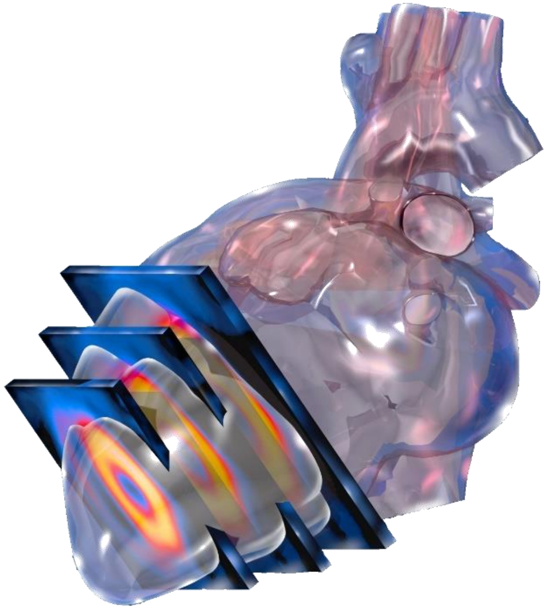
MASTER PhyMed

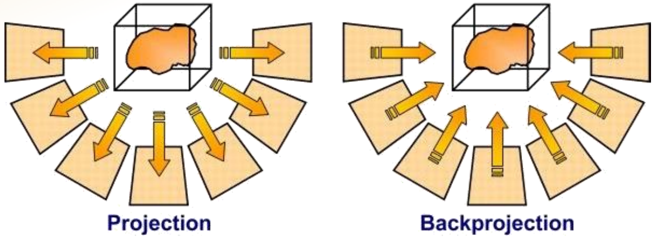
GMPH308 - Physique de l'imagerie médicale

TOMOGRAPHIE D'EMISSION

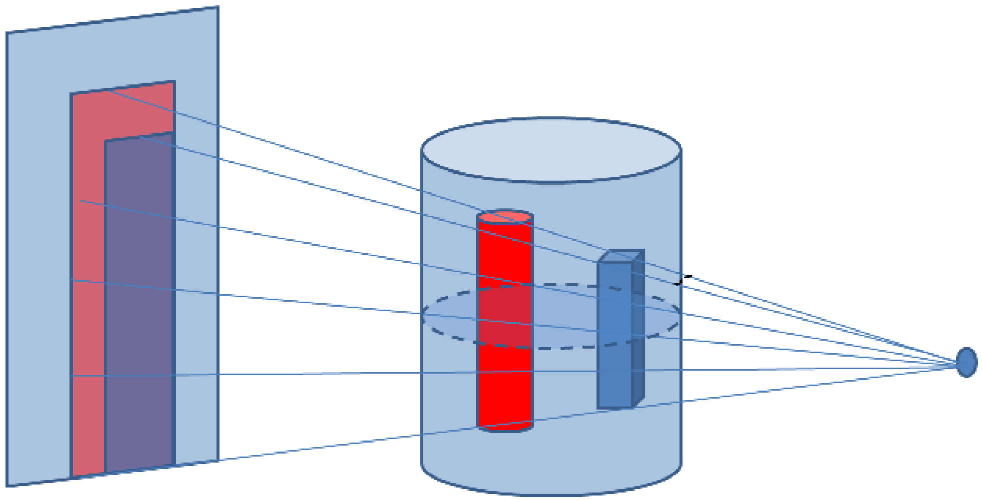


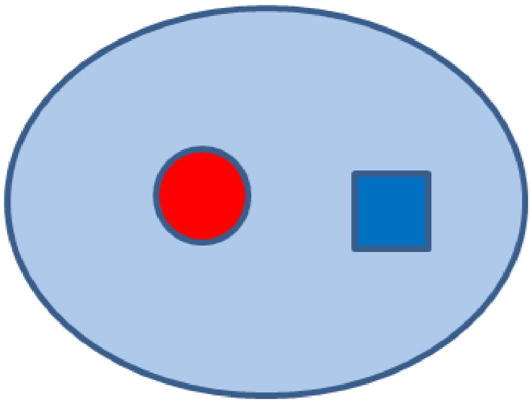
Fayçal Ben Bouallègue
faybenb@hotmail.com
http://scinti.etud.univ-montp1.fr

Tomographie



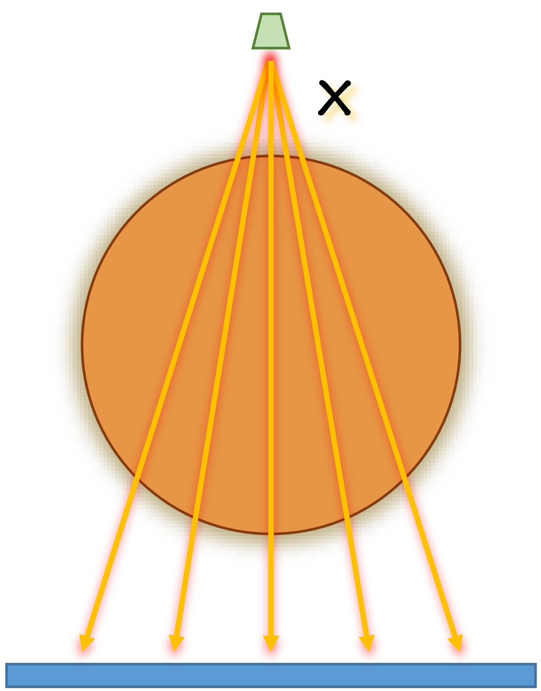
Projections → Coupes



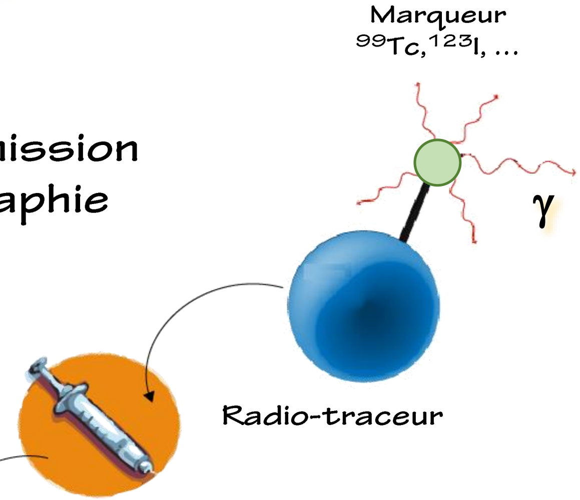


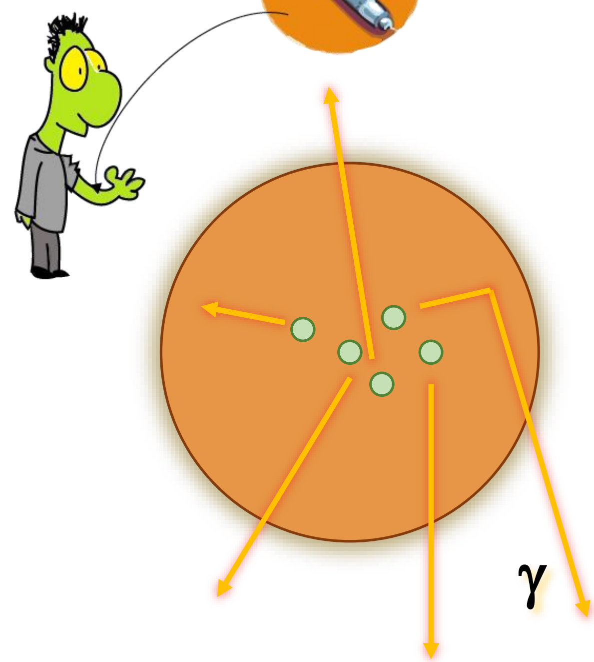
Tomographie

Tomographie de transmission



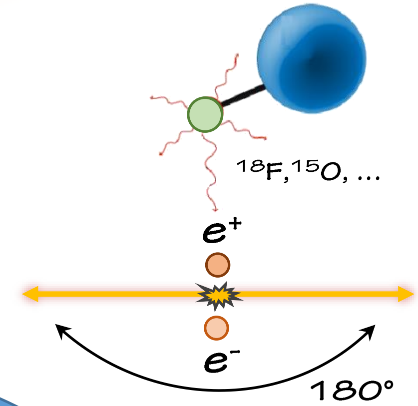
Tomographie d'émission
= tomo-scintigraphie



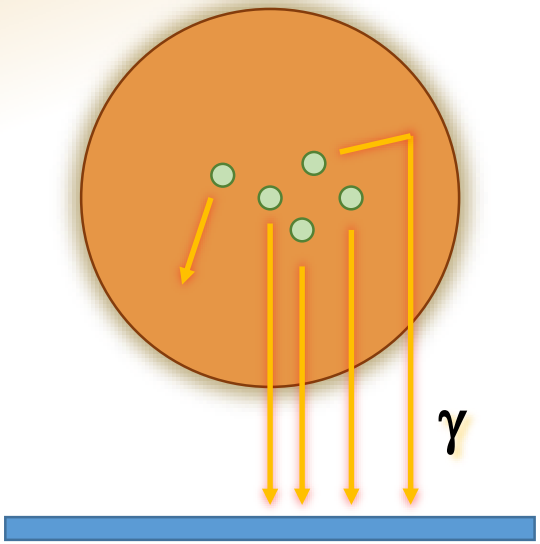


Tomographie

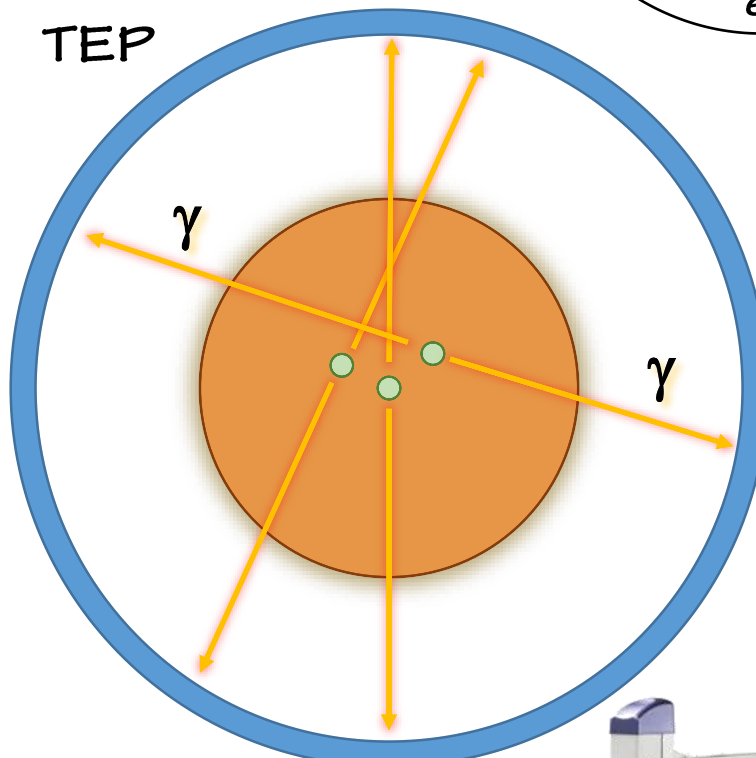
Tomographie d'émission



TEMP = SPECT

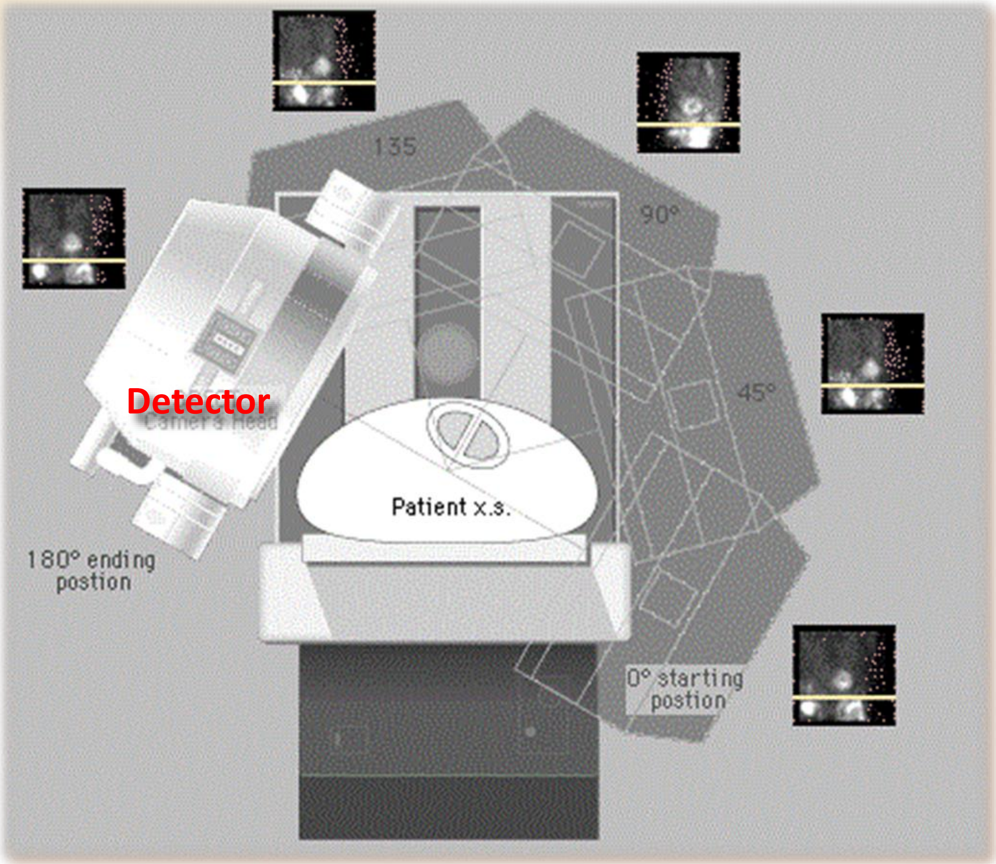


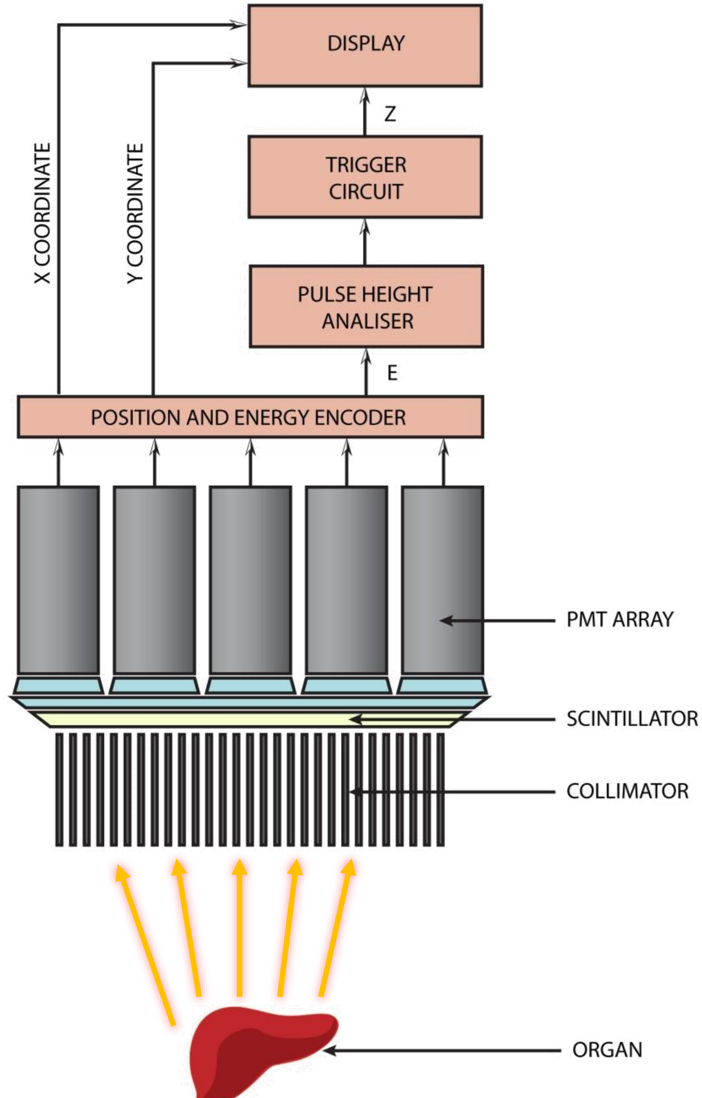
TEP



TEMP - acquisition

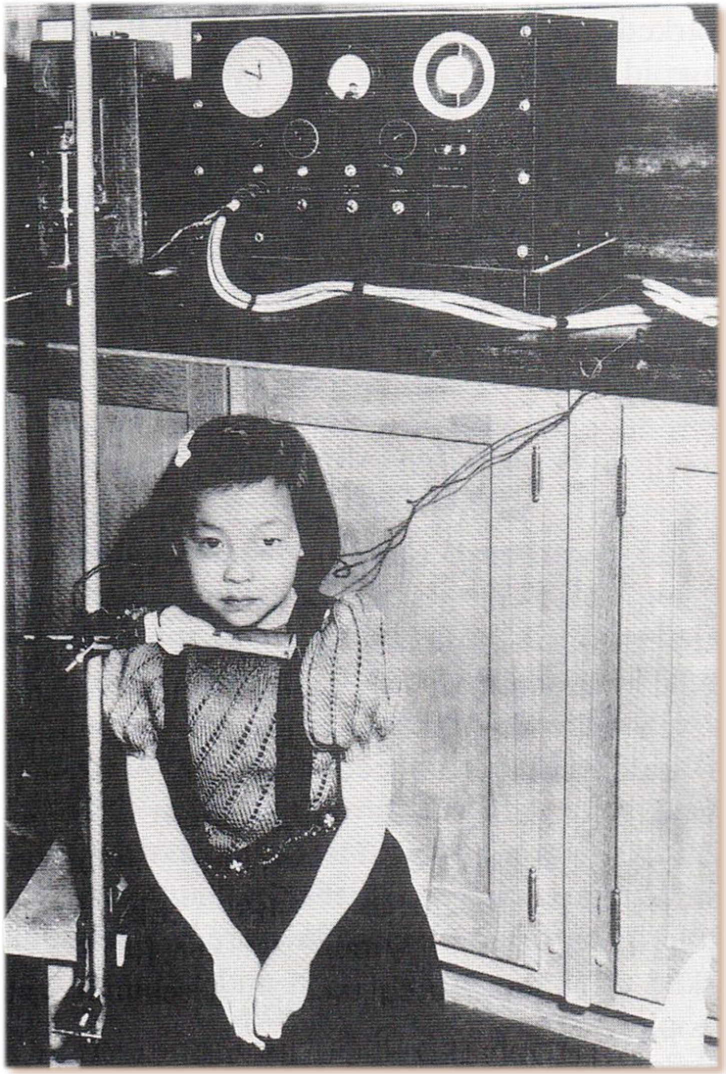
Gamma caméra d'Anger





TEMP - acquisition

Gamma caméra d'Anger
(1958)

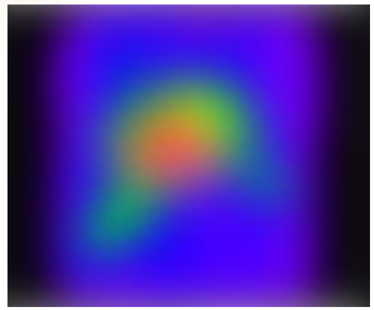


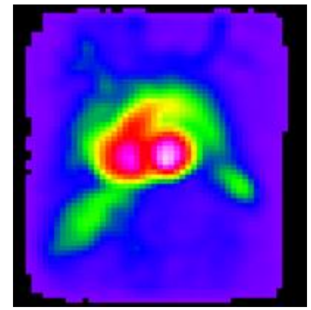
TEMP - acquisition

Gamma caméra d'Anger
(2010)

