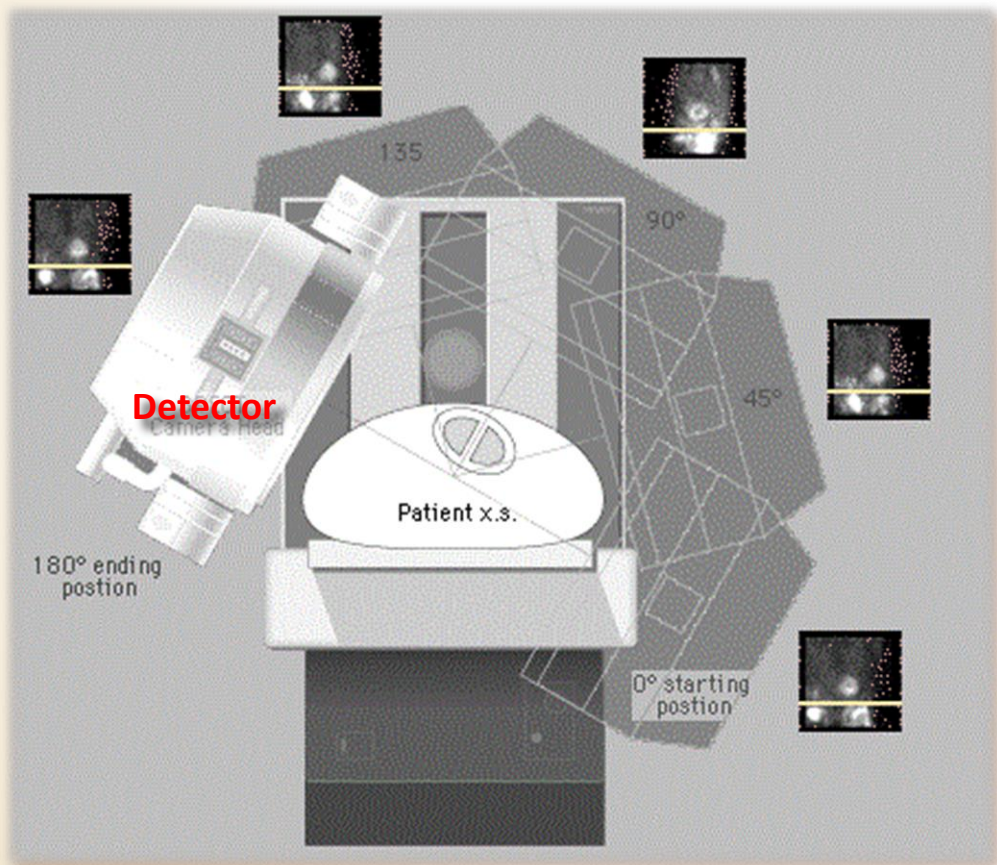


TEP



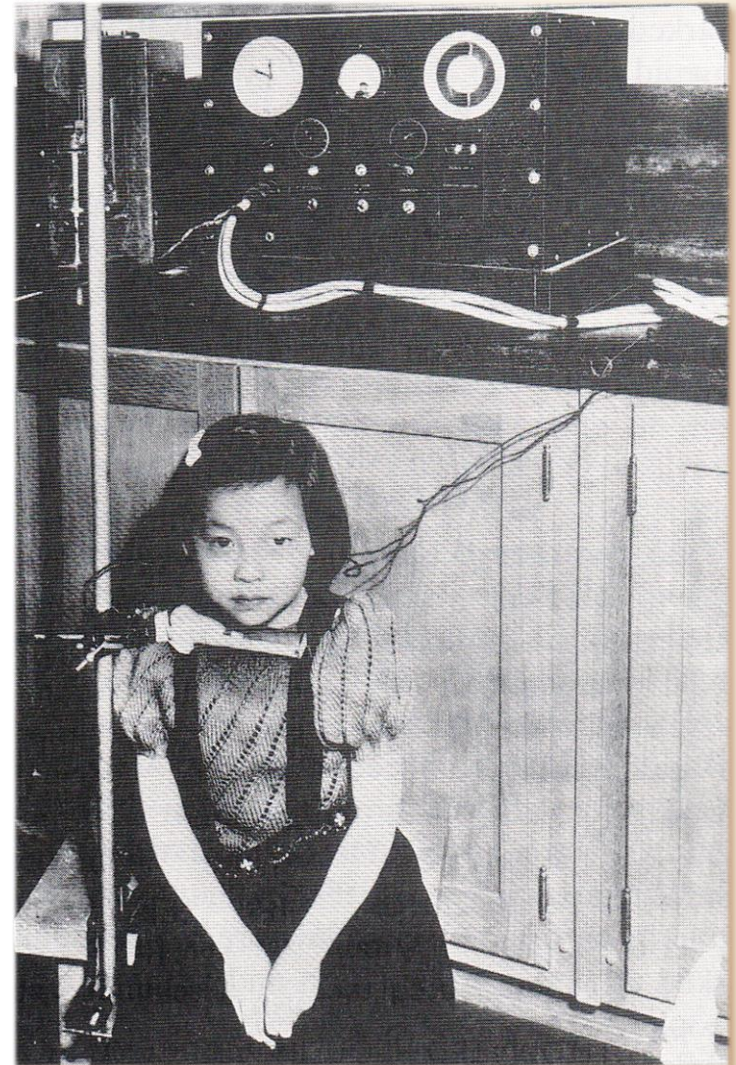
Acquisition

Gamma camera d'Anger



Acquisition

Gamma caméra d'Anger
(1958)



Acquisition

Gamma caméra d'Anger
(2010)



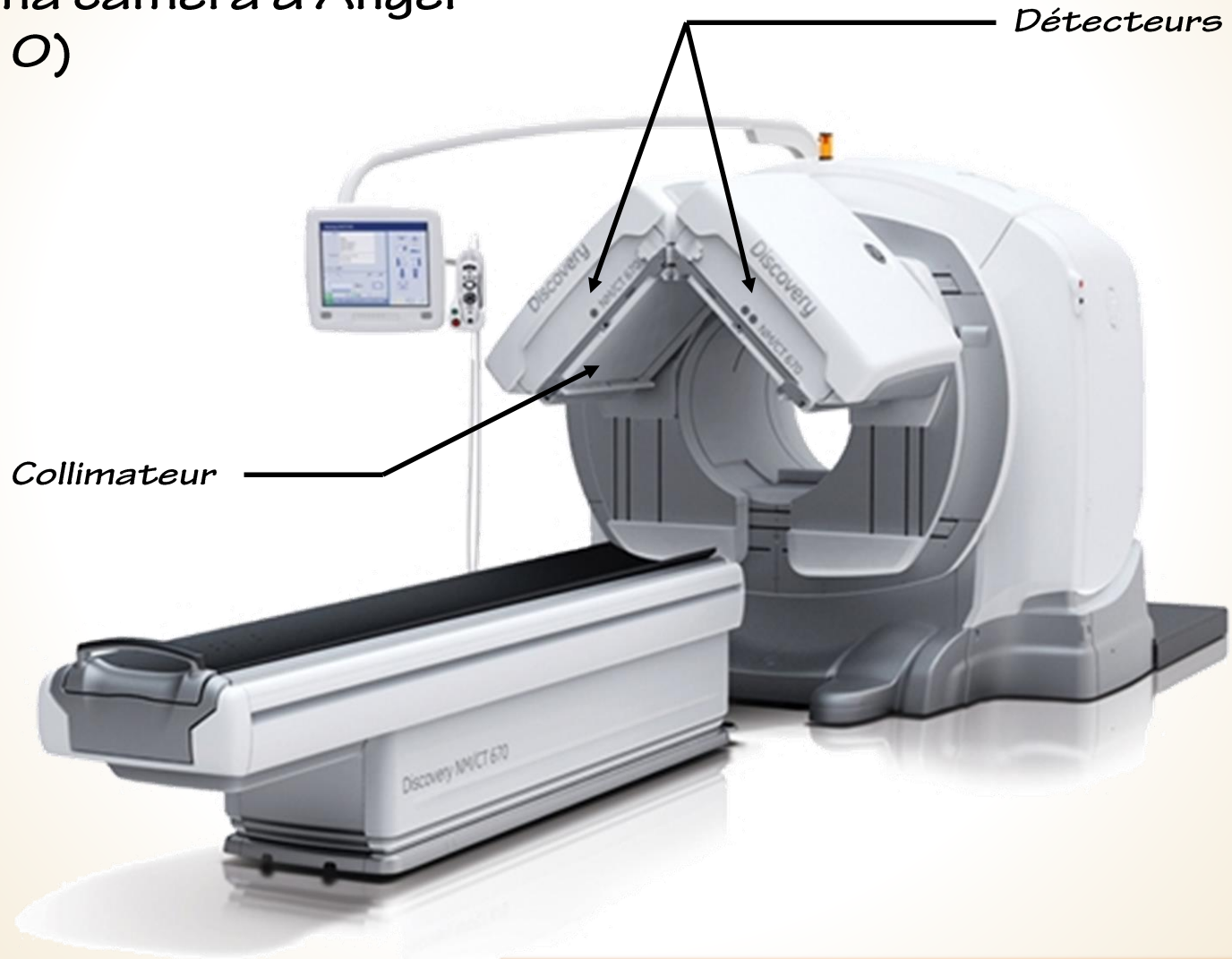
Acquisition

Gamma caméra d'Anger
(2010)



Acquisition

Gamma caméra d'Anger
(2010)



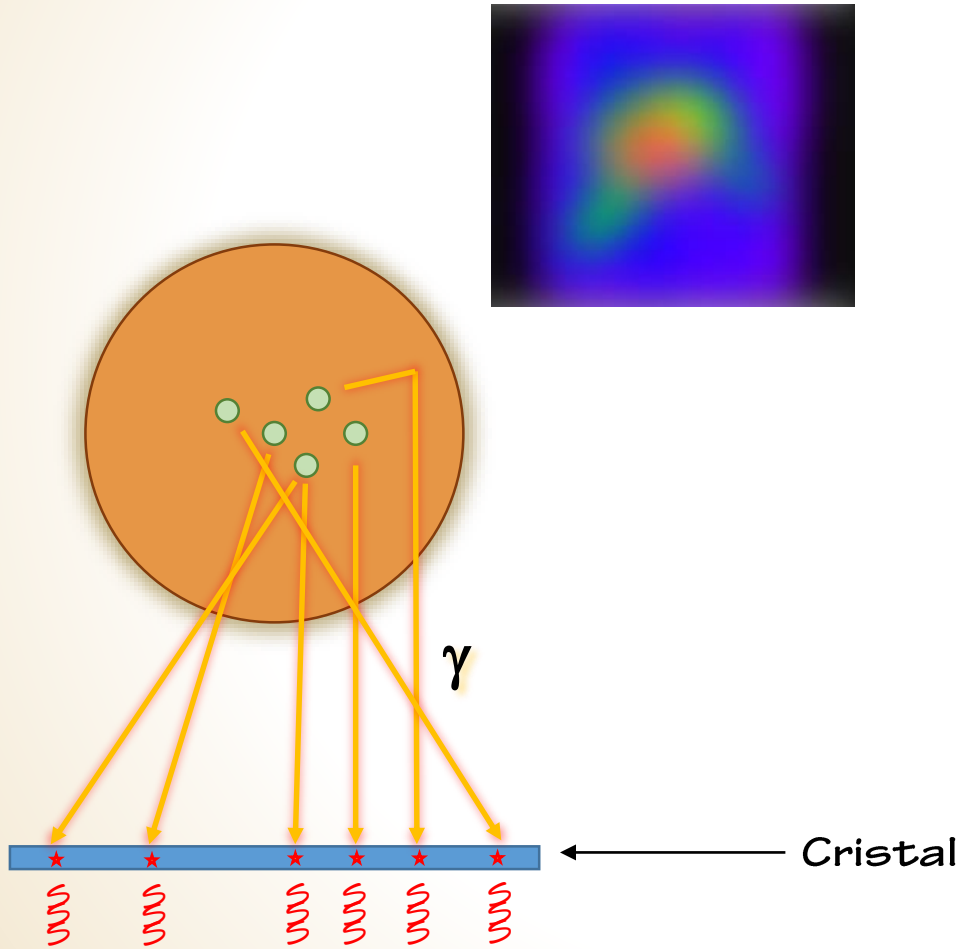
Acquisition

Gamma caméra d'Anger
(2010)



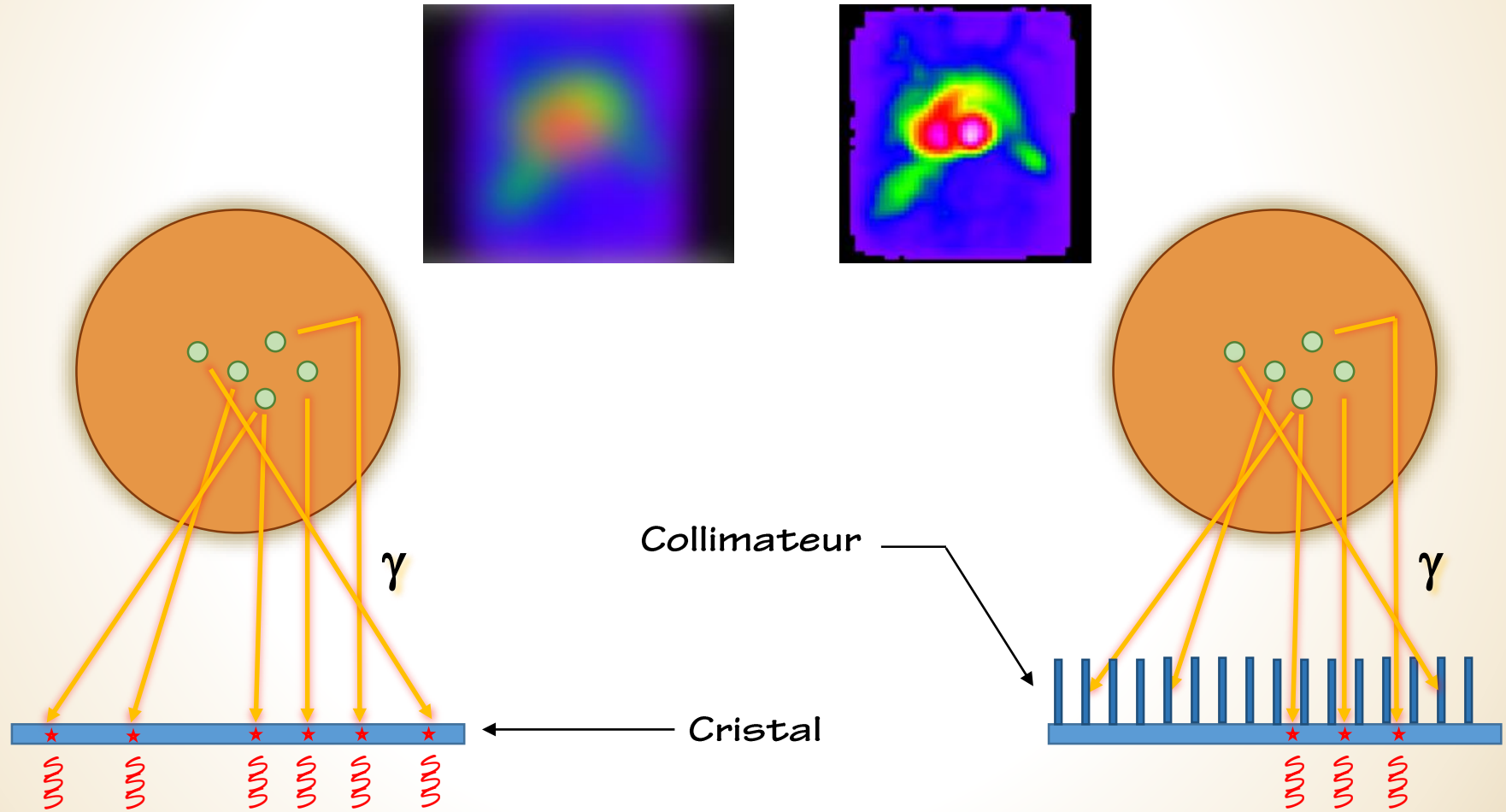
Acquisition

■ Collimation



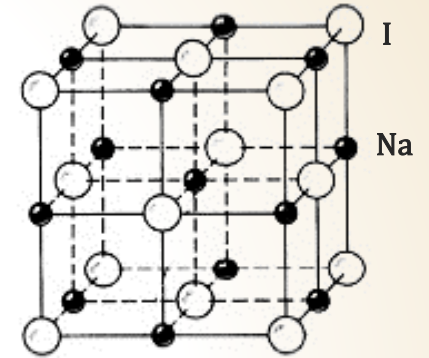
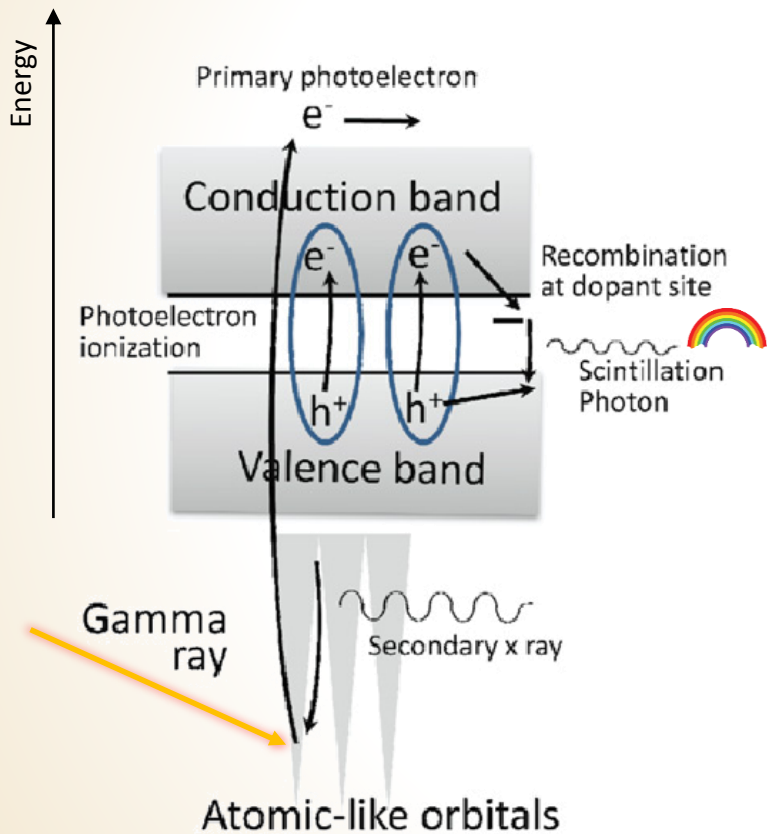
Acquisition

■ Collimation

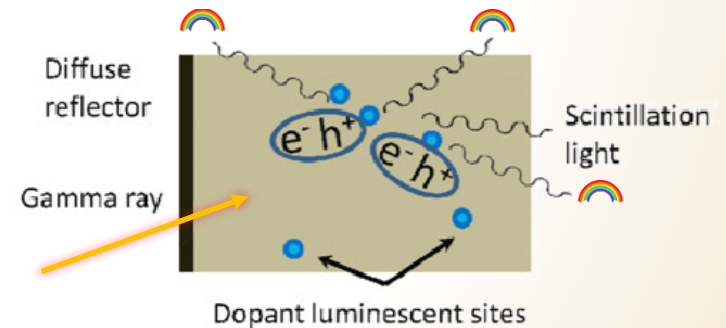


Acquisition

■ Scintillation

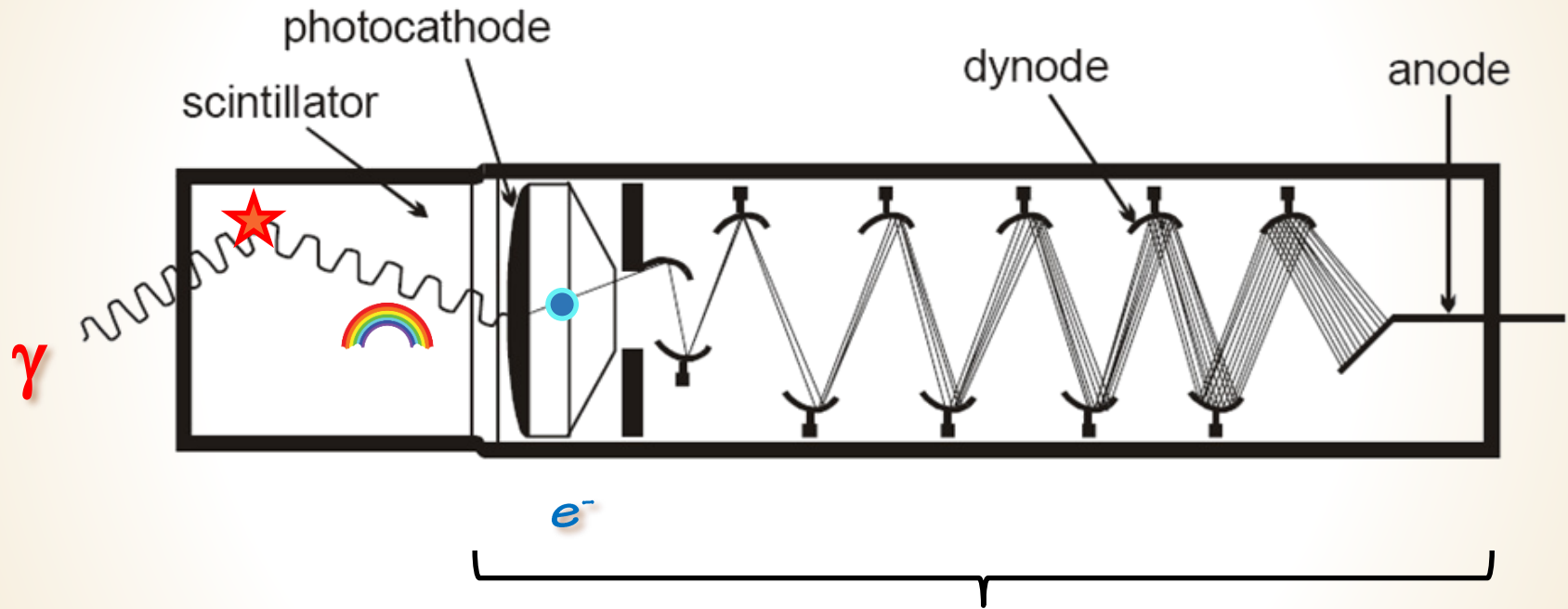


NaI(Tl)



