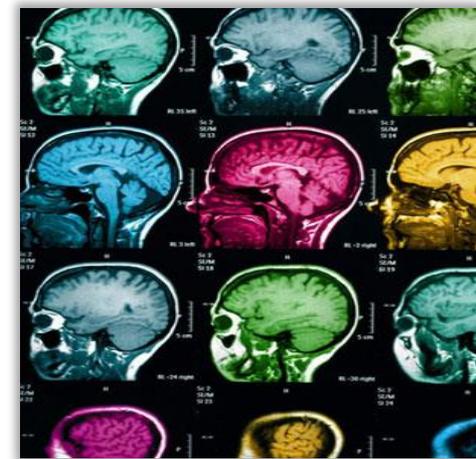


# MASTER PhyMed

GMPH308 - Physique de l'imagerie médicale

FORMATION des  
IMAGES X et  $\gamma$

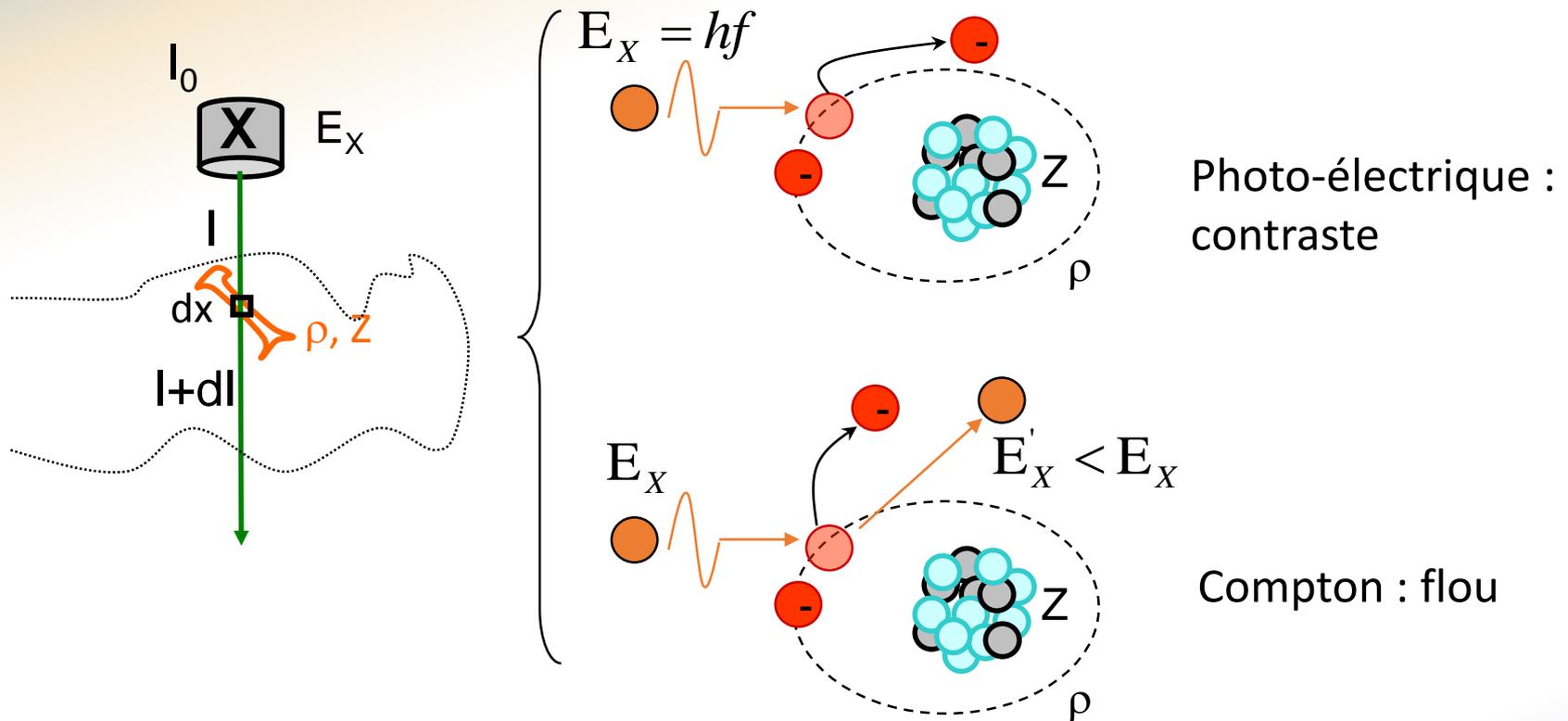


Fayçal Ben Bouallègue

faybenb@hotmail.com

<http://scinti.etud.univ-montp1.fr>

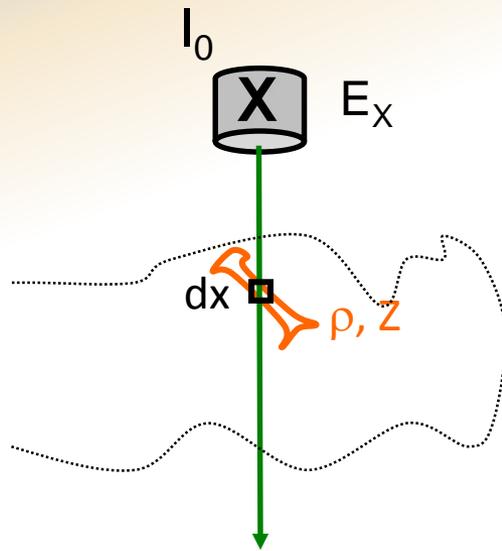
# IMAGERIE DE TRANSMISSION X



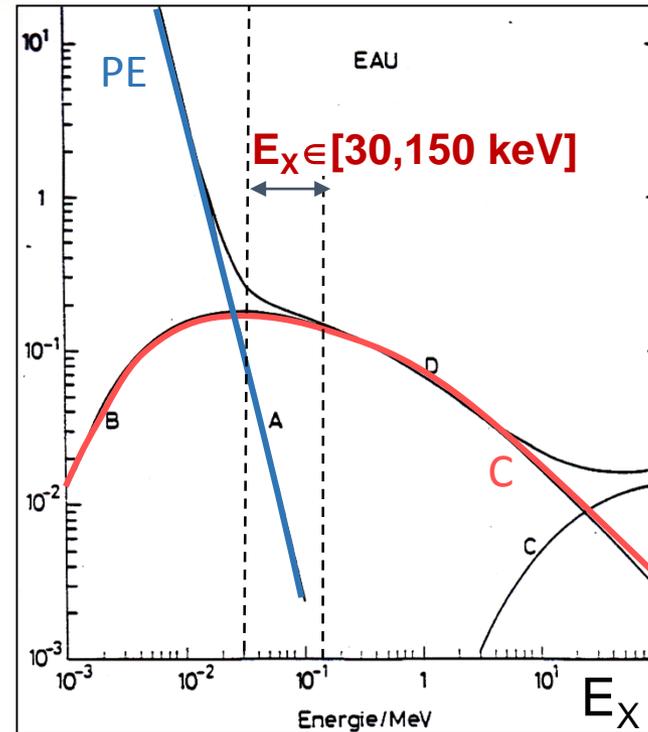
Interaction aléatoire  $\Rightarrow$   $dl / I = - \mu dx$

$\mu$  = coef. linéique d'atténuation  
= probabilité d'atténuation par cm

# IMAGERIE DE TRANSMISSION X



$\mu/\rho$   
 $\text{cm}^2/\text{g}$



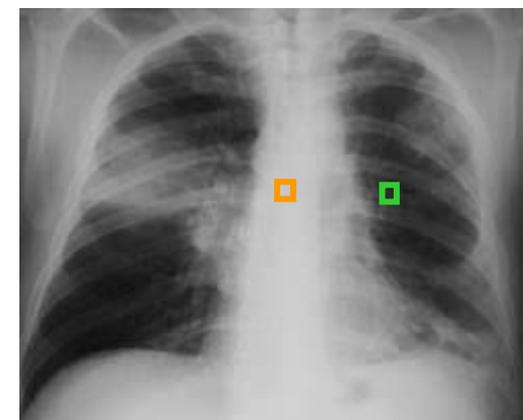
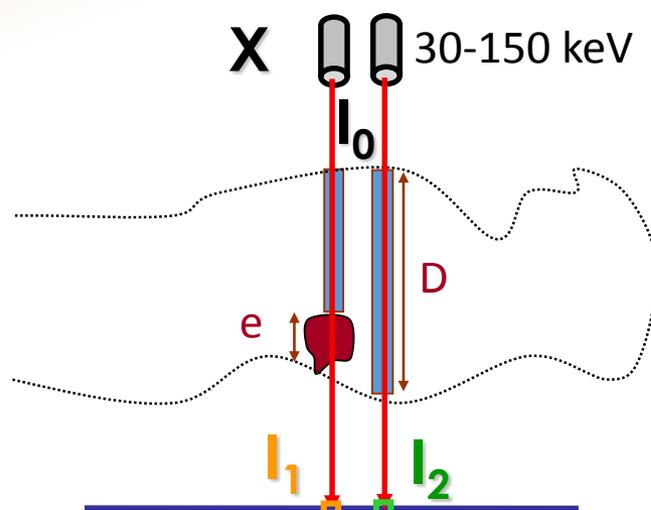
$\mu =$   
coef. linéique  
d'atténuation  
= probabilité  
d'atténuation  
par cm

$$\frac{dI}{I} = \mu dx \Rightarrow I = I_0 \cdot e^{-\mu \cdot x}$$

$$\mu_C \propto \rho$$

$$\mu_{PE} \propto \rho \cdot (Z/E_X)^3$$

# IMAGERIE DE TRANSMISSION X



$$I_1 = I_0 e^{-\mu(D-e) - \mu'e}$$

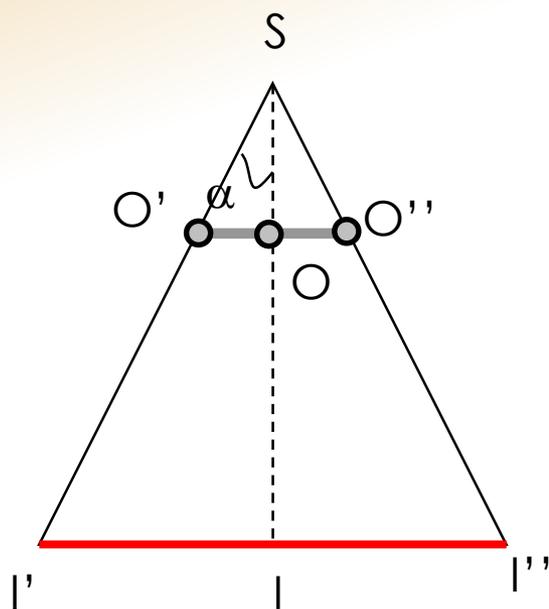
$$\approx I_0 (1 - \mu(D-e) - \mu'e)$$

$$I_2 = I_0 e^{-\mu D}$$

$$\approx I_0 (1 - \mu D)$$

$$C = \frac{|I_1 - I_2|}{I_1 + I_2} = \frac{|\mu' - \mu| \cdot e}{2} \propto \Delta\mu \Rightarrow C \propto \frac{e}{E_x^3} \Delta(\rho \cdot Z^3) : \text{différence de densité}$$

# GRANDISSEMENT EN IMAGERIE X



Source de X = S

Objet absorbant

Détecteur (film)

$$\tan \alpha = \frac{II'}{SI} = \frac{OO'}{SO}$$

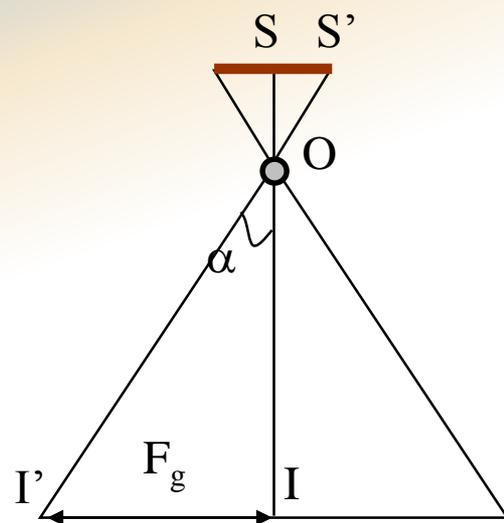


$$\frac{SI}{SO} = \frac{II'}{OO'} = G$$

Donc le grandissement G dépend :

- de la distance de la source de X au film (SI)
- de la **distance de la source à l'objet** (SO)

# FLOUS EN IMAGERIE X



Foyer  $F = SS'$

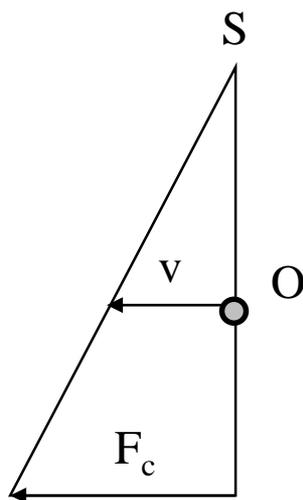
Objet absorbant

$$\text{tg}\alpha = \frac{IO'}{IO} = \frac{SS'}{SO}$$

$$\Rightarrow F_g = F \cdot \frac{IO}{SO} = F \cdot \left( \frac{IS - OS}{SO} \right)$$