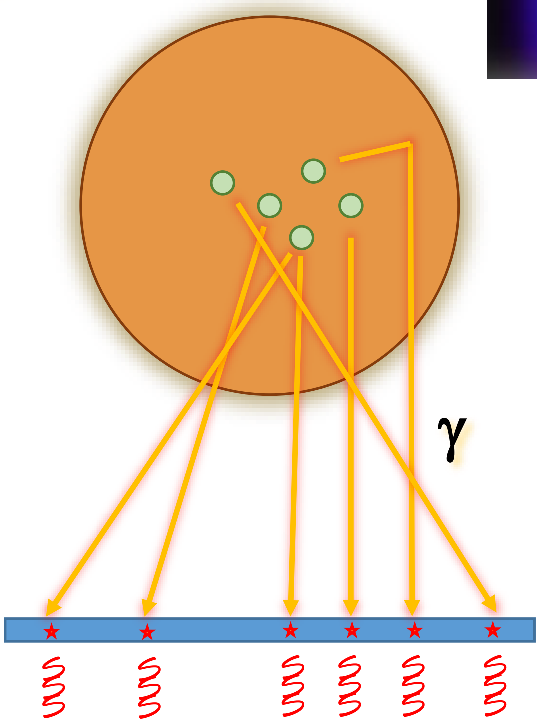
TEMP - acquisition

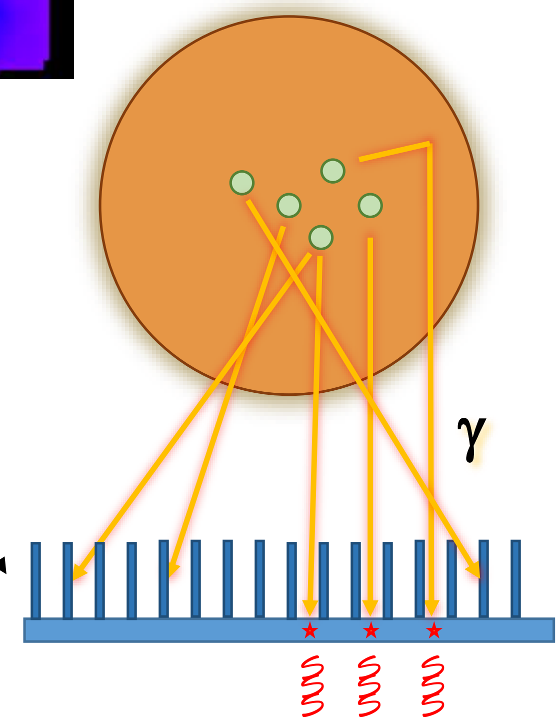
- Collimation





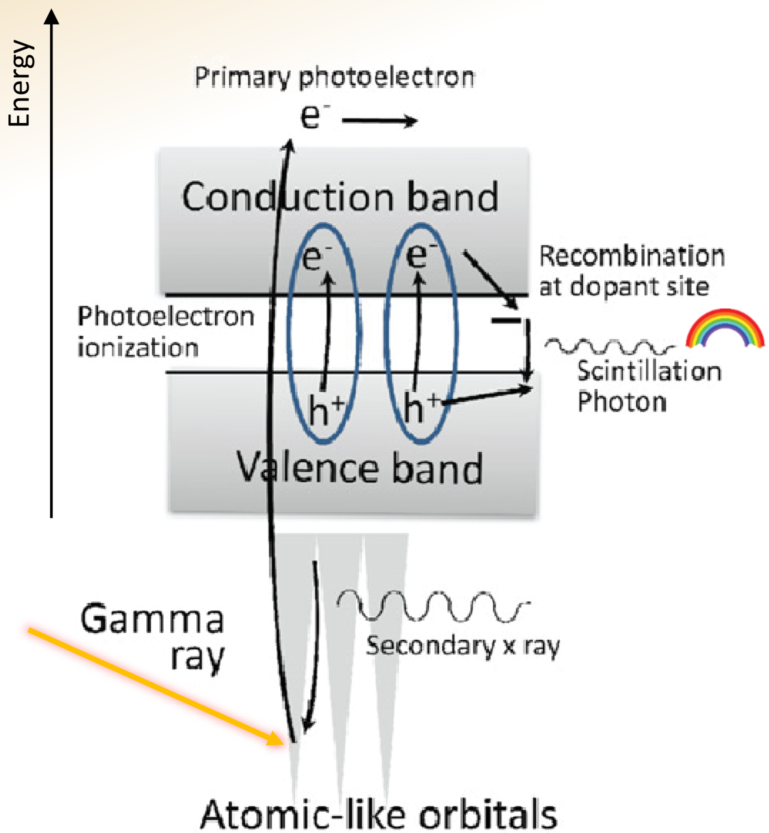
Compromis
résolution / sensibilité

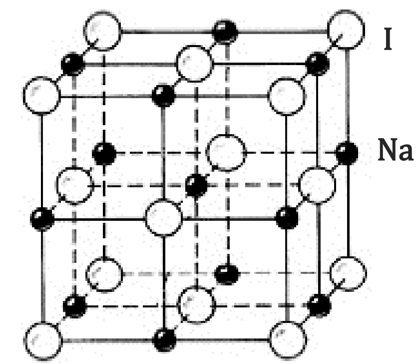




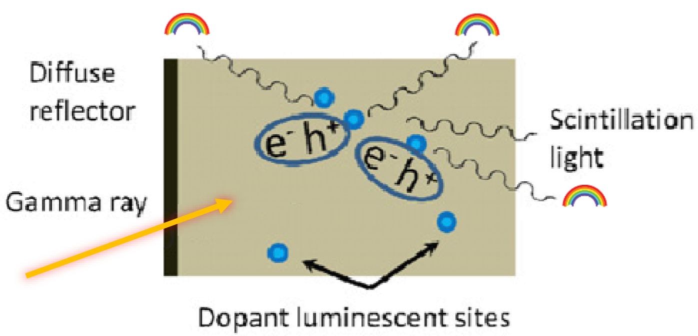
TEMP - acquisition

- Scintillation



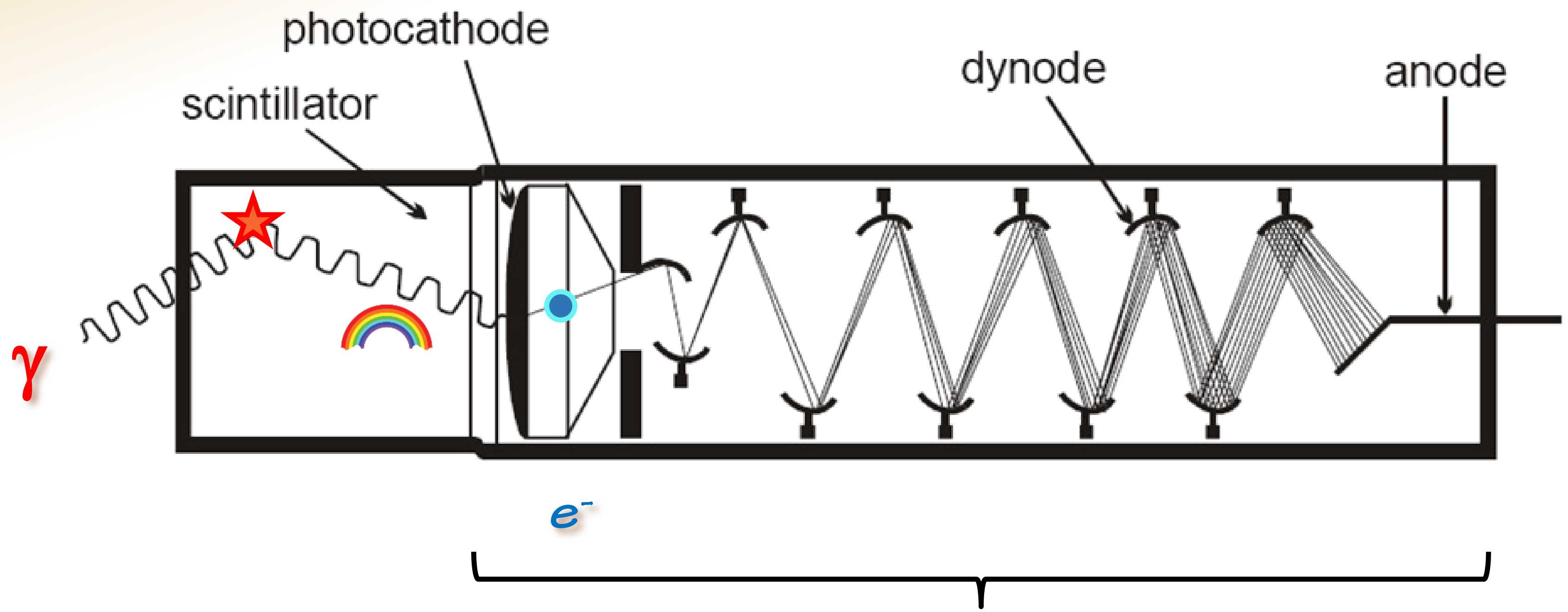


NaI(Tl)



TEMP - acquisition

- Amplification

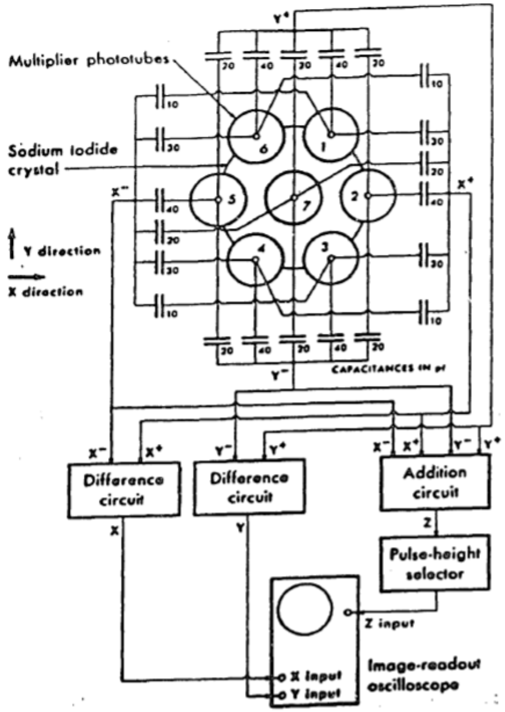


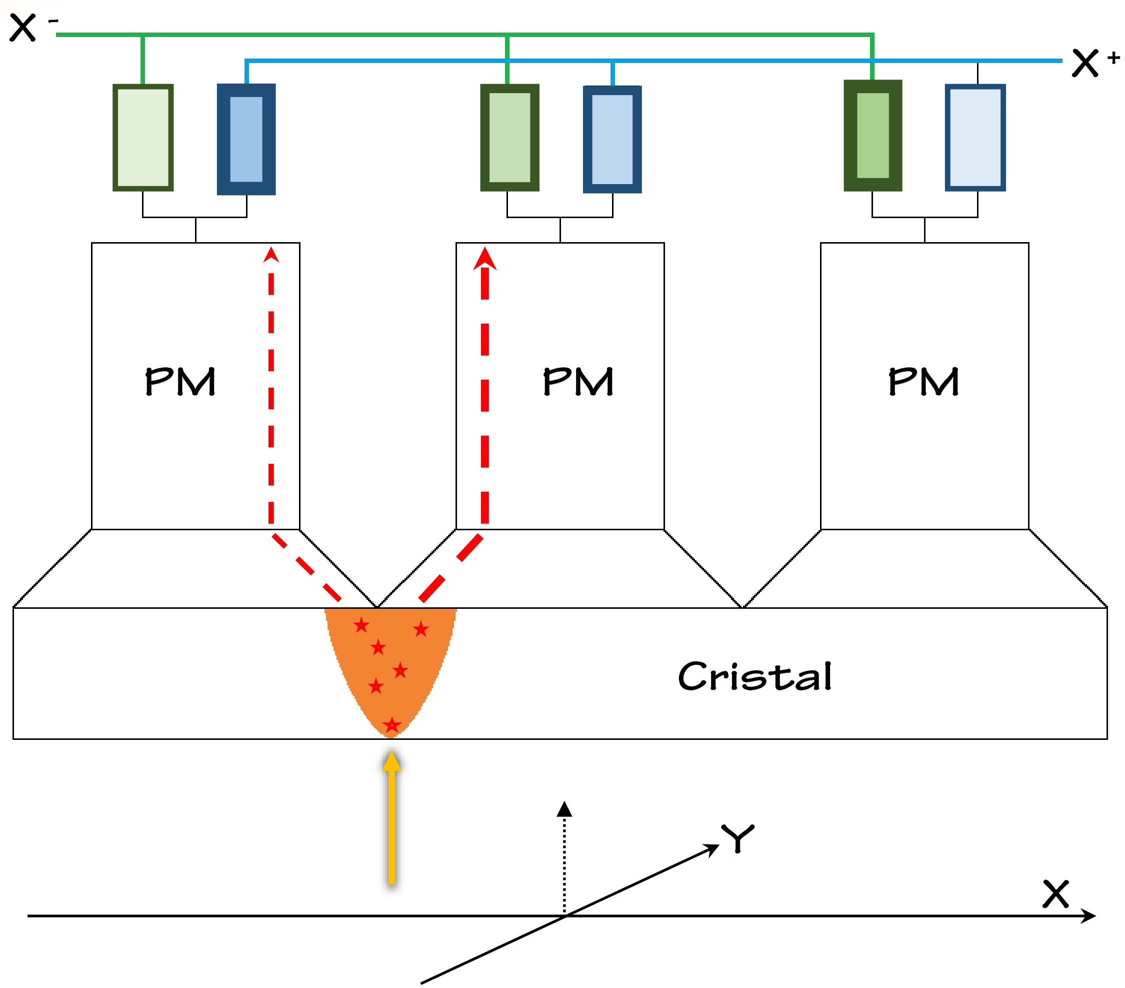
photomultiplier tube

TEMP - acquisition

- Localisation

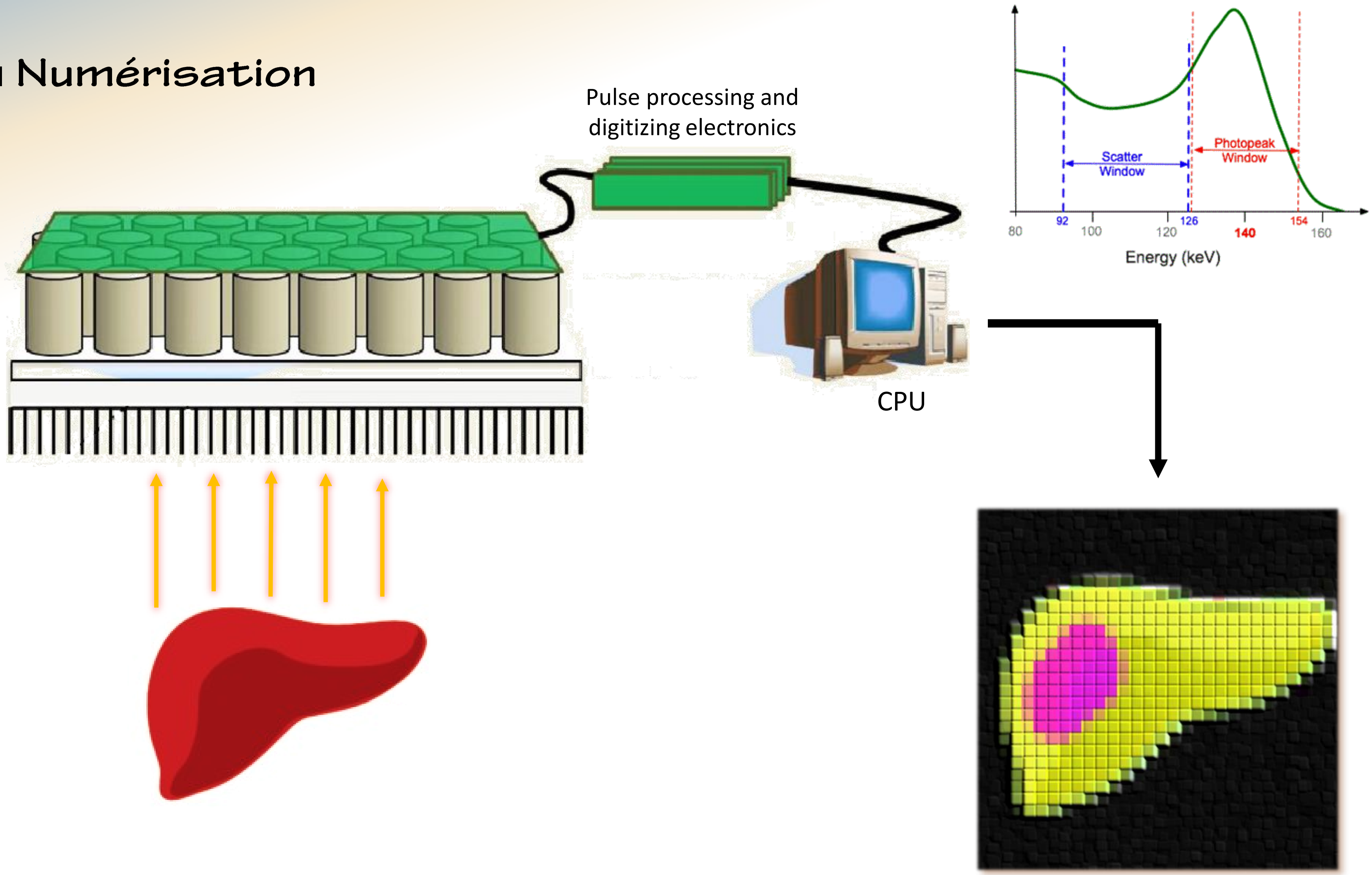
$$X = X^{+} - X^{-}$$
$$Y = Y^{+} - Y^{-}$$
$$E = X^{+} + X^{-} + Y^{+} + Y^{-}$$



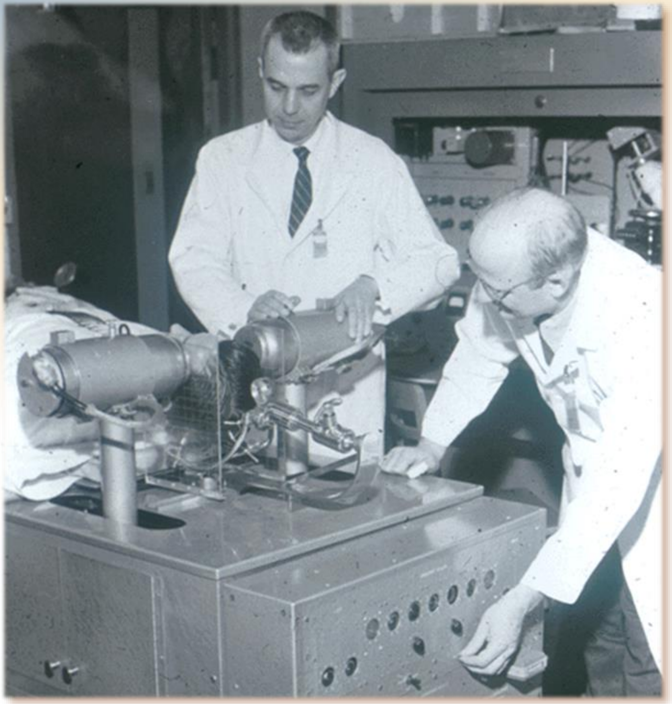


TEMP - acquisition

- Numérisation



TEP - acquisition



G Brownell & C Burnham
Boston 1952

The New England
Journal of Medicine

Volume 245 DECEMBER 6, 1951 Number 23

THE USES OF NUCLEAR DISINTEGRATION IN THE DIAGNOSIS AND TREATMENT OF BRAIN TUMOR*

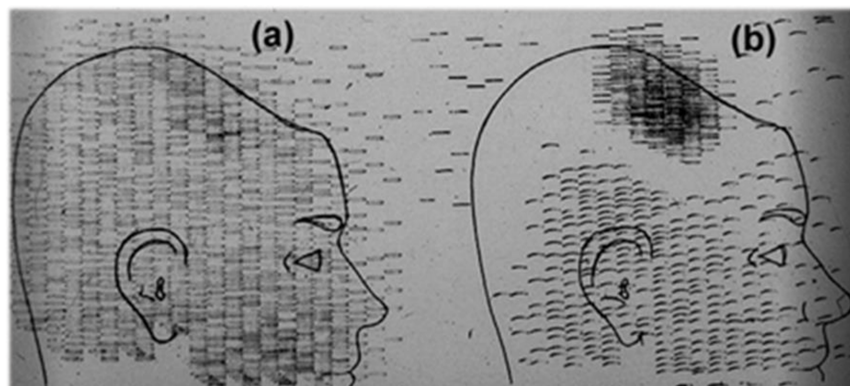
WILLIAM H. SWEET, M.D.†

BOSTON

IN THE utilization of isotopes to aid in the clinical management of intracranial tumors that has been developed at this hospital in the past few years, we have been aided by many workers. The time at which an elaborate addition to the hospital's investigative facilities is brought into action is perhaps a suitable time to draw attention to the diversity of talents that has been required for making some headway in such a limited field as that of brain tumors.

ascertain by the counting rate when neoplasm is entered and where its limits are.