Acquisition

■ Amplification



photomultiplier tube

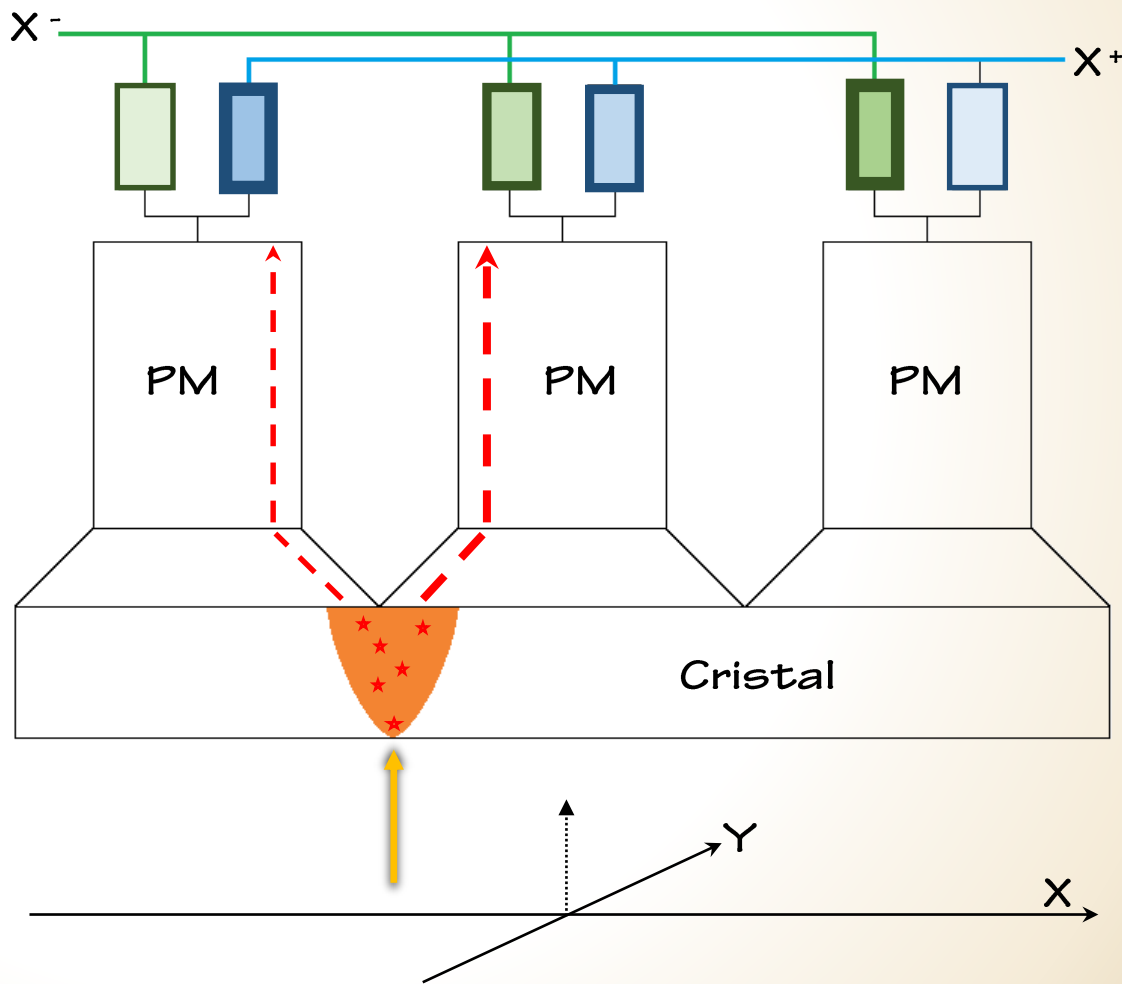
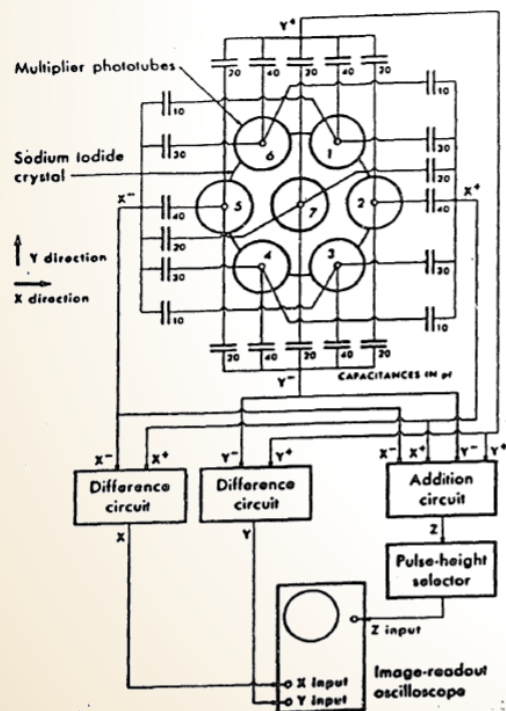
Acquisition

■ Localisation

$$X = X^+ - X^-$$

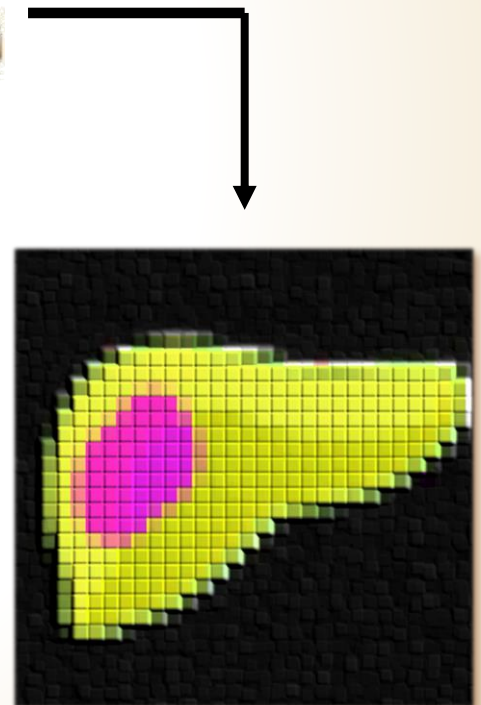
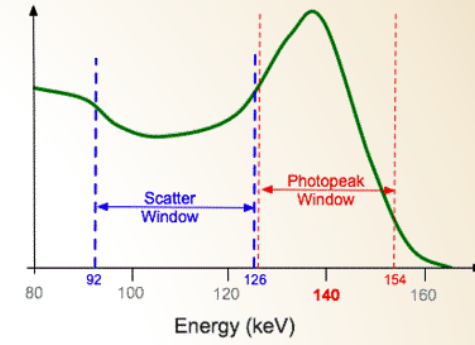
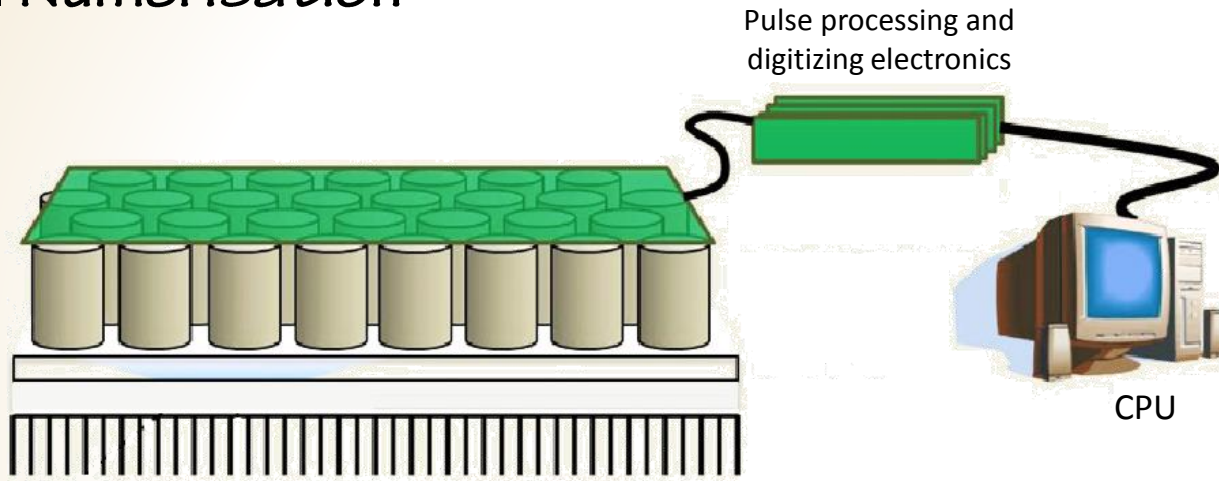
$$Y = Y^+ - Y^-$$