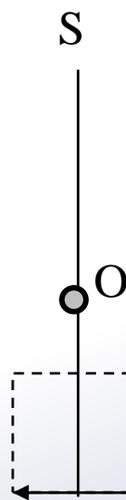
Flou géométrique :

$$F_g = F \cdot (G - 1)$$



Flou cinétique :

$$F_c = G \cdot v \cdot t$$

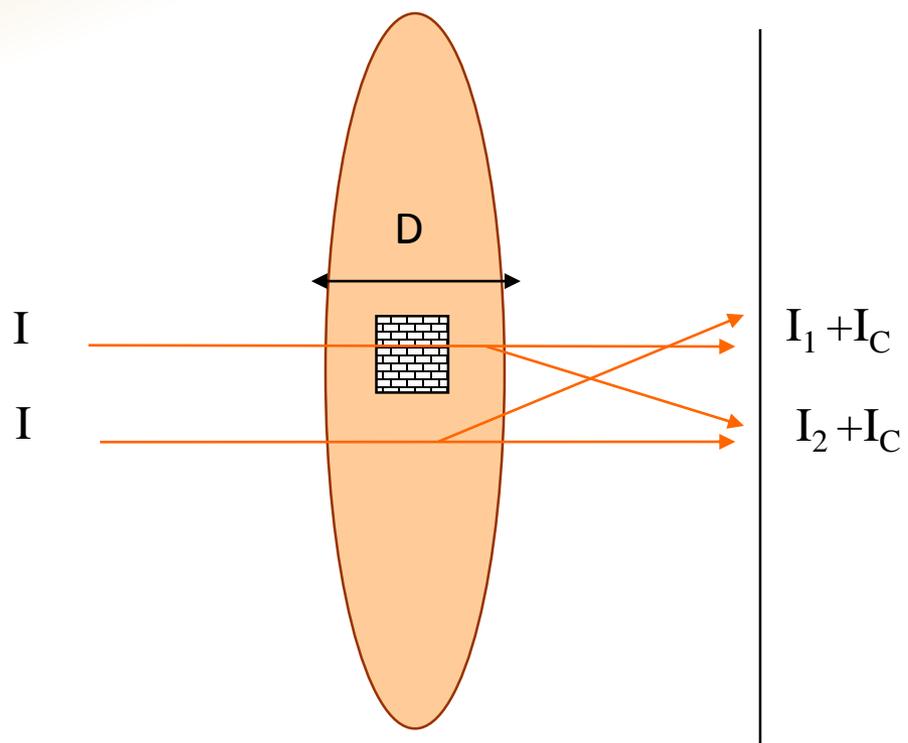


Flou d'écran :

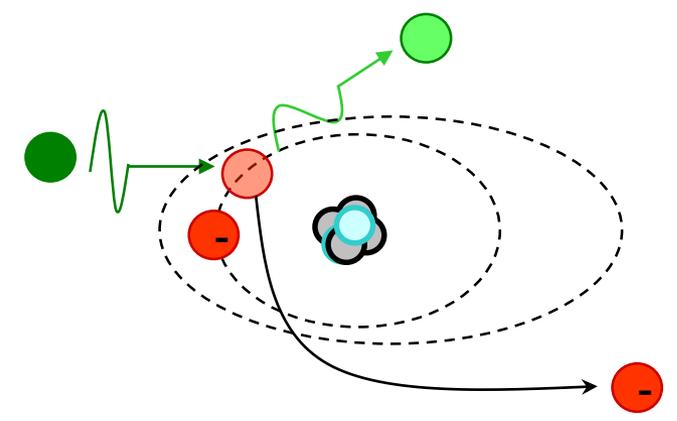
$$F_e = \text{Grain, pixel}$$

# FLOUS EN IMAGERIE X

Flou par diffusion Compton :



$$\mu_c \approx 5 \cdot \mu_{PE}$$



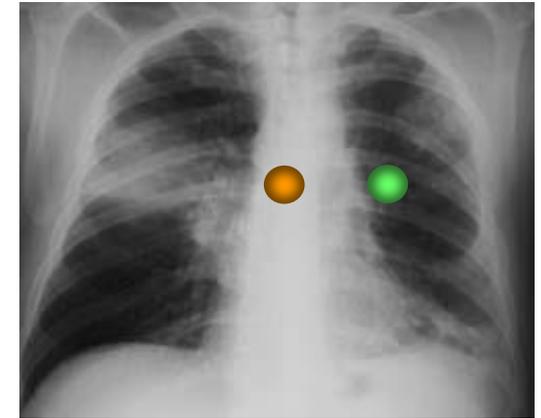
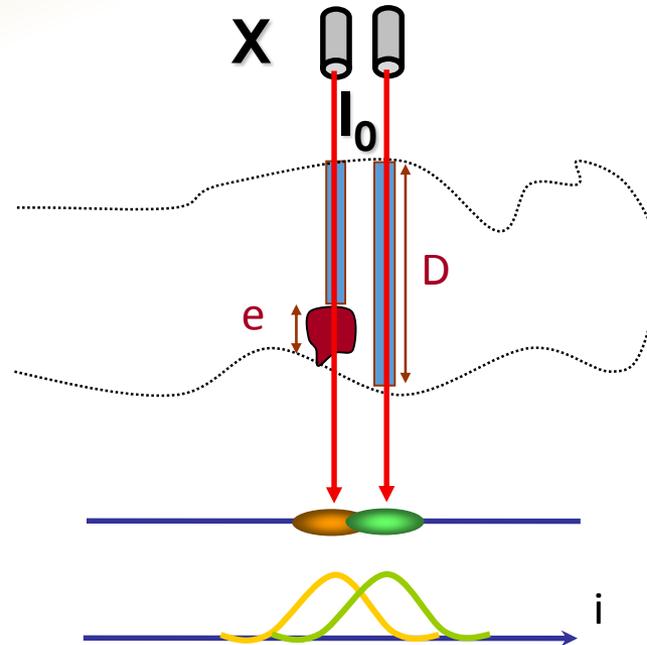
$$C = \frac{|I_1 - I_2|}{I_1 + I_2 + 2I_c}$$

- ✓ Compression :  $D \downarrow$
- ✓ Diaphragme
- ✓ Grille
- ✓ Radio-protection

# REPONSE EN IMAGERIE X 2D



ANALOGIQUE

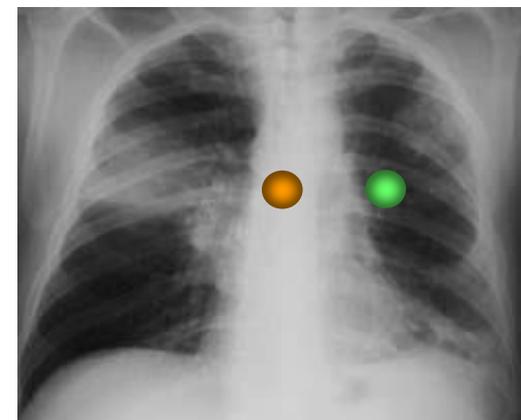
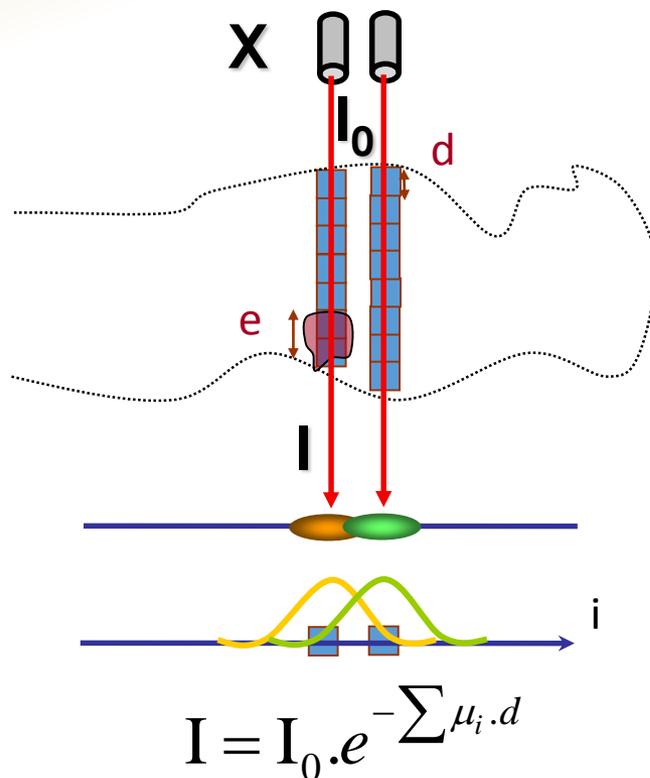


$$h(i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{i^2}{2\sigma^2}}$$

# REPONSE EN IMAGERIE X 2D



NUMERISE



$$I = I_0 \cdot e^{-\sum \mu_i \cdot d}$$

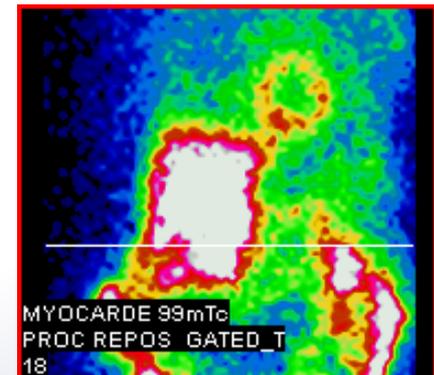
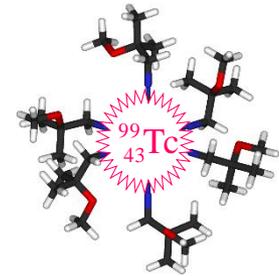
$$h(i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{i^2}{2\sigma^2}}$$

$$p = \mu_1 + \mu_2 + \dots + \mu_n$$

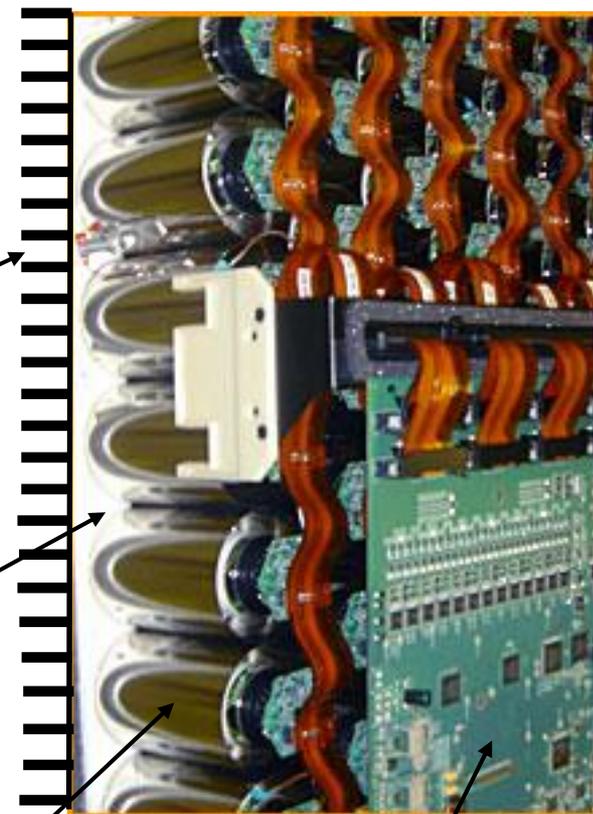
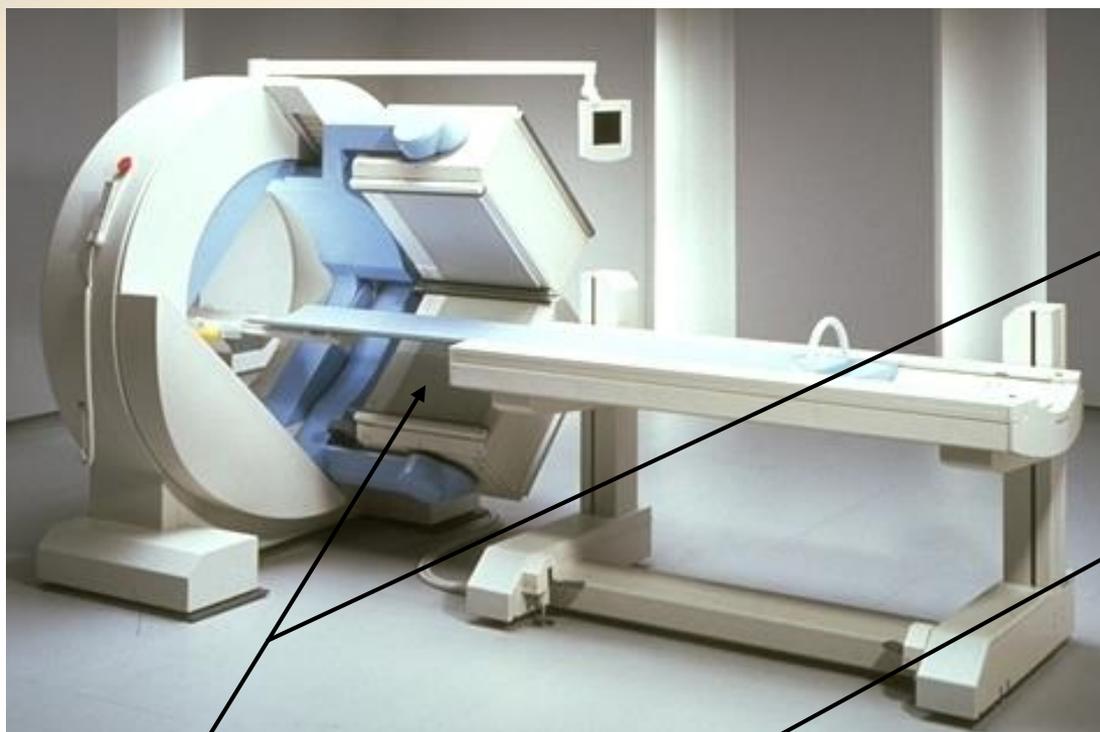
avec  $p = -\frac{1}{d} \ln \frac{I}{I_0}$

# IMAGERIE D'EMISSION GAMMA

- Principe : lier un isotope radioactif  $\gamma$  ou  $\beta^+$  (marqueur) à une molécule d'intérêt biologique (vecteur) pour obtenir un traceur.
- Produire une cartographie de la radioactivité (scintigraphie) pour suivre le devenir du traceur dans un organisme humain.