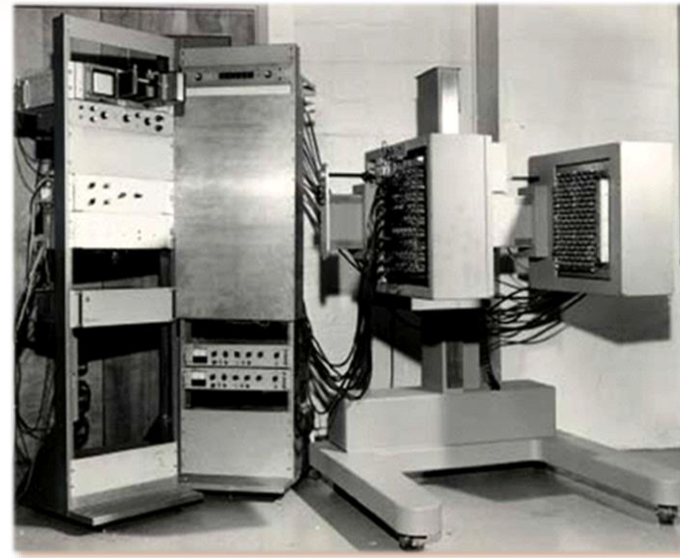
Such a probe counter was first designed by the physicist Dr. Charles Robinson, then of Wisconsin. Dr. Solomon promptly secured the aid of Dr. Robinson and his counter, which device we used in man in time to send a telegram with the data to