$$E = X^+ + X^- + Y^+ + Y^-$$

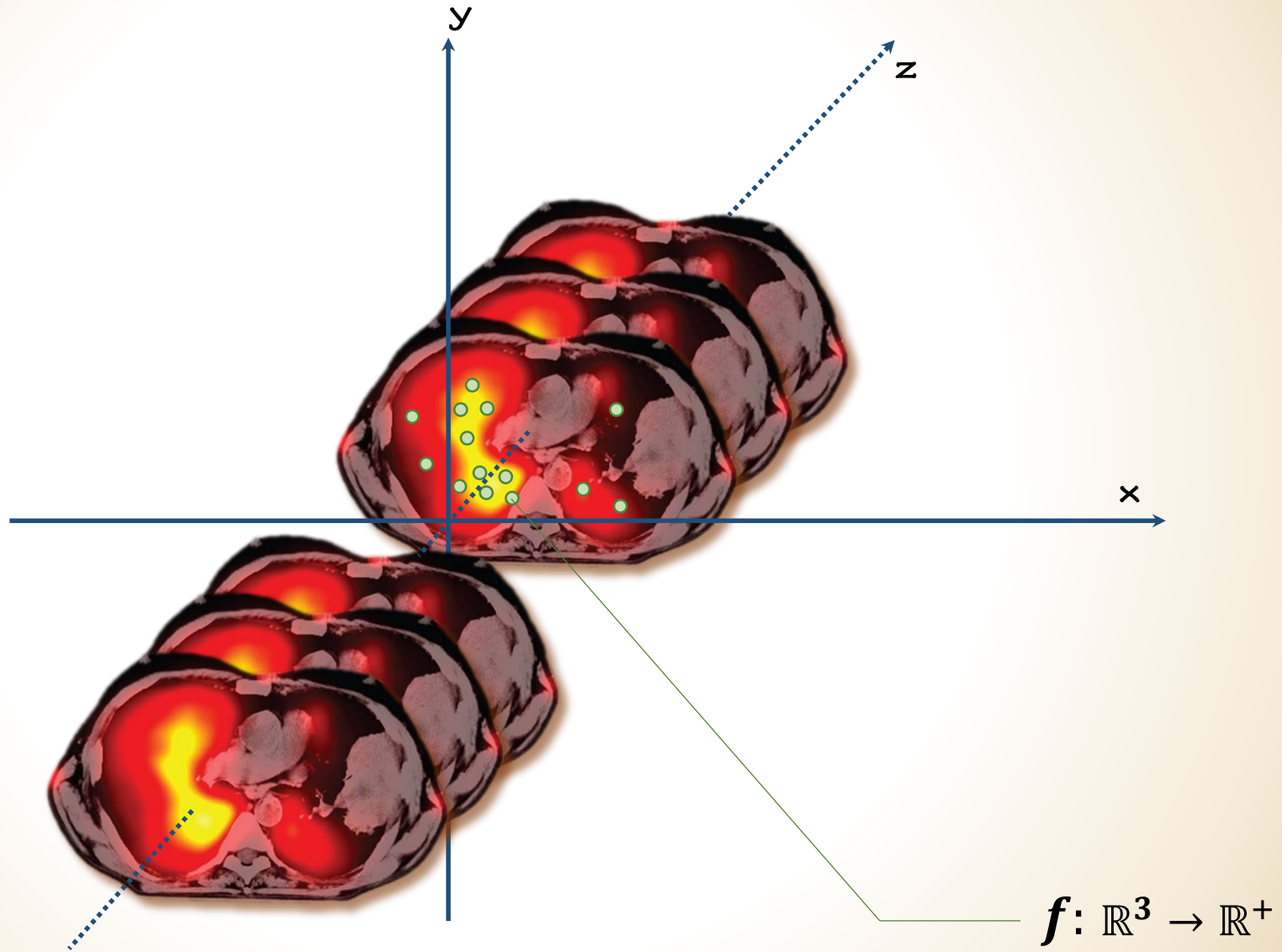


Acquisition

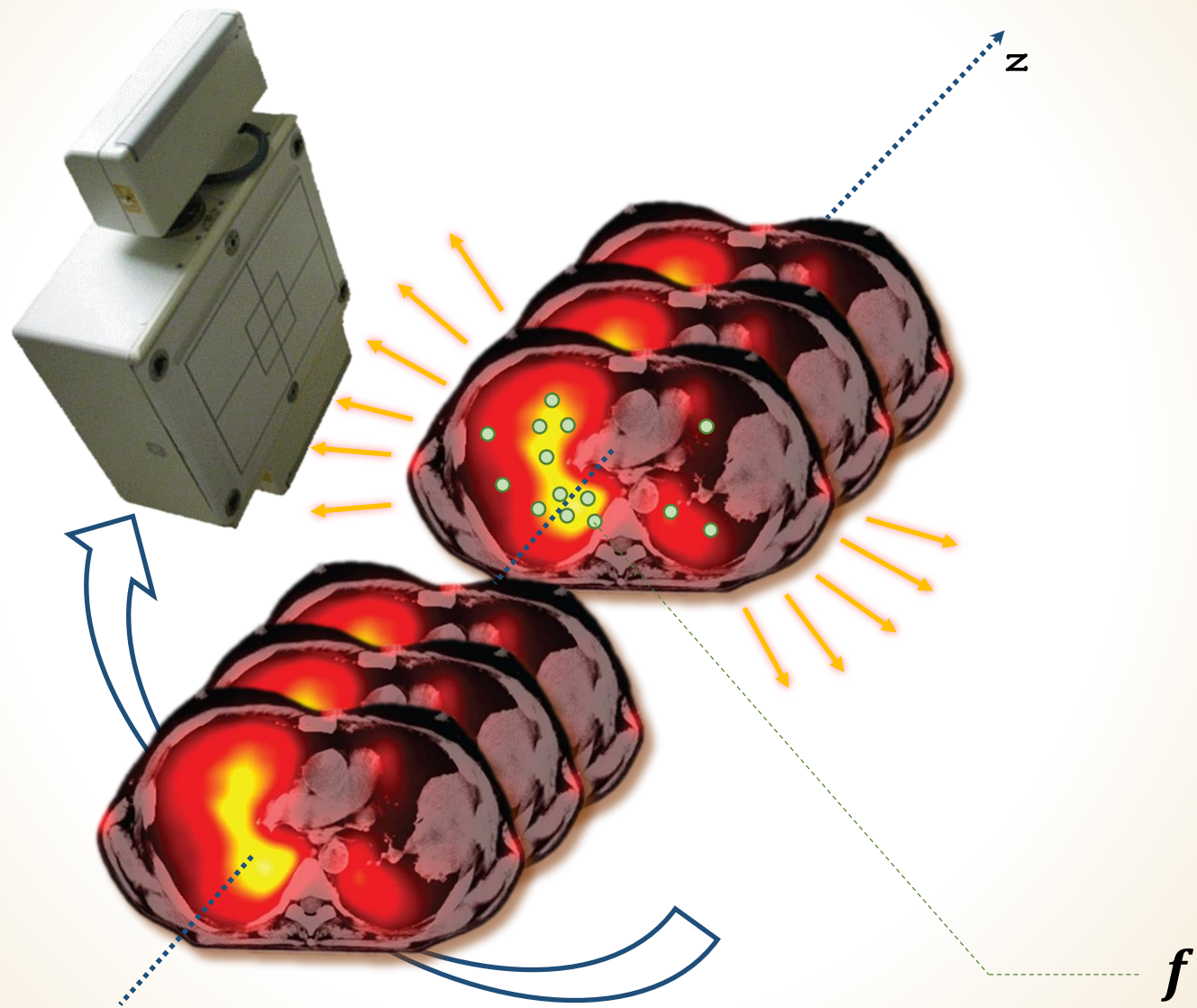
■ Numérisation



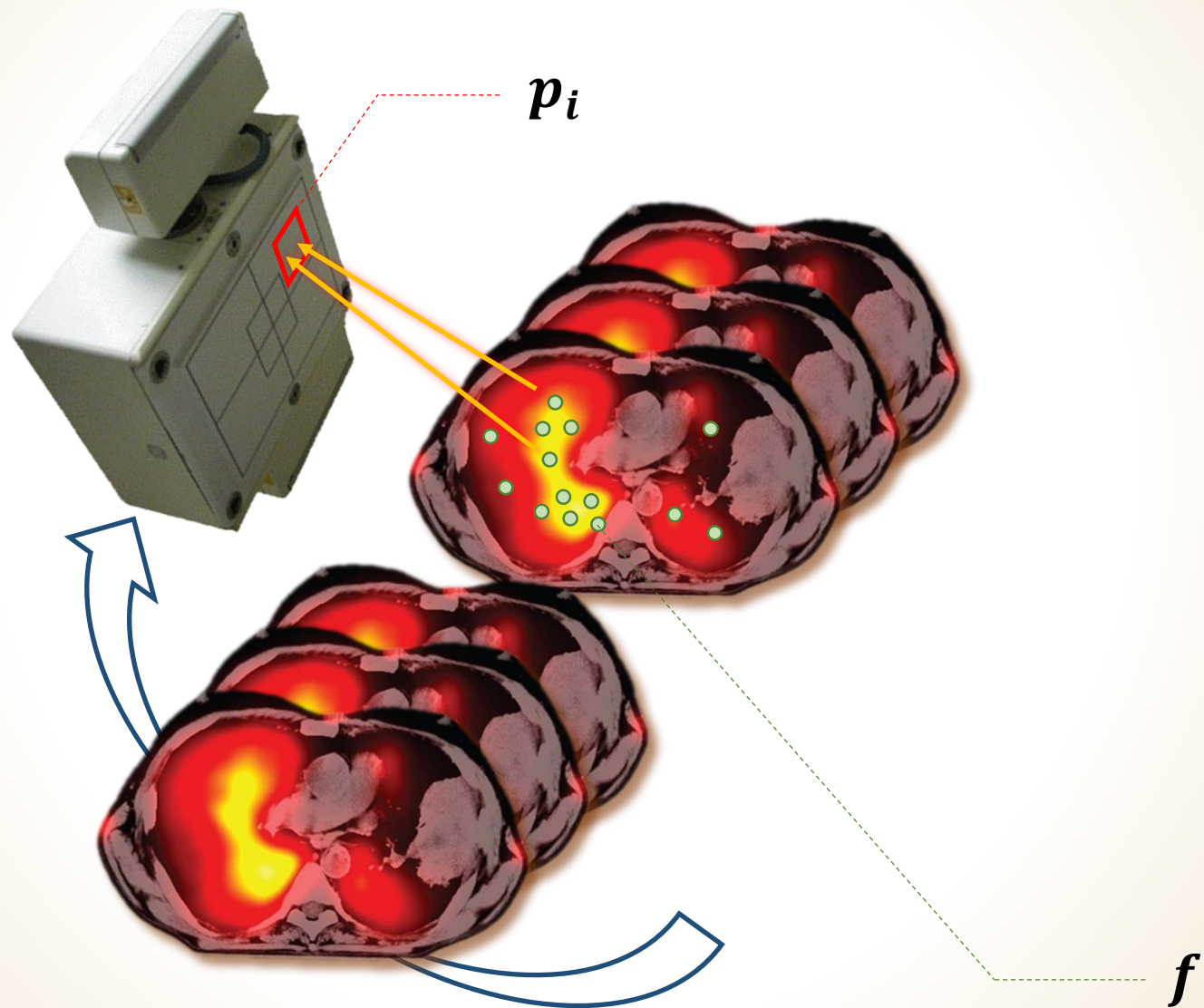
Problème tomographique



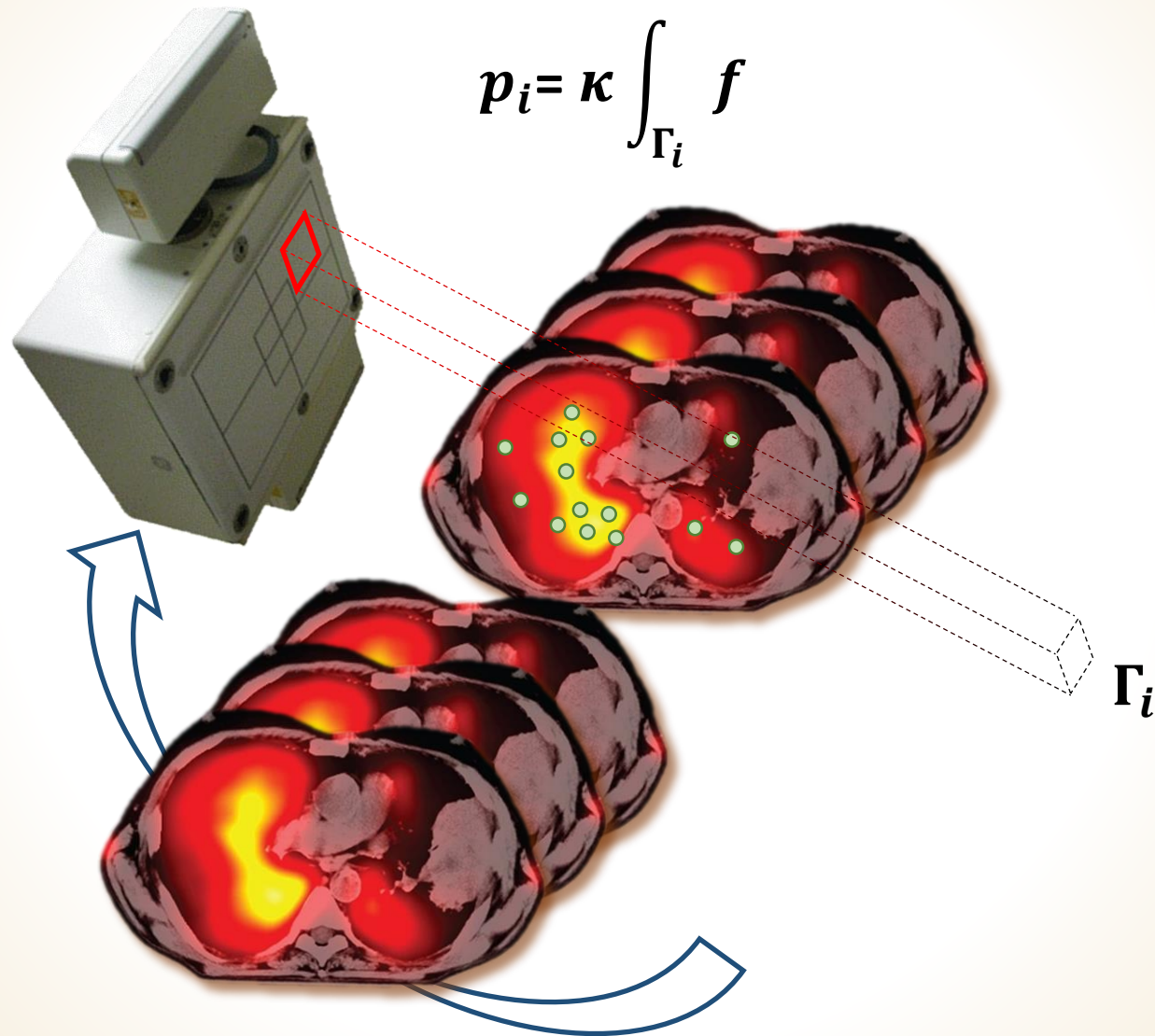
Problème tomographique



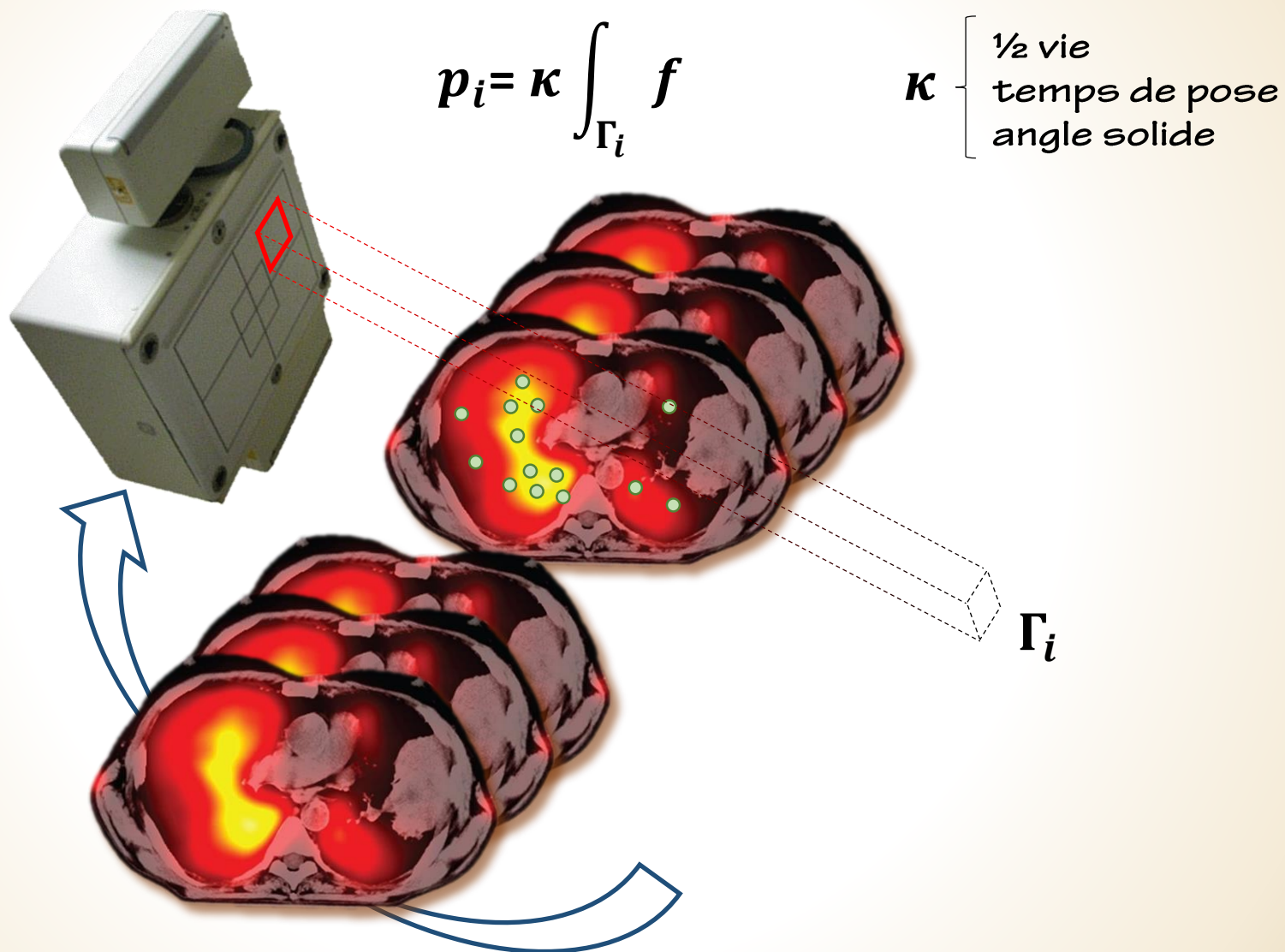
Problème tomographique



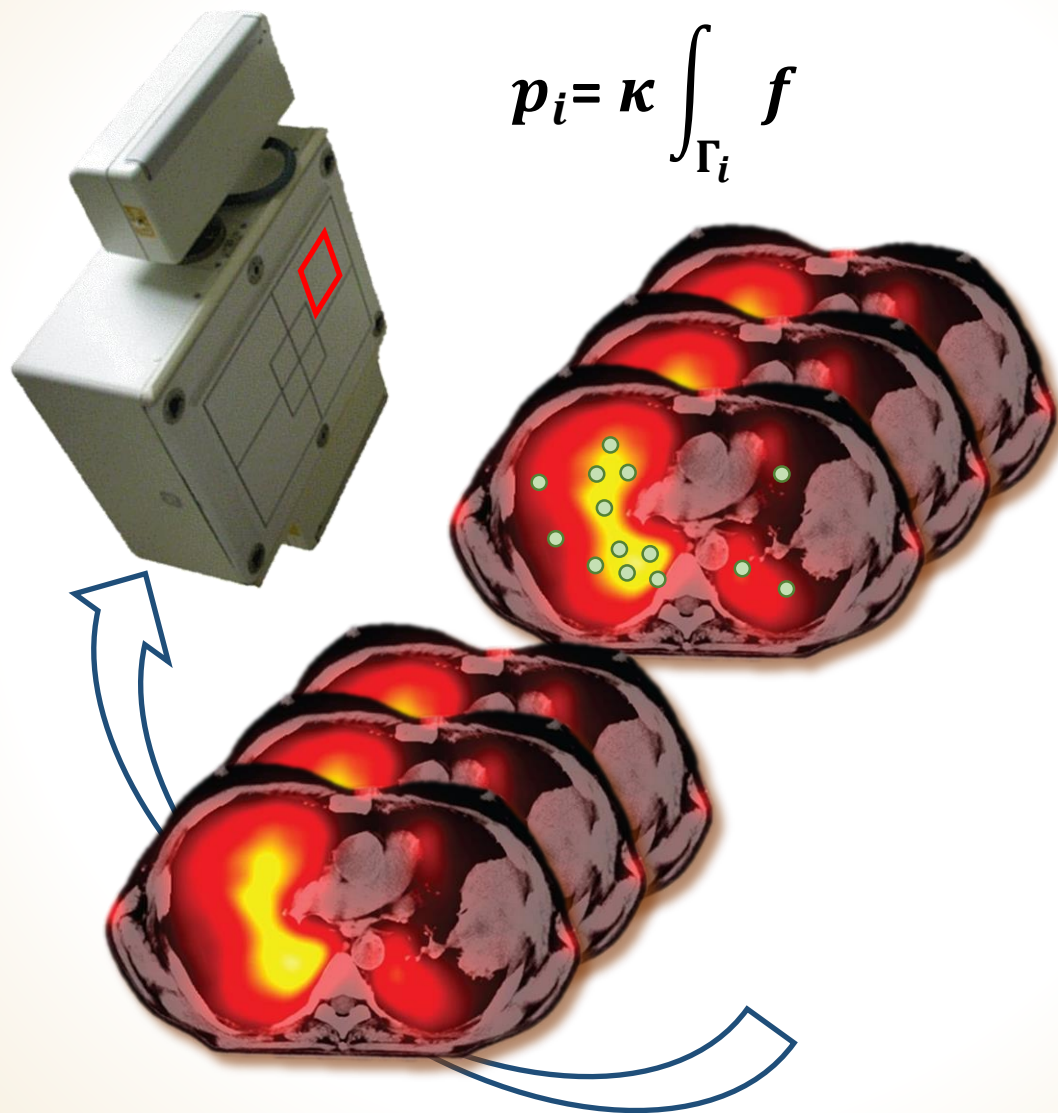
Problème tomographique



Problème tomographique



Problème tomographique



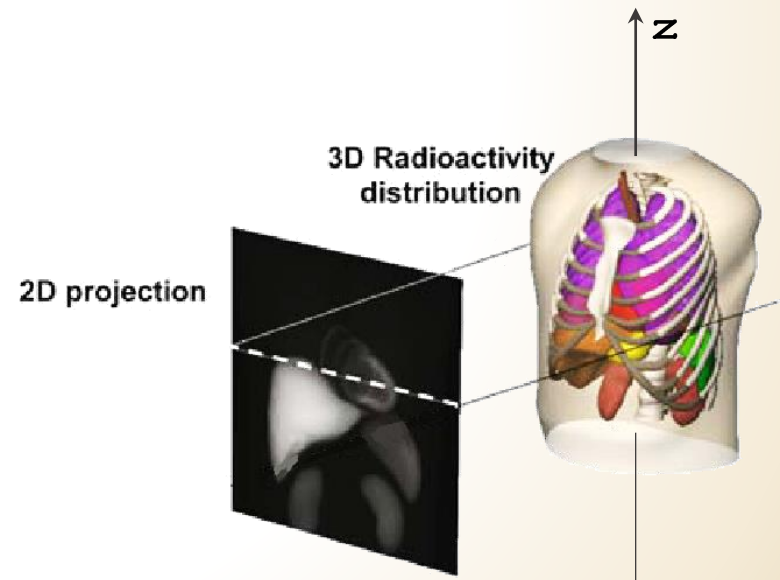
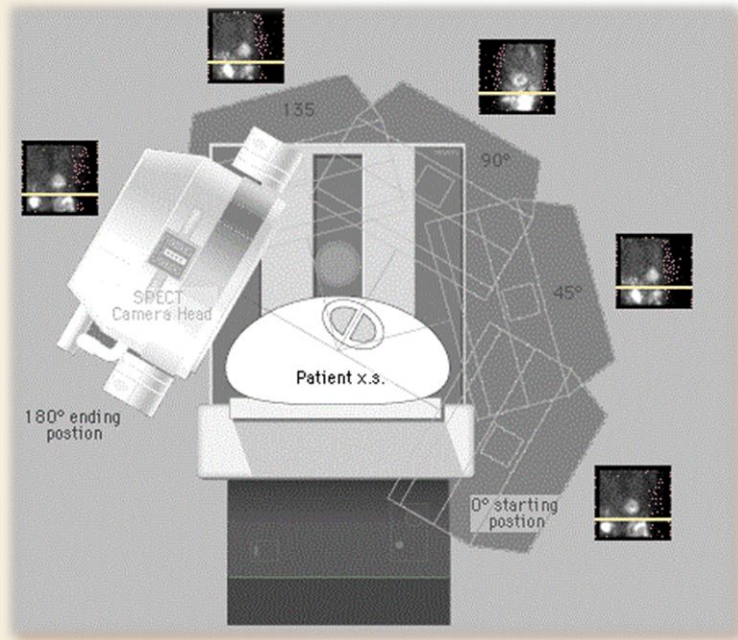
$p_i \quad (i = 1 \dots N)$



f

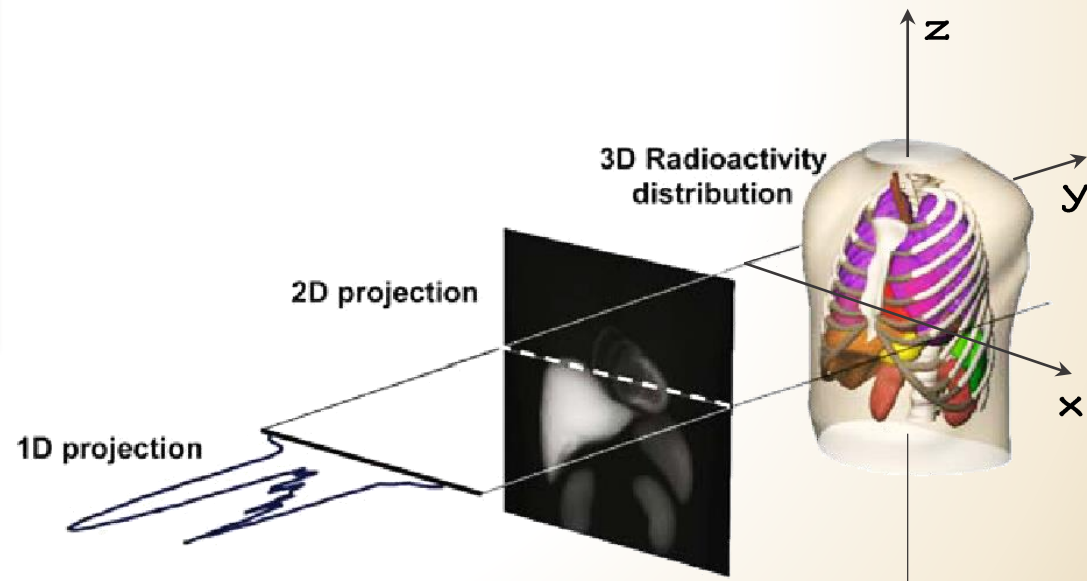
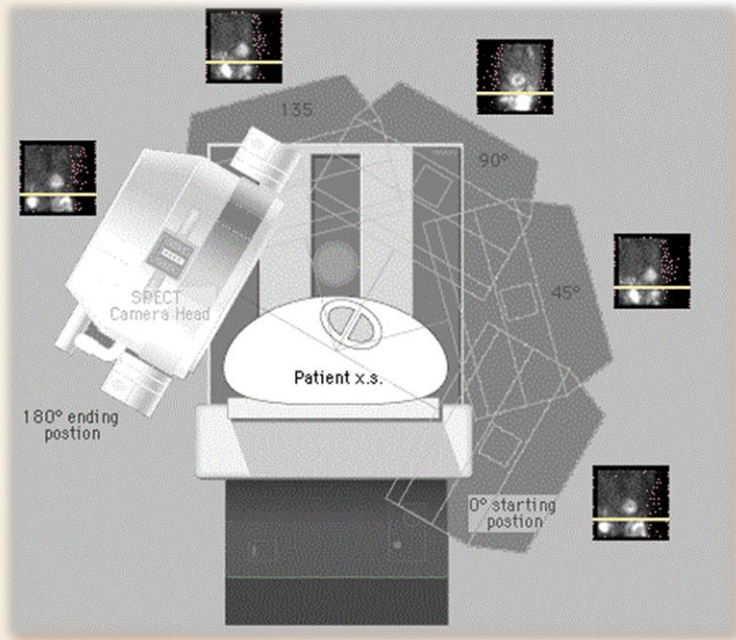
Reconstruction

■ Modèle analytique



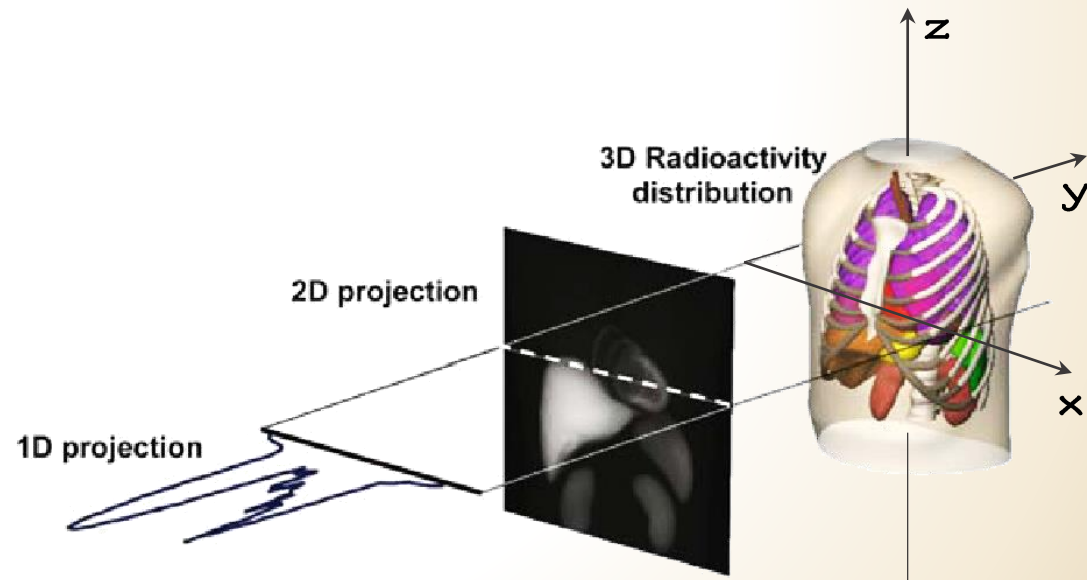
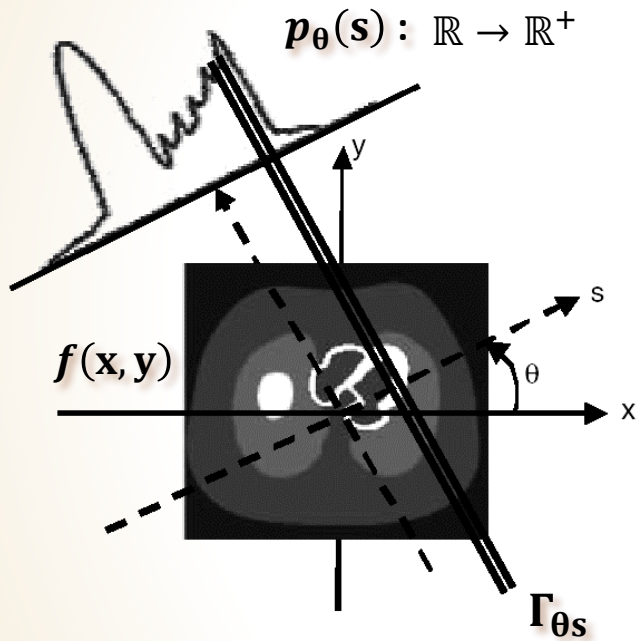
Reconstruction

■ Modèle analytique



Reconstruction

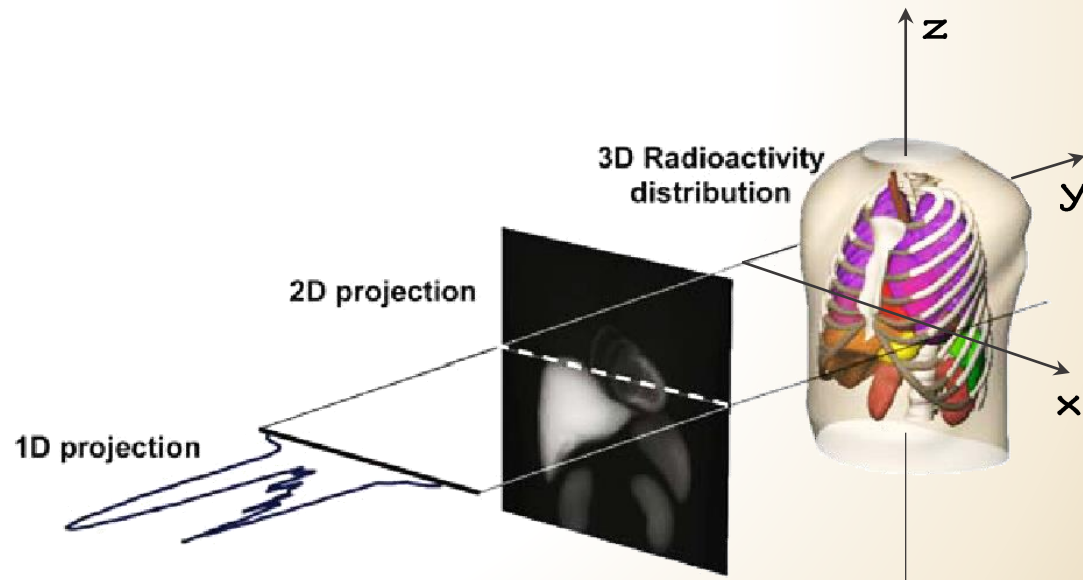
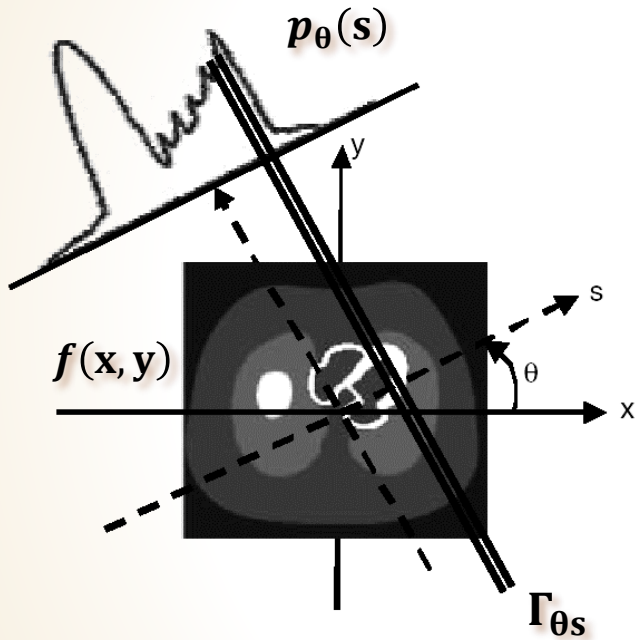
■ Modèle analytique



Reconstruction

■ Modèle analytique

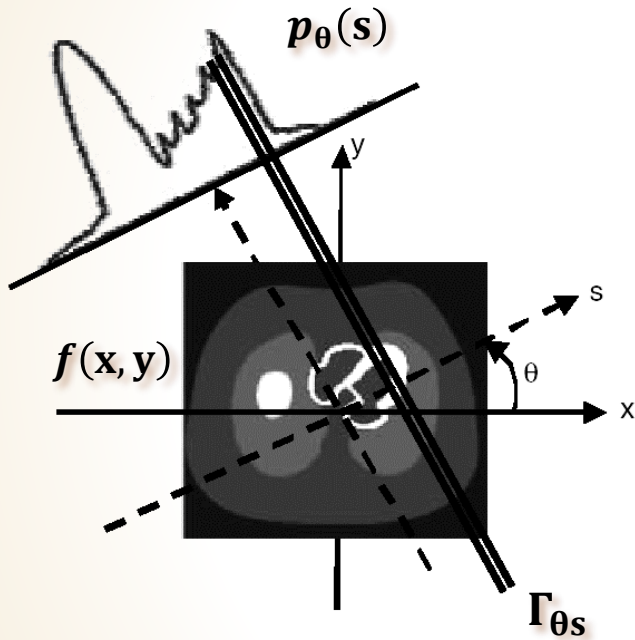
$$p_{\theta}(s) = \int_{\Gamma_{\theta s}} f(x, y)$$