# $\gamma$ -CAMERA



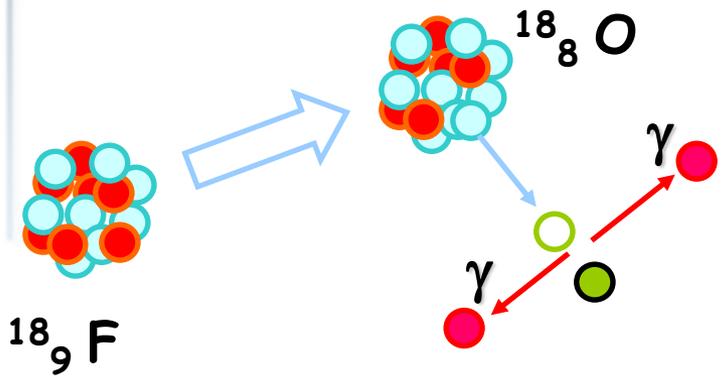
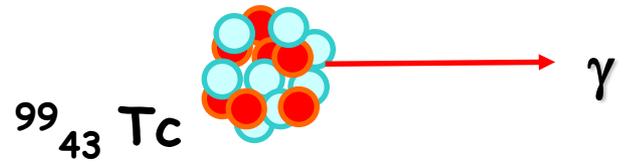
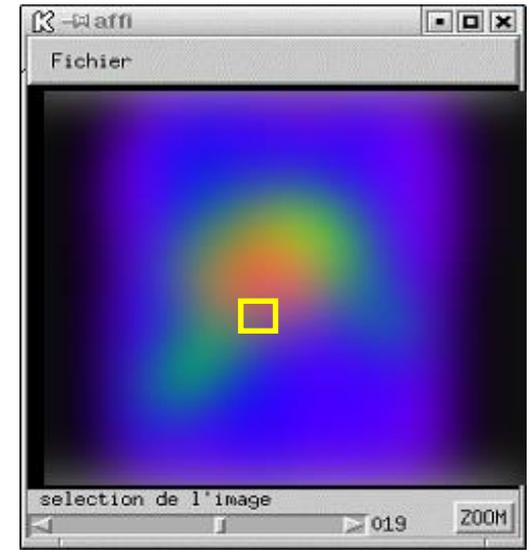
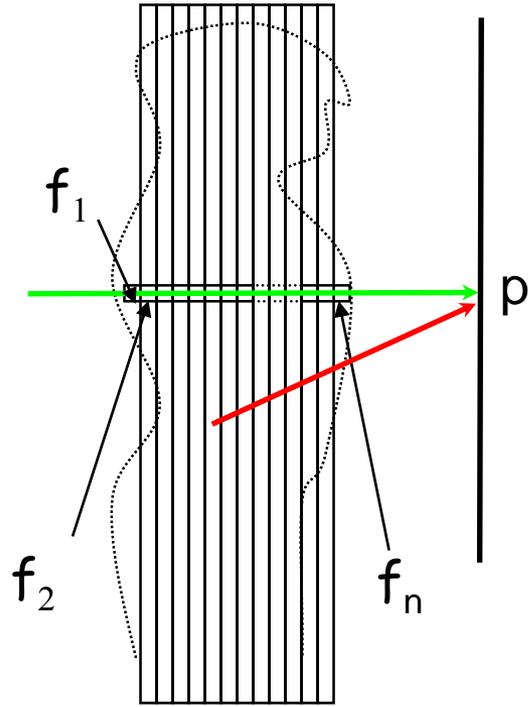
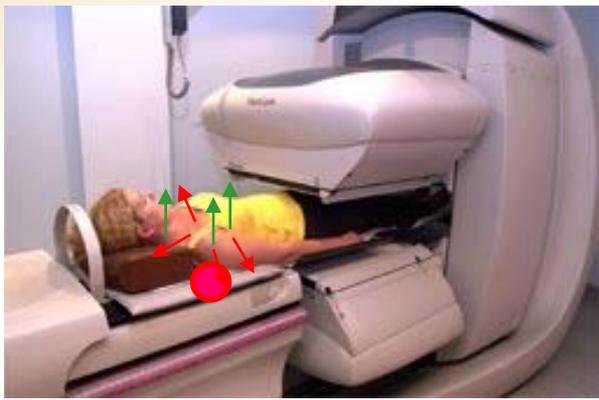
Collimateur

Scintillateur

Photo-multiplicateur

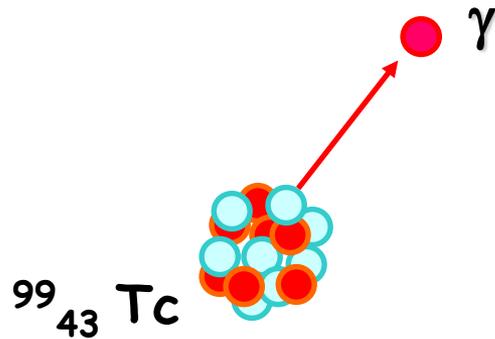
Localisation

# ACQUISITION 2D

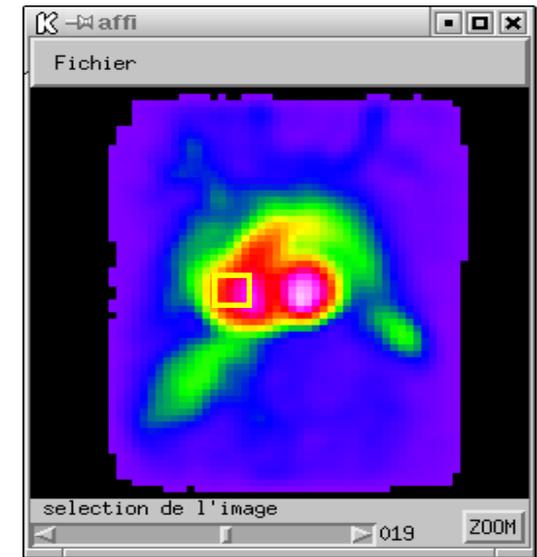
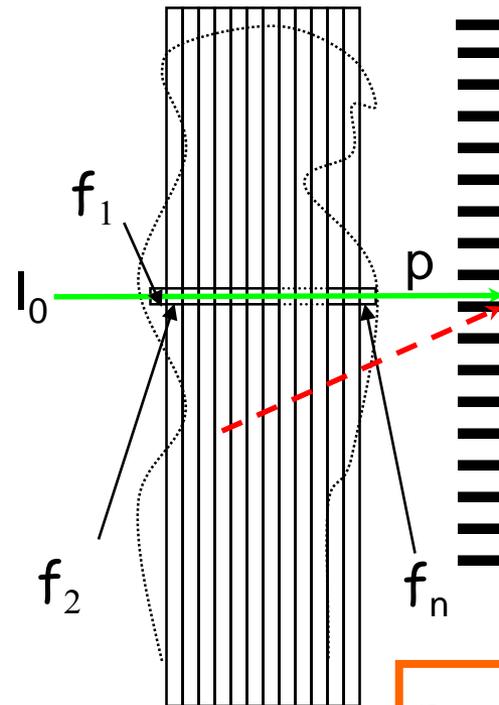


Isotopes émetteurs  $\gamma$  ou  $\beta^+$

# ACQUISITION 2D



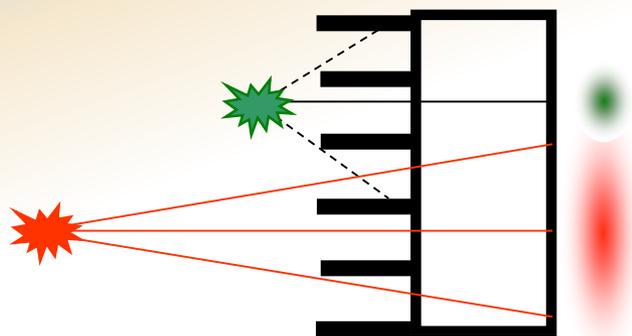
COLLIMATEUR



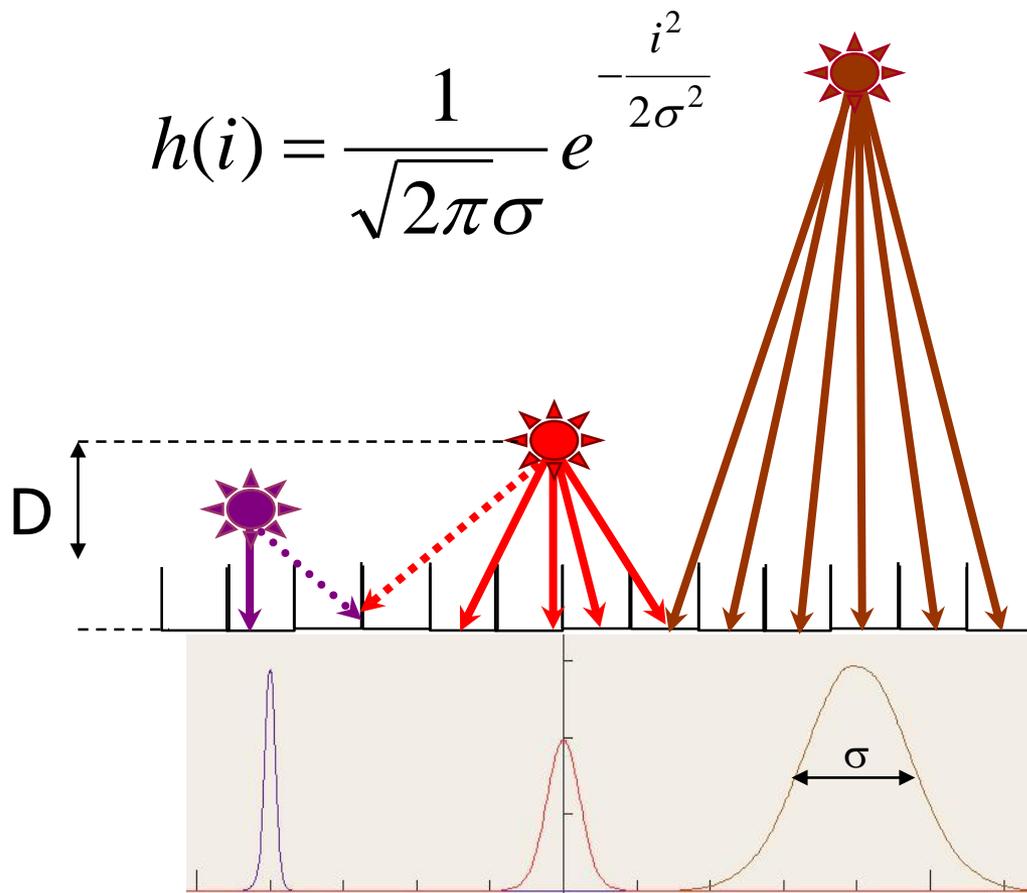
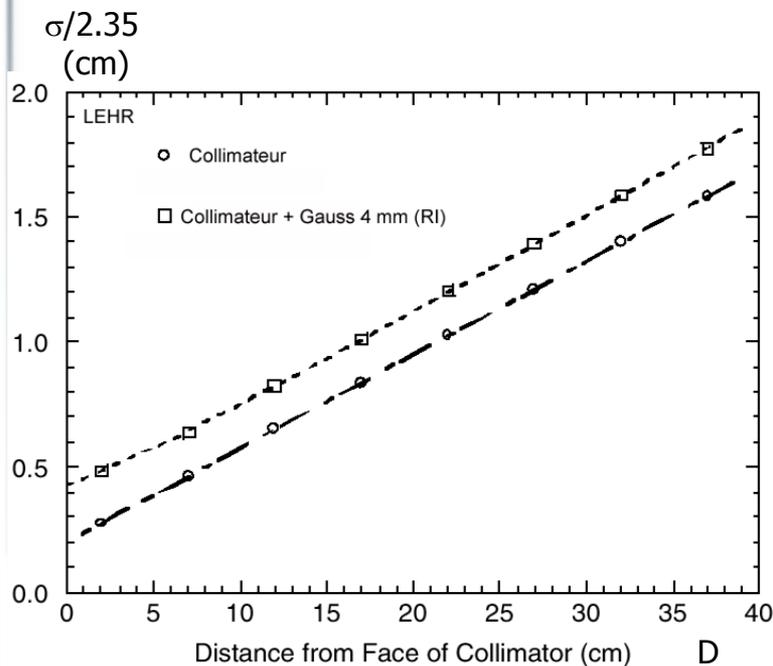
$$p = R_1 f_1 + R_2 f_2 + \dots + R_n f_n$$