TEP - acquisition

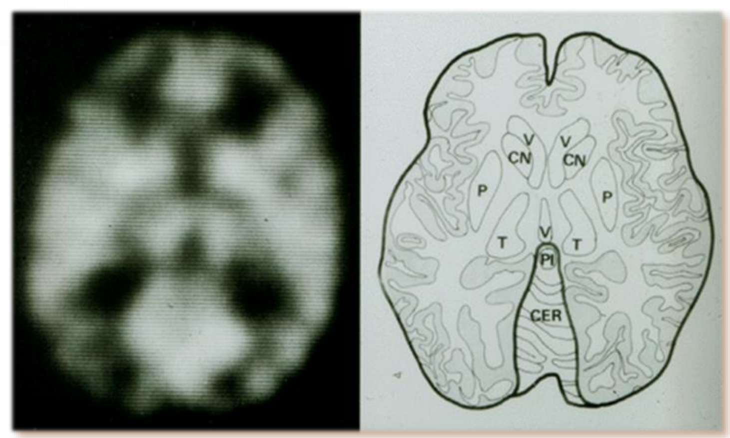
Multidétecteur
Mode tomographique

1975



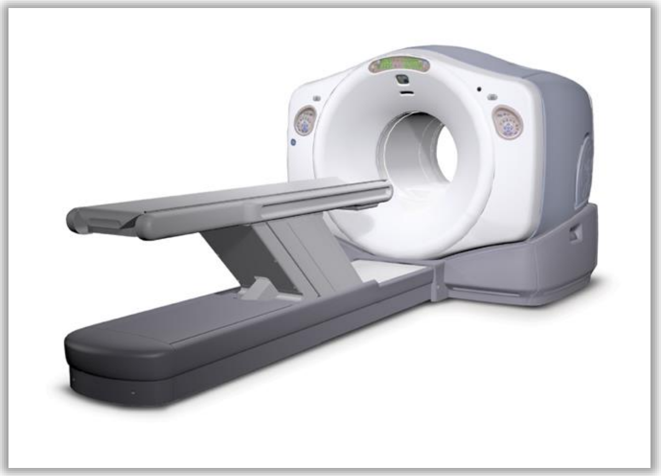
1970

^{18}FDG
1976



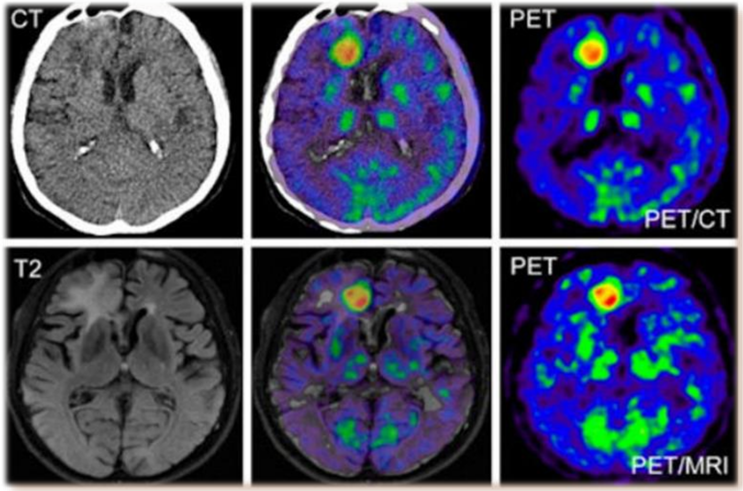
TEP - acquisition

Anneau de détecteurs
1980

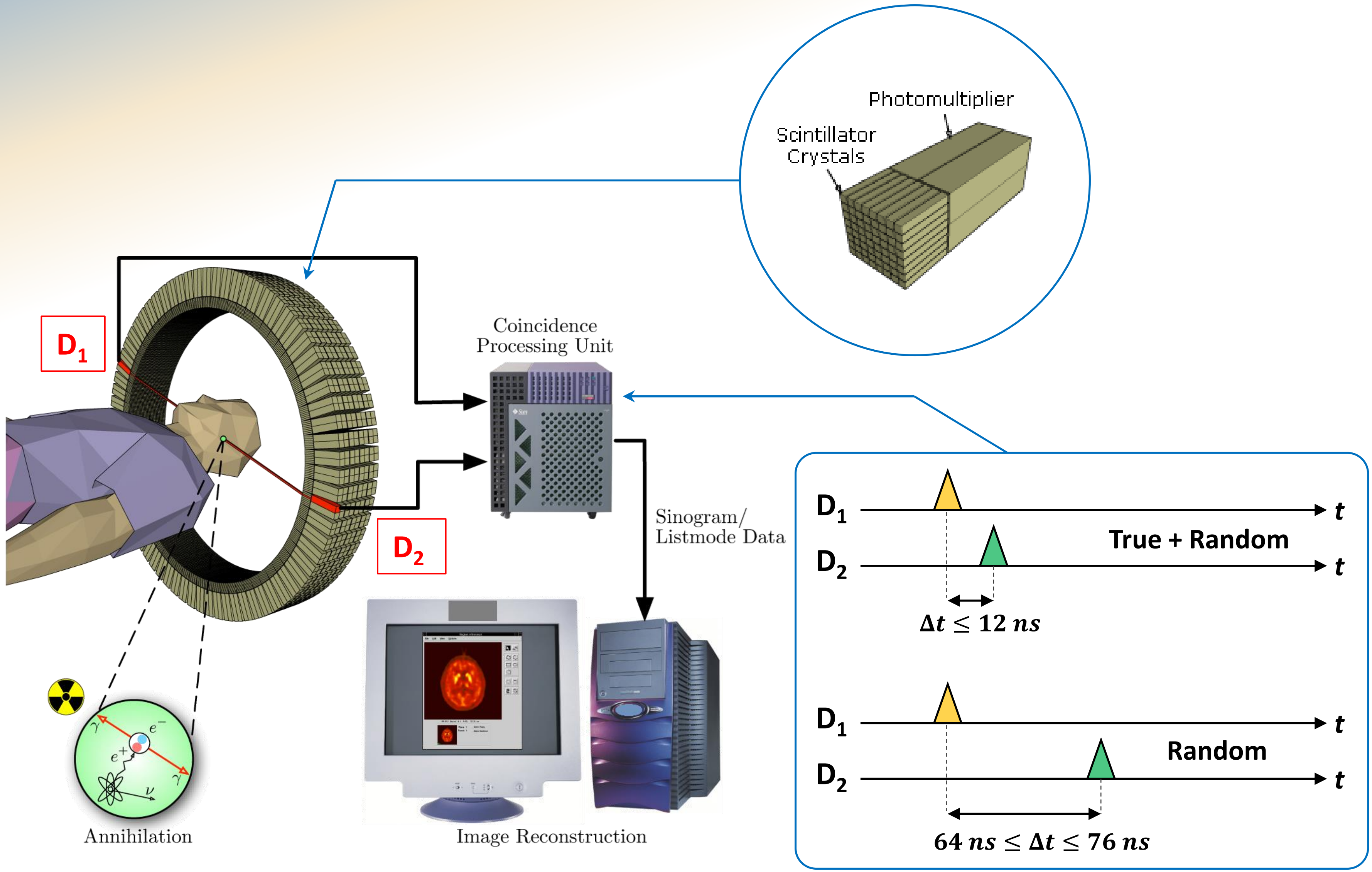


TEP-CT
1998

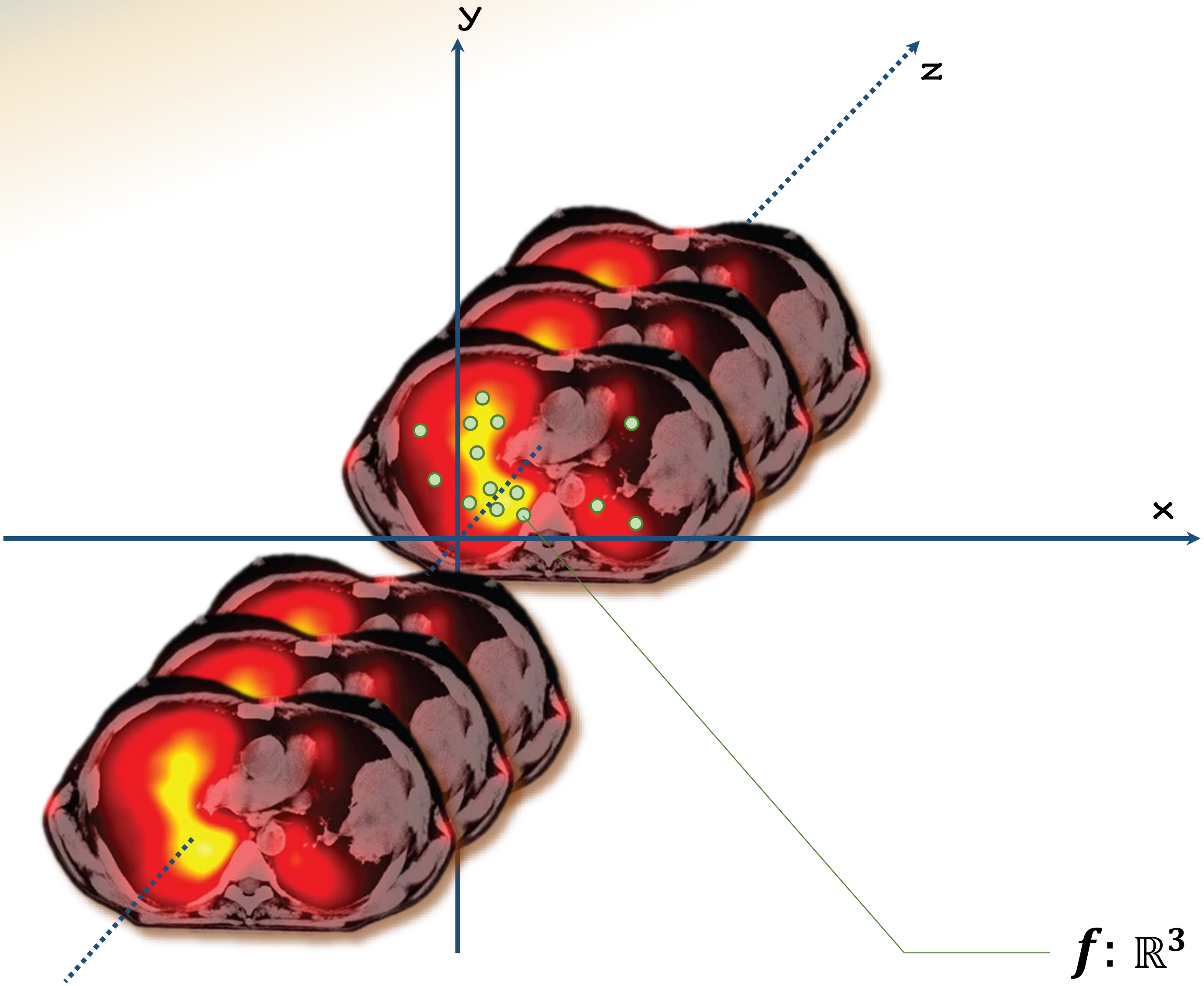
TEP-MR
2010



TEP - acquisition

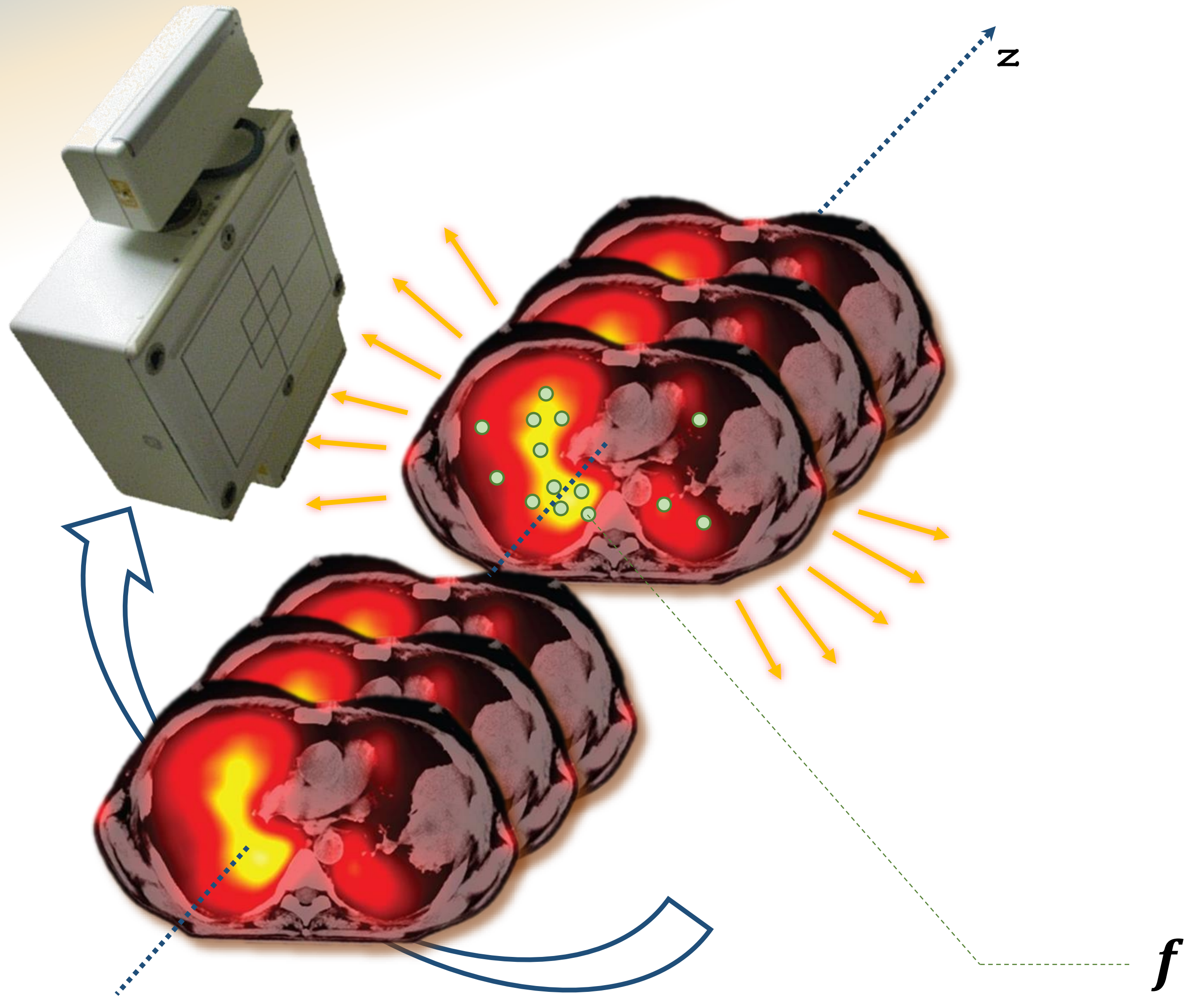


Problème tomographique

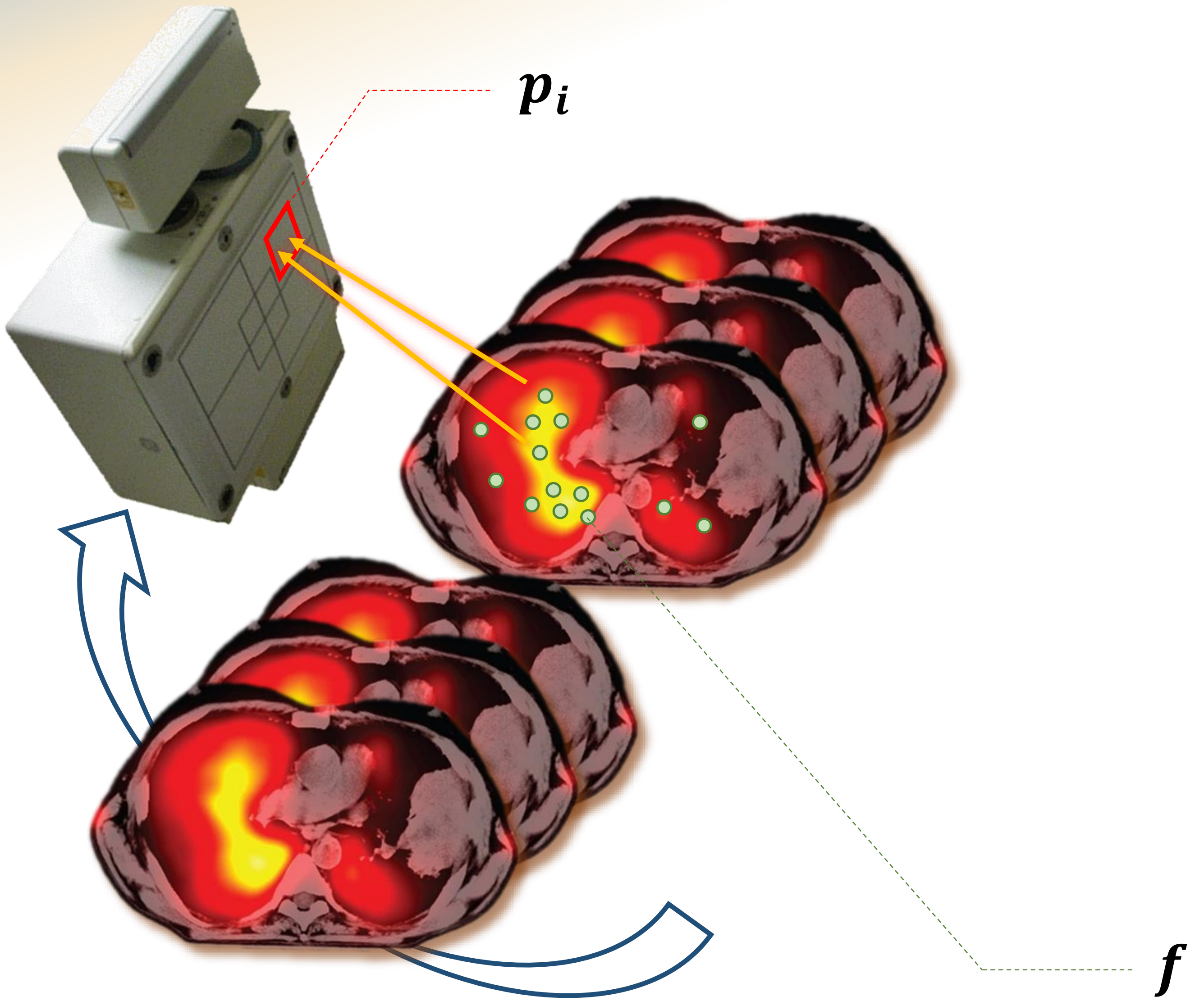


$\boldsymbol{f}: \mathbb{R}^3 \rightarrow \mathbb{R}^+$

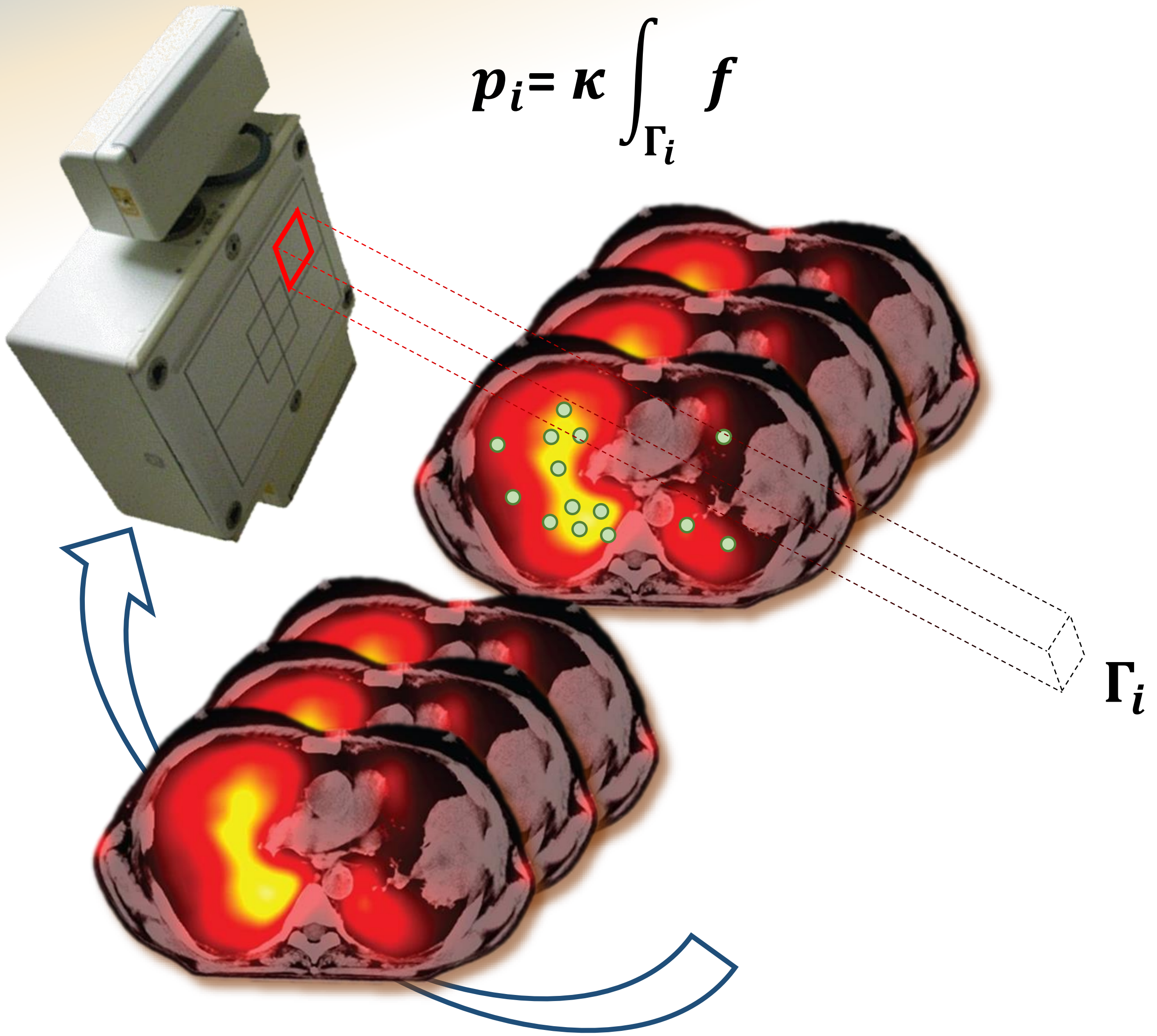
Problème tomographique



Problème tomographique

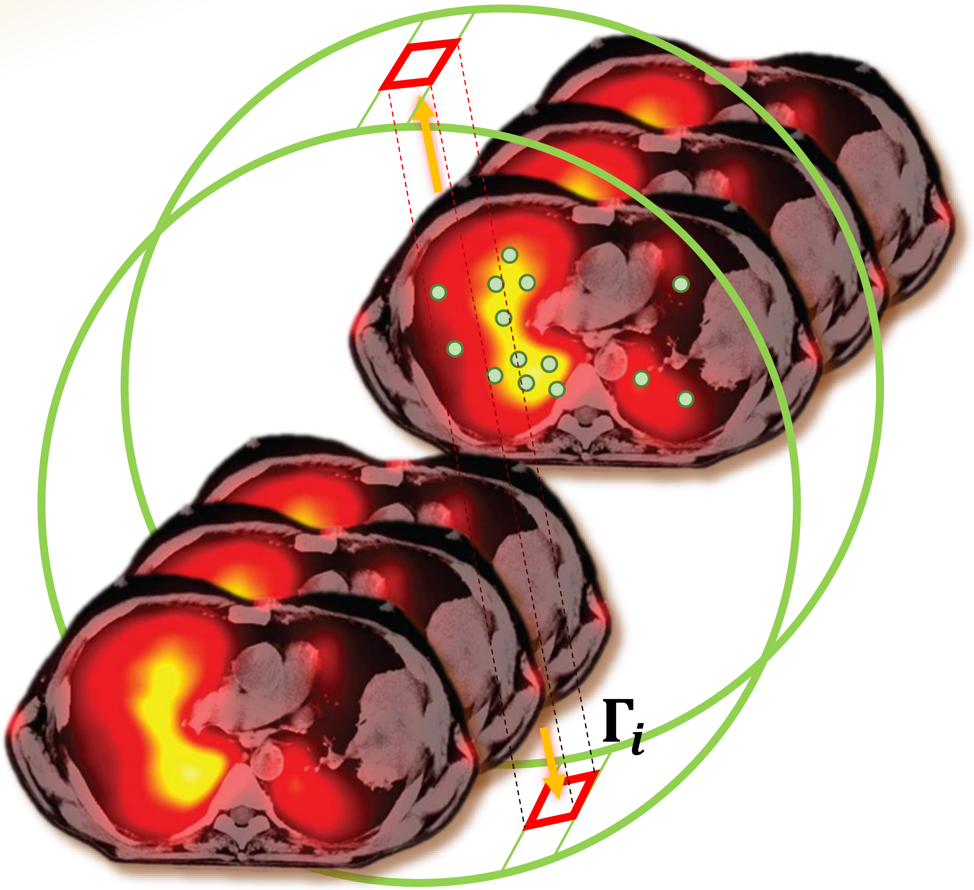


Problème tomographique



Problème tomographique

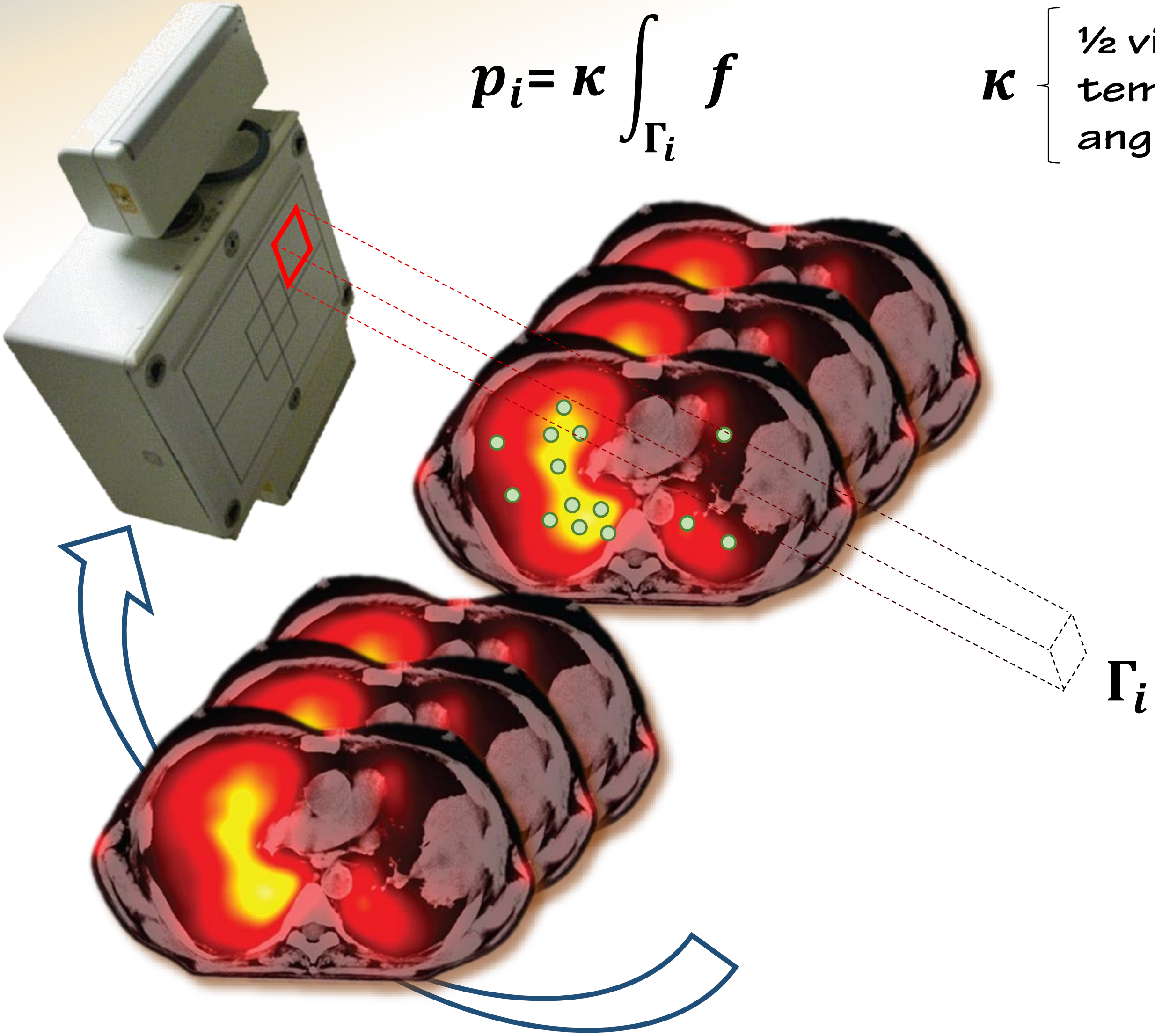
$$p_i = \kappa \int_{\Gamma_i} f$$



Problème tomographique

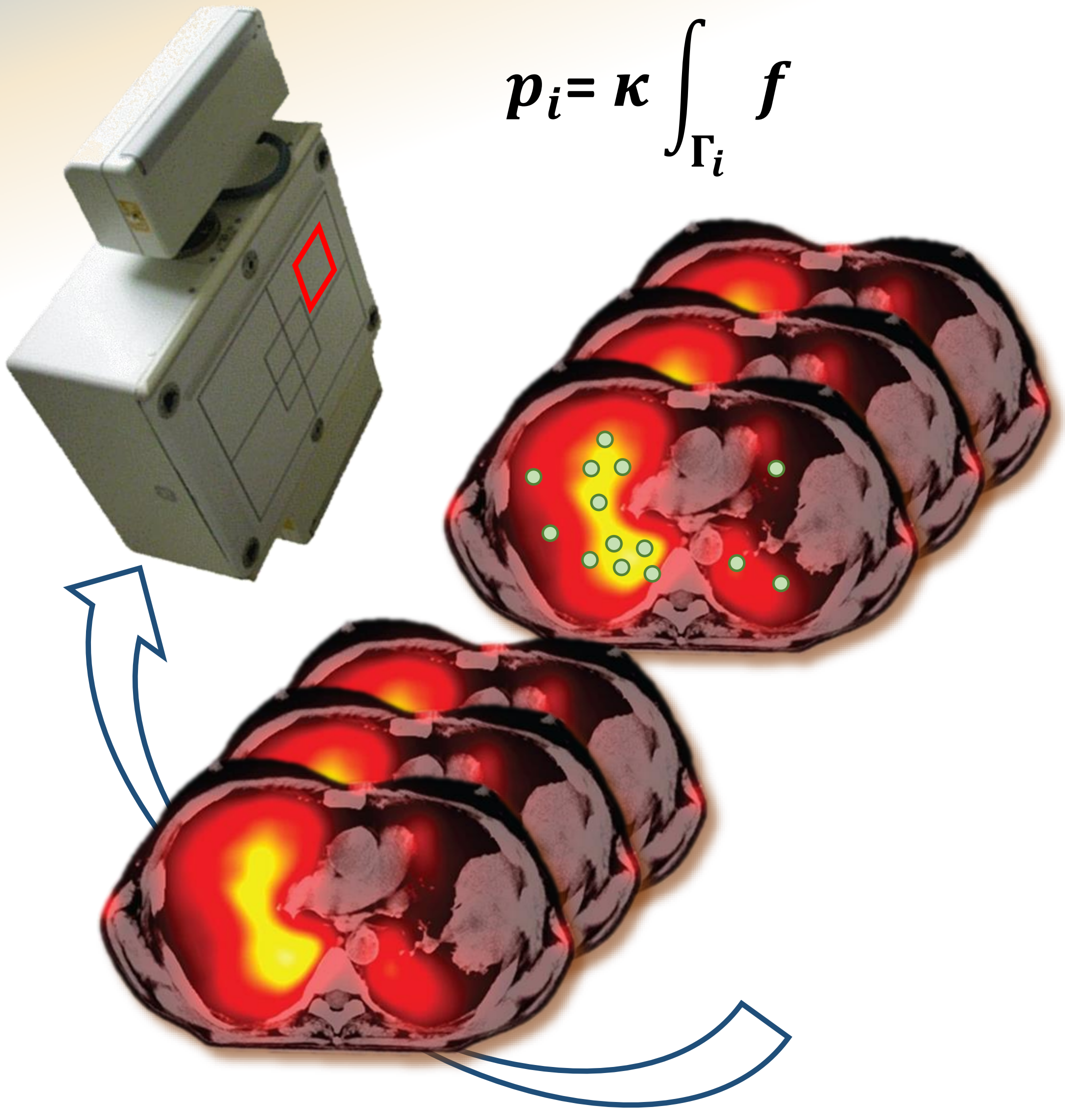
$$p_i = \kappa \int_{\Gamma_i} f$$

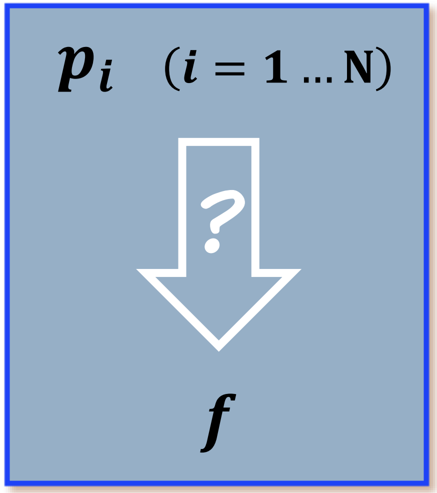
κ $\begin{cases} \text{½ vie} \\ \text{temps de pose} \\ \text{angle solide} \end{cases}$



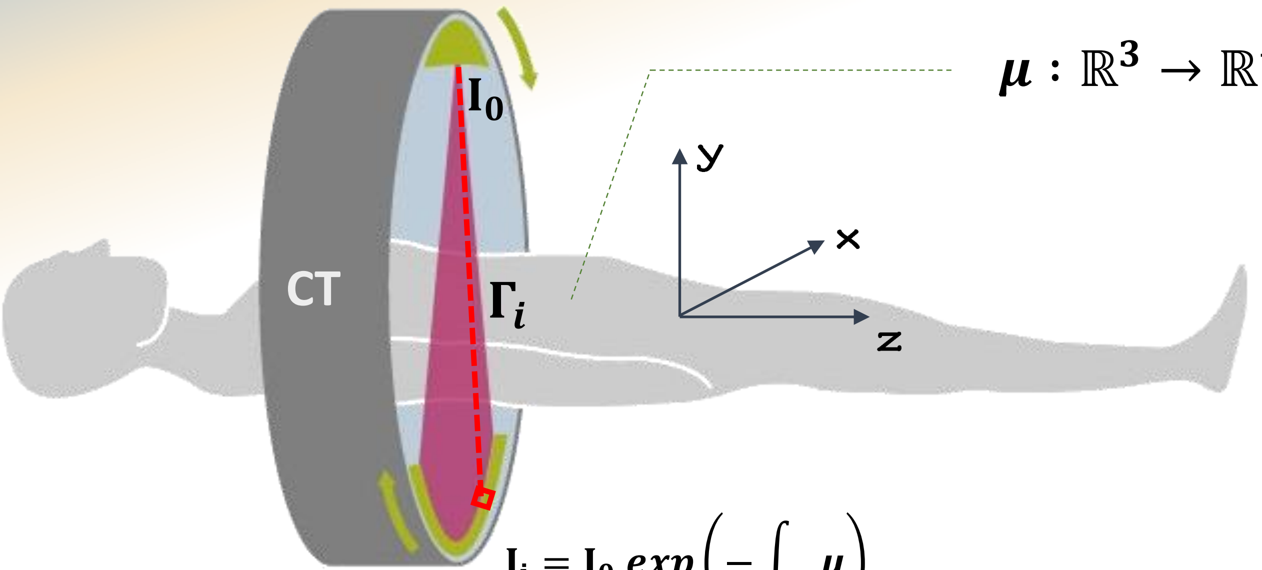
Problème tomographique

$$p_i = \kappa \int_{\Gamma_i} f$$





Problème tomographique



$$\mu : \mathbb{R}^3 \to \mathbb{R}^+$$

$$I_i = I_0 \, exp\left(-\int_{\Gamma_i} \mu\right)$$

$$p_i = -ln\left(\frac{I_i}{I_0}\right) = \int_{\Gamma_i} \mu$$

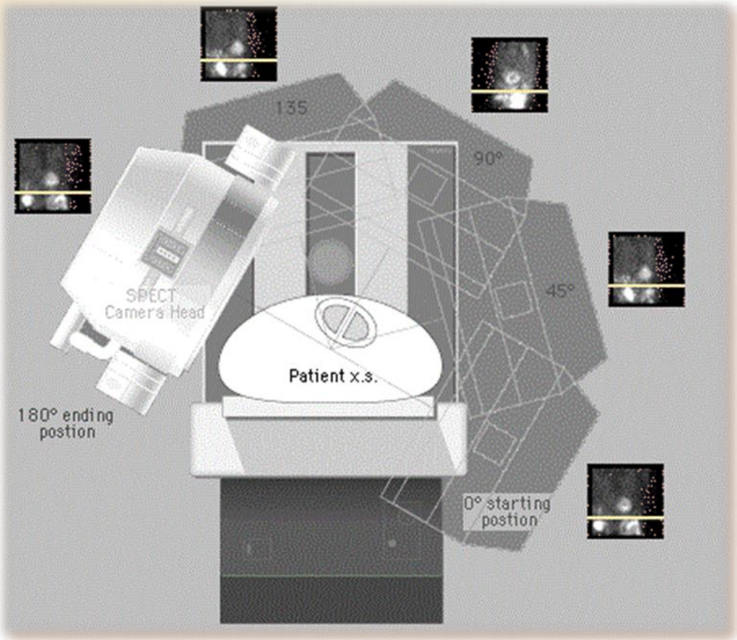
$p_i \quad (i = 1 \ldots N)$

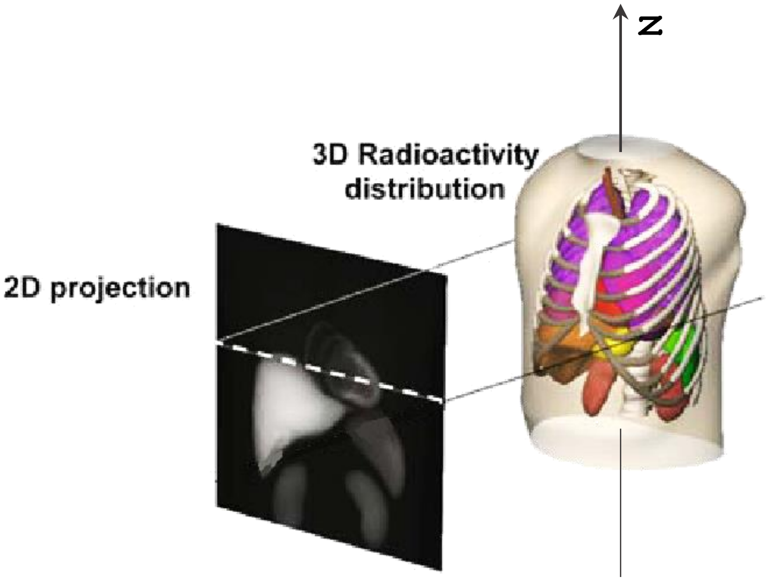
?