Reconstruction

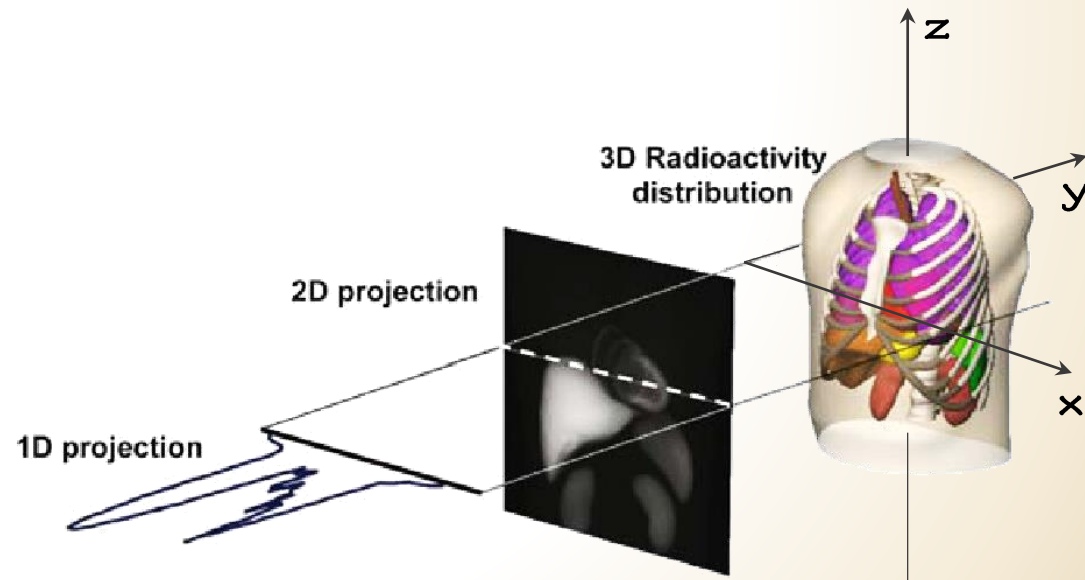
■ Modèle analytique

$$p_{\theta}(s) = \int_{\Gamma_{\theta s}} f(x, y)$$



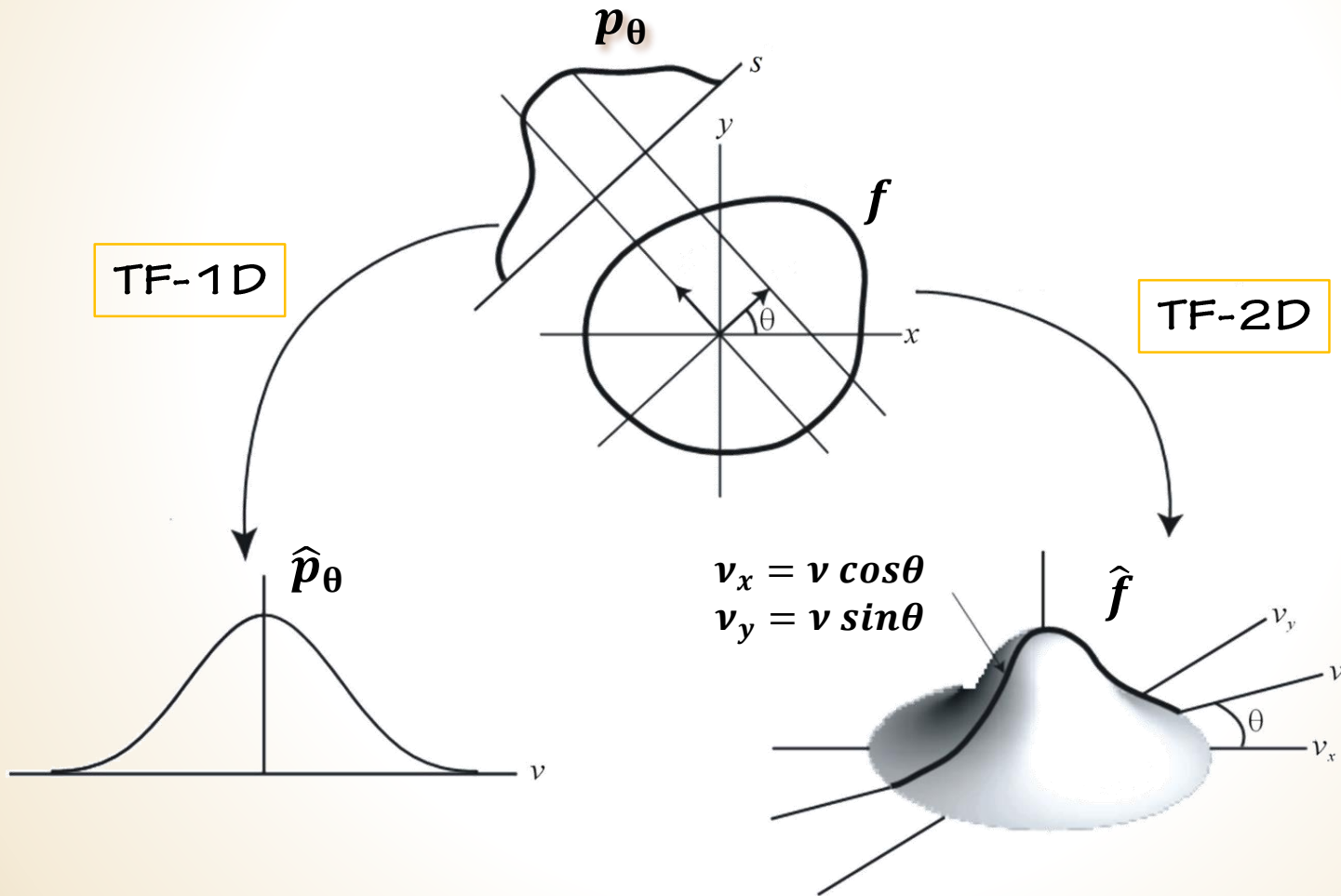
Projection (Radon) : $p = R f$

Rétro-projection : R^*



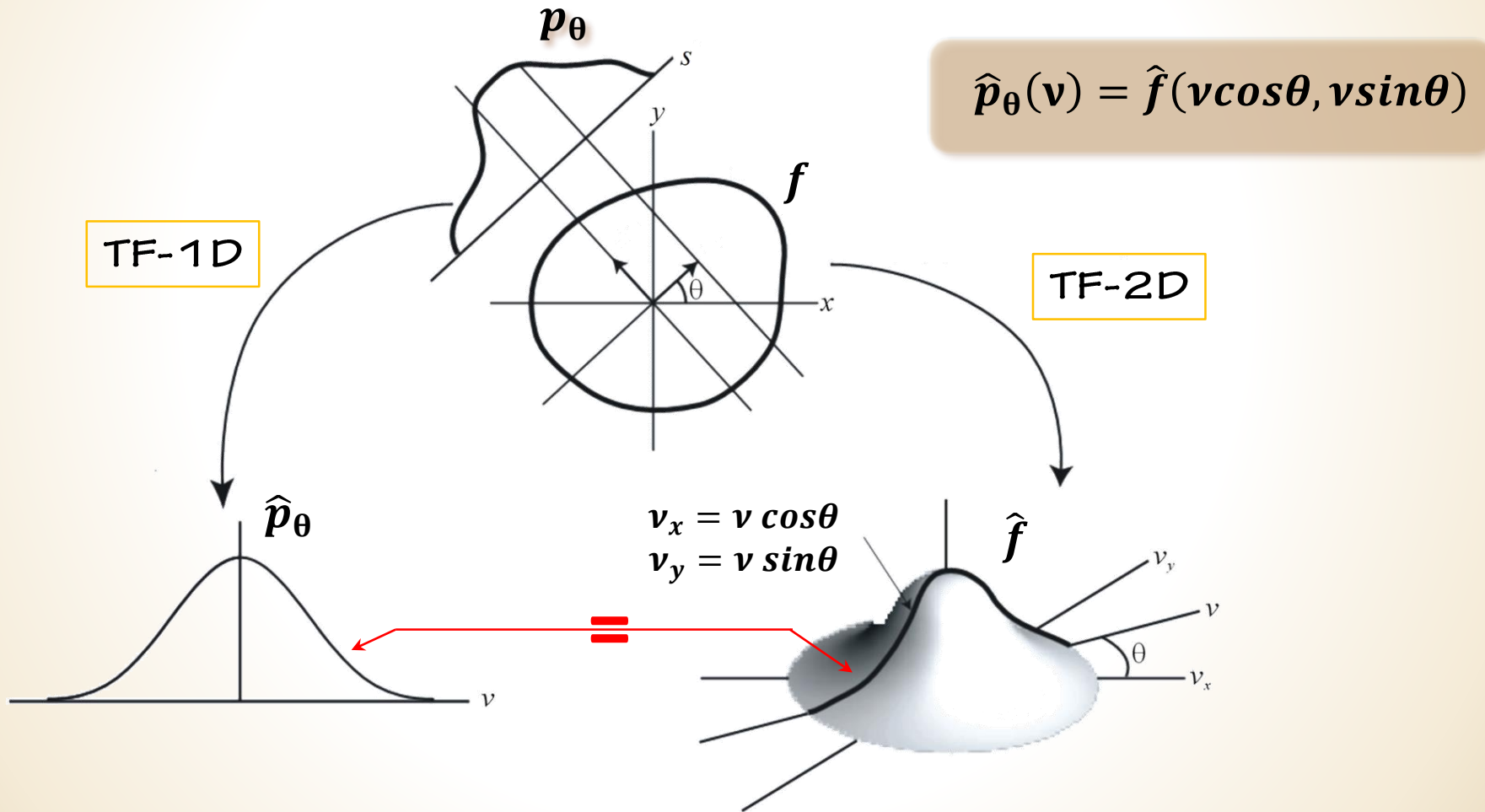
Reconstruction

■ Modèle analytique — Synthèse de Fourier



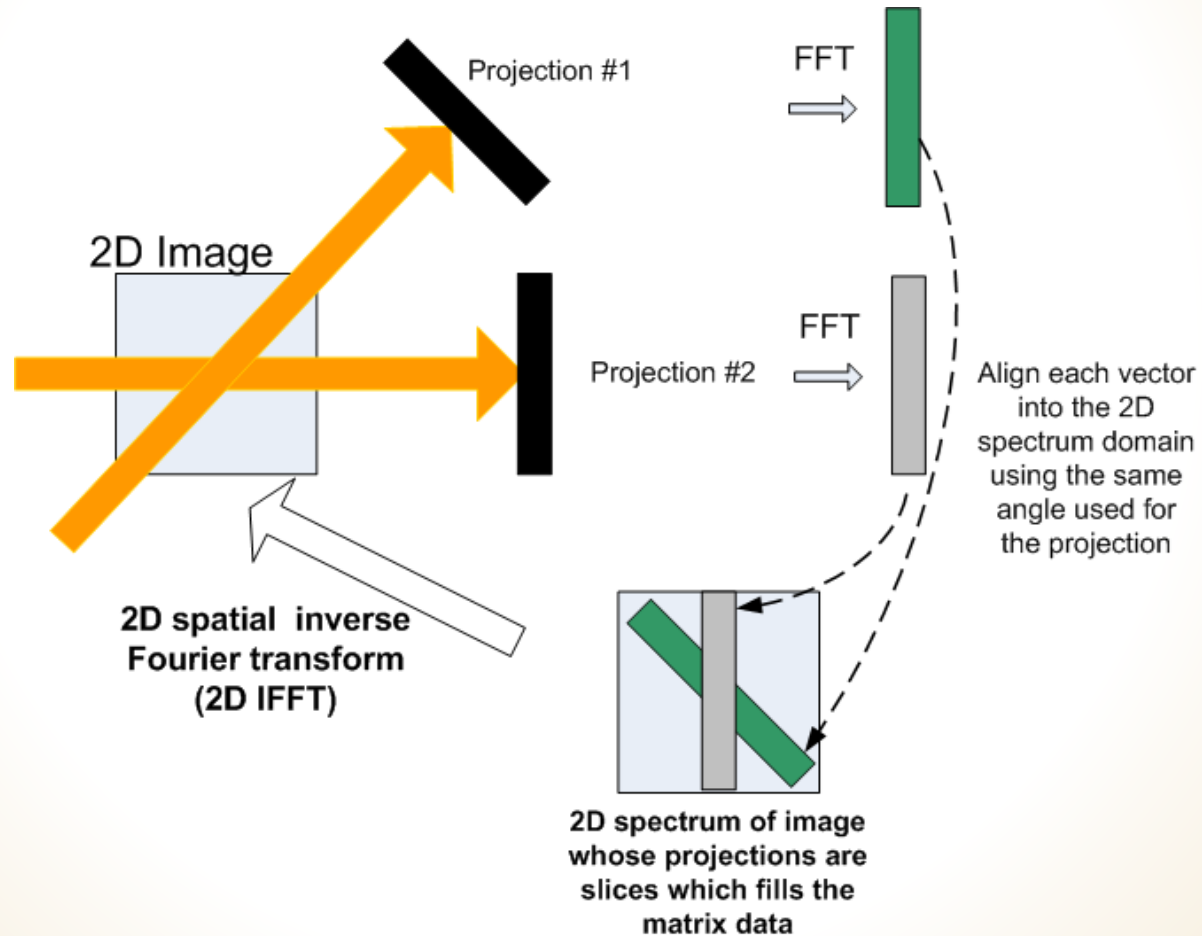
Reconstruction

■ Modèle analytique — Synthèse de Fourier



Reconstruction

■ Modèle analytique — Synthèse de Fourier



Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f} e^{i(xv_x + yv_y)} dv_x dv_y$$

$$\begin{aligned} v_x &= v \cos\theta \\ v_y &= v \sin\theta \end{aligned}$$

Reconstruction

■ Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f} e^{i(xv_x + yv_y)} dv_x dv_y$$

$$\begin{aligned} v_x &= v \cos\theta \\ v_y &= v \sin\theta \end{aligned}$$

$$f = \int d\theta \int \hat{f} e^{iv(x\cos\theta + y\sin\theta)} |v| dv$$

Reconstruction

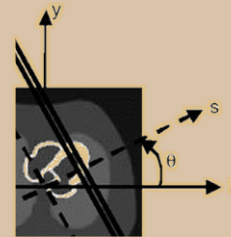
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$= s$



Reconstruction

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Reconstruction

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Reconstruction

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Reconstruction

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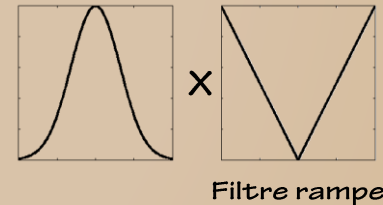
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Reconstruction

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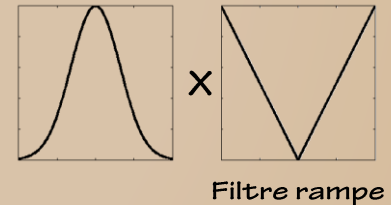
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TF inv - 1D



Reconstruction

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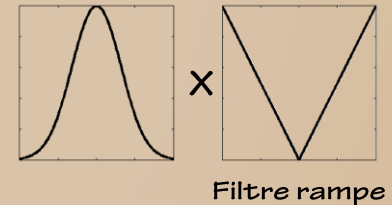
$$\hat{f}(v\cos\theta, v\sin\theta) = \hat{p}_\theta(v)$$

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Rétro-projection



Reconstruction

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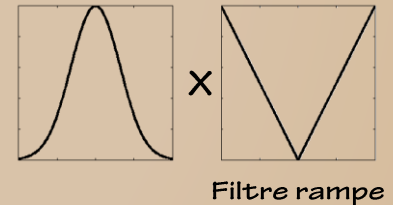
$$\hat{f}(v\cos\theta, v\sin\theta) = \hat{p}_\theta(v)$$

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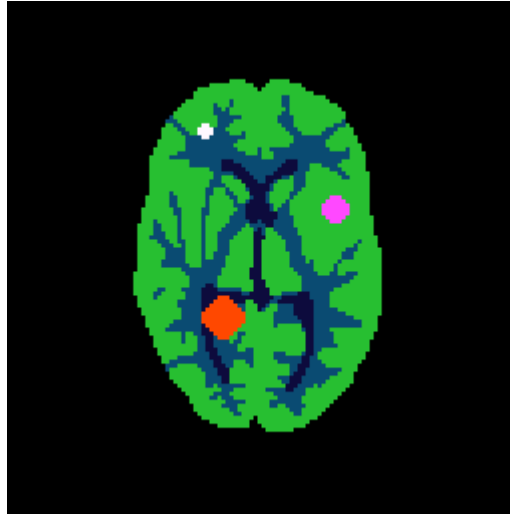
Rétro-projection



$$f = R^* Hp$$

Reconstruction

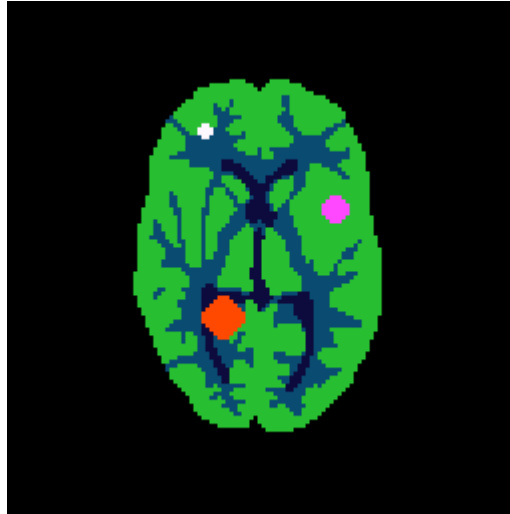
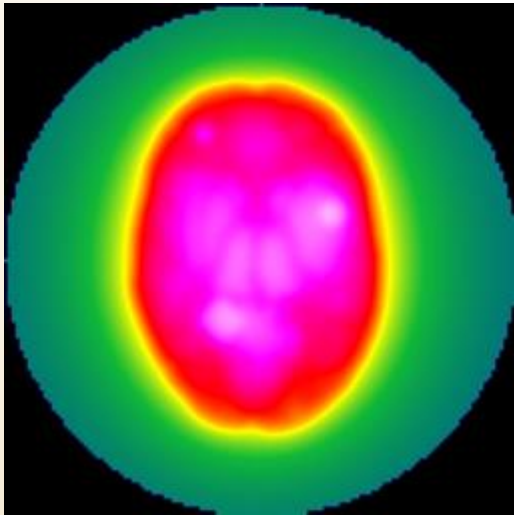
- Modèle analytique — Rétro-projection filtrée



Reconstruction

- Modèle analytique — Rétro-projection filtrée

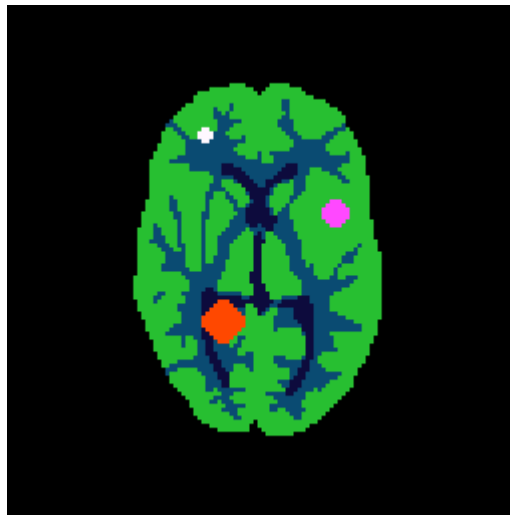
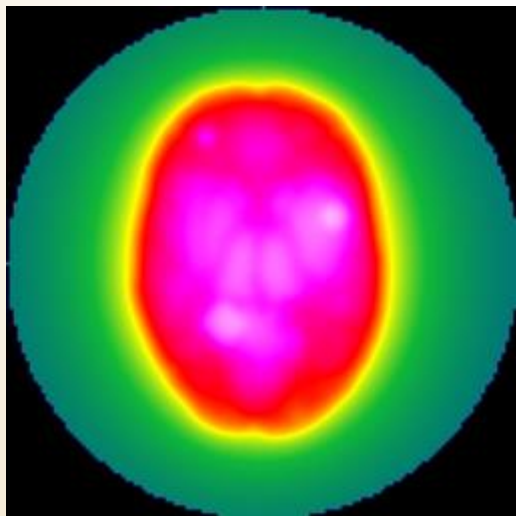
$$f = R^* p$$



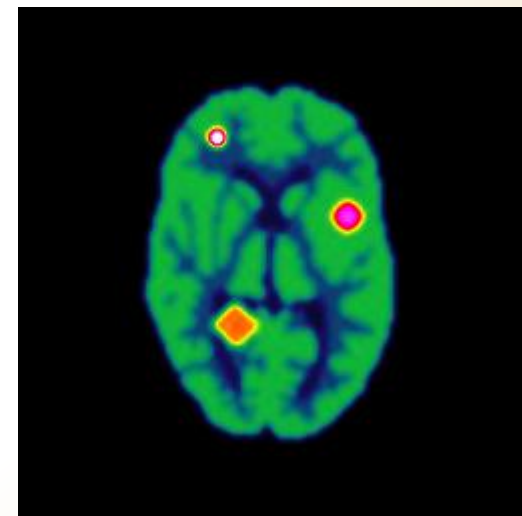
Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = R^* p$$



$$f = R^* H p$$

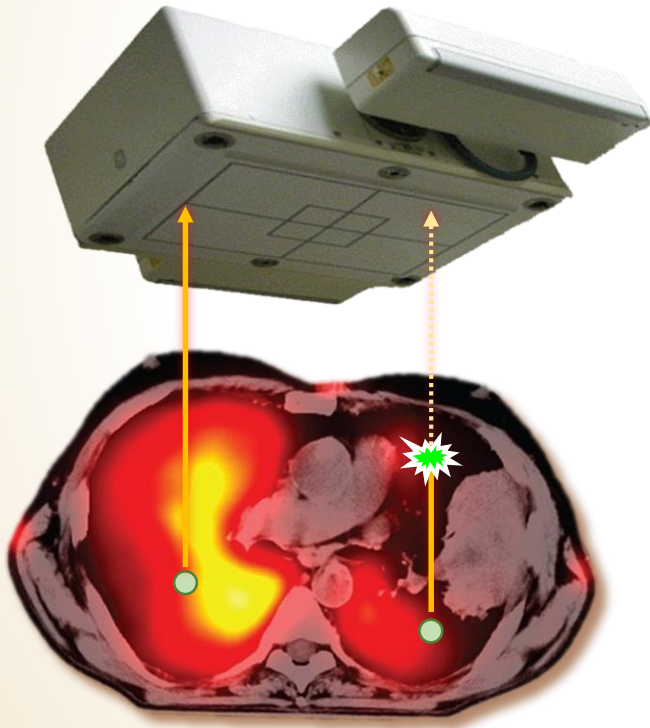


Reconstruction

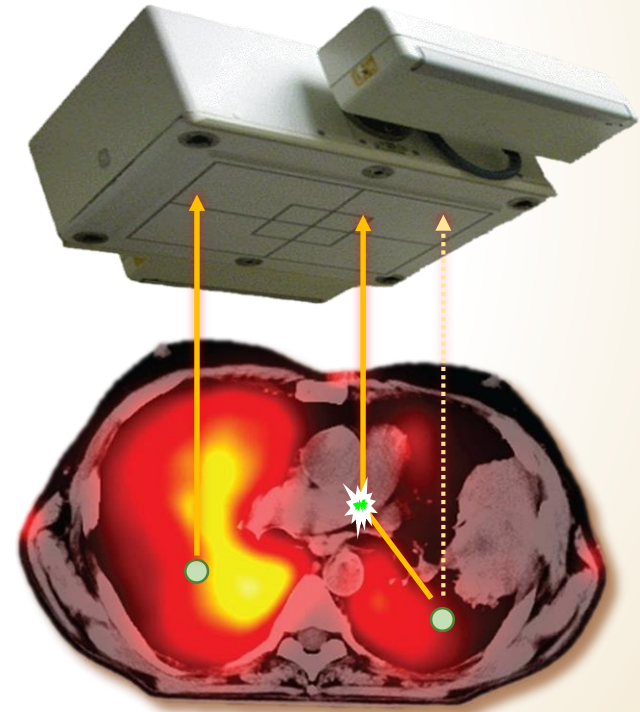
■ Modèle analytique — Limites

Interactions γ - matière

Absorption



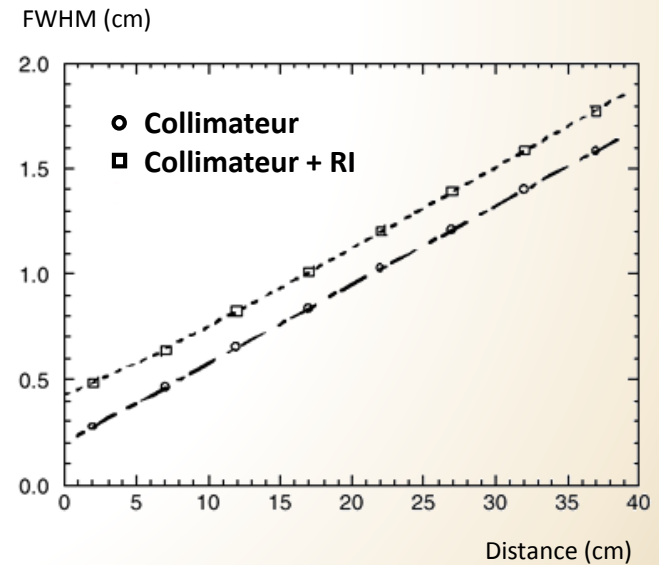
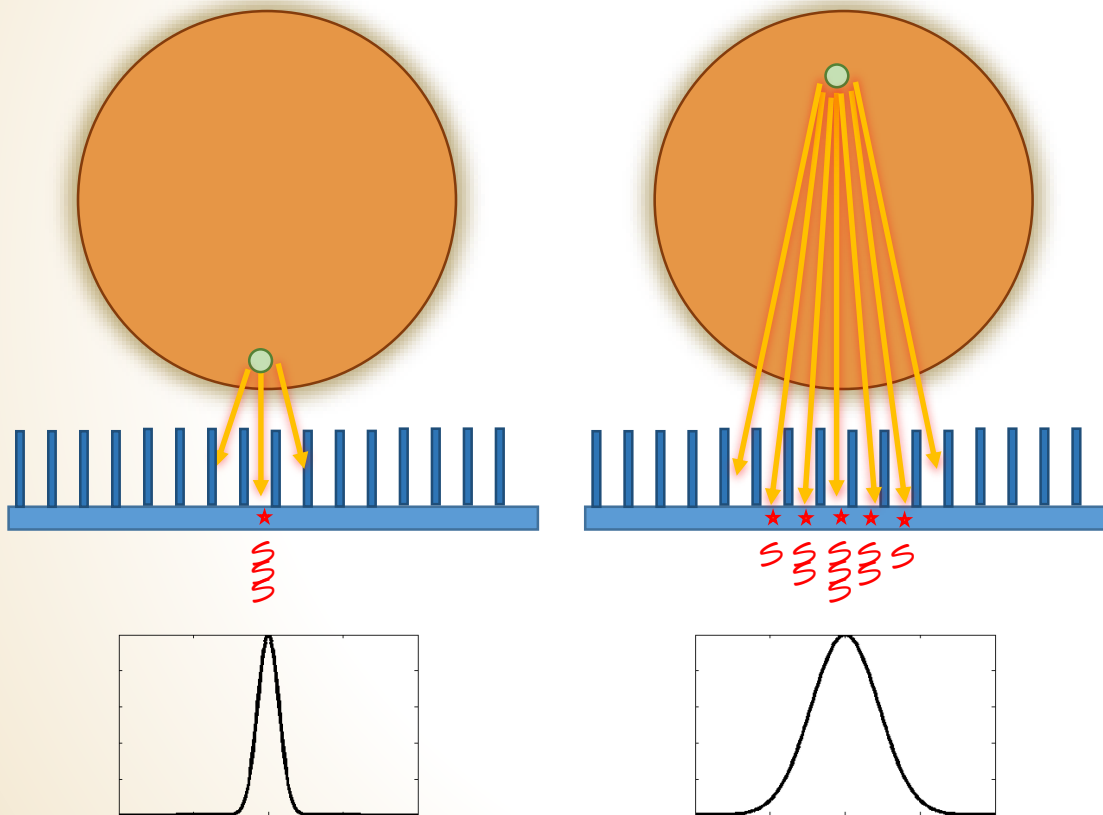
Diffusion