résolution  $\approx$  cm  
bruit de Poisson

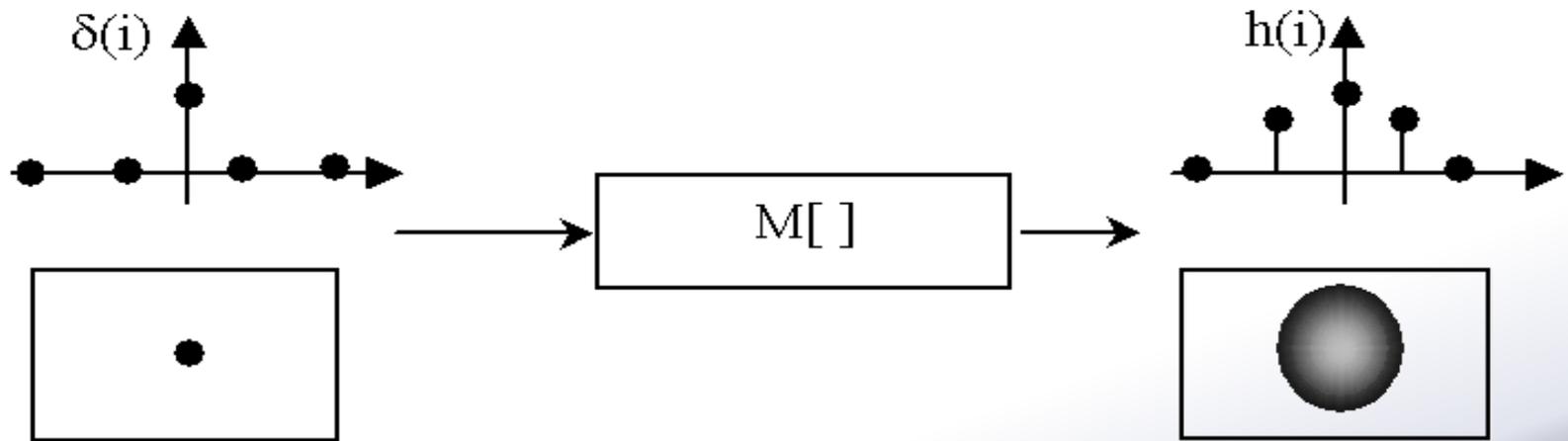
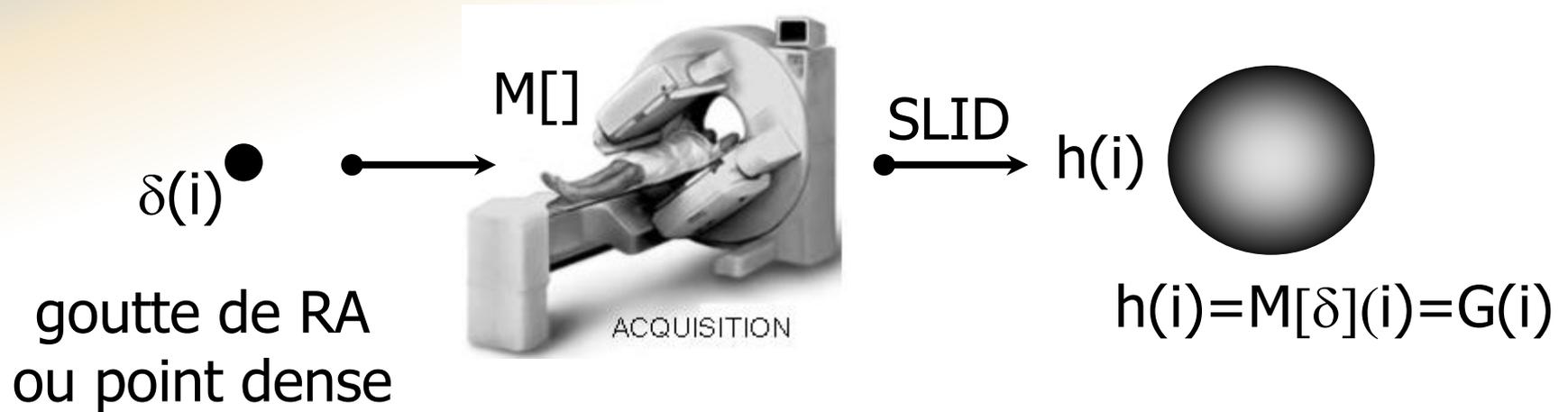
# FLOU LIE AU COLLIMATEUR



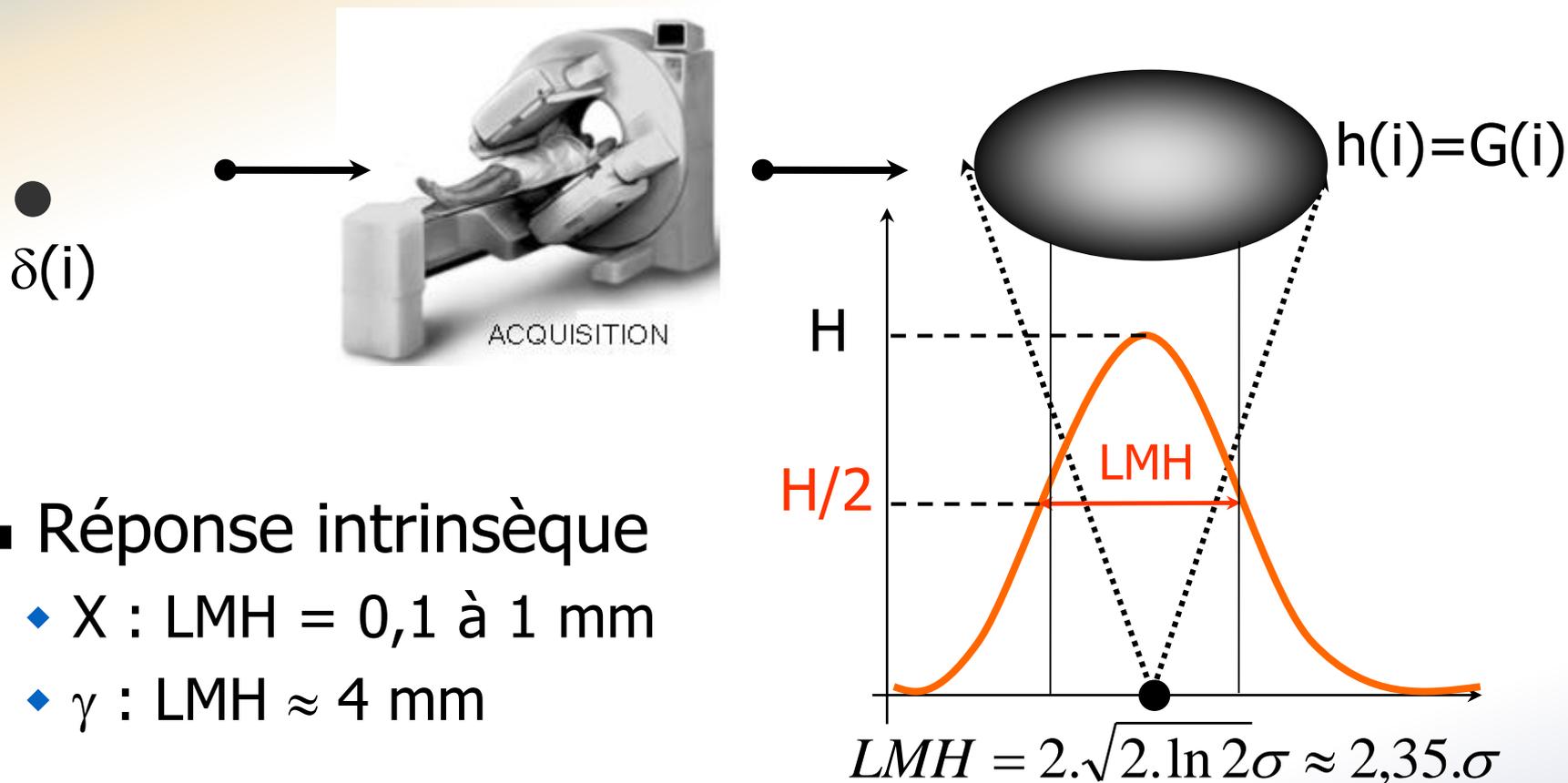
$$h(i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{i^2}{2\sigma^2}}$$