μ

Reconstruction

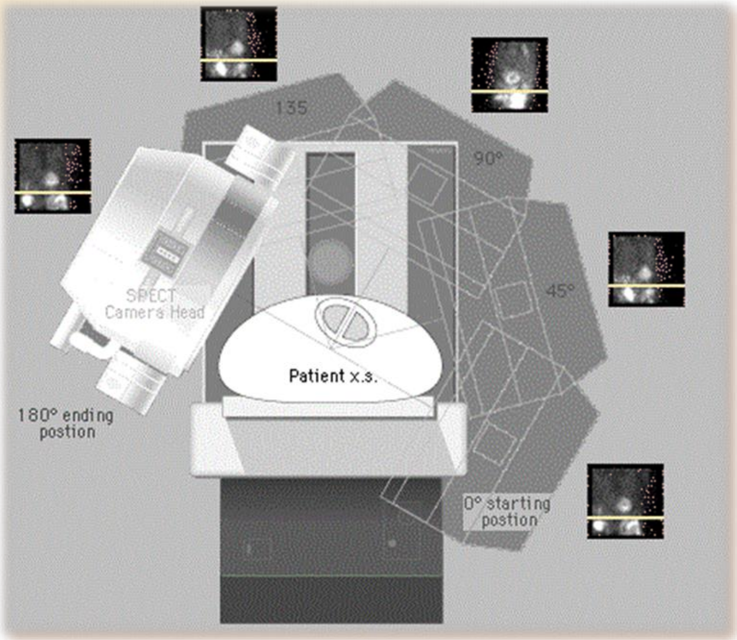
- Modèle analytique

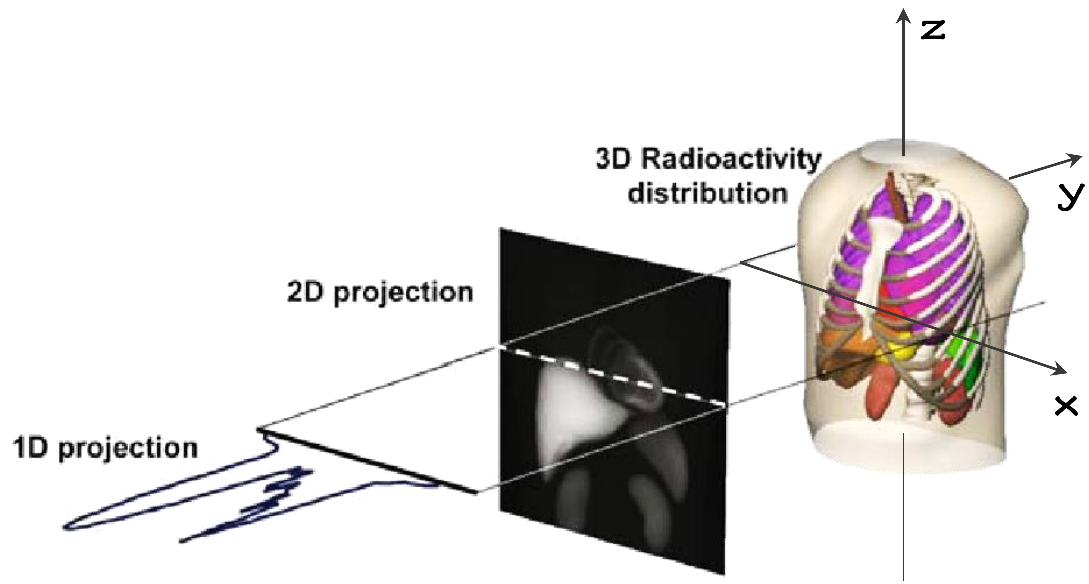




Reconstruction

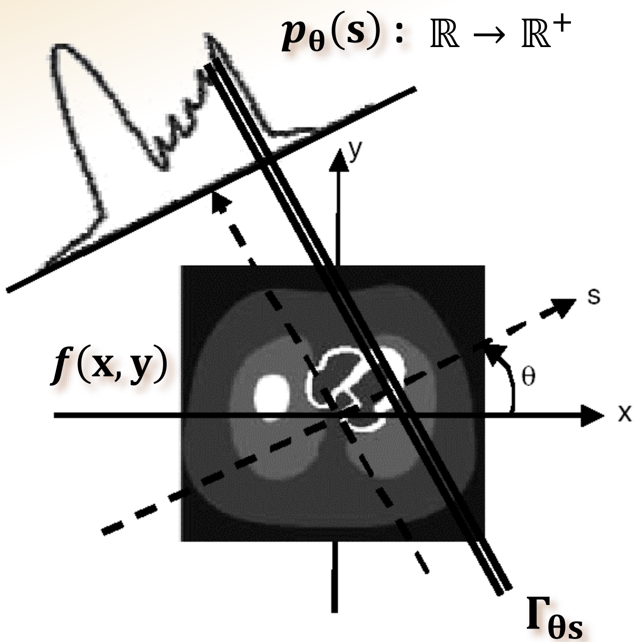
- Modèle analytique

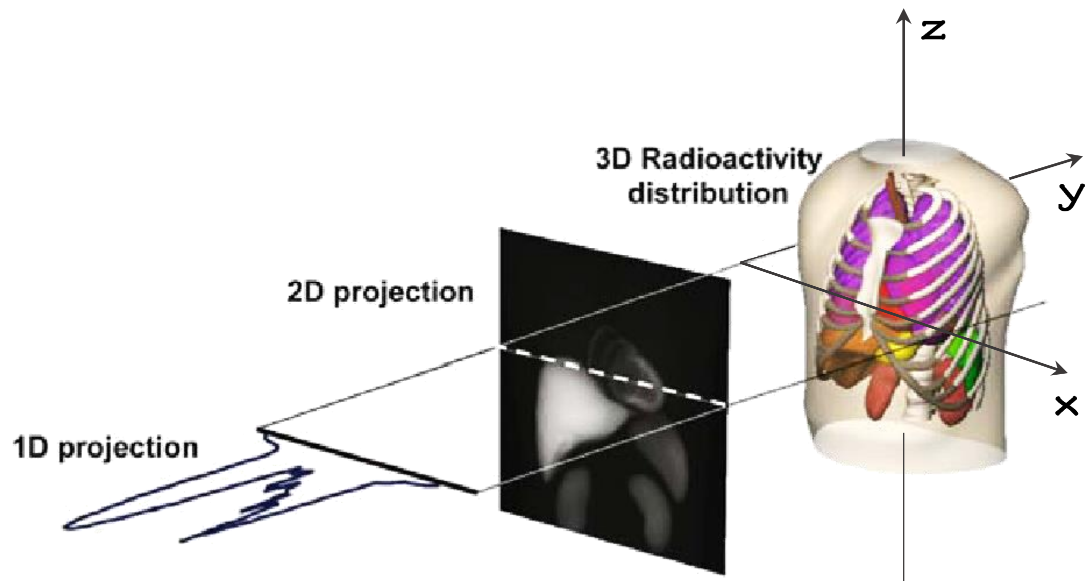




Reconstruction

- Modèle analytique

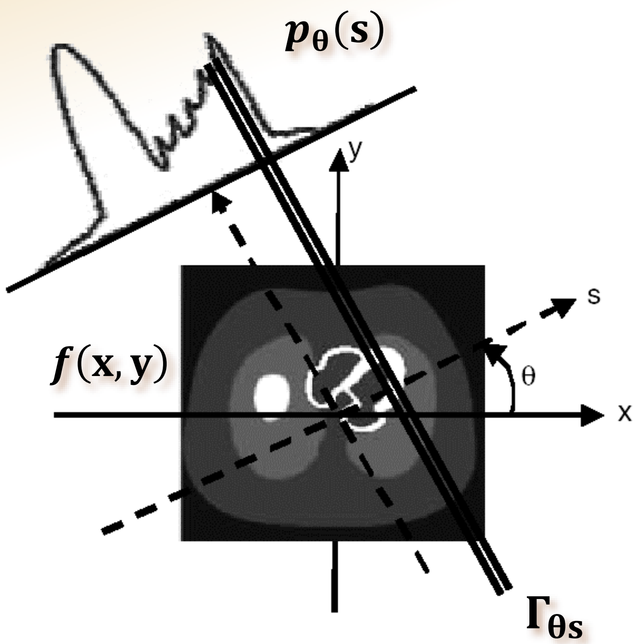


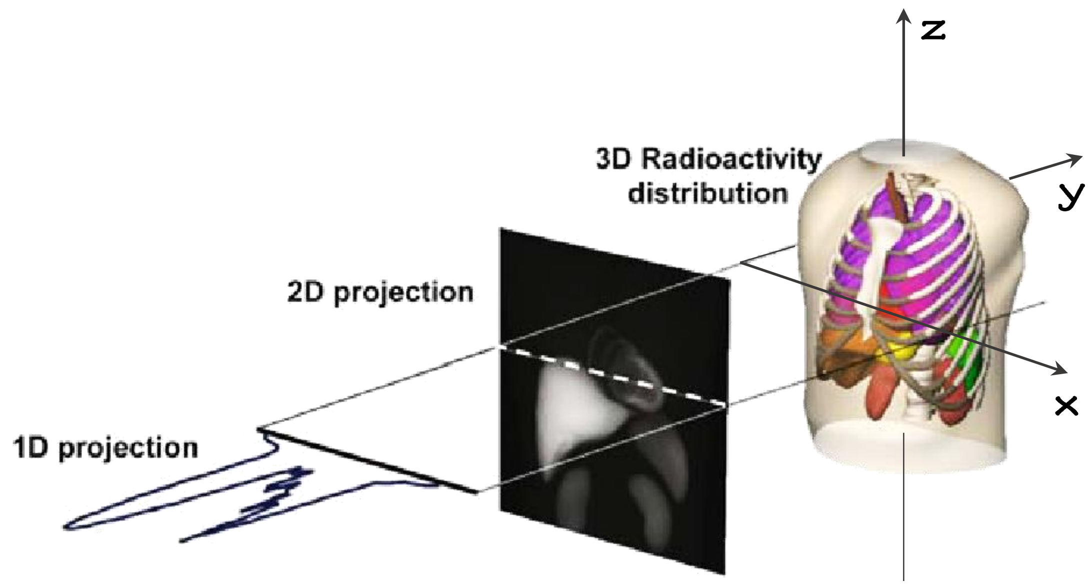


Reconstruction

- Modèle analytique

$$p_\theta(s) = \int_{\Gamma_{\theta s}} f(x,y)$$

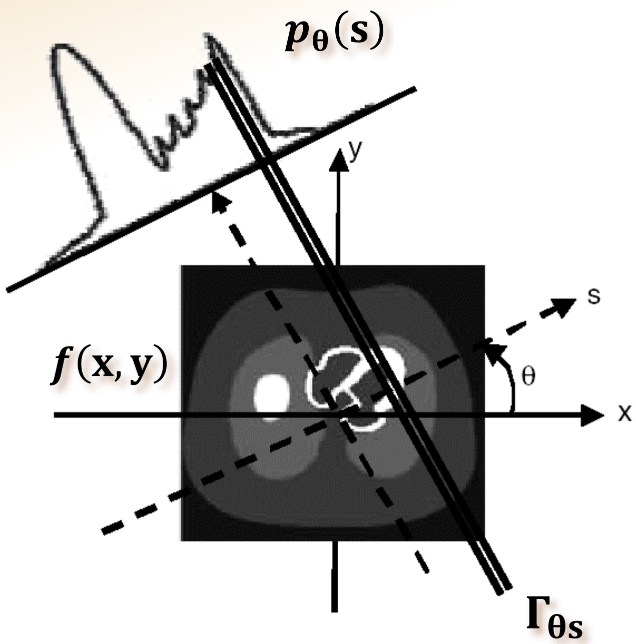


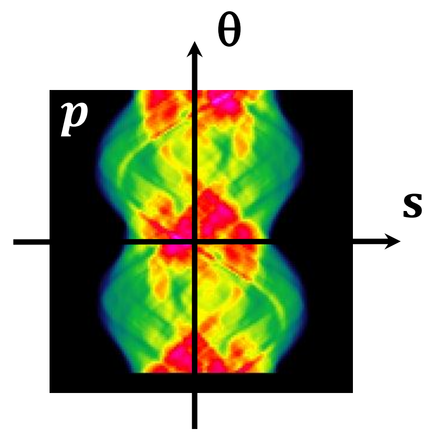


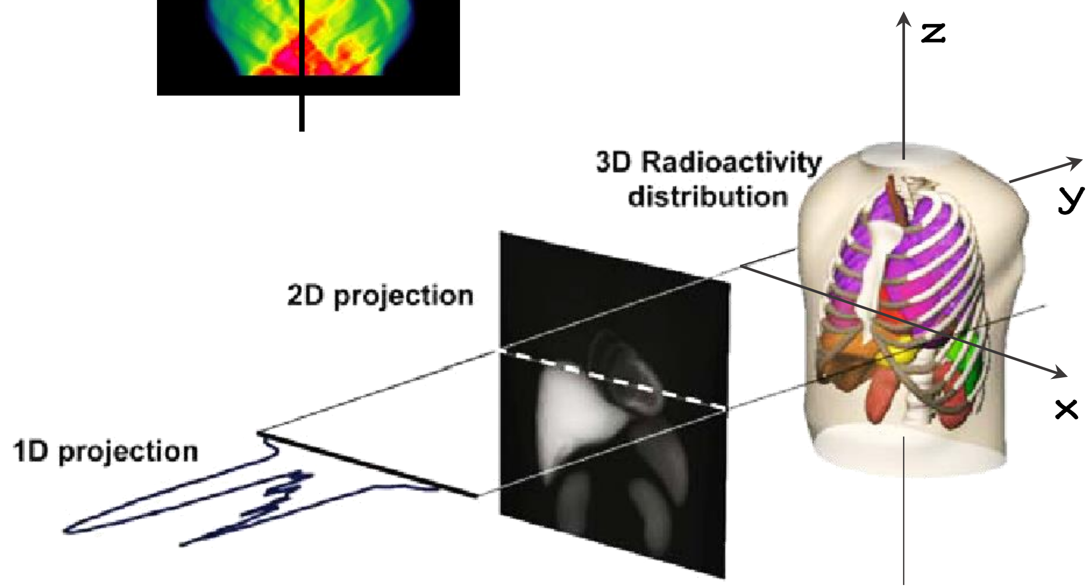
Reconstruction

- Modèle analytique

$$p_{\theta}(s) = \int_{\Gamma_{\theta s}} f(x, y)$$



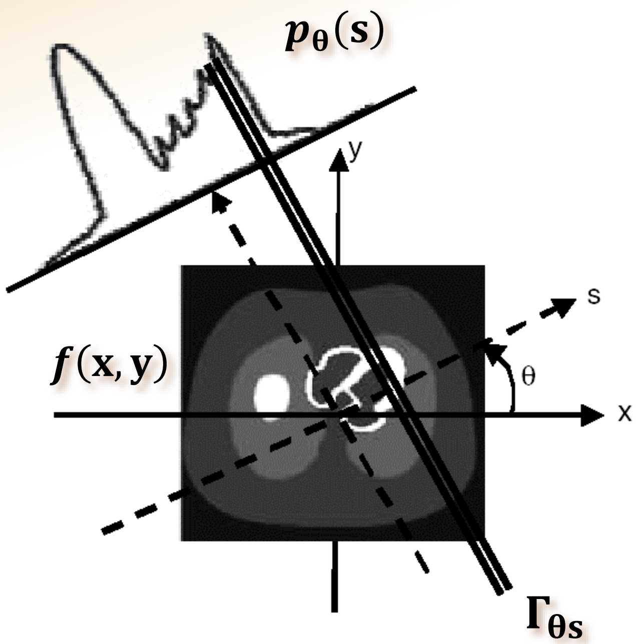




Reconstruction

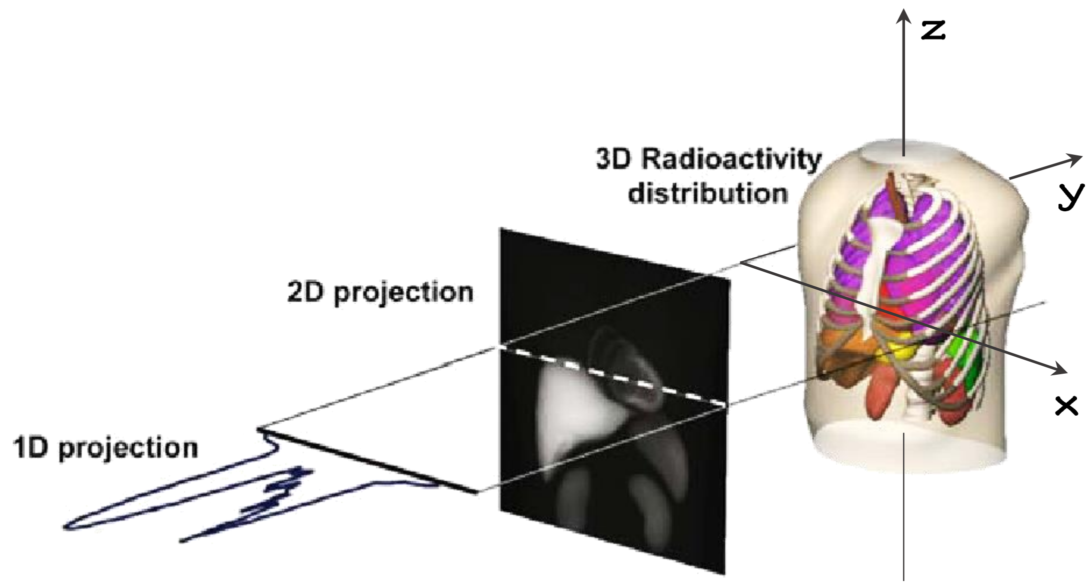
- Modèle analytique

$$p_\theta(s) = \int_{\Gamma_{\theta s}} f(x,y)$$



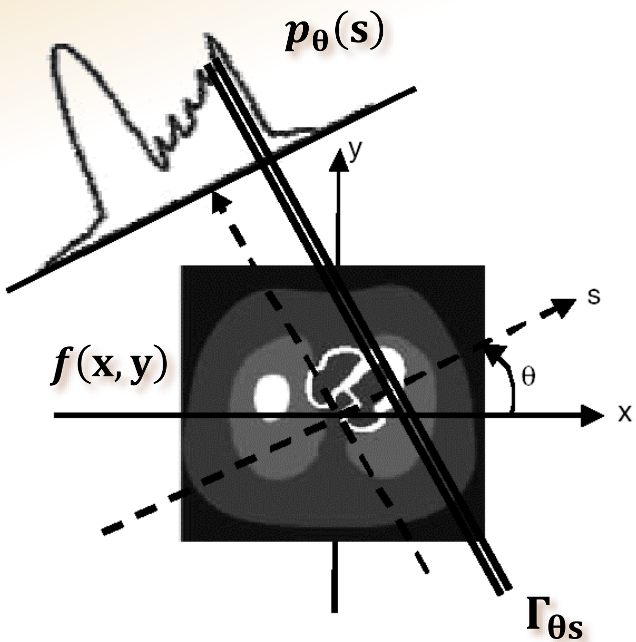
Projection (Radon) : $p = \mathrm{R}\, f$