Reconstruction

■ Modèle analytique — Limites

Réponse du détecteur



Reconstruction

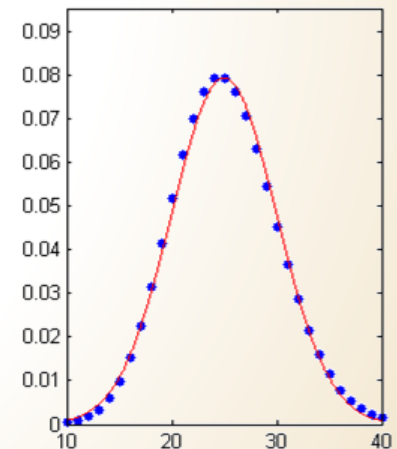
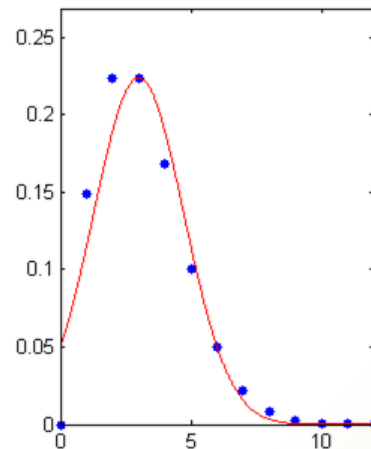
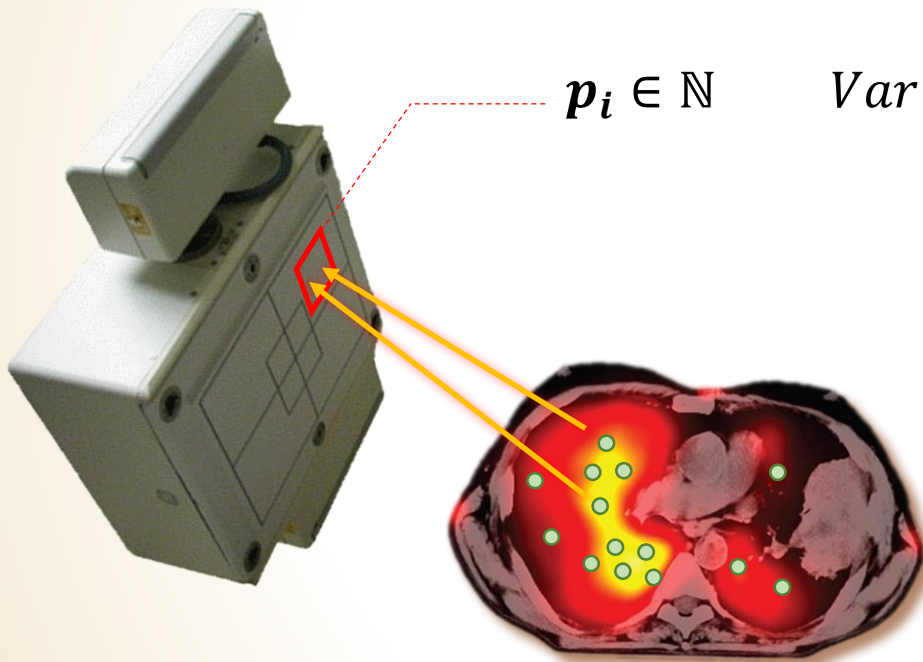
■ Modèle analytique — Limites

Bruit statistique

$$p_i \in \mathbb{N}$$

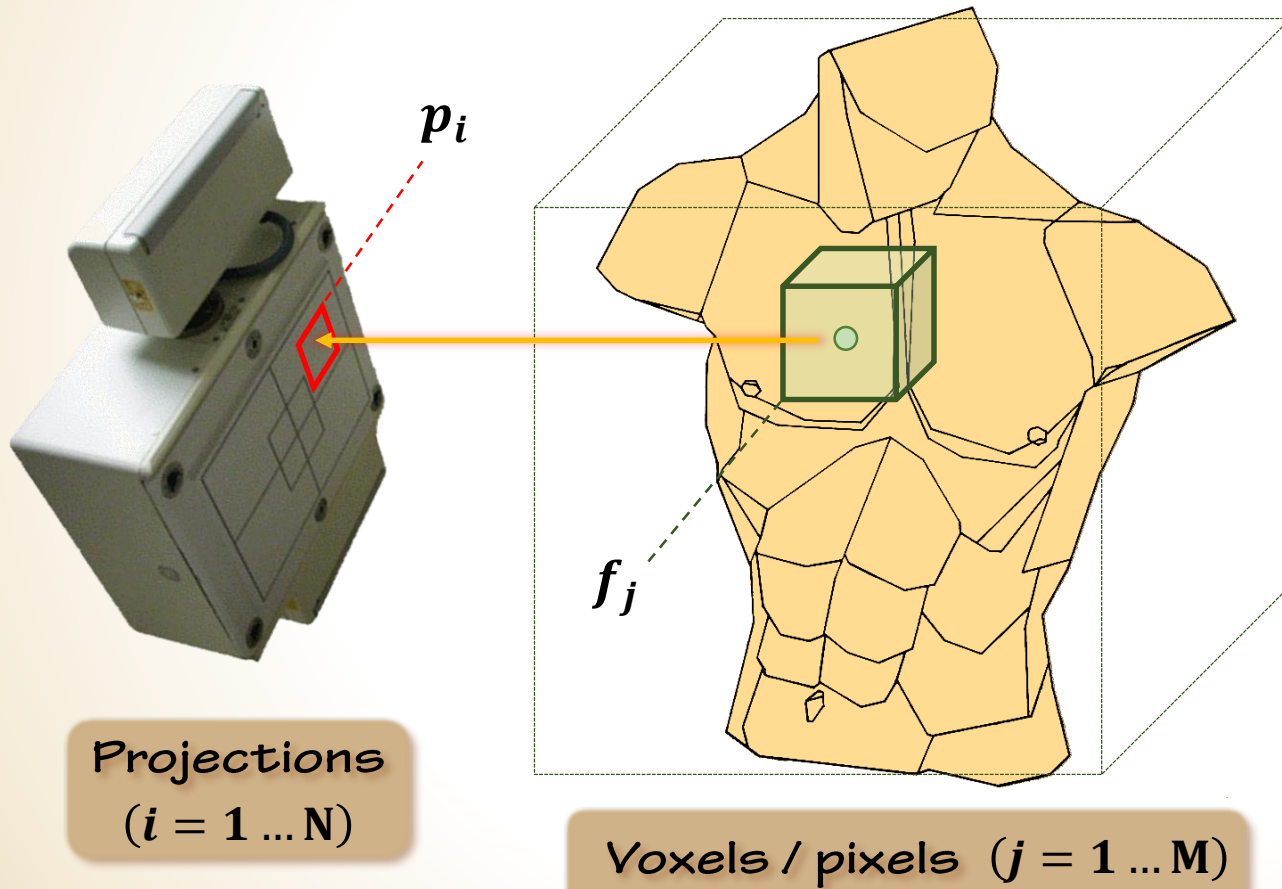
$$\text{Var}(p_i) = E(p_i)$$

$$\text{SNR} = \frac{E(p_i)}{\sigma(p_i)} \approx \sqrt{p_i}$$



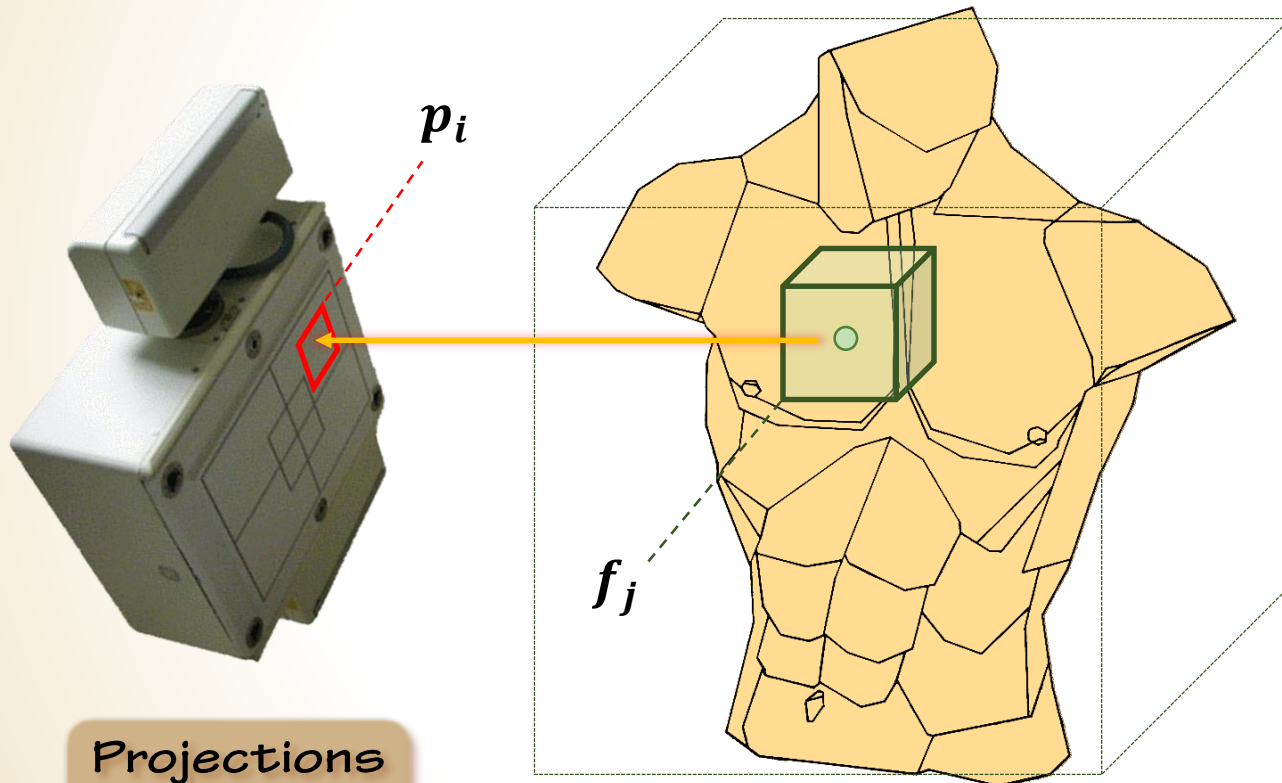
Reconstruction

■ Modèle algébrique



Reconstruction

■ Modèle algébrique



Projections
($i = 1 \dots N$)

Voxels / pixels ($j = 1 \dots M$)

Matrice système

$$\mathbf{R} \in \mathbb{R}^{N \times M}$$

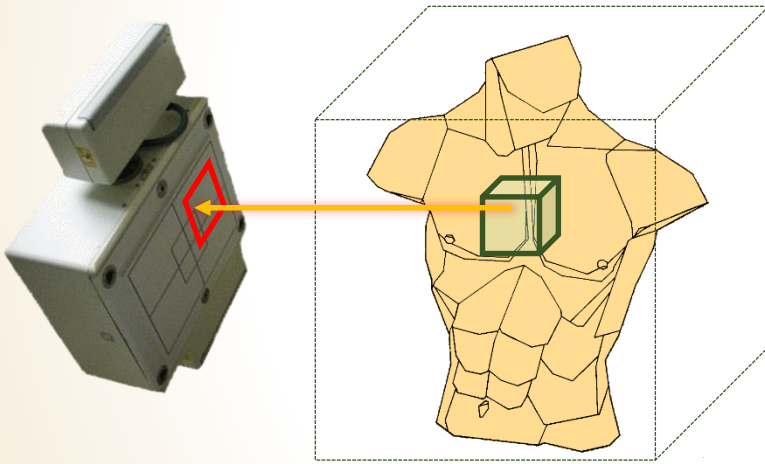
$$R_{ij} = \wp(j \rightarrow i)$$

- > Géométrie
- > Atténuation
- > Réponse du détecteur

Reconstruction

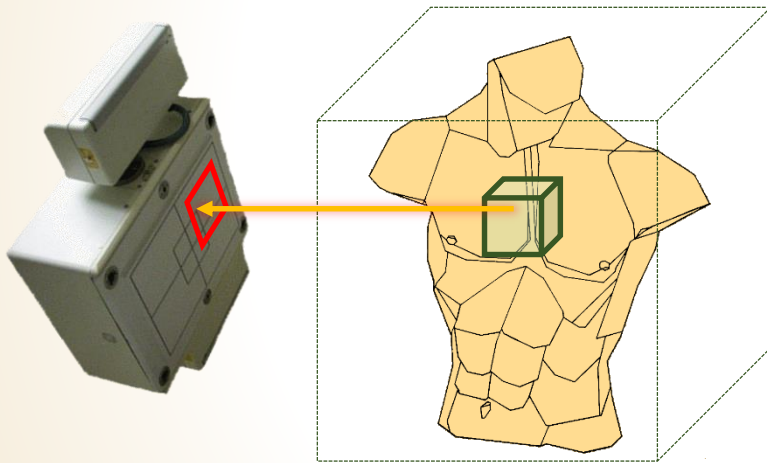
■ Modèle algébrique

$$p = Rf + s + n$$

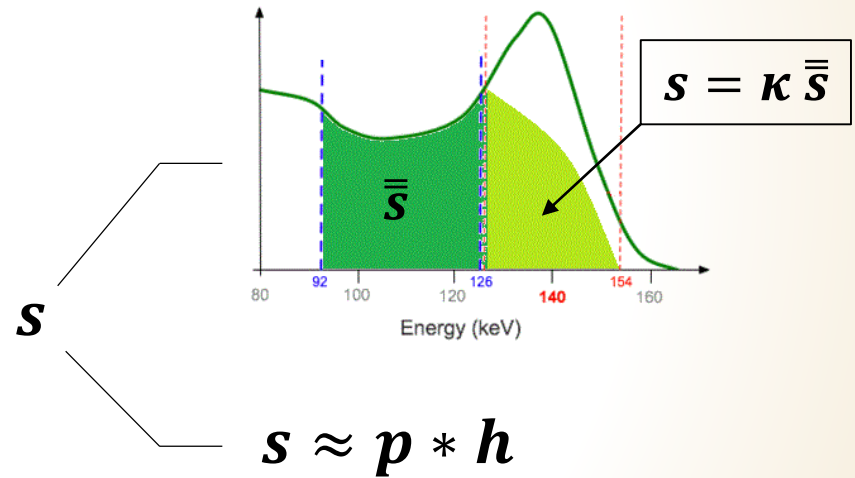


Reconstruction

■ Modèle algébrique

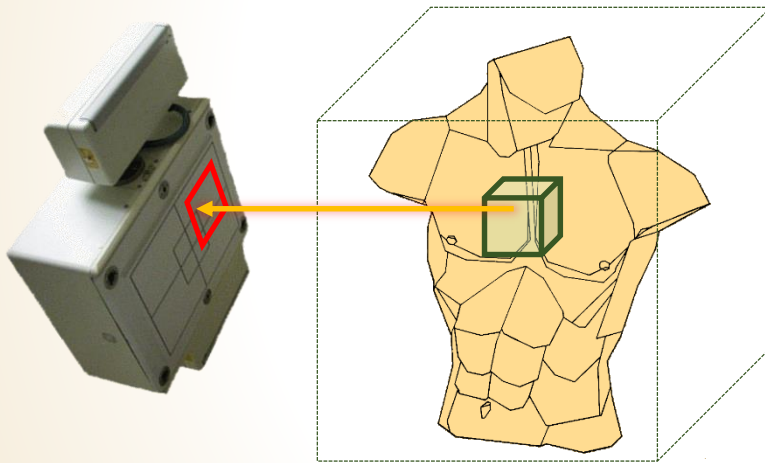


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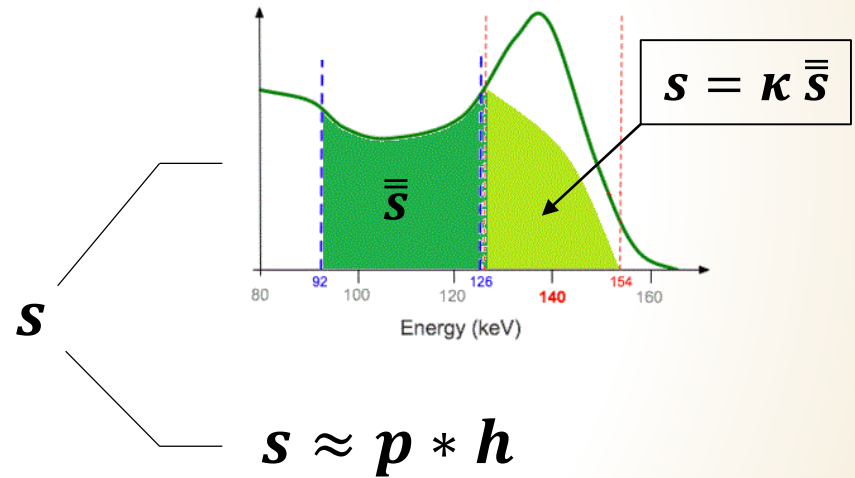


Reconstruction

■ Modèle algébrique



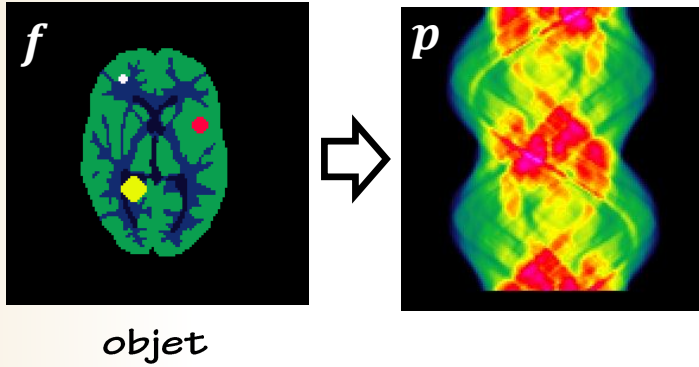
$$p = Rf + s + n$$



$$E(n) = 0 ; V(n) \approx p$$

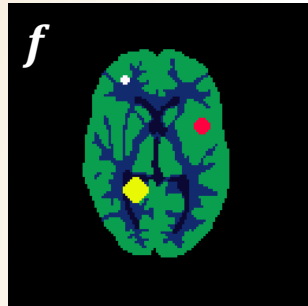
Reconstruction

■ Modèle algébrique

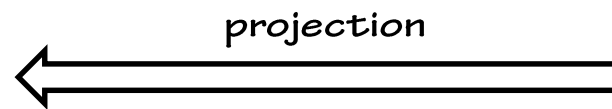
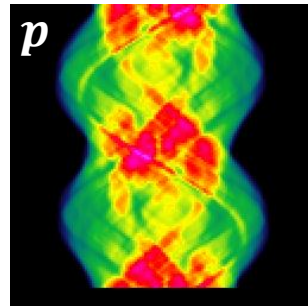