# REPONSE D'UNE CAMERA X, $\gamma$



# LARGEUR A MI-HAUTEUR



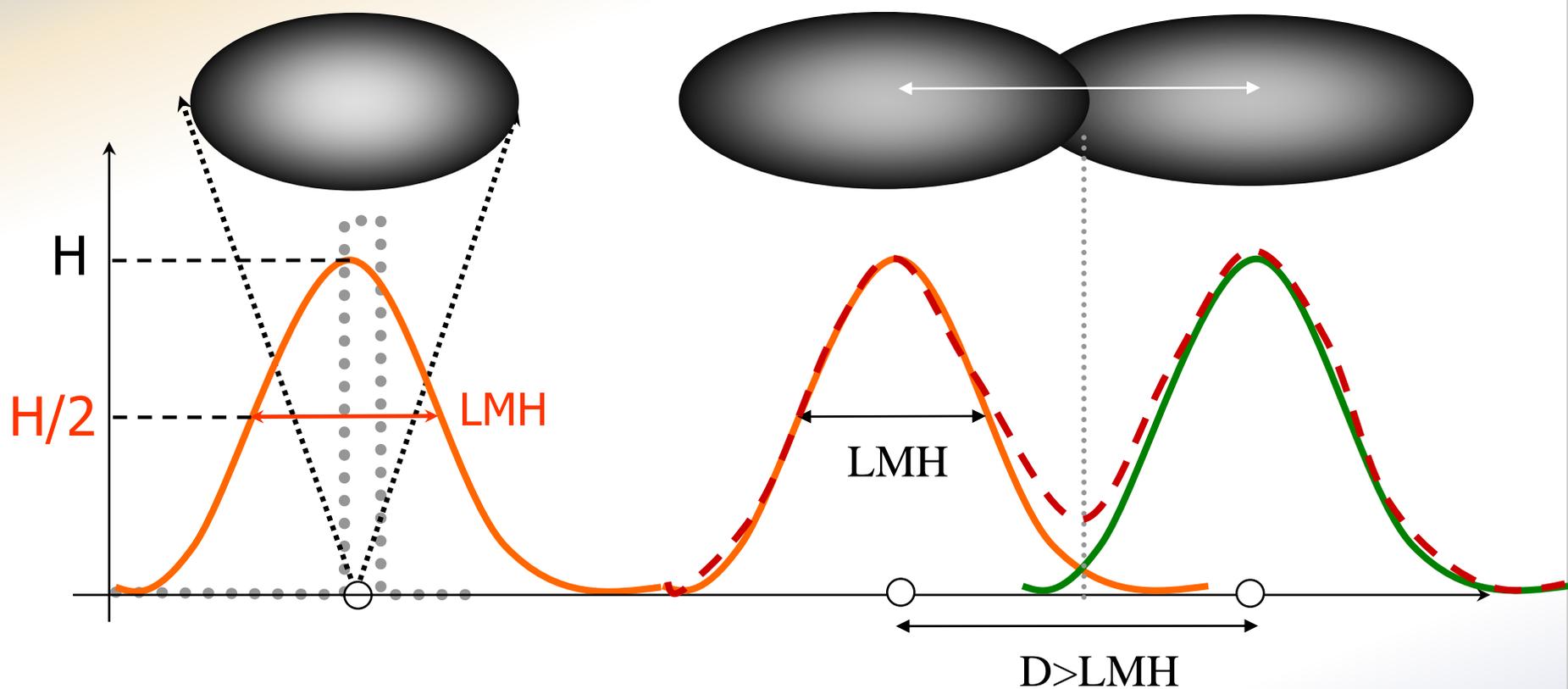
## ■ Réponse intrinsèque

- ◆ X : LMH = 0,1 à 1 mm
- ◆  $\gamma$  : LMH  $\approx$  4 mm

## ■ Réponse du collimateur (seulement pour l'imagerie $\gamma$ )

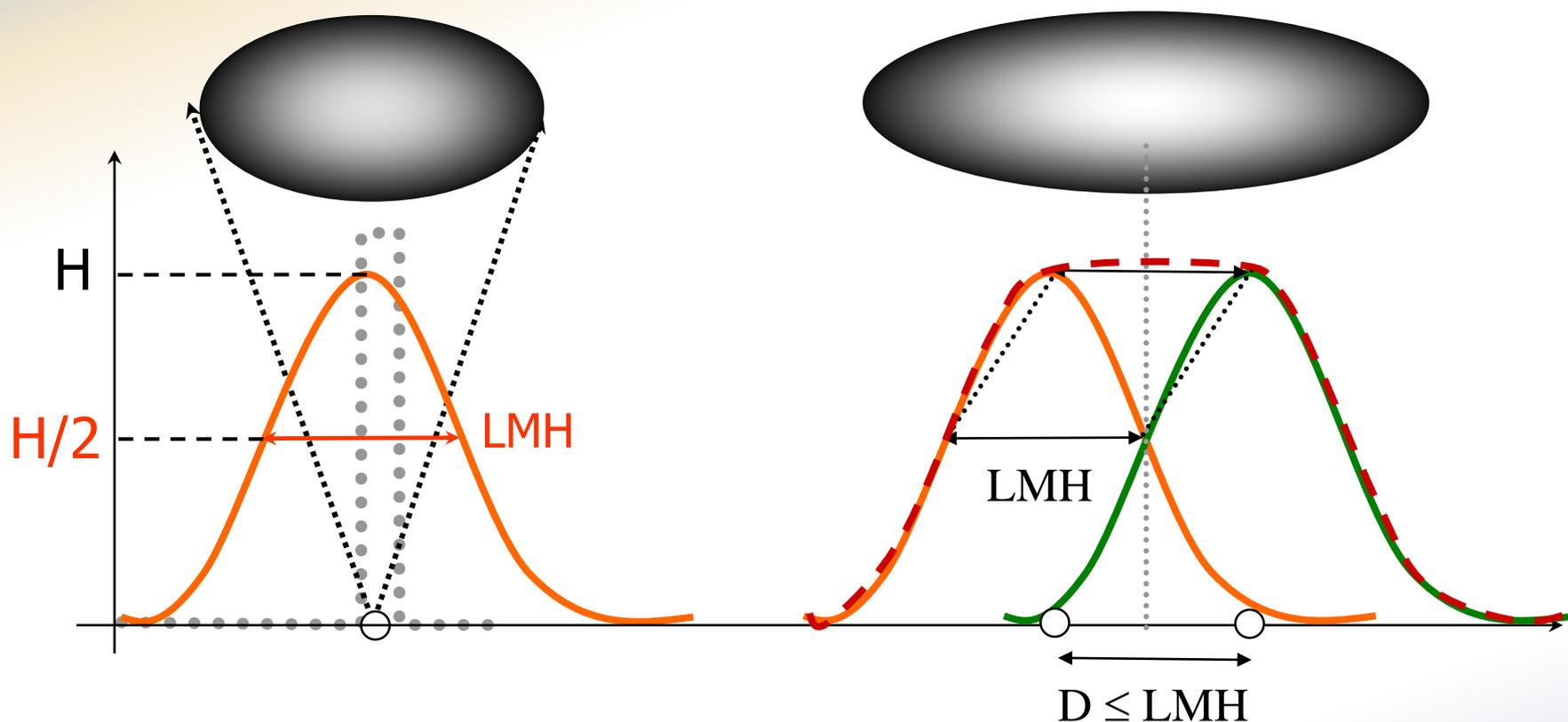
- ◆ LMH supplémentaire qui dépend de la distance source-détecteur

# INTERPRETATION



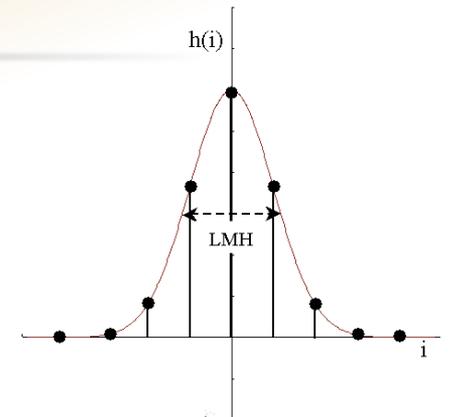
Si  $D > LMH$ , les deux objets sont discernables sur l'image

# INTERPRETATION

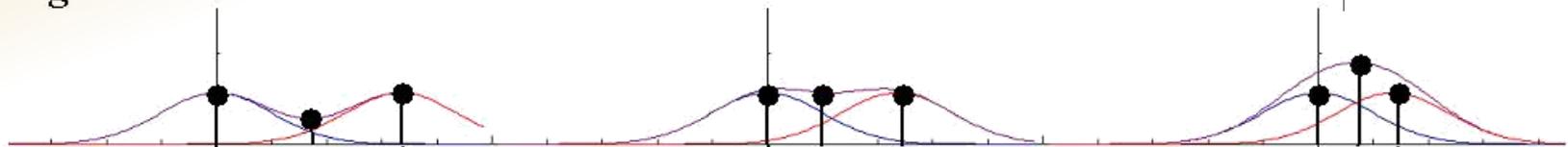


Si  $D < LMH$ , les deux images fusionnent :  
 $LMH = \text{résolution} = \text{pouvoir séparateur}$

# INTERPRETATION

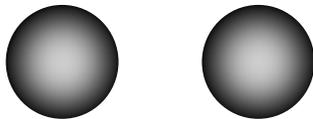


*Images :*

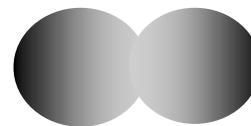


*Objets :*

$d > LMH$



$d = LMH$



$d < LMH$

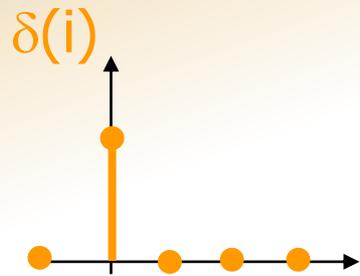


$LMH =$  pouvoir séparateur = résolution de l'imageur

$LMH =$  période minimale observable sur l'image

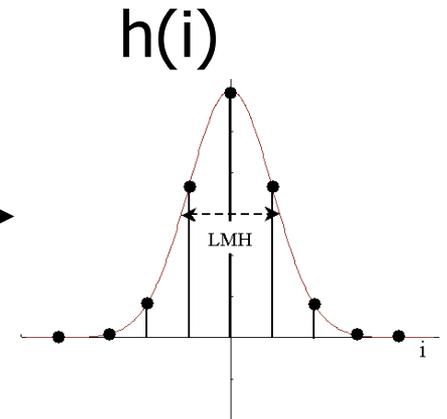
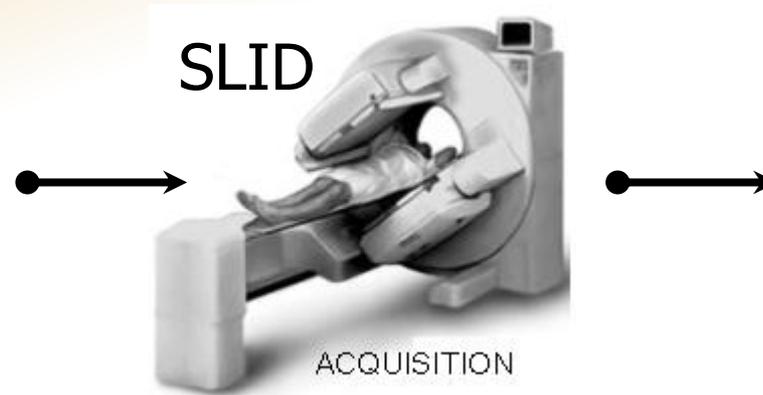
$1/LMH =$  fréquence spatiale maximale dans le signal

# FORMATION DE L'IMAGE



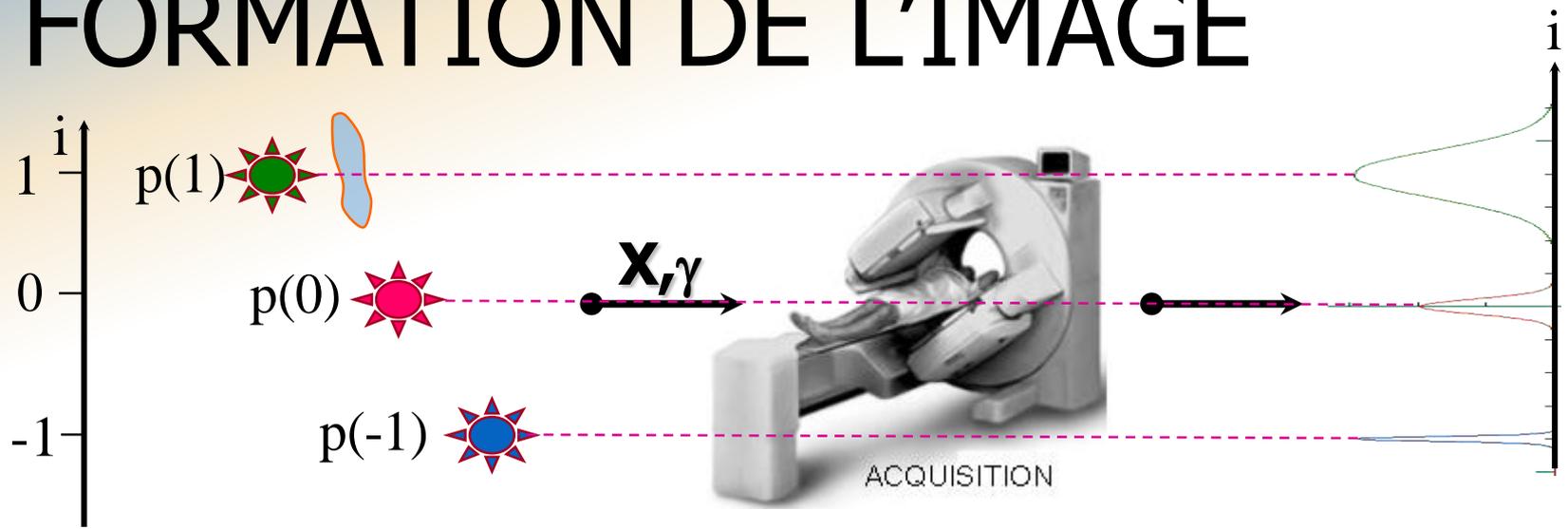
$$\delta(0) = 1$$

$$\delta(k) = 0 \text{ si } k \neq 0$$



$$h(i) = M[\delta](i)$$

# FORMATION DE L'IMAGE

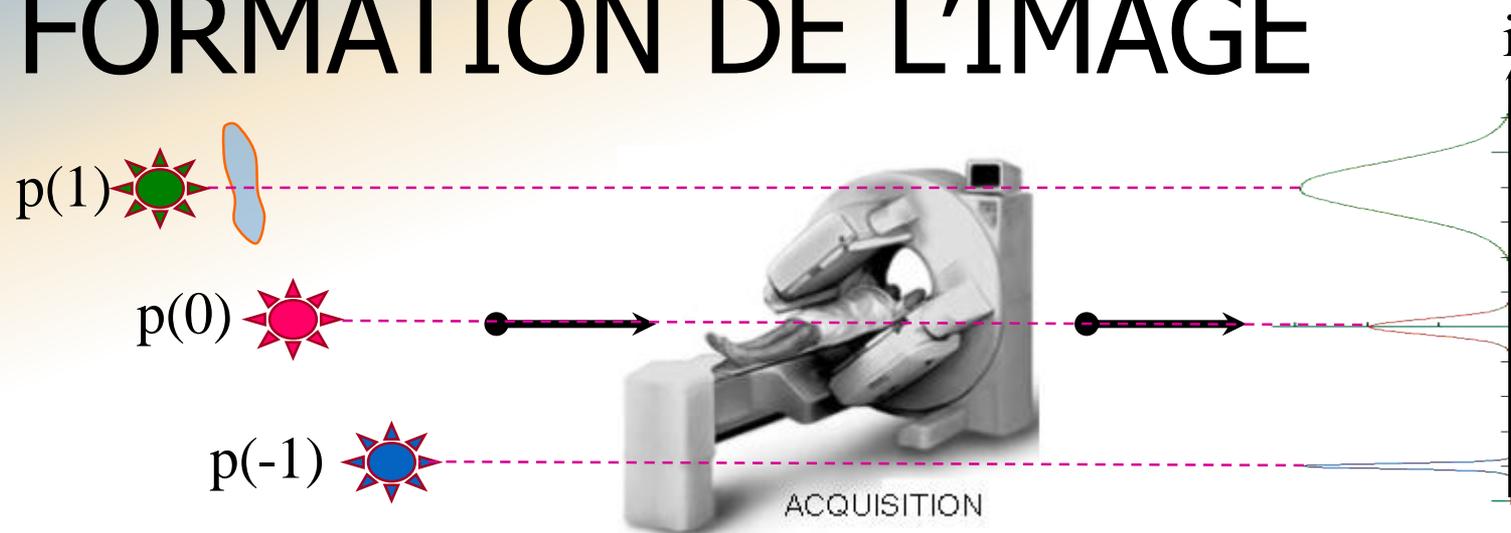


$$p(i) = \sum_{k=-\infty}^{+\infty} p(k) \cdot \underbrace{\delta(i-k)}_{= 0 \text{ sauf si } k=i \text{ où } \delta(0)=1}, \quad i \text{ fixé}$$

$$\sum_{k=-\infty}^{+\infty} p(k) \cdot \delta(i-k) = p(-1) \underbrace{\delta(i+1)}_0 + p(0) \underbrace{\delta(i)}_0 + p(1) \underbrace{\delta(i-1)}_{p(1)}$$

Exemple si  $i = 1$  :