Rétro-projection : R^*



Reconstruction

- Modèle analytique



Projection

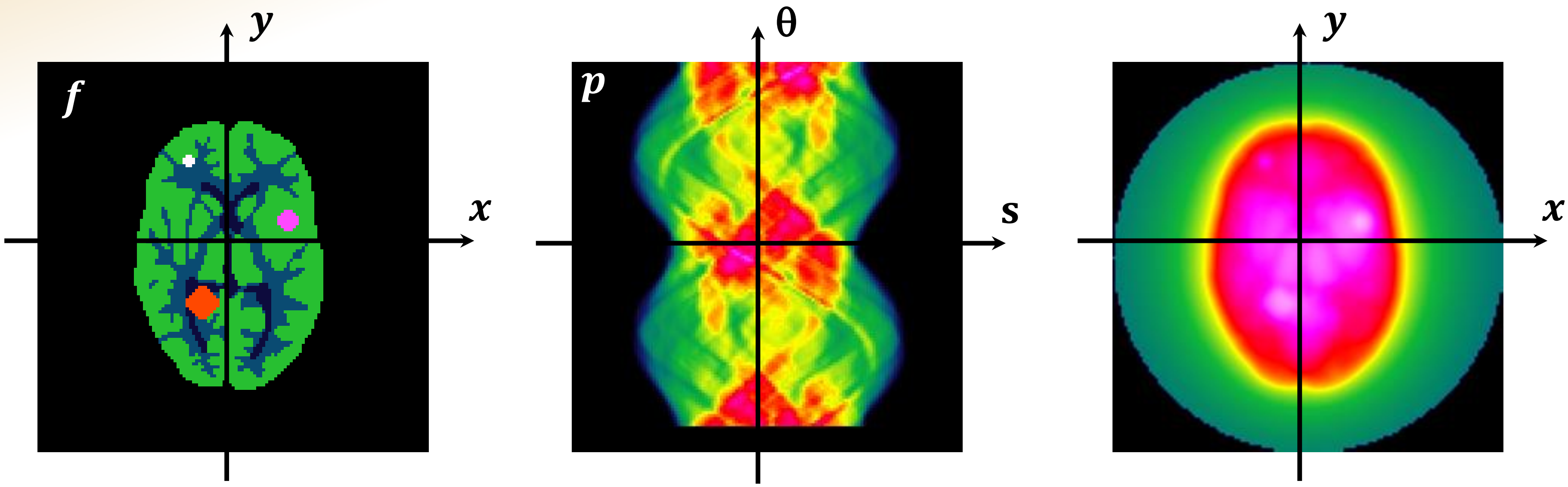
$$\mathrm{R}f_{\theta}(s) = \int_{\mathbb{R}} f(s\cos\theta - t\sin\theta , s\sin\theta + t\cos\theta)\, dt$$

Rétro-projection

$$\mathrm{R}^{*}p\,(x, y) = \int_{0}^{\pi} p_{\theta}(x\cos\theta + y\sin\theta)\, d\theta$$

Reconstruction

- Modèle analytique

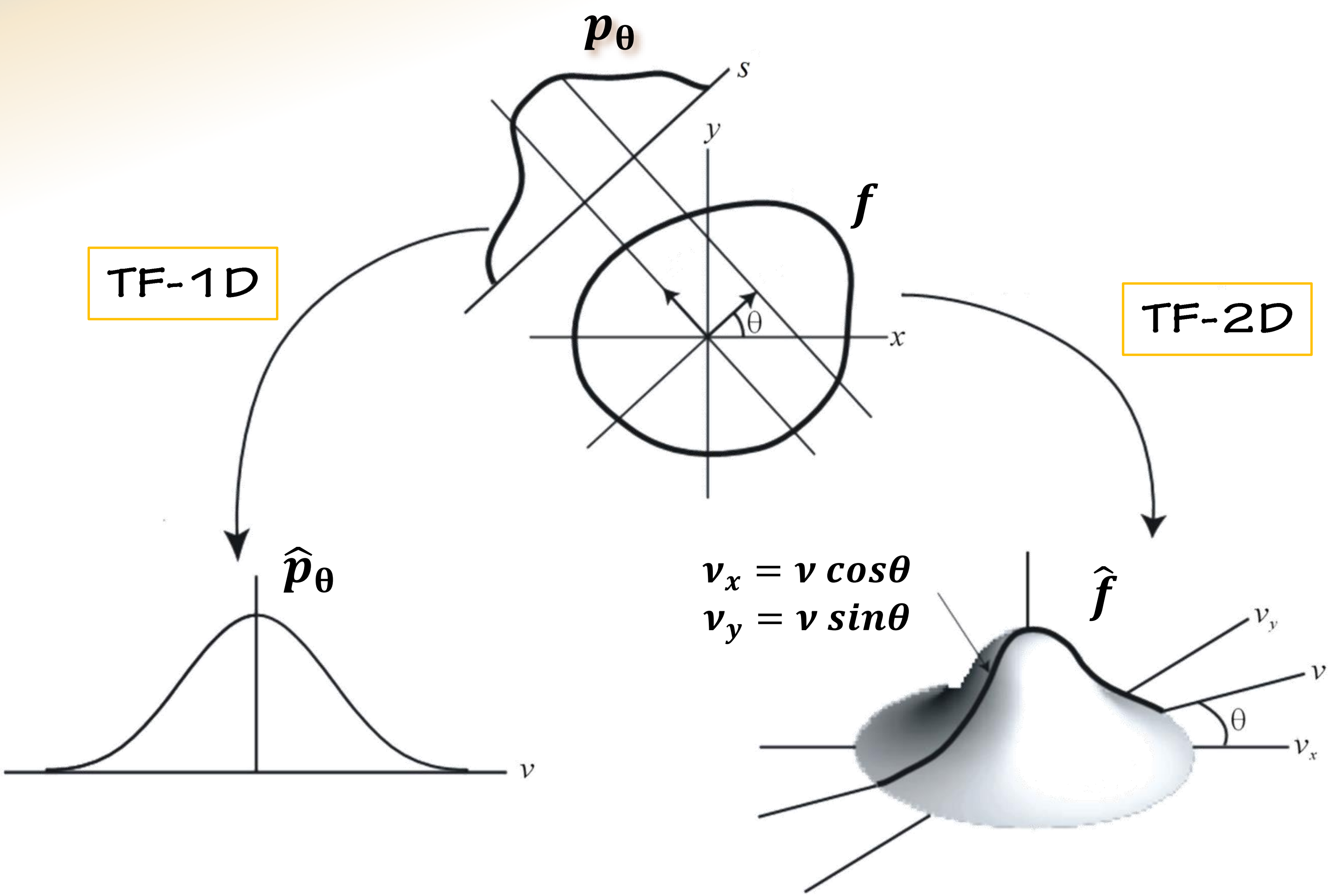


$\mathbf{R}\,f$

$\mathbf{R}^*\,p$

Reconstruction

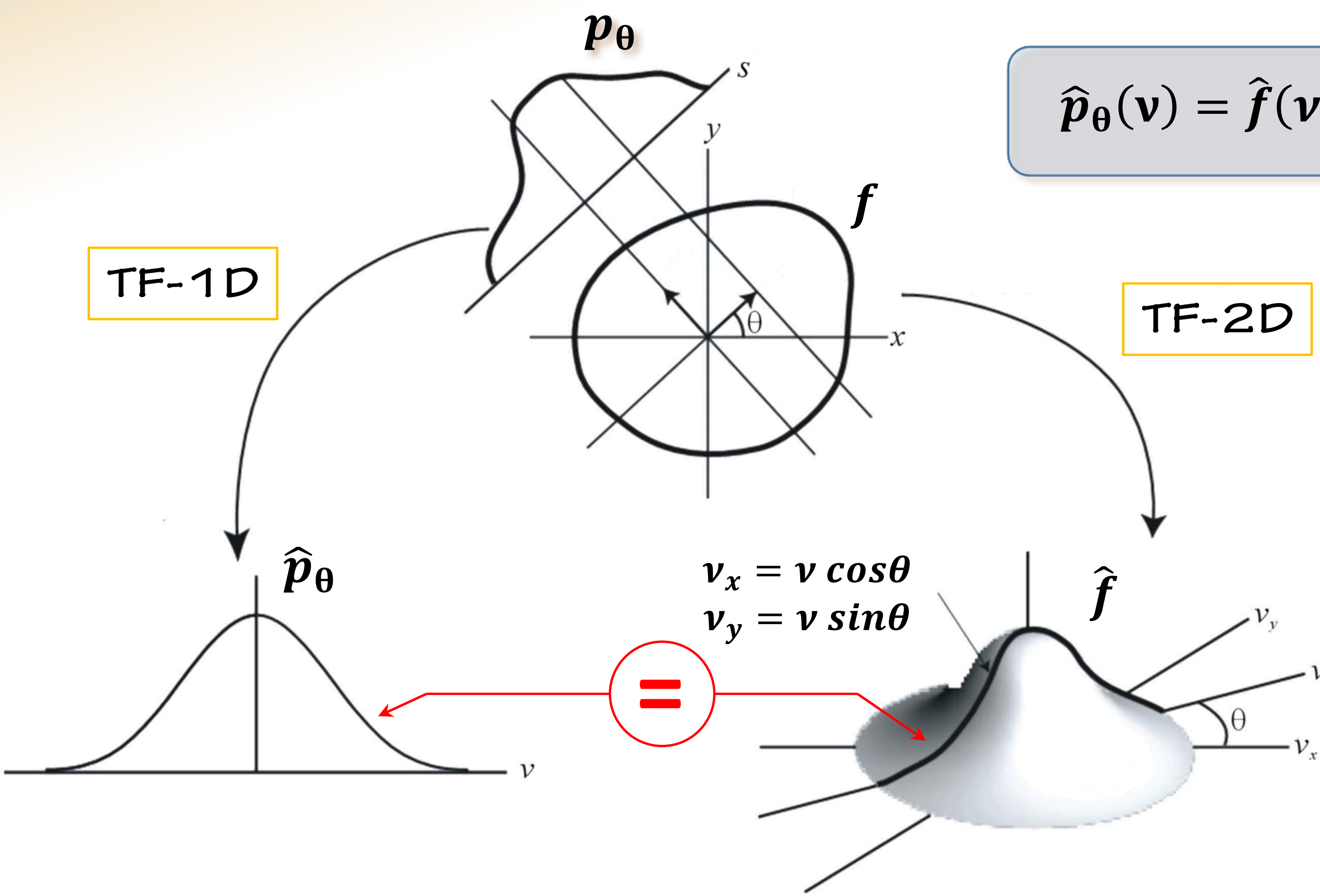
- Modèle analytique — Synthèse de Fourier



Reconstruction

- Modèle analytique — Synthèse de Fourier

$$\hat{p}_\theta(\nu) = \hat{f}(\nu cos\theta, \nu sin\theta)$$



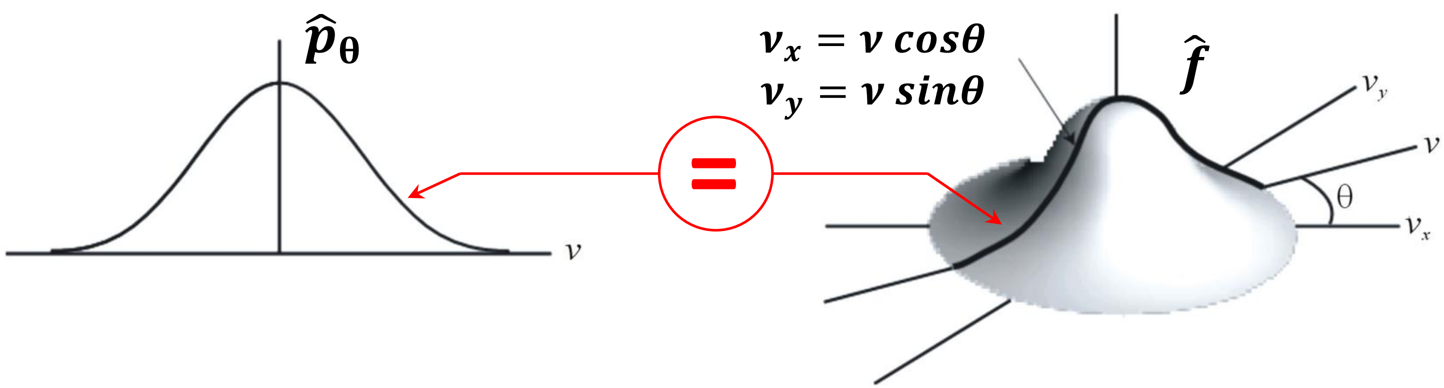
Reconstruction

- Modèle analytique — Synthèse de Fourier

$$\hat{p}_\theta(\nu) = \int_{\mathbb{R}} p_\theta(s)\, e^{-i\nu s}\, ds = \iint_{\mathbb{R}^2} f(s\cos\theta - t\sin\theta\,, s\sin\theta + t\cos\theta)\, e^{-i\nu s}\, ds\, dt$$

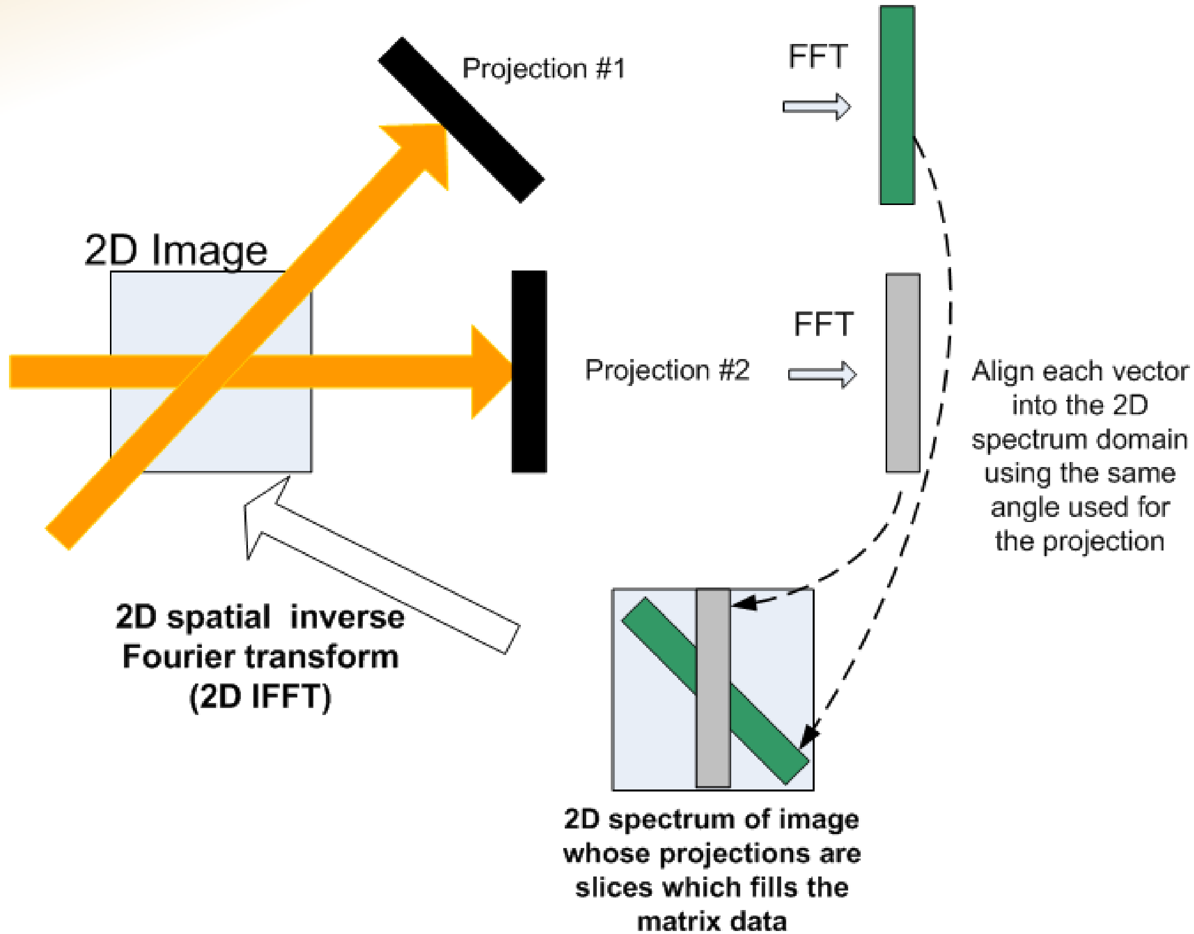
$$\begin{cases} x = s\cos\theta - t\sin\theta \\ y = s\sin\theta + t\cos\theta \end{cases} \qquad \begin{cases} s = x\cos\theta + y\sin\theta \\ t = x\sin\theta - y\cos\theta \end{cases}$$

$$= \iint_{\mathbb{R}^2} f(x,y)\, e^{-i\nu(x\cos\theta + y\sin\theta)}\, dx\, dy = \hat{f}(\nu\cos\theta, \nu\sin\theta)$$