Reconstruction

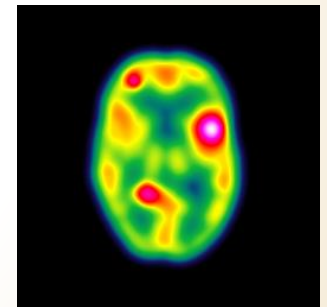
■ Modèle algébrique



objet



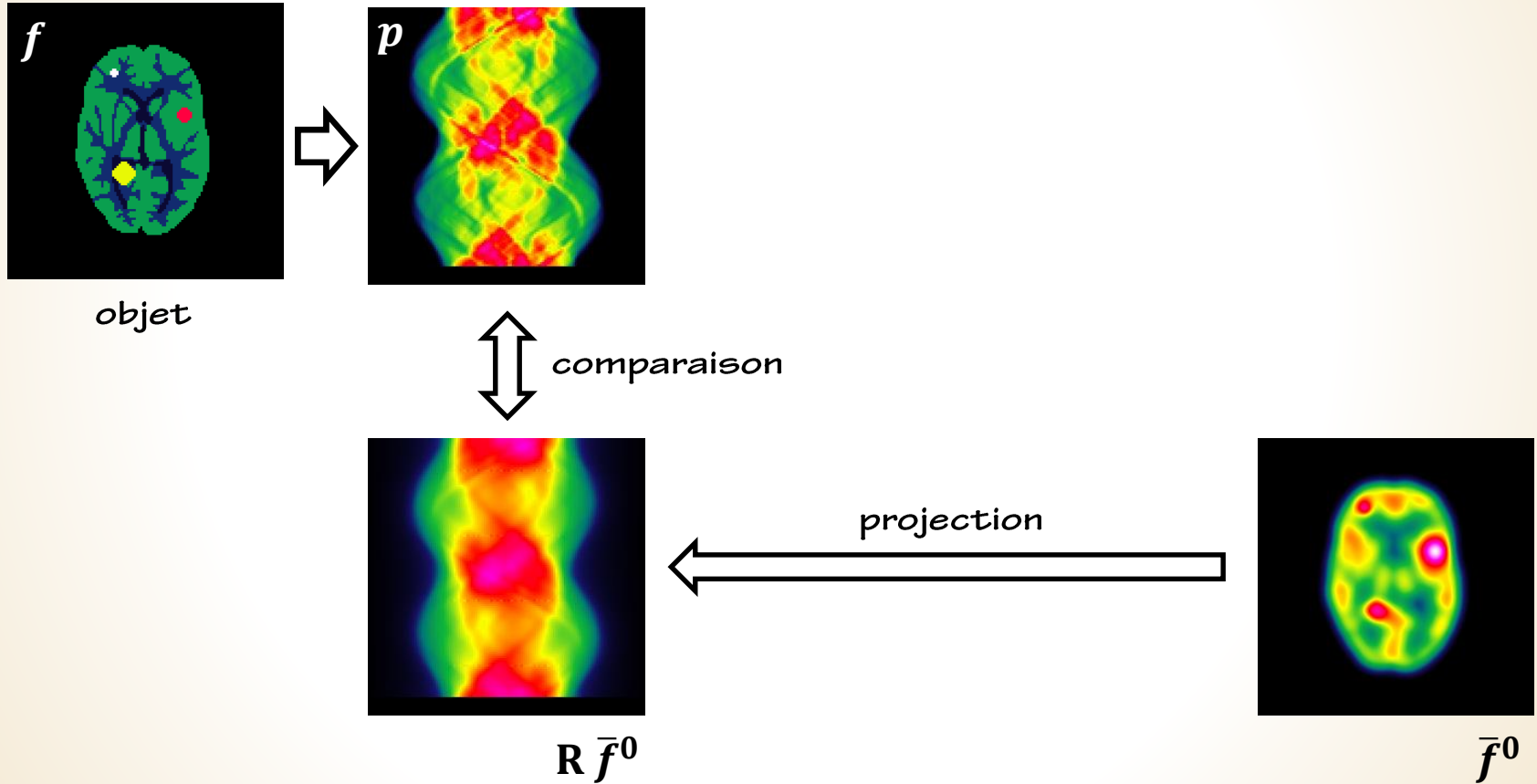
estimation



\bar{f}^0

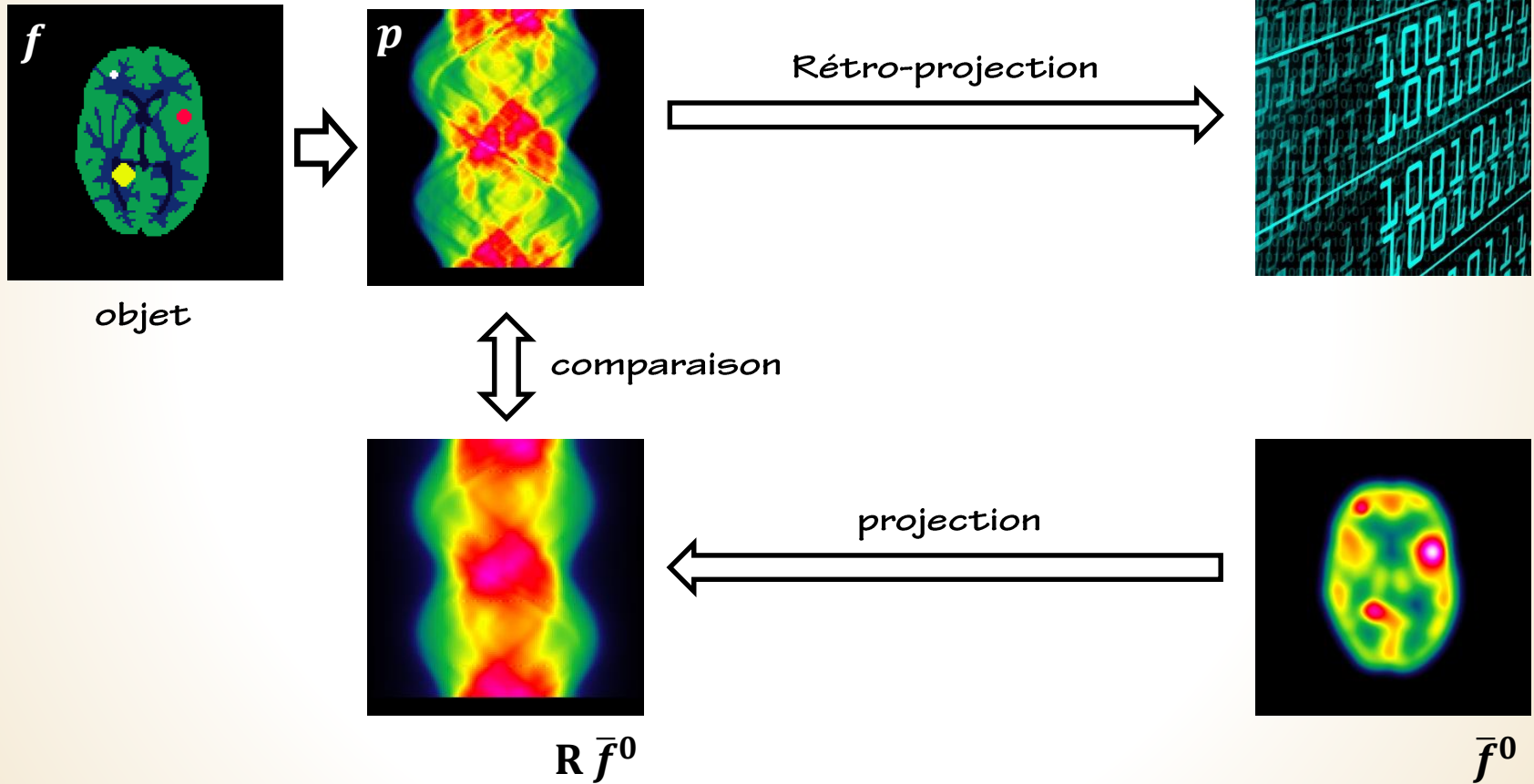
Reconstruction

■ Modèle algébrique



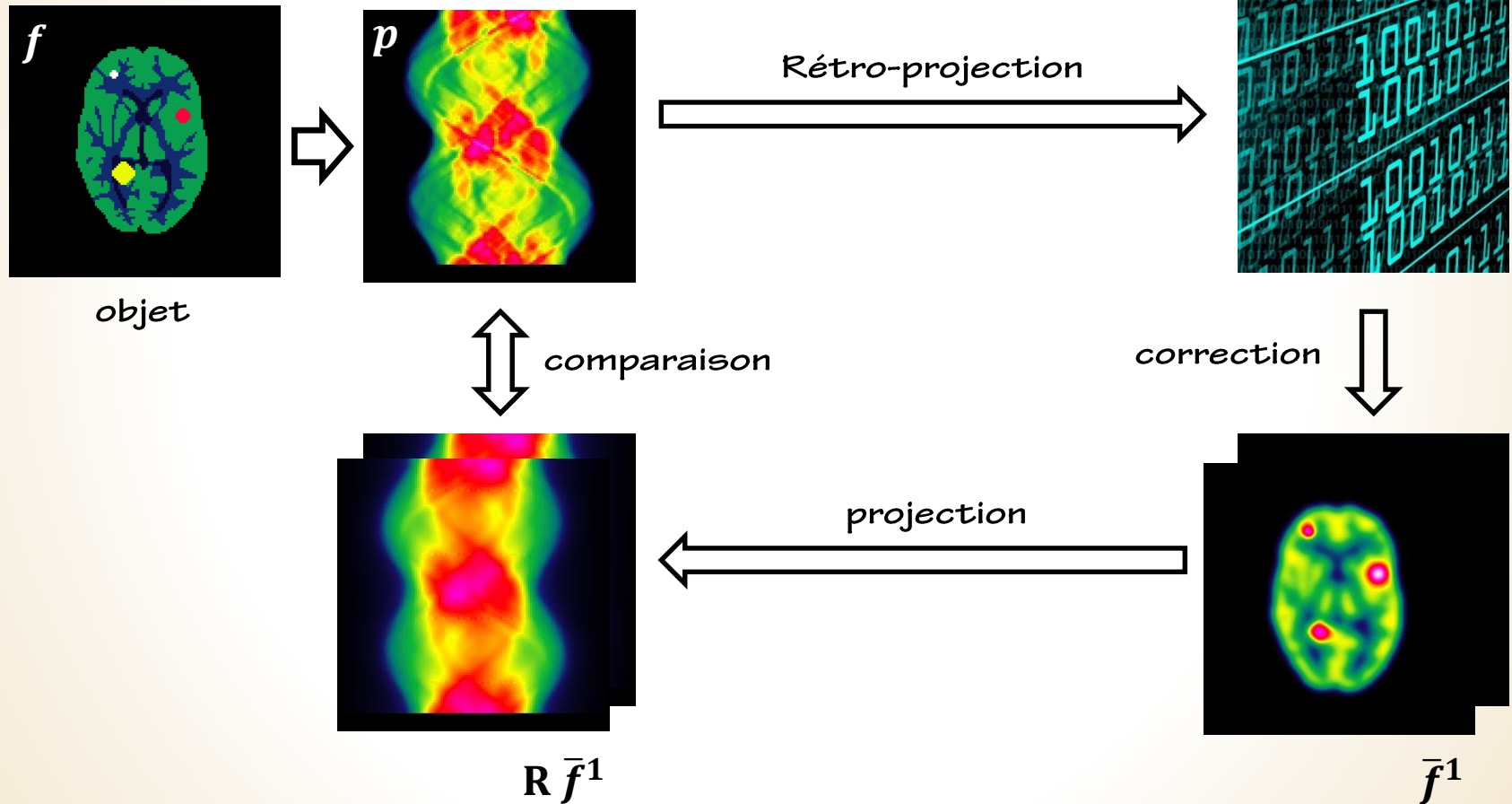
Reconstruction

■ Modèle algébrique



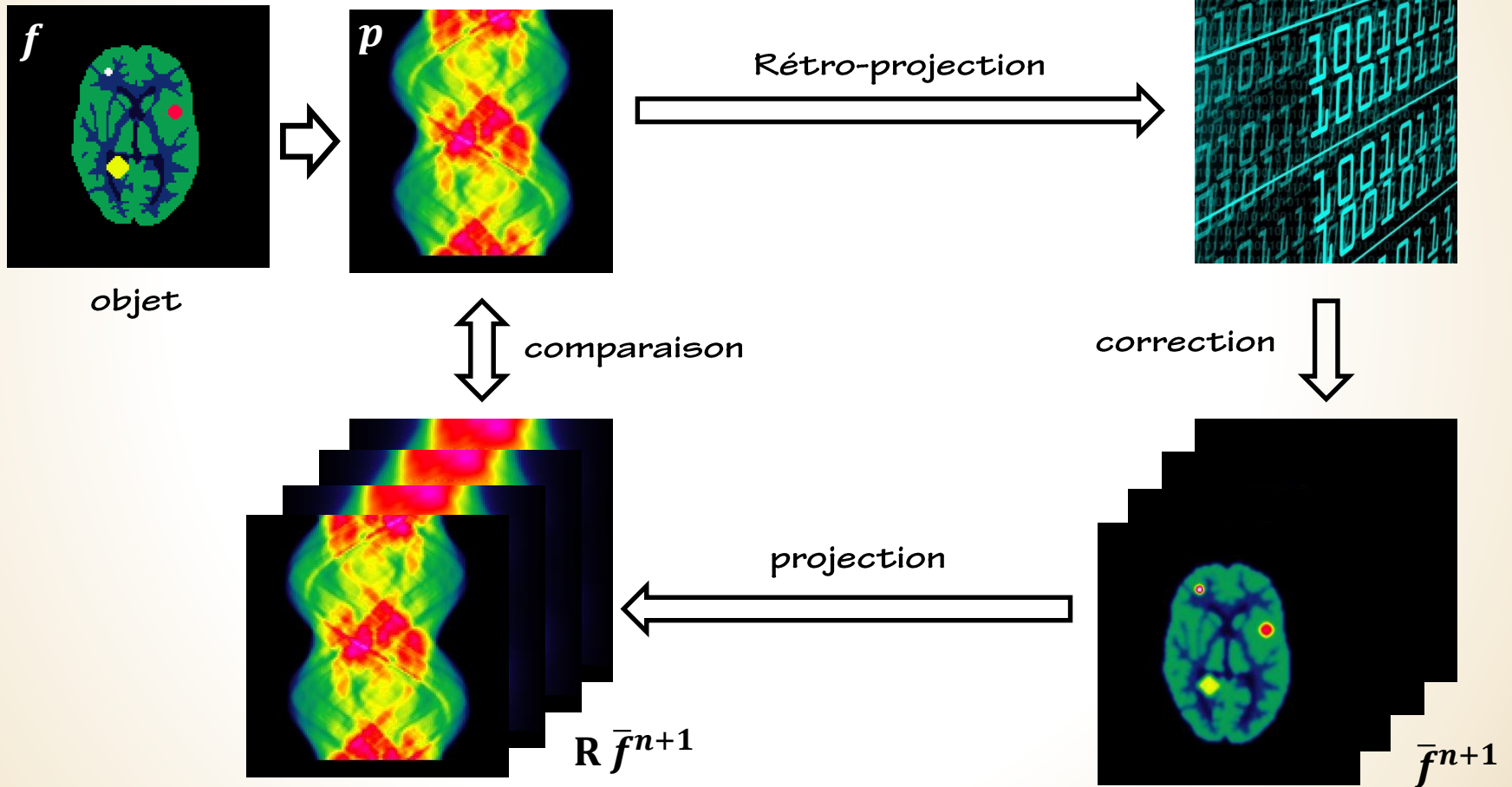
Reconstruction

■ Modèle algébrique



Reconstruction

■ Modèle algébrique



Reconstruction

■ Modèle algébrique



$$\bar{f} = \underset{f \in C}{\operatorname{argmin}} \{J(f)\}$$

$$J(f) = \|\mathbf{R}f - \mathbf{p}\|^2$$

Méthode **LS**

Reconstruction

■ Modèle algébrique



Méthode **LS**

$$\bar{f} = \underset{f \in C}{\operatorname{argmin}} \{J(f)\}$$

$$J(f) = \|\mathbf{R}f - \mathbf{p}\|^2$$

$$\bar{f} = (\mathbf{R}^* \mathbf{R})^{-1} \mathbf{R}^* \mathbf{p}$$

Reconstruction

■ Modèle algébrique



Méthode LS

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$$J(f) = \|\mathbf{R}f - \mathbf{p}\|^2$$

$$\bar{f} = (\mathbf{R}^* \mathbf{R})^{-1} \mathbf{R}^* \mathbf{p}$$

$$\dim(\mathbf{R}^* \mathbf{R}) = \sigma(10^5)$$

$$\kappa(\mathbf{R}^* \mathbf{R}) \gg$$

Reconstruction

■ Modèle algébrique



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$$\bar{f}^{n+1} = \bar{f}^n + \mathbf{R}^* (\mathbf{p} - \mathbf{R} \bar{f}^n)$$

Reconstruction

■ Modèle algébrique



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correction additive

Reconstruction

■ Modèle algébrique



Méthode LS

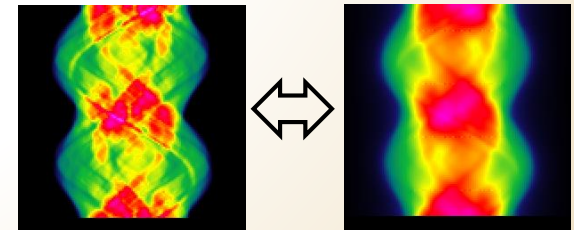
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$$\bar{f}^{n+1} = \bar{f}^n + \mathbf{R}^* (p - \mathbf{R} \bar{f}^n)$$

comparaison



Reconstruction

■ Modèle algébrique



Méthode LS

$$\bar{f} = \underset{f \in C}{\operatorname{argmin}} \{J(f)\}$$

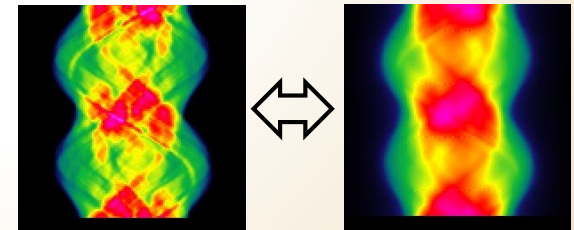
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rétro-projection

comparaison



Reconstruction

■ Modèle algébrique



$$\bar{f} = \underset{f \in C}{\operatorname{argmin}} \{J(f)\}$$

$$J(f) = -\log\{\varphi(p|f)\}$$

Méthode ML

Reconstruction

■ Modèle algébrique



Méthode ML

$$\bar{f} = \underset{f \in C}{\operatorname{argmin}} \{ J(f) \}$$

$$J(f) = -\log \{ \wp(\mathbf{p}|f) \}$$

$$\wp(\mathbf{p}|f) = \prod_i \frac{e^{-\check{p}_i} \check{p}_i^{p_i}}{p_i!}$$
$$\check{p}_i = (\mathbf{R}f)_i$$

Reconstruction

■ Modèle algébrique



Méthode ML

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{J(f)\}$$

$$J(f) = -\log\{\varphi(\mathbf{p}|f)\}$$

$$J(f) = \sum_i \{\mathbf{R}f_i - p_i \log(\mathbf{R}f_i)\}$$

$$\varphi(\mathbf{p}|f) = \prod_i \frac{e^{-\check{p}_i} \check{p}_i^{p_i}}{p_i!}$$
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Reconstruction

■ Modèle algébrique



Méthode ML

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{J(f)\}$$

$$J(f) = -\log\{\varphi(p|f)\}$$

$$J(f) = \sum_i \{Rf_i - p_i \log(Rf_i)\}$$

$$\bar{f}^{n+1} = \bar{f}^n \times \left(\mathbf{R}^* \frac{p}{\mathbf{R} \bar{f}^n} \right)$$

Reconstruction

■ Modèle algébrique



Méthode ML

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correction multiplicative

Reconstruction

■ Modèle algébrique



Méthode ML

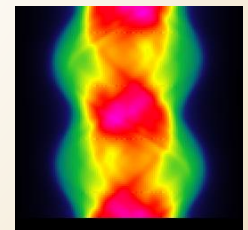
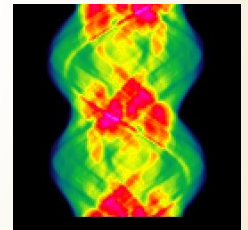
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comparaison



Reconstruction

■ Modèle algébrique



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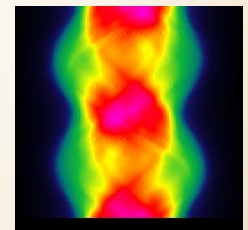
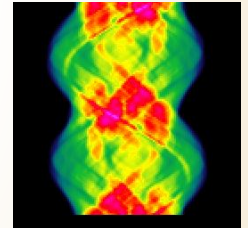
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rétro-projection

comparaison



Régularisation

$$p = Rf$$

R est mal conditionné : $\kappa(\mathbf{R}) \gg$

Régularisation

$$\mathbf{p} = \mathbf{R}f$$

\mathbf{R} est mal conditionné : $\kappa(\mathbf{R}) \gg$

$$\frac{\|\Delta f\|}{\|f\|} \leq \kappa(\mathbf{R}) \frac{\|\Delta p\|}{\|p\|} \approx \sigma\left(\frac{1}{\sqrt{p}}\right)$$

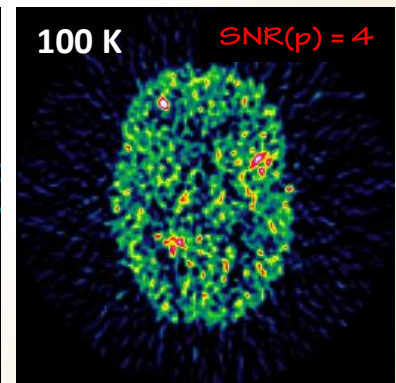
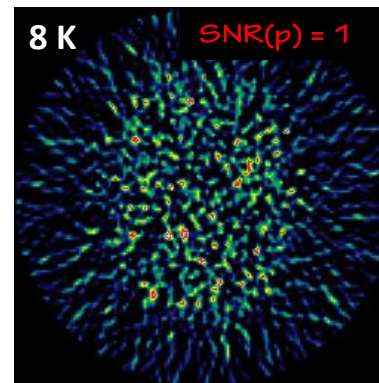
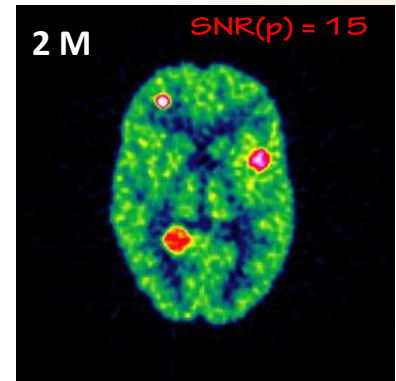
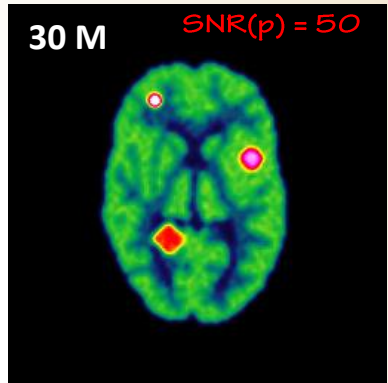
Régularisation

$$p = Rf$$

R est mal conditionné : $\kappa(\mathbf{R}) \gg$

$$\frac{\|\Delta f\|}{\|f\|} \leq \kappa(\mathbf{R}) \frac{\|\Delta p\|}{\|p\|} \approx \sigma\left(\frac{1}{\sqrt{p}}\right)$$