# FORMATION DE L'IMAGE

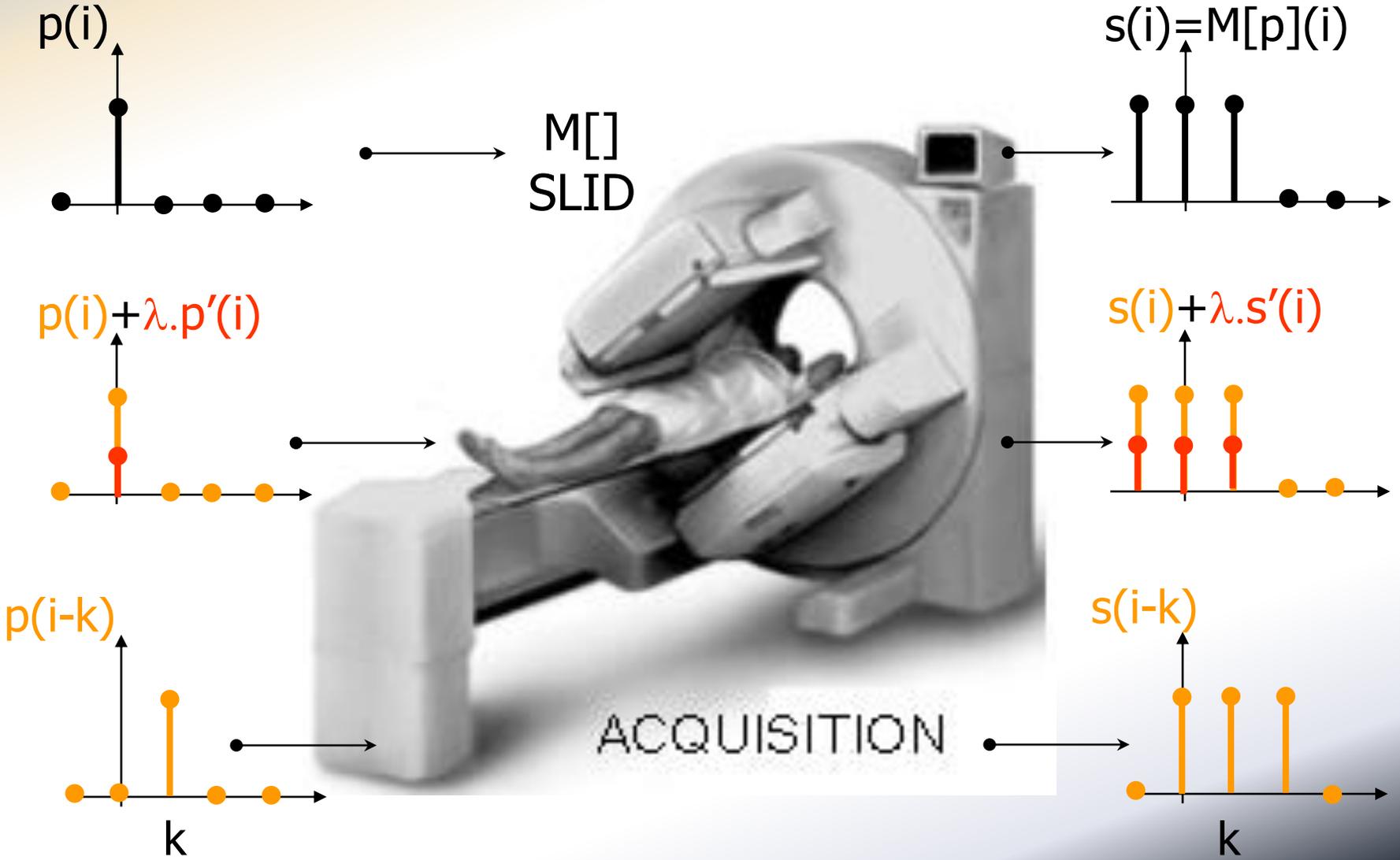


$$p(i) = \sum_{k=-\infty}^{+\infty} p(k) \cdot \underbrace{\delta(i-k)}_{= 0 \text{ sauf si } k=i}, \quad i \text{ fixé} \quad \Rightarrow \quad s(i) = M[p](i) = ?$$

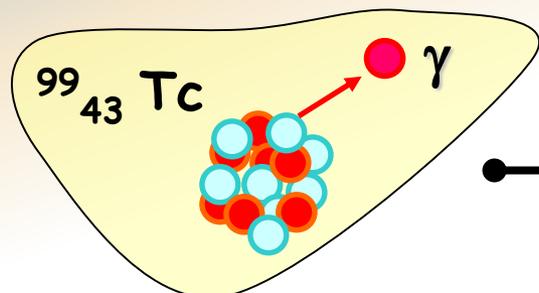
$$\delta(0) = 1$$

Pour déterminer  $s$ , il faut faire des hypothèses sur  $M$ , donc sur les caractéristiques de la caméra...

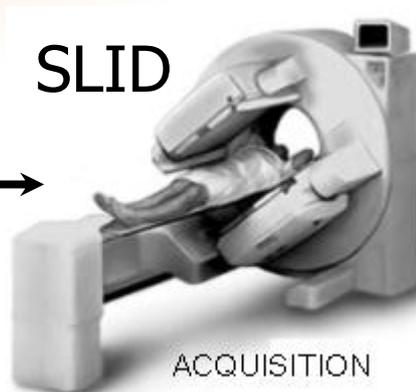
# CAMERA $\approx$ LINEAIRE & INVARIANTE



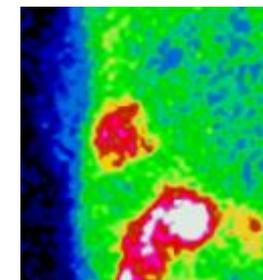
# FORMATION DE L'IMAGE



SLID



ACQUISITION



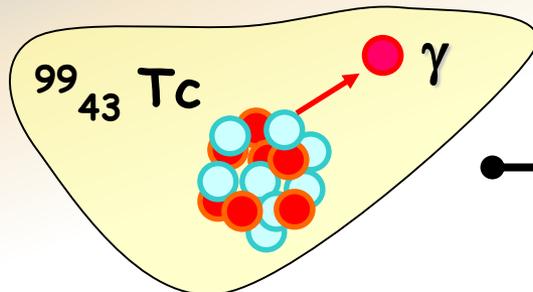
$$p(i) = \sum_{k=-\infty}^{+\infty} p(k) \cdot \delta(i - k)$$

$$h(i) = M[\delta](i)$$

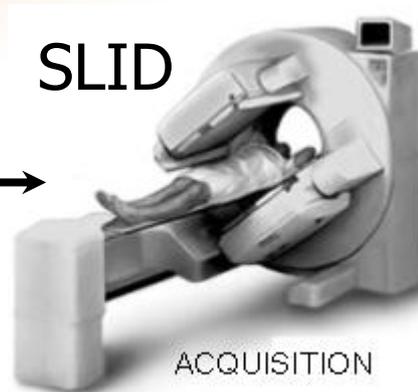
$$s(i) = M[p](i) = ?$$

$$s(i) = M\left[\sum_{k=-\infty}^{+\infty} p(k) \cdot \delta(i - k)\right]$$

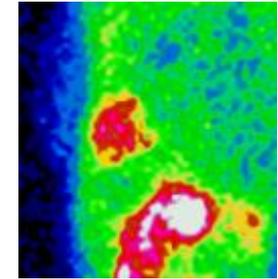
# FORMATION DE L'IMAGE



$$p(i) = \sum_{k=-\infty}^{+\infty} p(k) \cdot \delta(i - k)$$



$$h(i) = M[\delta](i)$$

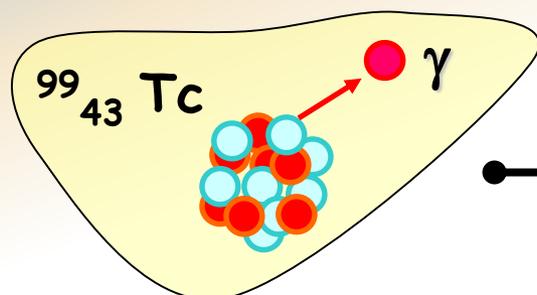


$$s(i) = M[p](i) = ?$$

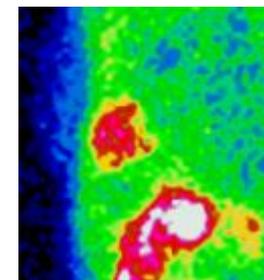
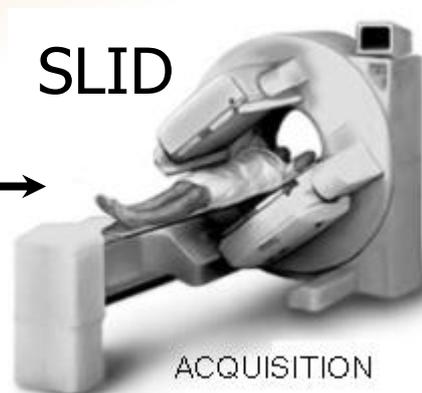
$$s(i) = M\left[\sum_{k=-\infty}^{+\infty} p(k) \cdot \delta(i - k)\right] = \sum_{k=-\infty}^{+\infty} p(k) \cdot M[\delta(i - k)]$$

linéarité

# FORMATION DE L'IMAGE



$$p(i) = \sum_{k=-\infty}^{+\infty} p(k) \cdot \delta(i-k)$$



$$s(i) = M[p](i) = ?$$

$$h(i) = M[\delta](i)$$

$$s(i) = M\left[\sum_{k=-\infty}^{+\infty} p(k) \cdot \delta(i-k)\right] = \sum_{k=-\infty}^{+\infty} p(k) \cdot \overbrace{M[\delta(i-k)]}^{h(i-k)} = \sum_{k=-\infty}^{+\infty} p(k) \cdot h(i-k)$$

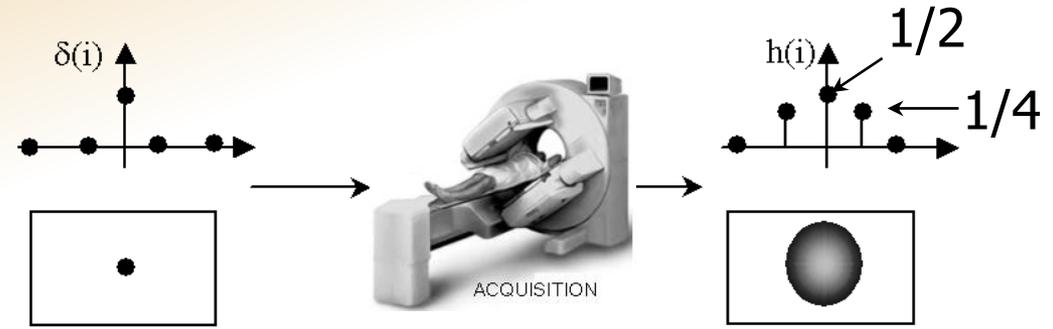
Invariance dans le décalage

$$s(i) = \sum_{k=-\infty}^{+\infty} p(k) \cdot h(i-k) = \sum_{k=-\infty}^{+\infty} h(k) \cdot p(i-k) = (p * h)(i)$$

produit de convolution

# EXEMPLE

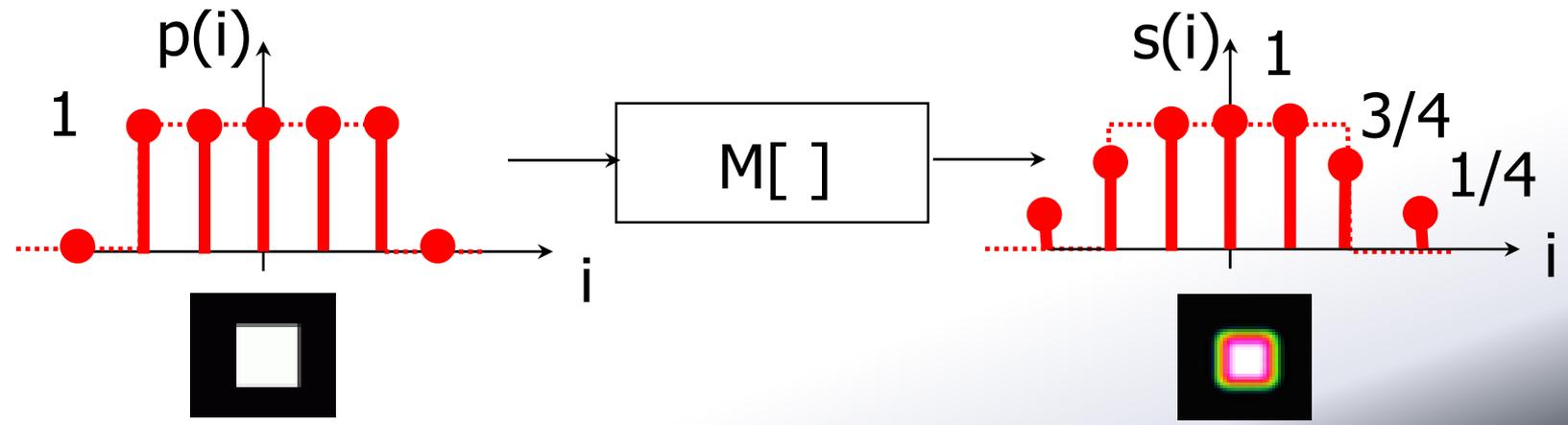
$$s(i) = \sum_{k=-1}^{+1} h(k).p(i-k)$$



$$s(i) = h(-1).p(i+1) + h(0).p(i) + h(1).p(i-1)$$

$$s(i) = \frac{1}{4}p(i+1) + \frac{1}{2}p(i) + \frac{1}{4}p(i-1) = \frac{2.p(i) + p(i+1) + p(i-1)}{4}$$

s = moyenne pondérée par h de la grandeur physique p



# THEOREME DE CONVOLUTION

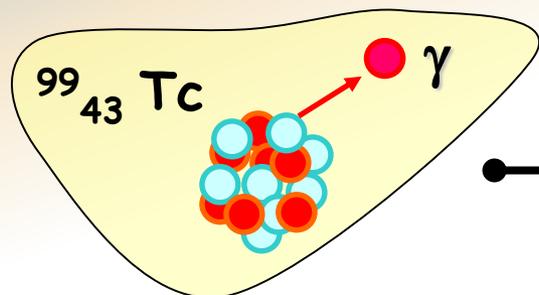
$$p(i) = e^{j.v.i} \rightarrow \boxed{M[\ ]} \longrightarrow s(i) = \sum_{k=-\infty}^{+\infty} h(k).p(i-k)$$

$$s(i) = \sum_{k=-\infty}^{k=+\infty} h(k).e^{j.v.(i-k)} = \underbrace{e^{j.v.i}}_{p(i)} \underbrace{\sum_{k=-\infty}^{k=+\infty} h(k).e^{-j.v.k}}_{\hat{h}(v)}$$