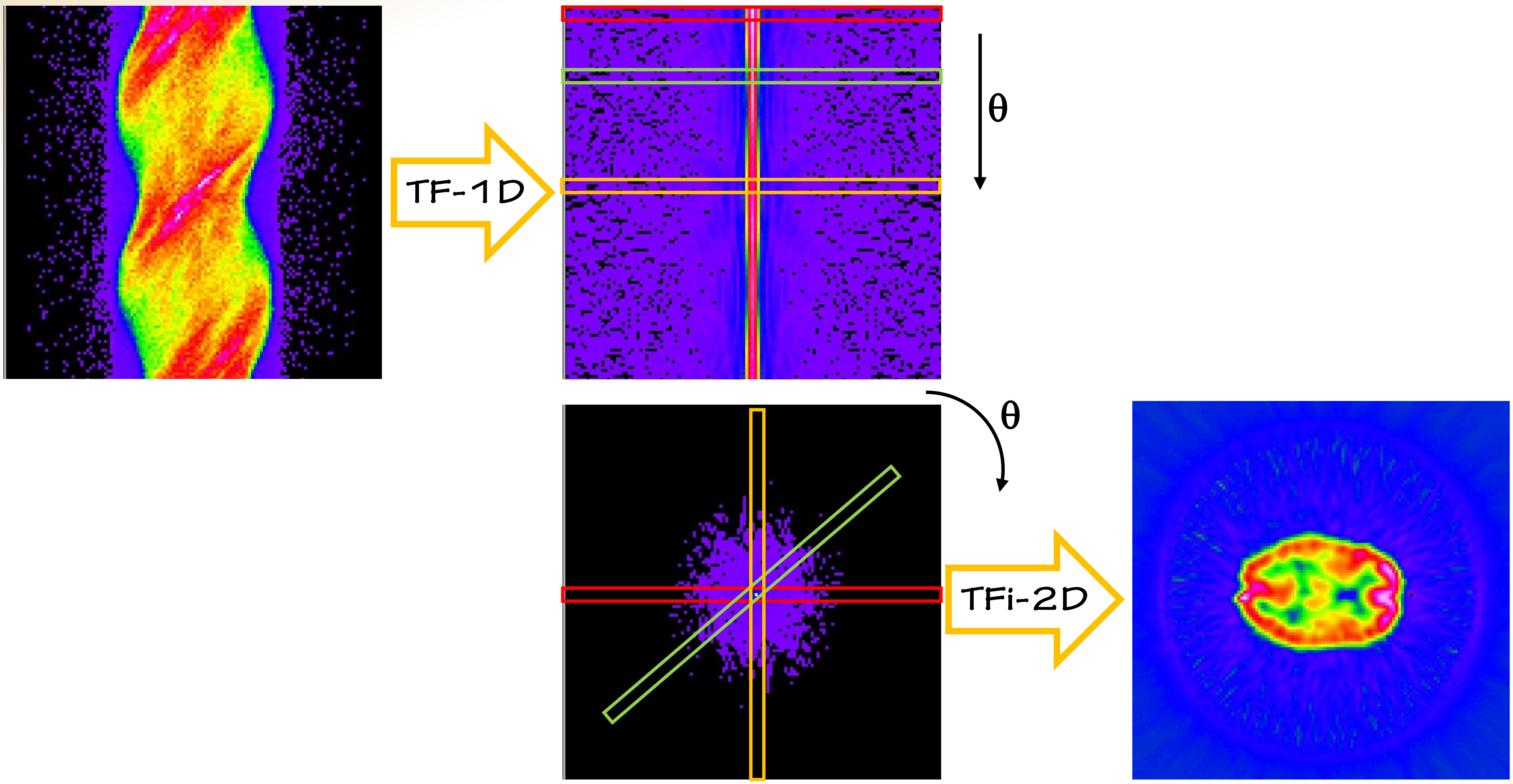
Reconstruction

- Modèle analytique — Synthèse de Fourier



Reconstruction

- Modèle analytique — Synthèse de Fourier



Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu \cos\theta$$
$$\nu_y = \nu \sin\theta$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$\boldsymbol{f} = \iint_{\mathbb{R}^2} \hat{\boldsymbol{f}}\, \boldsymbol{e}^{i(x\nu_x + y\nu_y)}\, \boldsymbol{d\nu_x d\nu_y}$$

$$\nu_x = \nu\, cos\theta$$
$$\nu_y = \nu\, sin\theta$$

$$\boldsymbol{f} = \int \boldsymbol{d\theta} \int \hat{\boldsymbol{f}}\, \boldsymbol{e}^{i\nu(xcos\theta + ysin\theta)}\, |\boldsymbol{\nu}|\, \boldsymbol{d\nu}$$

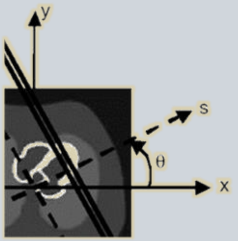
Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu \cos\theta$$
$$\nu_y = \nu \sin\theta$$

$$f = \int d\theta \int \hat{f}\, e^{i\nu(\underbrace{x\cos\theta + y\sin\theta}_{= s})}\, |\nu|\, d\nu$$



Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu \cos\theta$$
$$\nu_y = \nu \sin\theta$$

$$f = \int d\theta \int \boxed{\hat{f}}\, e^{i\nu(x\cos\theta + y\sin\theta)}\, |\nu|\, d\nu$$

$$\hat{f}(\nu\cos\theta, \nu\sin\theta) = \hat{p}_\theta(\nu)$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(xv_x + yv_y)}\, dv_x dv_y$$

$$v_x = v\, cos\theta$$
$$v_y = v\, sin\theta$$

$$f = \int d\theta \int \boxed{\hat{f}}\, e^{iv(xcos\theta + ysin\theta)}\, |v|\, dv$$

$$\hat{f}(vcos\theta, vsin\theta) = \hat{p}_\theta(v)$$

$$f = \int d\theta \int dv\, e^{isv}\, |v|\, \hat{p}$$

Reconstruction

■ Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu\, cos\theta$$
$$\nu_y = \nu\, sin\theta$$

$$f = \int d\theta \int \boxed{\hat{f}}\, e^{i\nu(xcos\theta + ysin\theta)}\, |\nu|\, d\nu$$

$$\hat{f}(\nu cos\theta, \nu sin\theta) = \hat{p}_\theta(\nu)$$

$$f = \int d\theta \int d\nu\, e^{is\nu}\, |\nu|\, \hat{p}$$

$$\widehat{\mathrm{H}p} = |\nu|\, \hat{p}$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(xv_x + yv_y)}\, dv_x dv_y$$

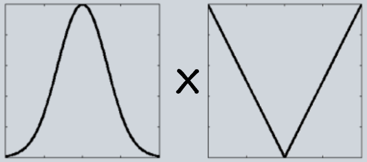
$$v_x = v\, cos\theta$$
$$v_y = v\, sin\theta$$

$$f = \int d\theta \int \boxed{\hat{f}}\, e^{iv(xcos\theta + ysin\theta)}\, |v|\, dv$$

$$\hat{f}(vcos\theta, vsin\theta) = \hat{p}_\theta(v)$$

$$f = \int d\theta \int dv\, e^{isv} |v|\, \hat{p}$$

$$\widehat{\mathrm{H}p} = |v|\, \hat{p}$$



Filtre rampe

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu\, cos\theta$$
$$\nu_y = \nu\, sin\theta$$

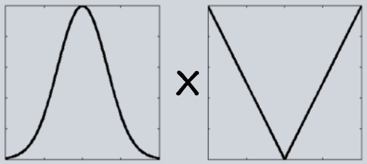
$$f = \int d\theta \int \boxed{\hat{f}}\, e^{i\nu(xcos\theta + ysin\theta)}\, |\nu|\, d\nu$$

$$\hat{f}(\nu cos\theta, \nu sin\theta) = \hat{p}_\theta(\nu)$$

$$f = \int d\theta \boxed{\int d\nu\, e^{is\nu}}\, |\nu|\, \hat{p}$$

$$\widehat{\mathrm{H}p} = |\nu|\, \hat{p}$$

TF inv - 1D



Filtre rampe

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu\, cos\theta$$
$$\nu_y = \nu\, sin\theta$$

$$f = \int d\theta \int \boxed{\hat{f}}\, e^{i\nu(xcos\theta + ysin\theta)}\, |\nu|\, d\nu$$

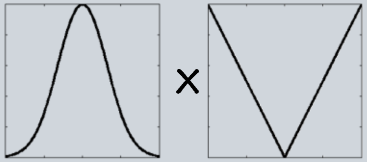
$$\hat{f}(\nu cos\theta, \nu sin\theta) = \hat{p}_\theta(\nu)$$

$$f = \boxed{\int d\theta} \boxed{\int d\nu\, e^{is\nu}} |\nu|\, \hat{p}$$

$$\widehat{\mathrm{H}p} = |\nu|\, \hat{p}$$

TF inv - 1D

Rétro-projection



Filtre rampe

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu \cos\theta$$
$$\nu_y = \nu \sin\theta$$

$$f = \int d\theta \int \hat{f}\, e^{i\nu(x\cos\theta + y\sin\theta)}\, |\nu|\, d\nu$$

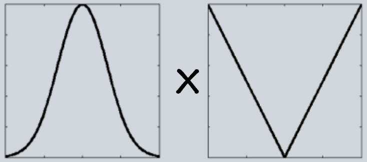
$$\hat{f}(\nu\cos\theta, \nu\sin\theta) = \hat{p}_\theta(\nu)$$

$$f = \int d\theta \int d\nu\, e^{is\nu} |\nu|\, \hat{p}$$

$$\widehat{\mathrm{H}p} = |\nu|\, \hat{p}$$

TF inv -1D

Rétro-projection



Filtre rampe

$$f = \mathrm{R}^* \mathrm{H}p$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$\mathbf{H}\boldsymbol{p} = \mathbf{TF}^{-1}\left(\widehat{\boldsymbol{h}}\,\widehat{\boldsymbol{p}}\right)$$

$$\widehat{\boldsymbol{h}}(\boldsymbol{\nu}) = |\boldsymbol{\nu}|$$

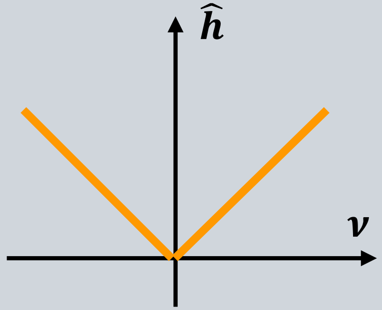
$$\mathbf{H}\boldsymbol{p} = \boldsymbol{h} * \boldsymbol{p}$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$\mathbf{H}p = \mathrm{TF}^{-1}\left(\hat{h}\,\hat{p}\right)$$

$$\hat{h}(\nu) = |\nu|$$



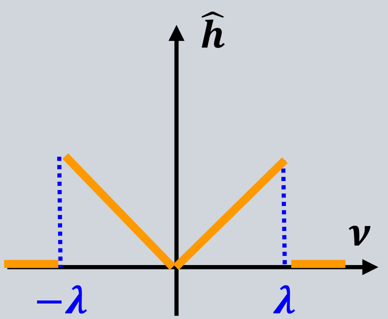
$$\mathbf{H}p = h * p$$

$$h(s) = \int_{\mathbb{R}} \hat{h}(\nu)\ e^{i\nu s}\, d\nu$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$\mathbf{H}p = \mathrm{TF}^{-1}(\hat{h}\,\hat{p})$$



$$\mathbf{H}p = h * p$$

$$h(s) = \int_{-\lambda}^{\lambda} \hat{h}(\nu)\; e^{i\nu s}\, d\nu$$

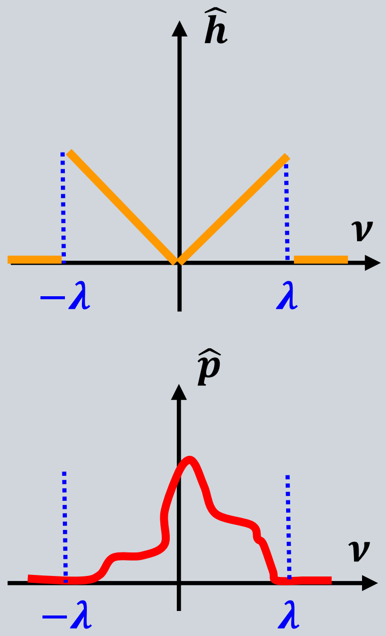
Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$\mathbf{H}p = \mathrm{TF}^{-1}\left(\hat{h}\,\hat{p}\right)$$

$$\mathbf{H}p = h * p$$

$$h(s) = \int_{-\lambda}^{\lambda} \hat{h}(\nu)\; e^{i\nu s}\, d\nu$$



$$\lambda \geq \nu_{max}$$
$$= 2\pi\, F_{max} = \pi$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

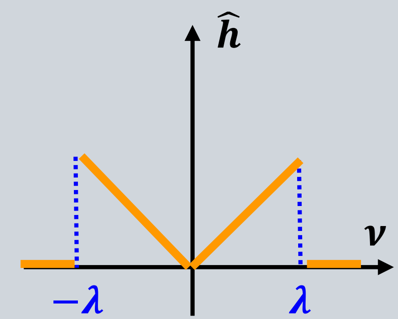
$$\mathbf{H}p = \mathrm{TF}^{-1}\left(\hat{h}\,\hat{p}\right)$$

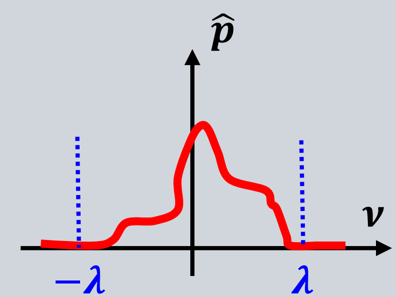
$$\mathbf{H}p = h * p$$

$$h(s) = \int_{-\lambda}^{\lambda} \hat{h}(\nu)\; e^{i\nu s}\, d\nu$$

$$= -\int_{-\pi}^{0} \nu\, e^{i\nu s}\, d\nu + \int_{0}^{\pi} \nu\, e^{i\nu s}\, d\nu$$

$$= \frac{2\pi}{s} sin(\pi s) + \frac{2}{s^2}\left(cos(\pi s) - 1\right)$$



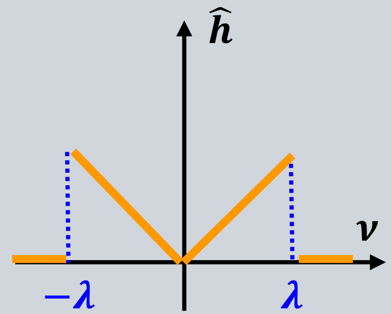


$$\lambda \geq \nu_{max}$$
$$= 2\pi\, F_{max} = \pi$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

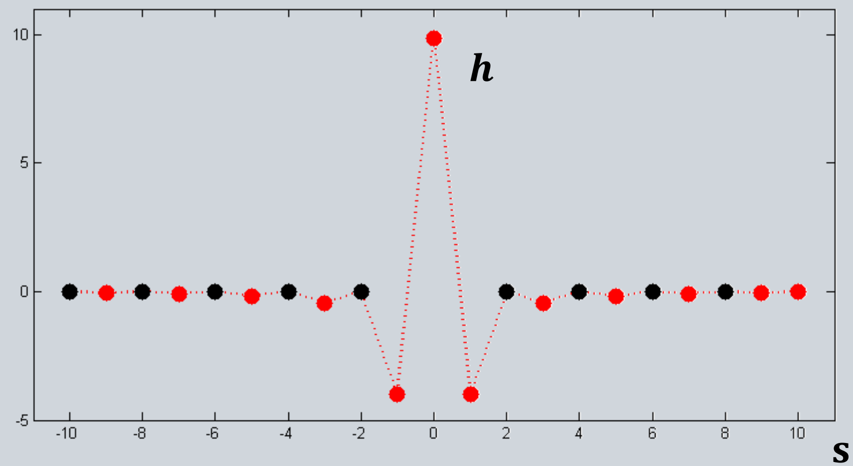
$$\mathrm{H}p = \mathrm{TF}^{-1}\left(\hat{h}\,\hat{p}\right)$$



$$\mathrm{H}p = h * p$$

$$h(s) = \int_{-\lambda}^{\lambda} \hat{h}(\nu)\; e^{i\nu s}\, d\nu$$

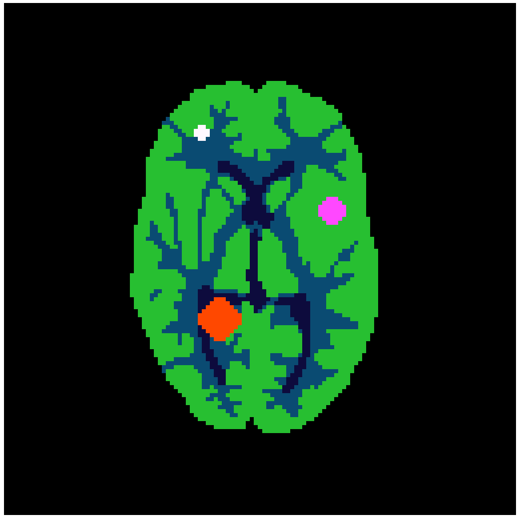
$$= \begin{cases} \pi^2 \quad en\;\; 0 \\ 0 \quad pour\; s\; pair \\ -\dfrac{4}{s^2} \quad pour\; s\; impair \end{cases}$$



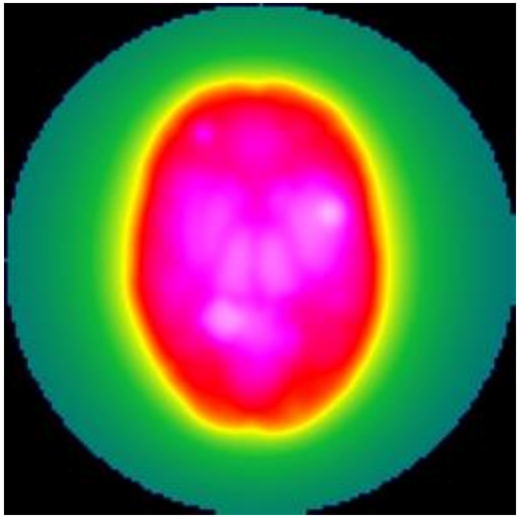
© Ramachandran-Lakshminarayan (RAM-LAK)