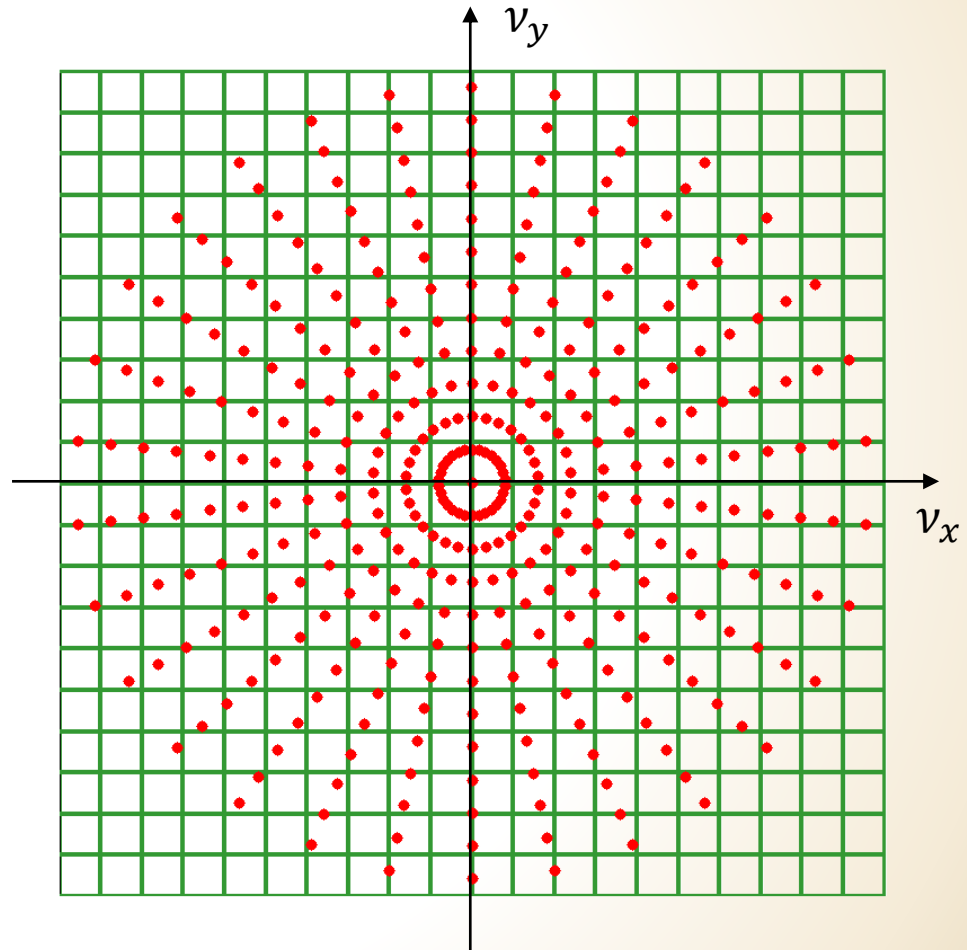
⇒ Sensibilité au bruit



Régularisation

■ Reconstruction analytique

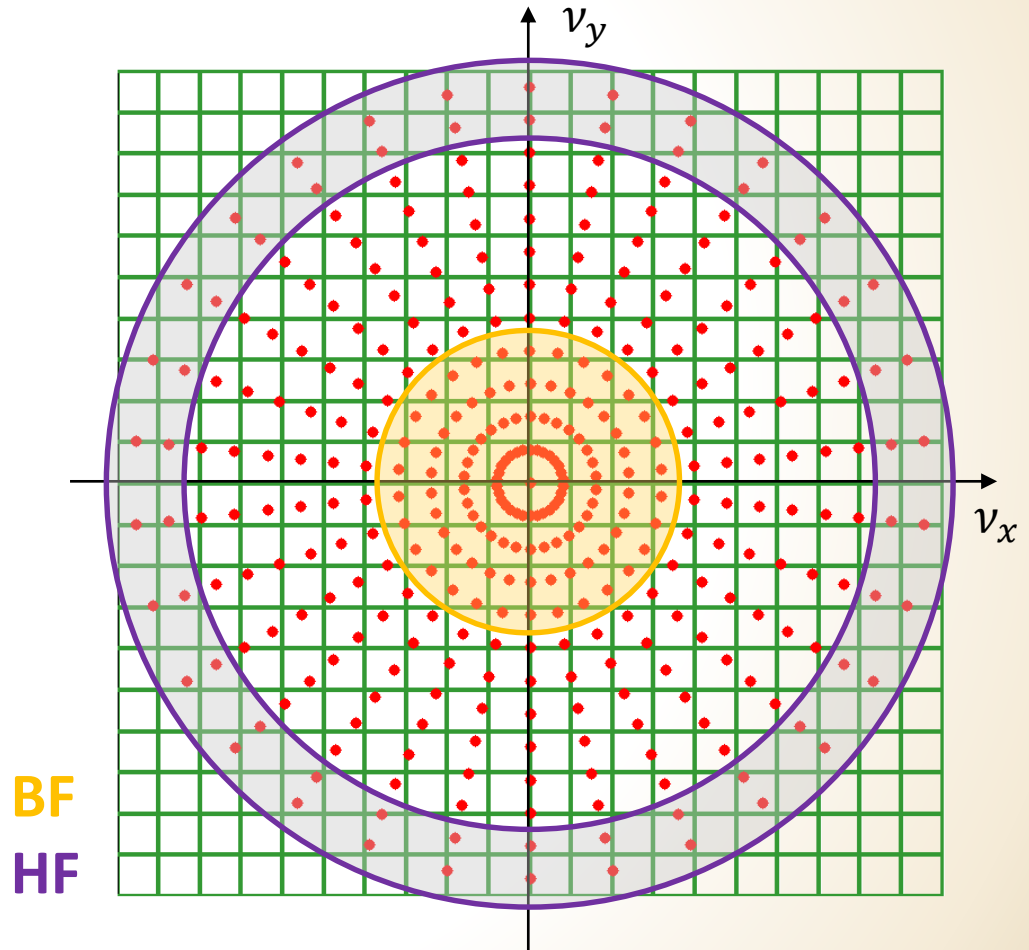
$$\hat{f}(v\cos\theta, v\sin\theta) = \hat{p}_\theta(v)$$



Régularisation

■ Reconstruction analytique

$$\hat{f}(v \cos \theta, v \sin \theta) = \hat{p}_\theta(v)$$

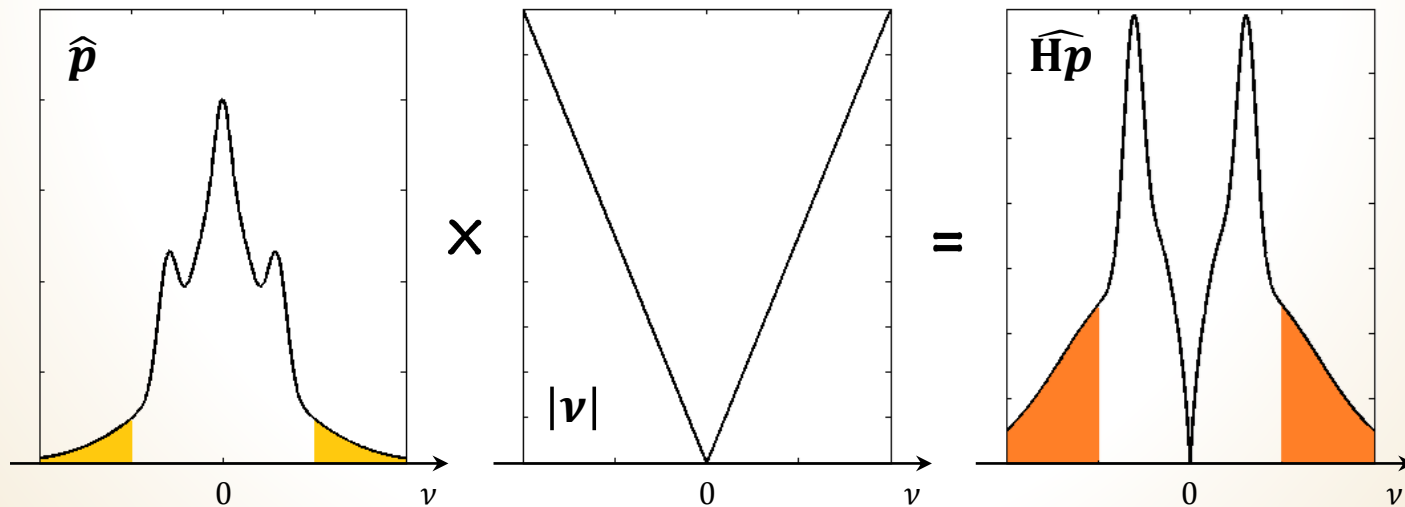
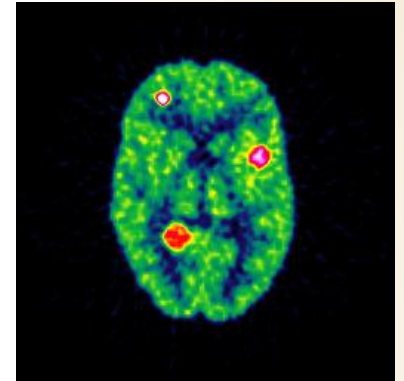


Régularisation

■ Reconstruction analytique

$$f = R^* H p$$

$$\widehat{H} p = |v| \widehat{p}$$

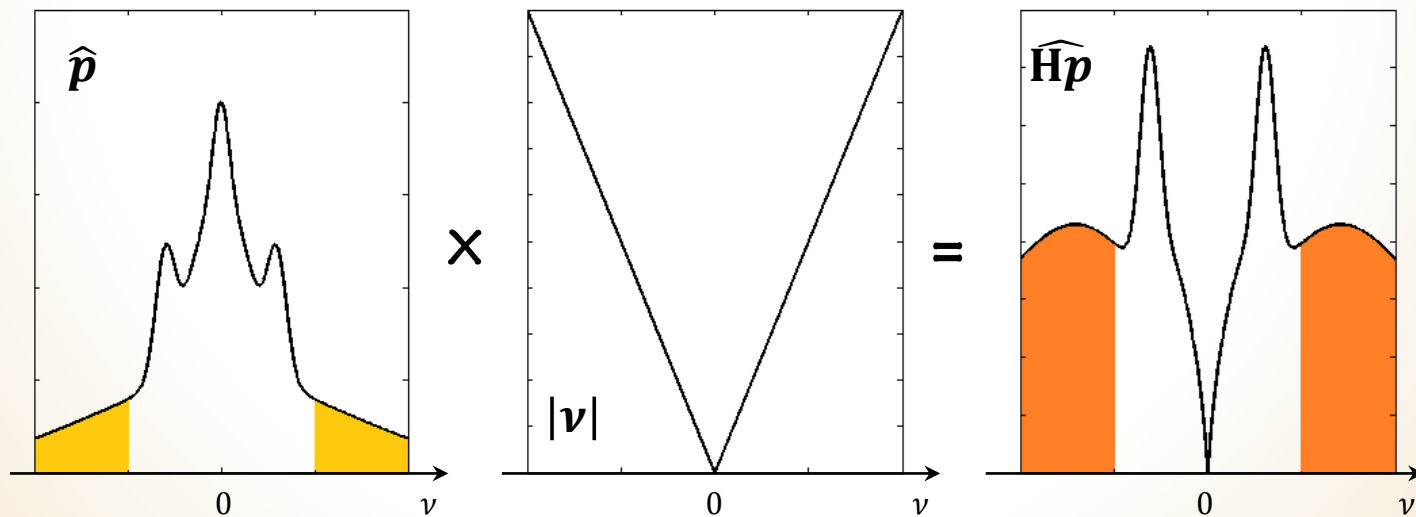
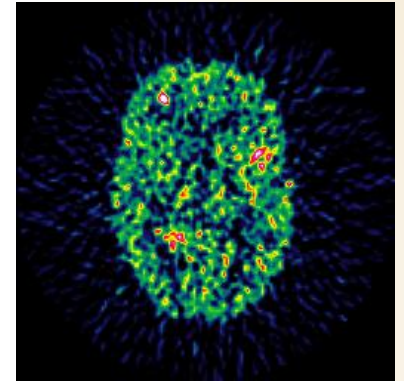


Régularisation

■ Reconstruction analytique

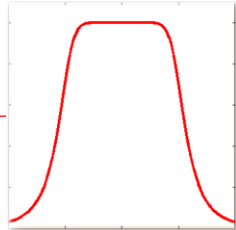
$$f = R^* H p$$

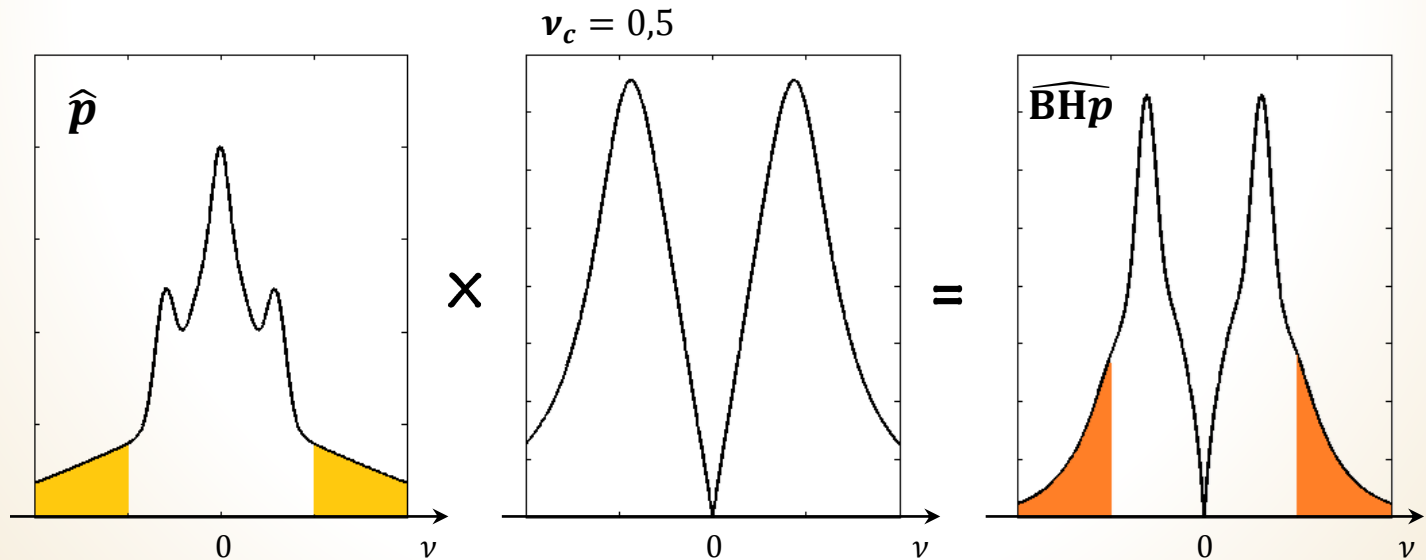
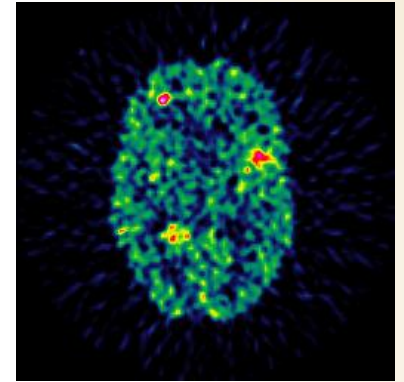
$$\widehat{H} p = |v| \widehat{p}$$



Régularisation

■ Reconstruction analytique

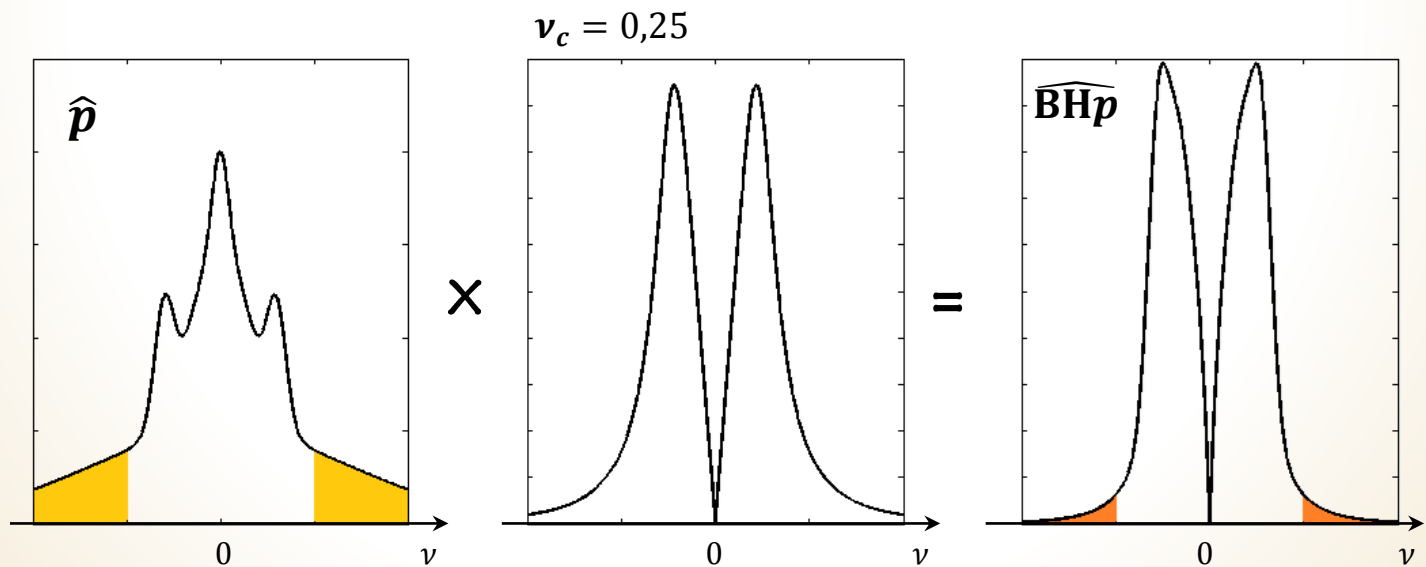
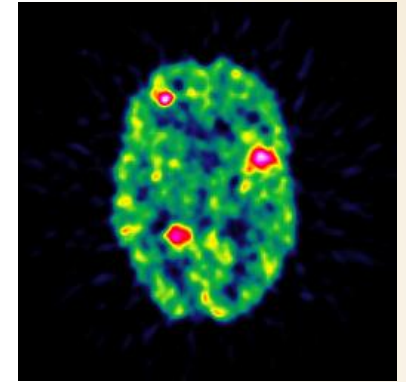
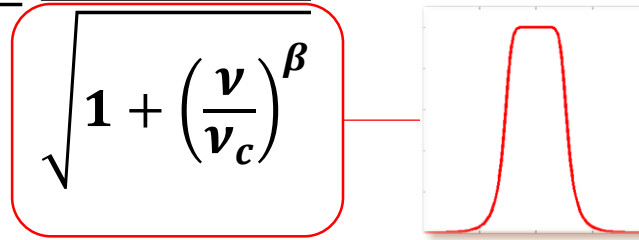
$$f = R^* \text{BH}p \quad \widehat{\text{BH}p} = \frac{|\nu| \hat{p}}{\sqrt{1 + \left(\frac{\nu}{\nu_c}\right)^\beta}}$$




Régularisation

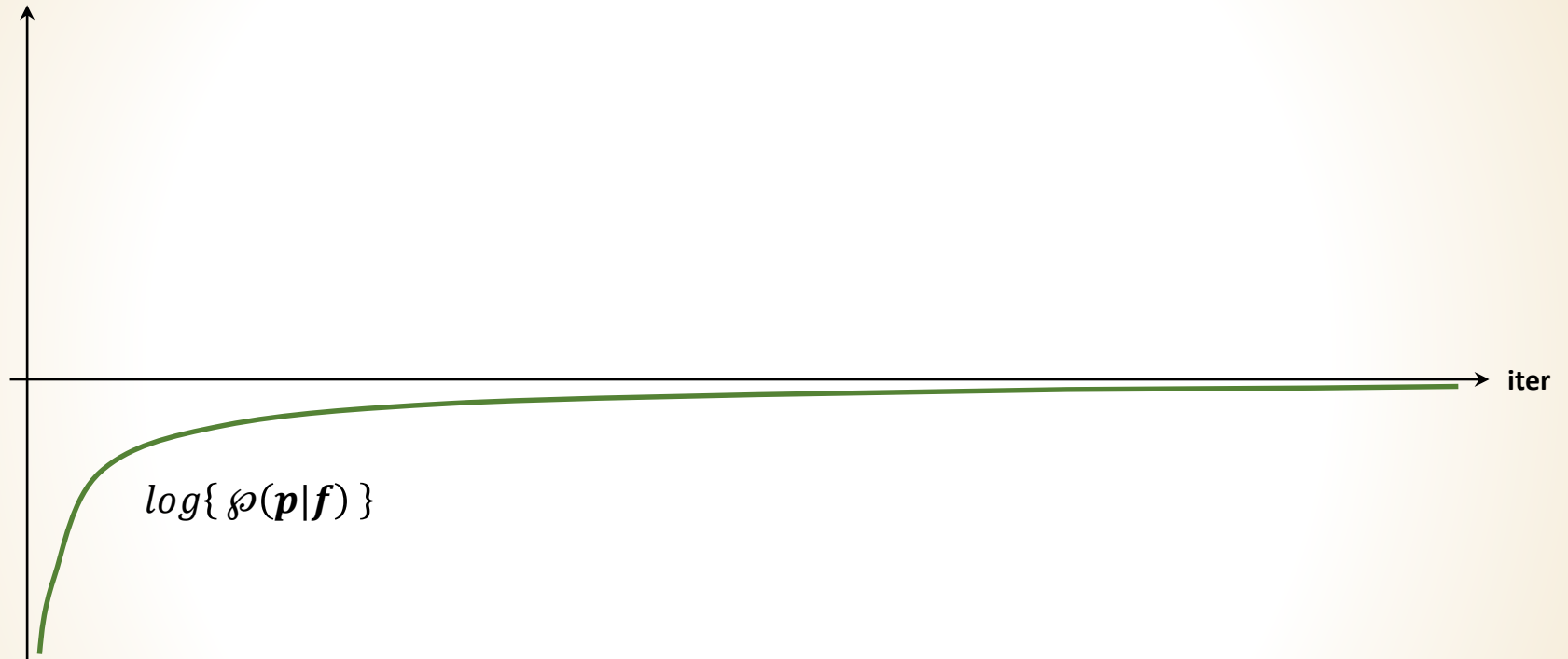
■ Reconstruction analytique

$$f = R^* \widehat{BHp} \quad \widehat{BHp} = \frac{|v| \widehat{p}}{\sqrt{1 + \left(\frac{v}{v_c}\right)^\beta}}$$



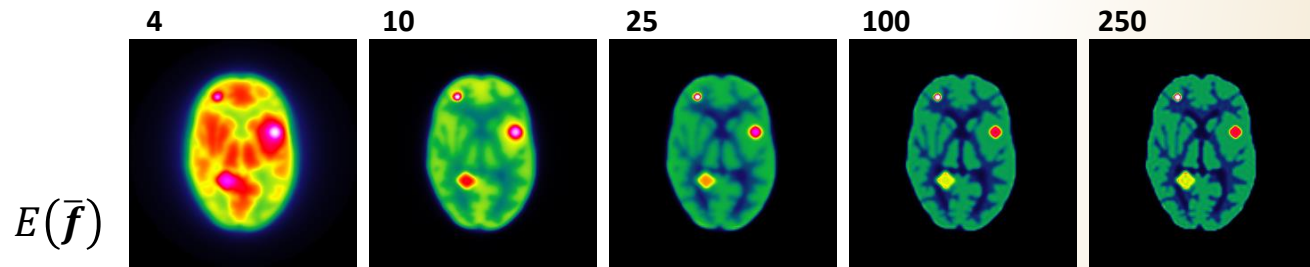
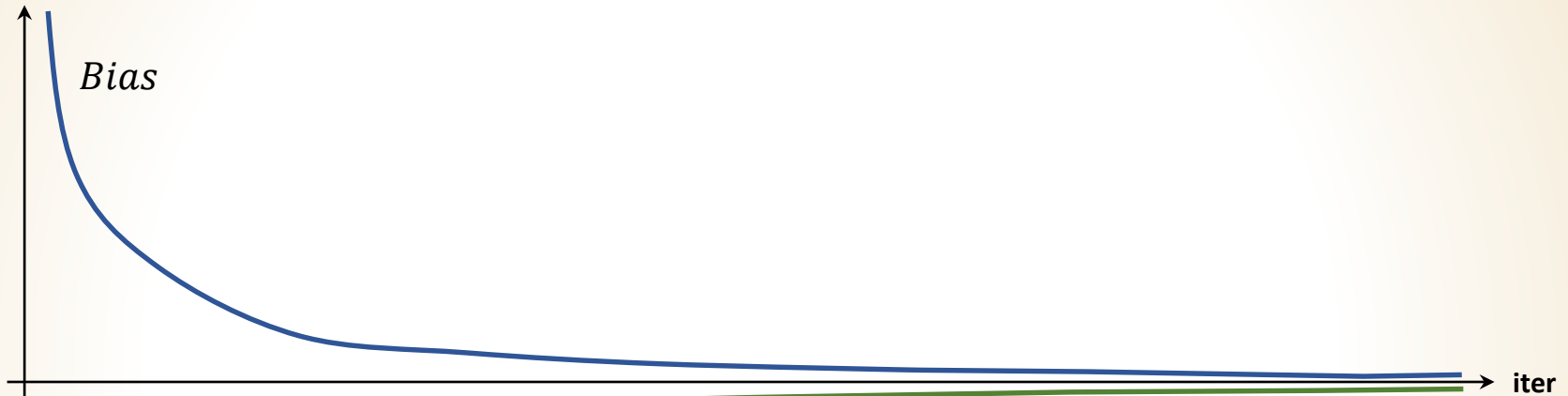
Régularisation