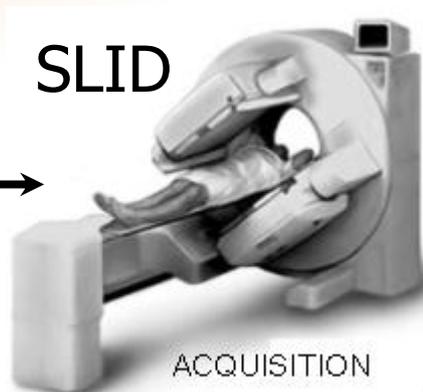
$$p(i) = e^{j.v.i} \implies s(i) = \hat{h}(v).p(i)$$

Un SLID agit sur l'harmonique  $v$  en l'amplifiant par la **réponse en fréquence** en  $v$  :  $\hat{h}(v)$

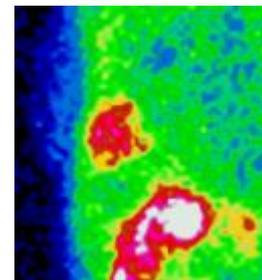
# REPONSE IMPULSIONNELLE / EN FREQUENCE



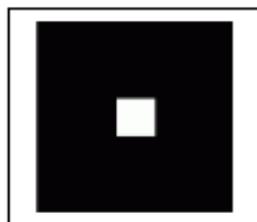
$p(i)$



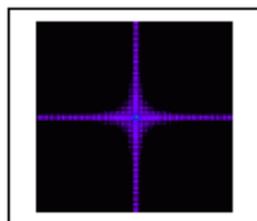
$s(i)$



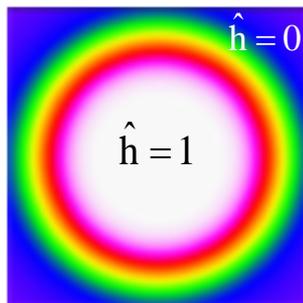
$$s(i) = \sum_{k=-\infty}^{+\infty} h(k).p(i-k) = (p * h)(i)$$



TF



$\hat{p}(v)$

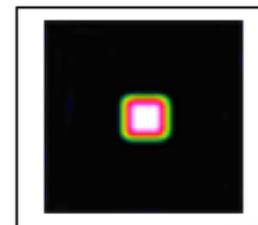


$\hat{h} = 0$

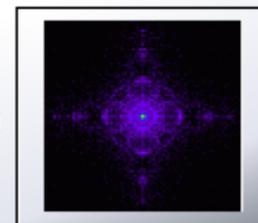
$\hat{h} = 1$

$s(i) = p(i) * h(i)$

TF<sup>-1</sup>

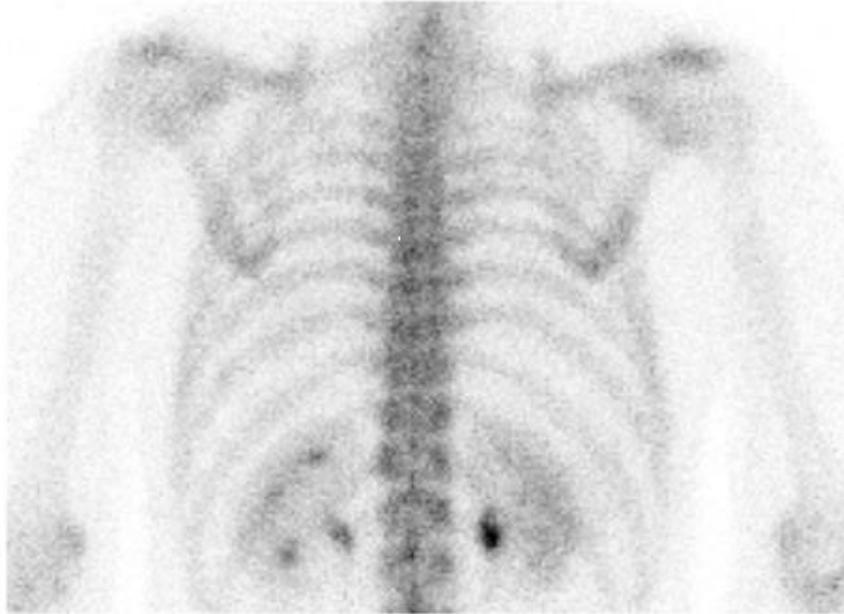


$\hat{s}(v) = \hat{p}(v) \cdot \hat{h}(v)$

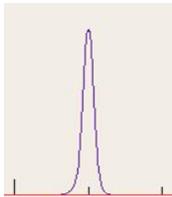


# RESOLUTION ET DISTANCE ( $\gamma$ )

$s$  = moyenne pondérée par  $h$  de la grandeur physique  $p$

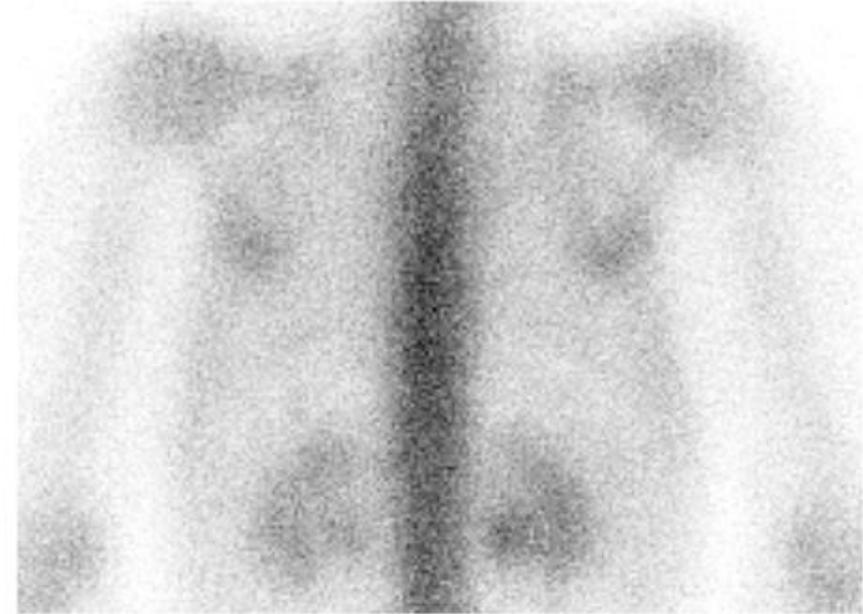


h

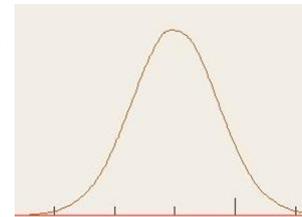


FP  
AU CONTACT

DT



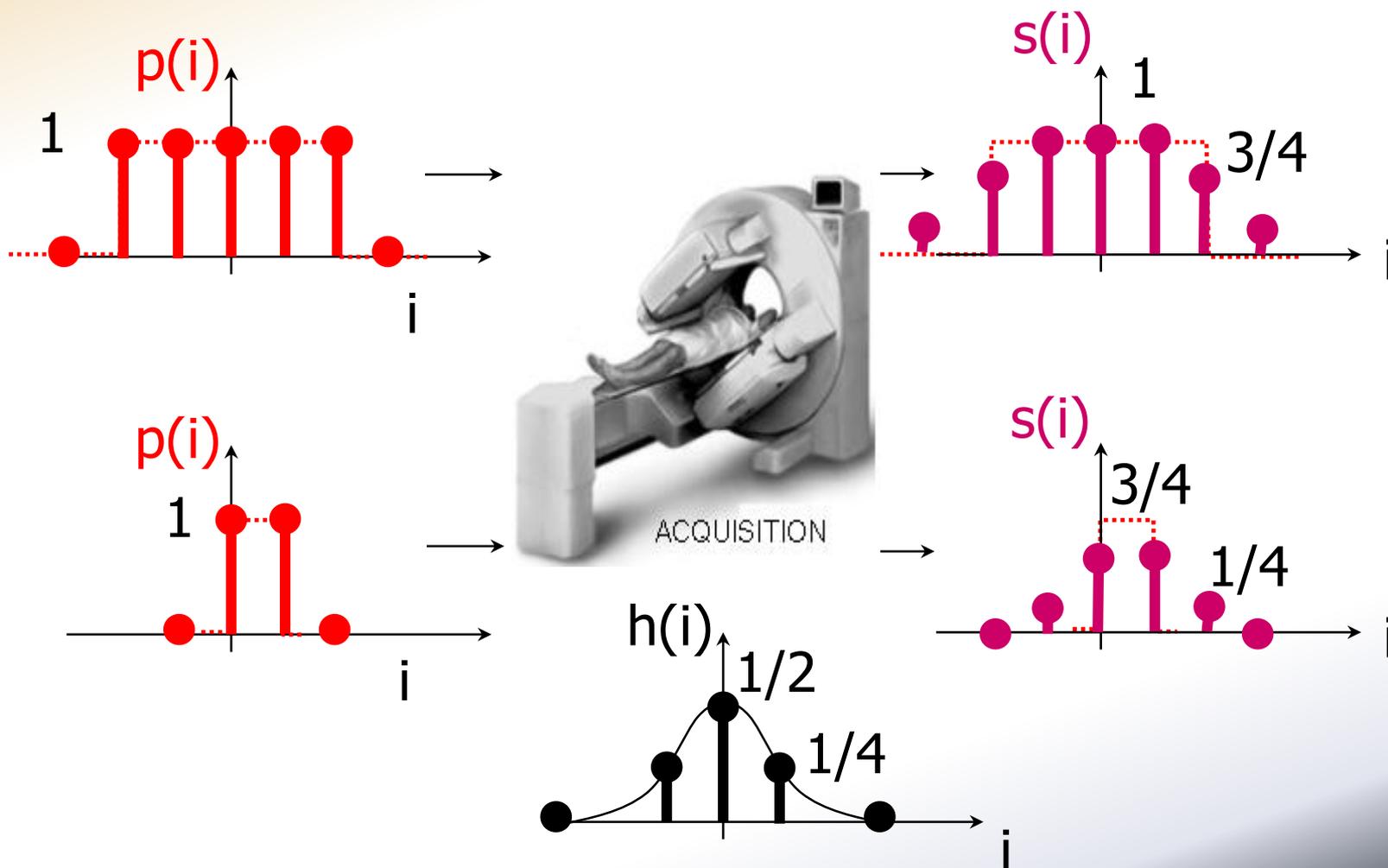
h



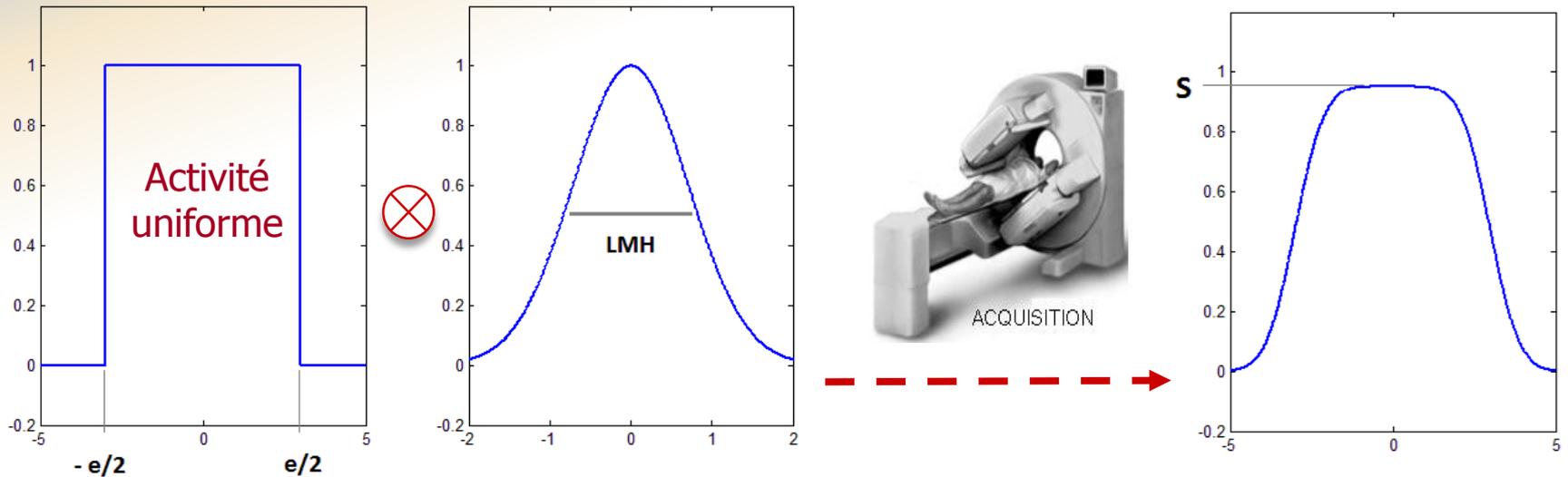
50 cm

DT

# EFFET DE VOLUME PARTIEL



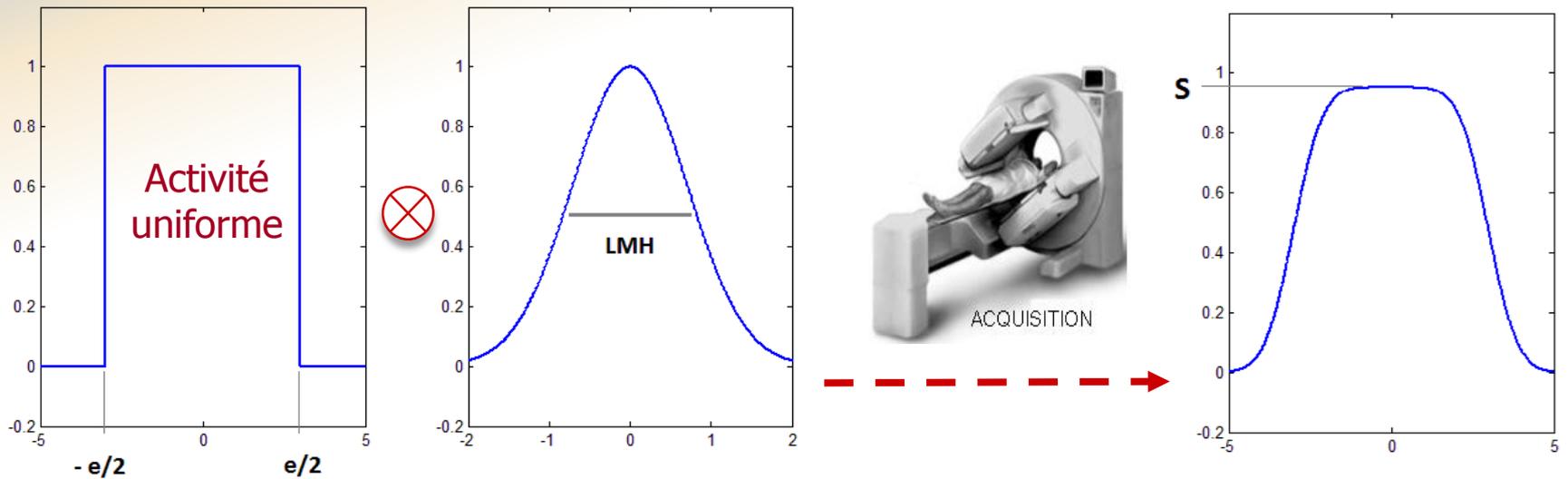
# EFFET DE VOLUME PARTIEL



$$S = \int_{-e/2}^{e/2} G(x) dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-e/2}^{e/2} e^{-\frac{x^2}{2\sigma^2}} dx$$

$$LMH = 2 \cdot \sqrt{2 \cdot \ln 2} \sigma \approx 2,35 \cdot \sigma$$

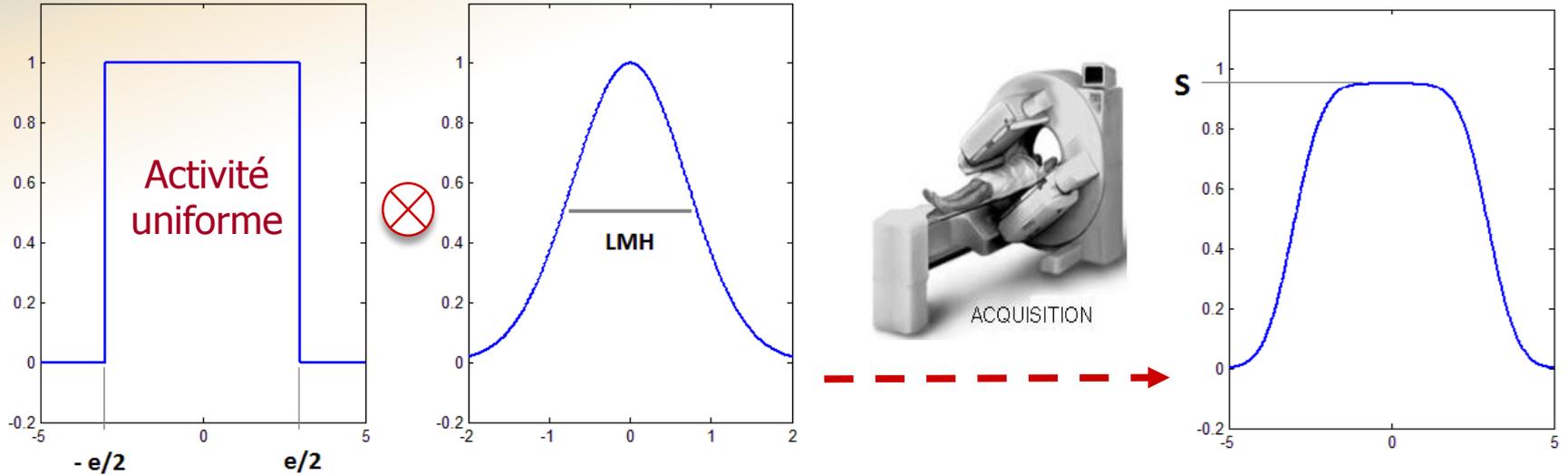
# EFFET DE VOLUME PARTIEL



$$S = \int_{-e/2}^{e/2} G(x) dx = \frac{2\sqrt{\ln 2 / \pi}}{LMH} \int_{-e/2}^{e/2} e^{-\frac{4 \cdot \ln 2}{LMH^2} x^2} dx$$

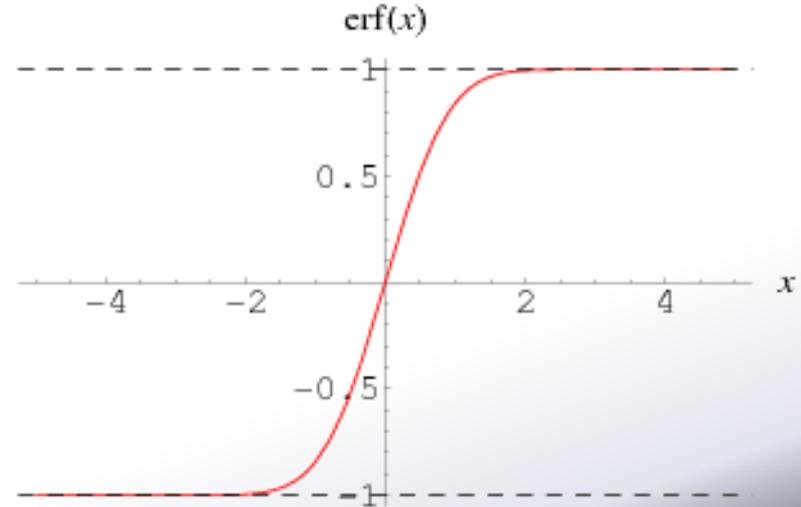
$$LMH = 2 \cdot \sqrt{2 \cdot \ln 2} \sigma \approx 2,35 \cdot \sigma$$

# EFFET DE VOLUME PARTIEL

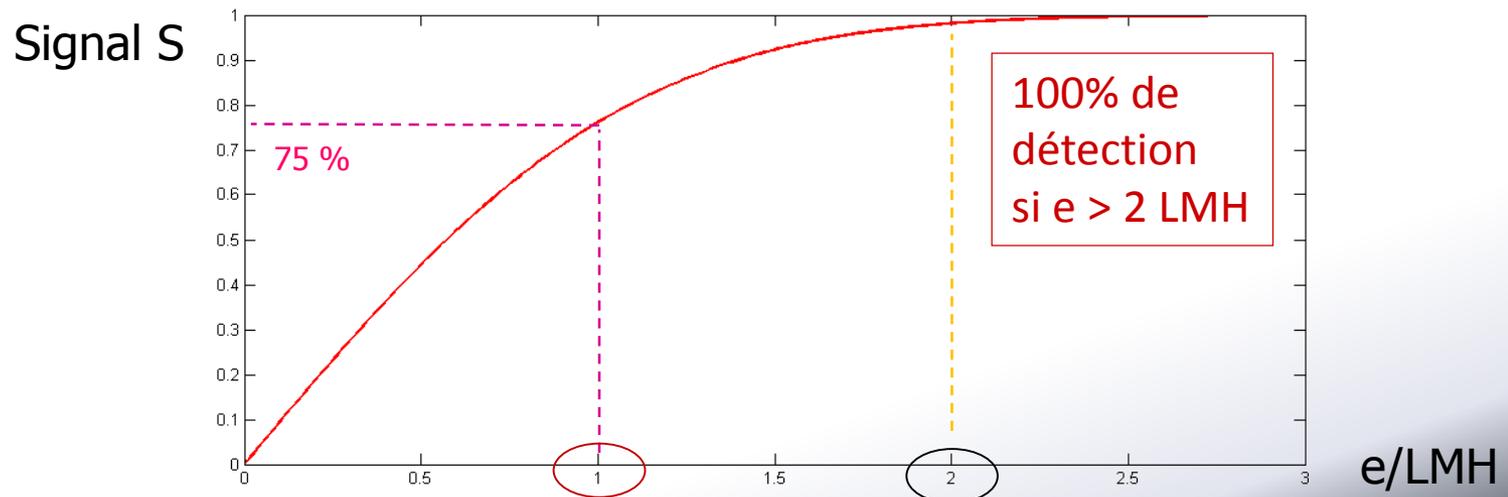
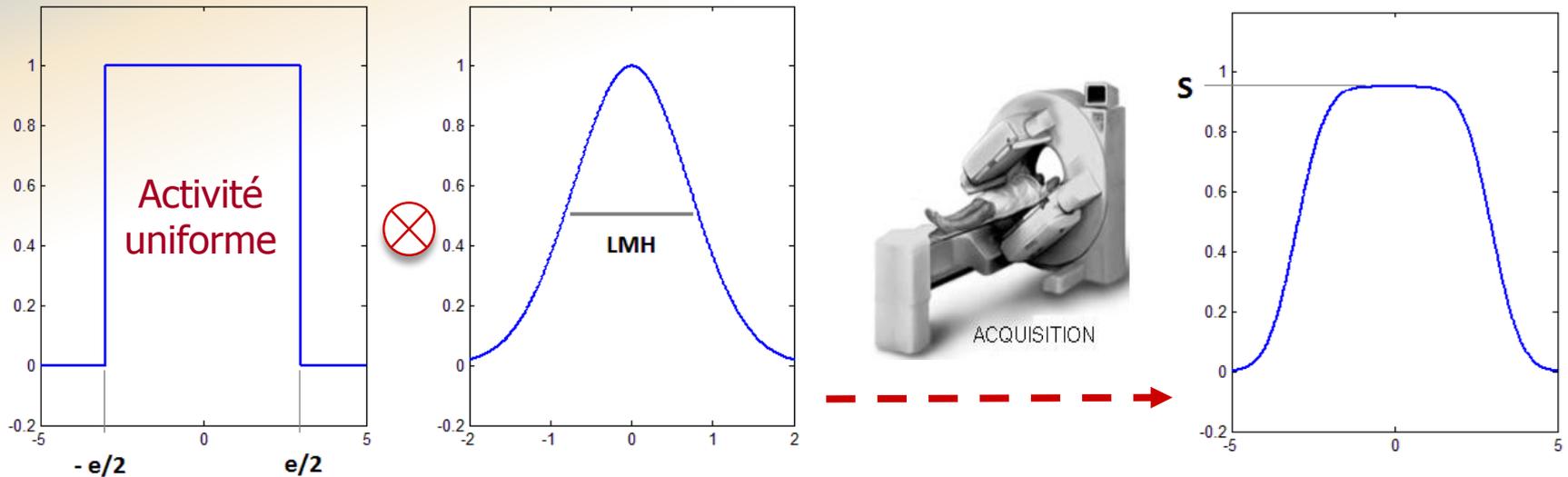


$$S = \int_{-e/2}^{e/2} G(x) dx$$

$$= \frac{1}{LMH} \operatorname{erf} \left( \sqrt{\ln 2} \frac{e}{LMH} \right)$$



# EFFET DE VOLUME PARTIEL



# IMPORTANCE DE LA LMH

- LMH forme l'image par convolution
- LMH = **Pouvoir séparateur** = résolution
- LMH = **Période minimale** des harmoniques
- LMH **limite les fréquences** dans l'image
- LMH conditionne l'effet de volume partiel
  - **100 % de détection si  $e > 2 \times \text{LMH}$**
- LMH conditionne l'échantillonnage (cf. suite)

# RESOLUTIONS EN MEDECINE