■ Reconstruction itérative



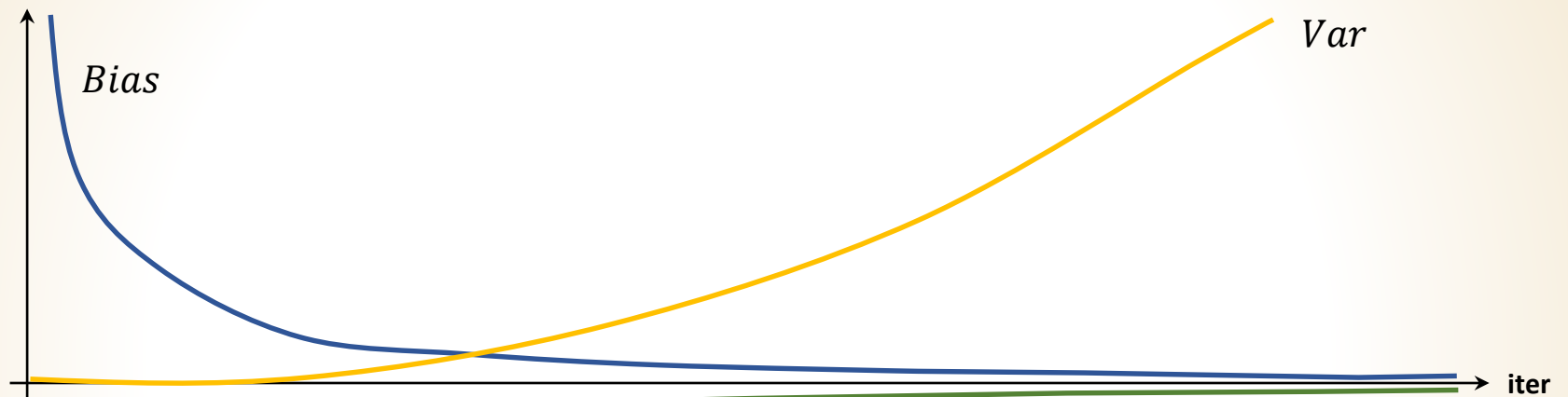
Régularisation

■ Reconstruction itérative



Régularisation

■ Reconstruction itérative



$\log\{\varphi(p|f)\}$

4

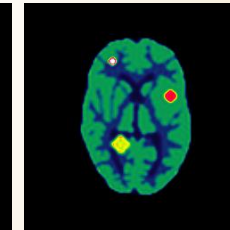
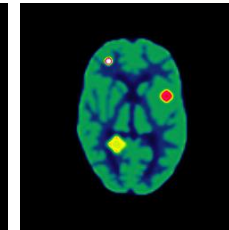
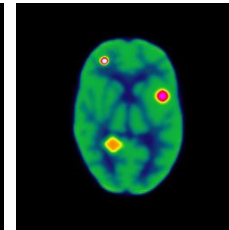
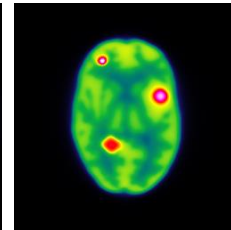
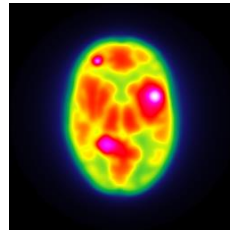
10

25

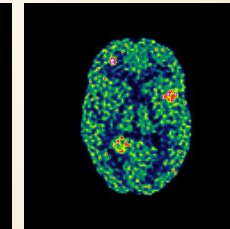
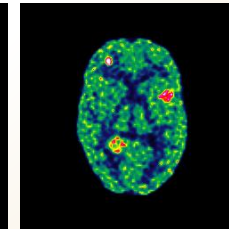
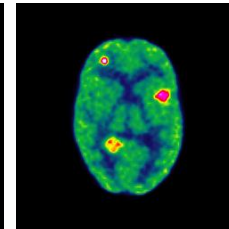
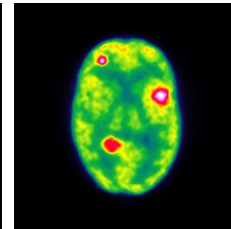
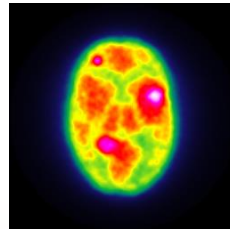
100

250

$E(\bar{f})$

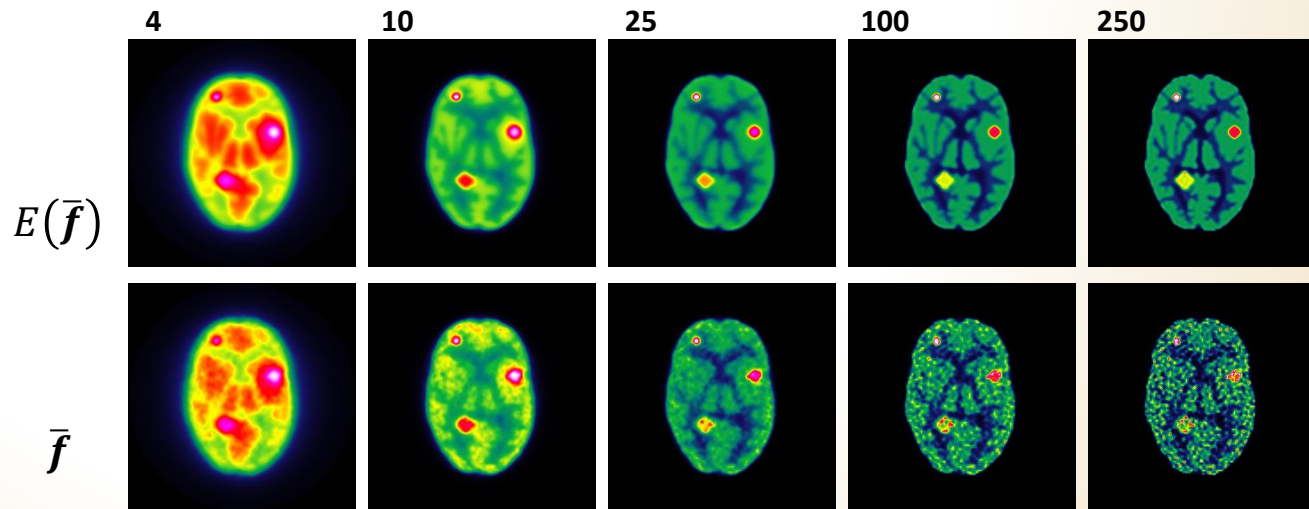
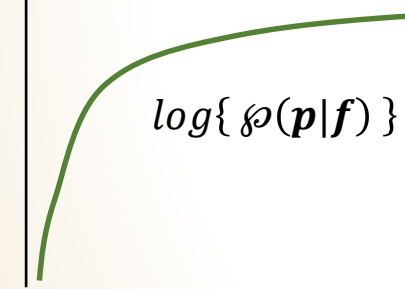
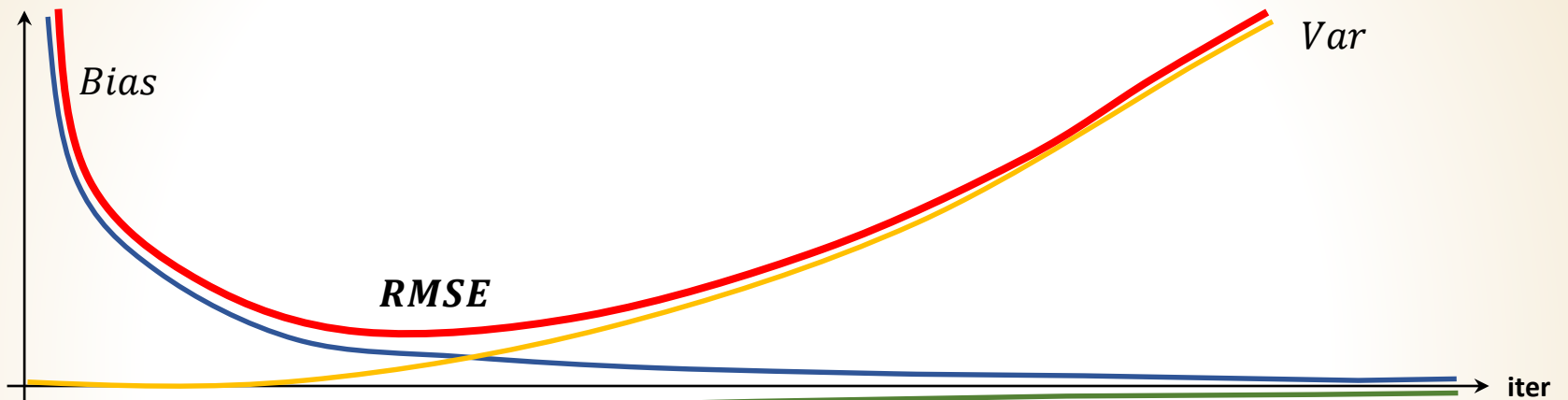


\bar{f}



Régularisation

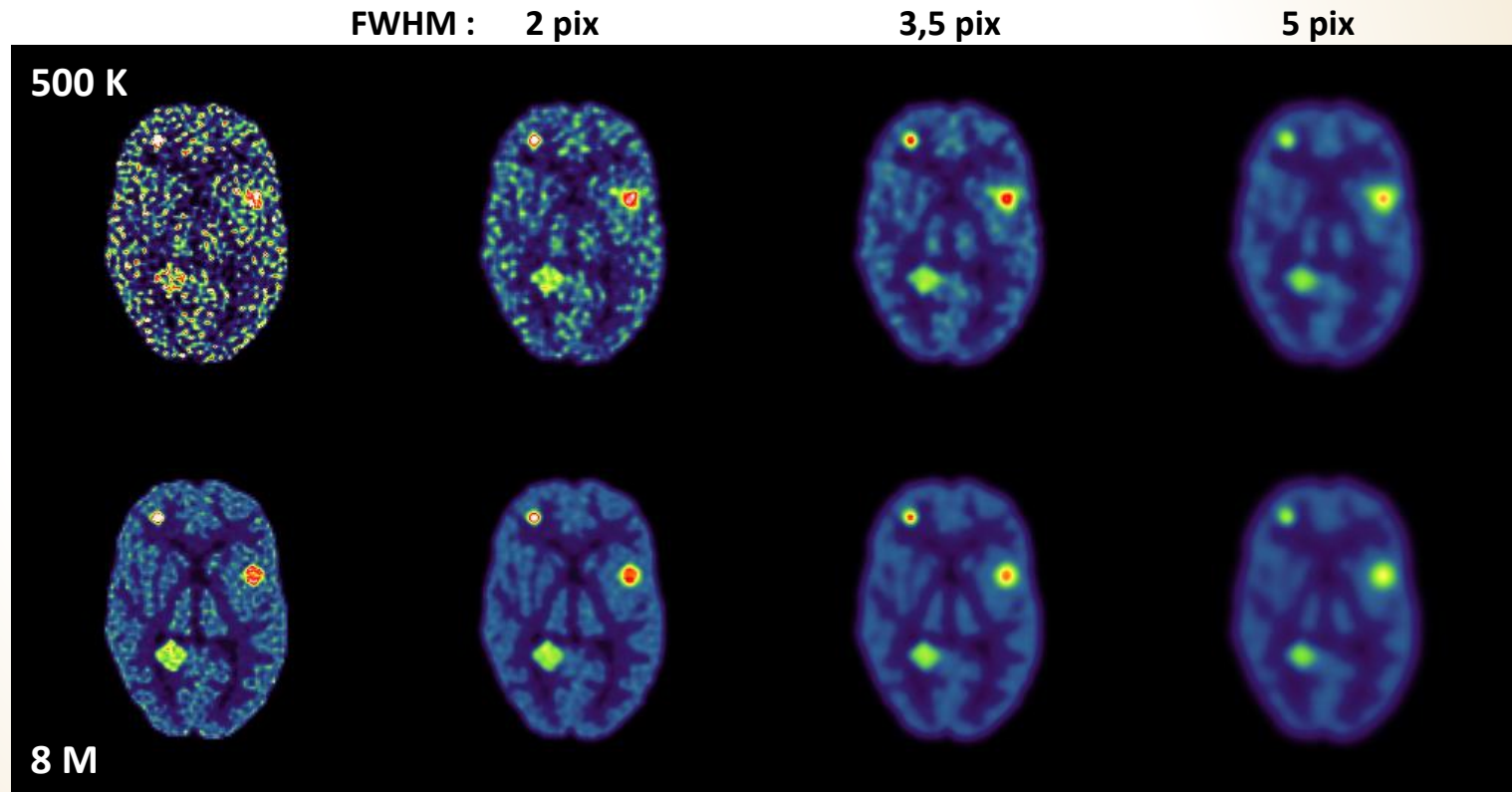
■ Reconstruction itérative



Régularisation

■ Reconstruction itérative

Post-filtrage



Régularisation

■ Reconstruction itérative

Tikhonov

$$\bar{\mathbf{f}} = \underset{\mathbf{f} \in \mathcal{C}}{\operatorname{argmin}} \{J(\mathbf{f})\}$$

$$J(\mathbf{f}) = \|\mathbf{R}\mathbf{f} - \mathbf{p}\|^2 + \rho(\mathbf{f})$$

Régularisation

■ Reconstruction itérative

Tikhonov

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{J(f)\}$$

$$J(f) = \|Rf - p\|^2 + \rho(f)$$

Adéquation
Surjectivité
↓ biais

Régularisation

■ Reconstruction itérative

Tikhonov

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{J(f)\}$$

$$J(f) = \|Rf - p\|^2 + \rho(f)$$

Adéquation
Surjectivité
↓ biais

Régularisation
Injectivité
↓ variance

Régularisation

■ Reconstruction itérative

Tikhonov

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{J(f)\}$$

$$J(f) = \|Rf - p\|^2 + \beta \|f\|^2$$

Régularisation

■ Reconstruction itérative

Tikhonov

$$\bar{\mathbf{f}} = \underset{\mathbf{f} \in \mathcal{C}}{\operatorname{argmin}} \{J(\mathbf{f})\}$$

$$J(\mathbf{f}) = \|\mathbf{R}\mathbf{f} - \mathbf{p}\|^2 + \beta \|\mathbf{f}\|^2$$

$$\bar{\mathbf{f}} = (\mathbf{R}^* \mathbf{R} + \beta \mathbf{I})^{-1} \mathbf{R}^* \mathbf{p}$$

$$\bar{\mathbf{f}}^{n+1} = (1 - \beta) \bar{\mathbf{f}}^n + \mathbf{R}^* (\mathbf{p} - \mathbf{R} \bar{\mathbf{f}}^n)$$

Régularisation

■ Reconstruction itérative

MAP

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{ J(f) \}$$

Posterior

$$J(f) = -\log\{ \wp(f|p) \}$$

Régularisation

■ Reconstruction itérative

MAP

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{ J(f) \}$$

Posterior

$$J(f) = -\log \{ \wp(f|\mathbf{p}) \}$$

$$\wp(f|\mathbf{p}) = \frac{\wp(\mathbf{p}|f) \wp(f)}{\wp(\mathbf{p})}$$

Régularisation

■ Reconstruction itérative

MAP

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{ J(f) \}$$

Posterior

$$J(f) = -\log \{ \wp(f|\mathbf{p}) \}$$

$$\wp(f|\mathbf{p}) = \frac{\wp(\mathbf{p}|f) \wp(f)}{\wp(\mathbf{p})}$$

$$J(f) = -\log \{ \wp(\mathbf{p}|f) \} - \log \{ \wp(f) \}$$

Régularisation

■ Reconstruction itérative

MAP

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{ J(f) \}$$

Posterior

$$J(f) = -\log\{ \wp(f|p) \}$$

Likelihood

$$J(f) = -\log\{ \wp(p|f) \} - \log\{ \wp(f) \}$$

Adéquation
Surjectivité
↓ biais

Régularisation

■ Reconstruction itérative

MAP

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{ J(f) \}$$

Posterior

$$J(f) = -\log\{ \wp(f|p) \}$$

Likelihood

Prior

$$J(f) = -\log\{ \wp(p|f) \} - \log\{ \wp(f) \}$$

Adéquation
Surjectivité
↓ biais

Régularisation
Injectivité
↓ variance

Régularisation

■ Reconstruction itérative

MAP

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{ J(f) \}$$

$$J(f) = -\log\{ \overset{\text{Posterior}}{\wp(f|p)} \}$$

$$J(f) = -\overset{\text{Likelihood}}{\log\{ \wp(p|f) \}} - \overset{\text{Prior}}{\log\{ \wp(f) \}}$$

$$\wp(f) = \kappa e^{-\beta U}$$

Régularisation

■ Reconstruction itérative

MAP

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{ J(f) \}$$

Posterior

$$J(f) = -\log \{ \wp(f|p) \}$$

Likelihood

Prior

$$J(f) = -\log \{ \wp(p|f) \} - \log \{ \wp(f) \}$$

$$\wp(f) = \kappa e^{-\beta U}$$

$$U = \sum_{i,j} w_{ij} \|f_i - f_j\|^2$$

Régularisation

■ Reconstruction itérative

MAP

$$\bar{f} = \underset{f \in \mathcal{C}}{\operatorname{argmin}} \{ J(f) \}$$

Posterior

$$J(f) = -\log \{ \wp(f|p) \}$$

Likelihood

Prior

$$J(f) = -\log \{ \wp(p|f) \} - \log \{ \wp(f) \}$$

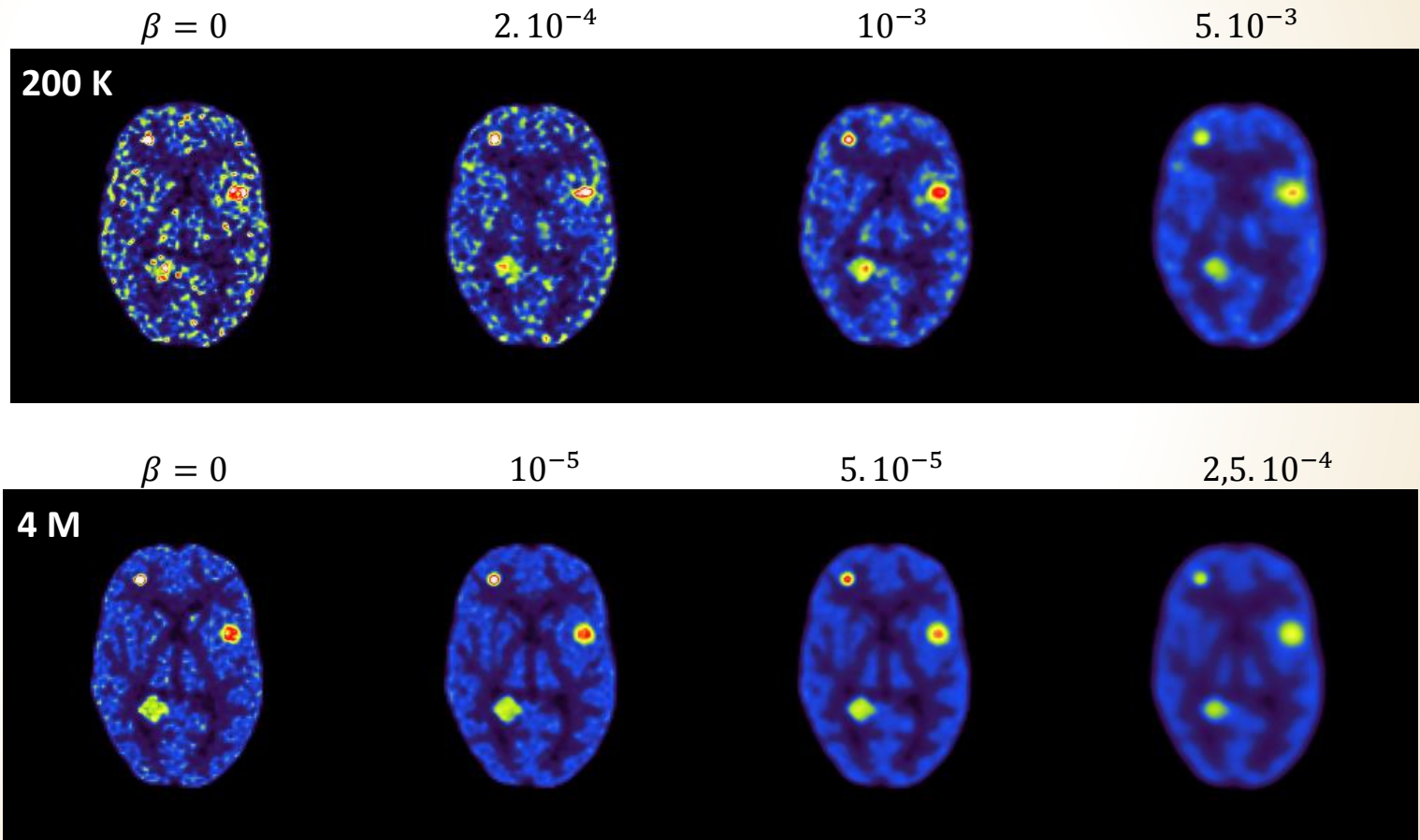
$$\wp(f) = \kappa e^{-\beta U}$$

$$\bar{f}^{n+1} = \frac{\bar{f}^n}{1 + \beta \nabla U} \times \left(\mathbf{R}^* \frac{p}{\mathbf{R} \bar{f}^n} \right)$$

Régularisation

■ Reconstruction itérative

MAP



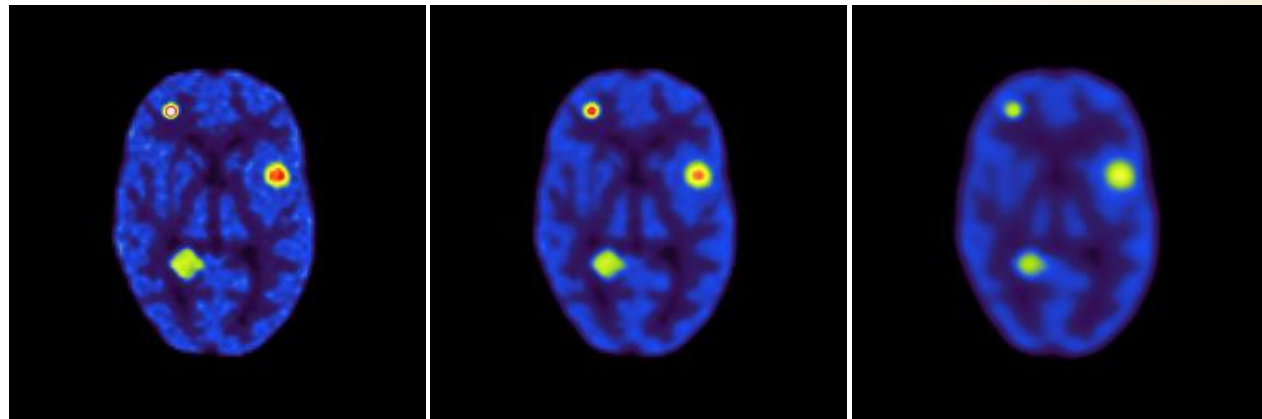
Régularisation

■ Reconstruction itérative

MAP

Quad. prior

$$U = \sum_{i,j} w_{ij} \|f_i - f_j\|^2$$



Median prior

$$U = \sum_{i,j} w_{ij} |f_i - f_j|$$

« edge-preserving »