Reconstruction

- Modèle analytique — Rétro-projection filtrée

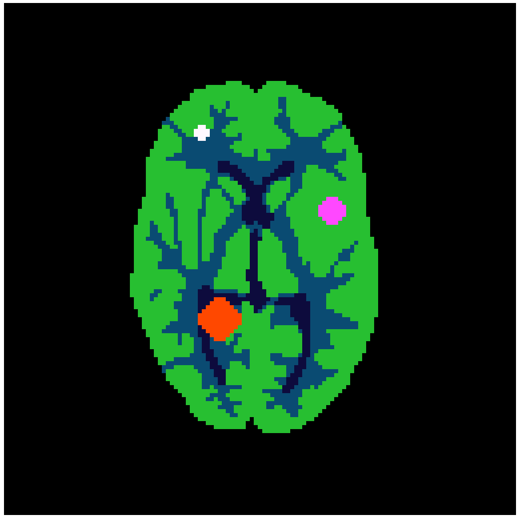


$$f = R^{*} p$$

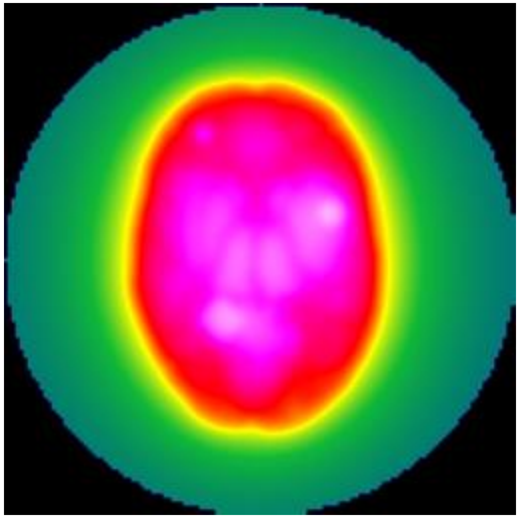


Reconstruction

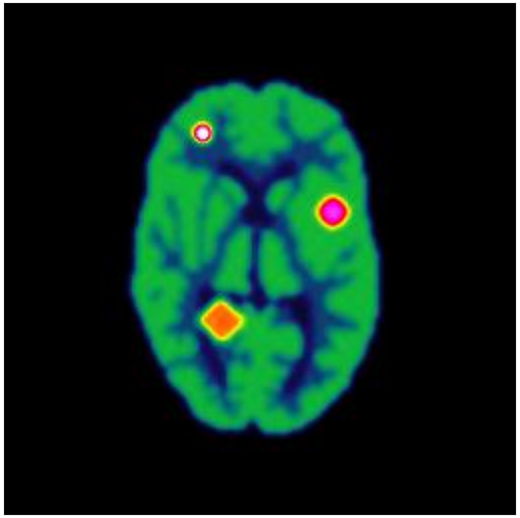
- Modèle analytique — Rétro-projection filtrée



$$\boldsymbol{f} = \mathbf{R}^* \boldsymbol{p}$$



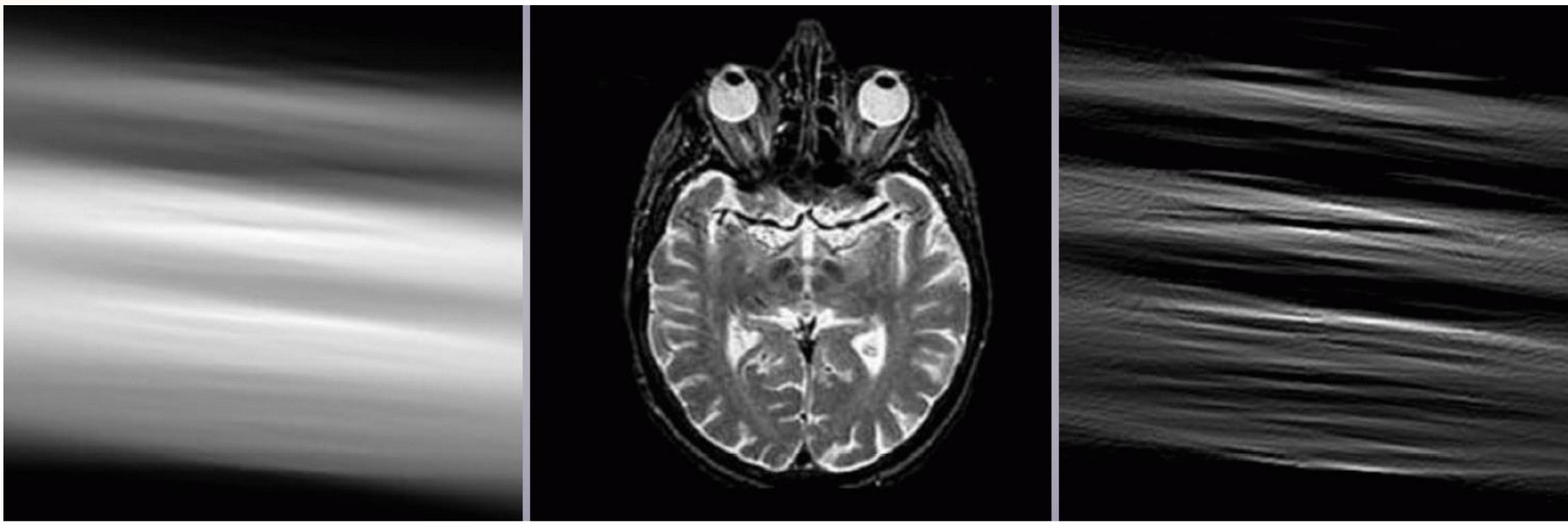
$$\boldsymbol{f} = \mathbf{R}^* \mathbf{H}\boldsymbol{p}$$



Reconstruction

- Modèle analytique — Rétro-projection filtrée

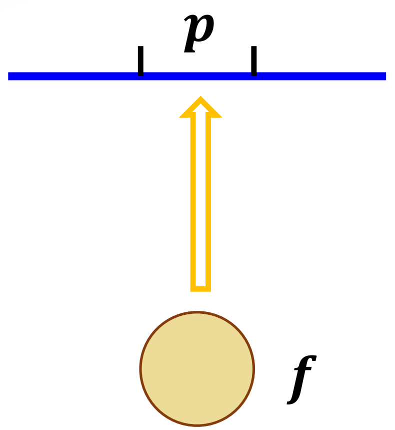
$$\boldsymbol{f} = \mathbf{R}^{*}\,\boldsymbol{p} \qquad\qquad \boldsymbol{f} = \mathbf{R}^{*}\,\mathbf{H}\boldsymbol{p}$$



Reconstruction

- Modèle analytique — Limites

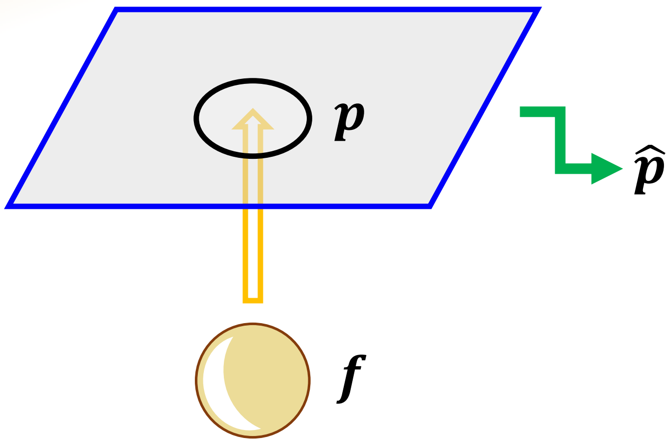
Troncature des données



Reconstruction

- Modèle analytique — Limites

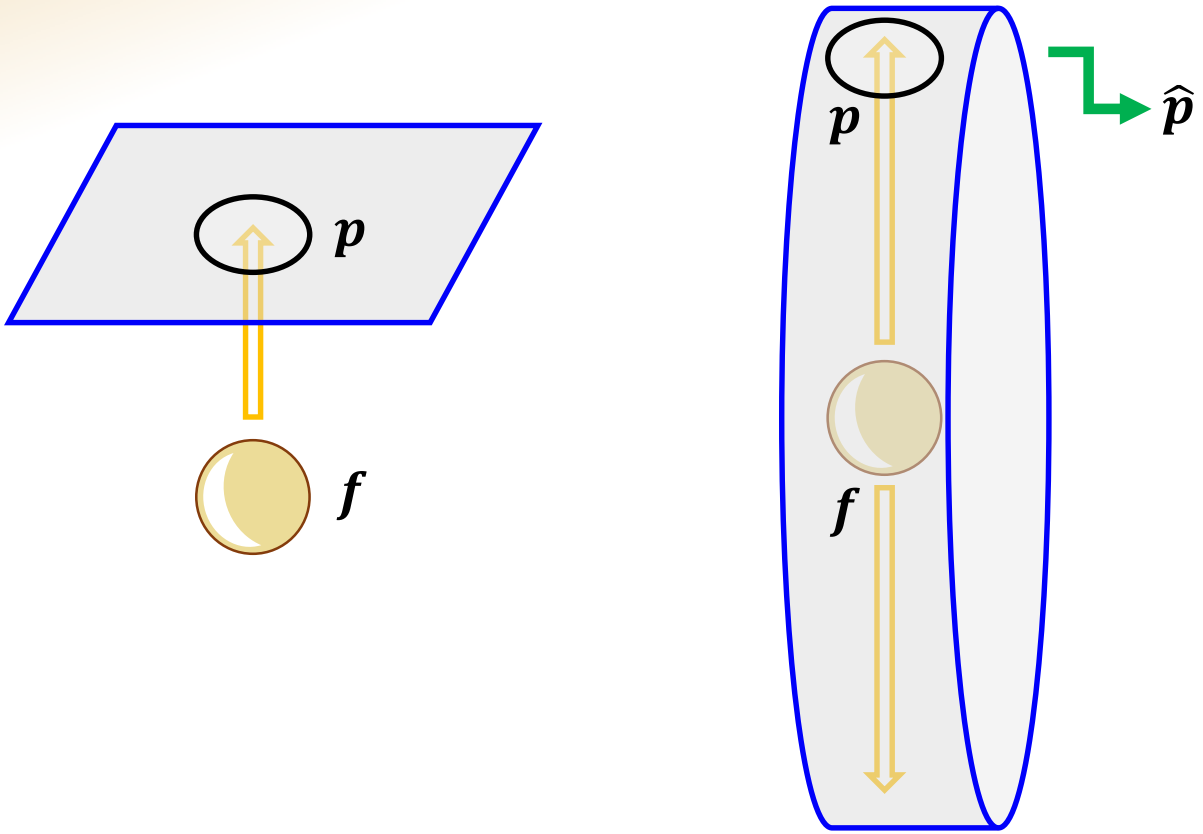
Troncature des données



Reconstruction

- Modèle analytique — Limites

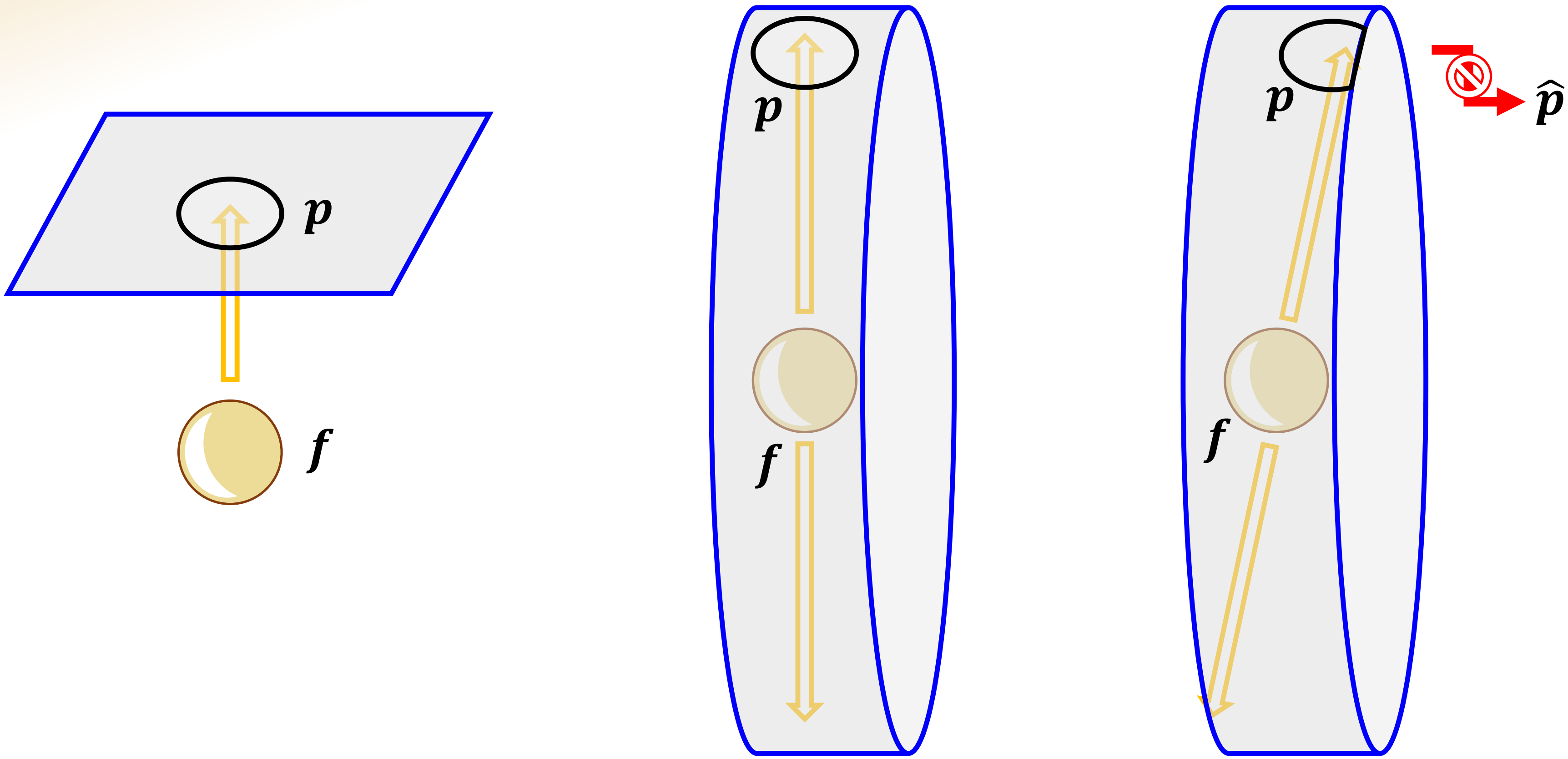
Troncature des données



Reconstruction

- Modèle analytique — Limites

Troncature des données

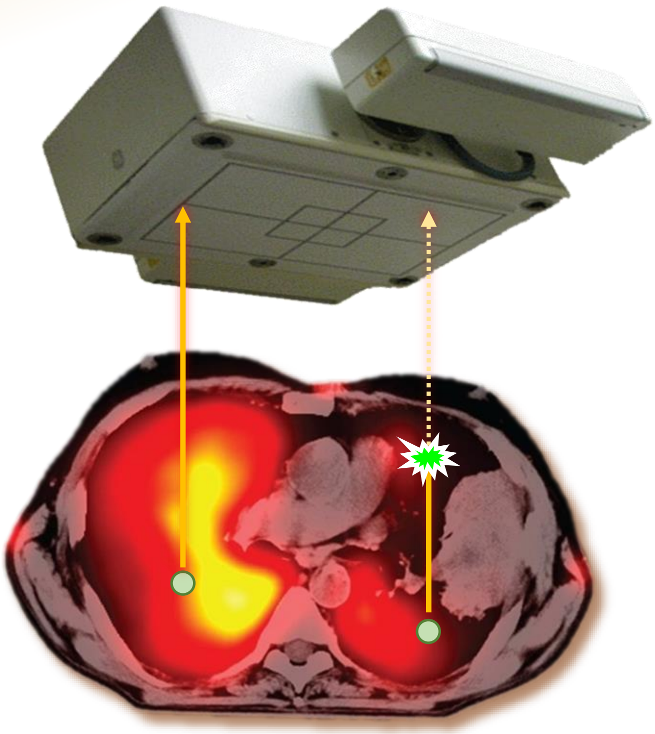


Reconstruction

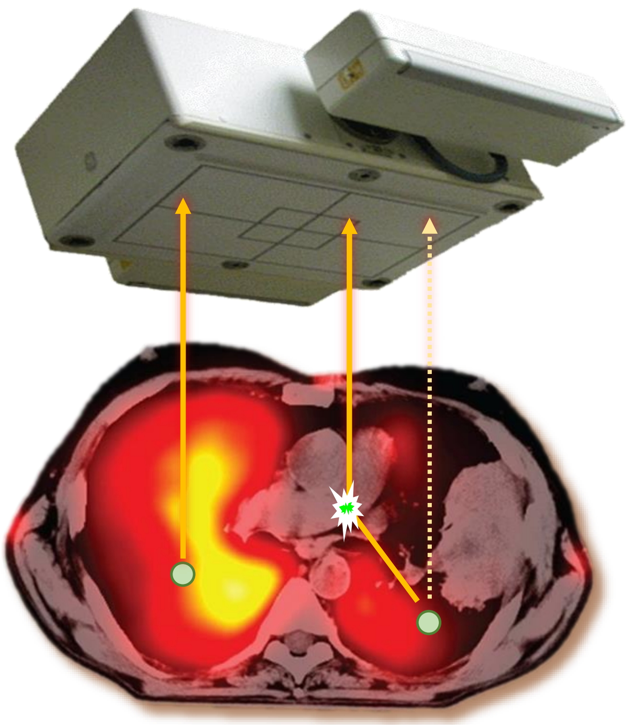
■ Modèle analytique — Limites

Interactions γ - matière

Absorption



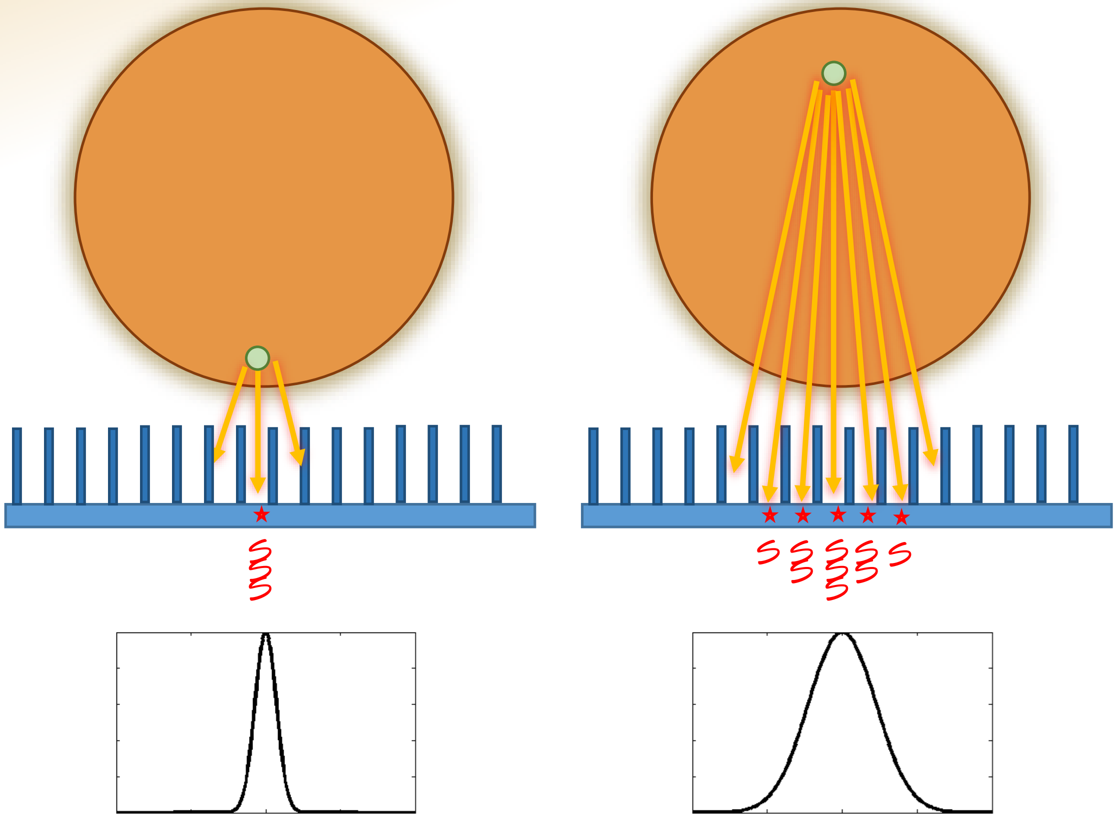
Diffusion

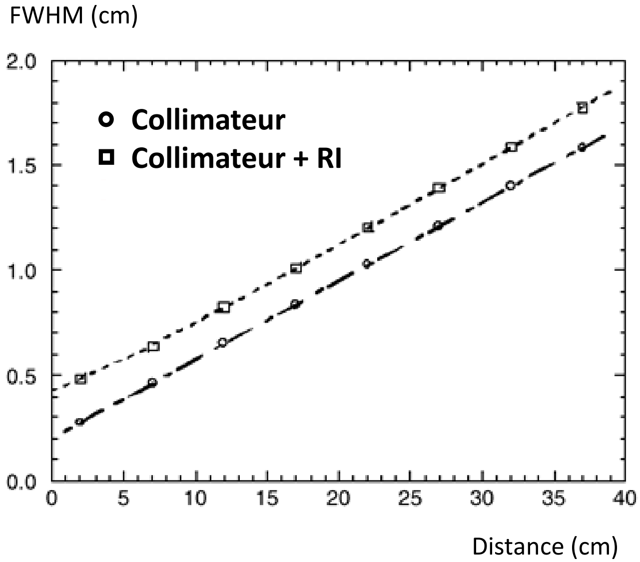


Reconstruction

- Modèle analytique — Limites

Réponse du détecteur





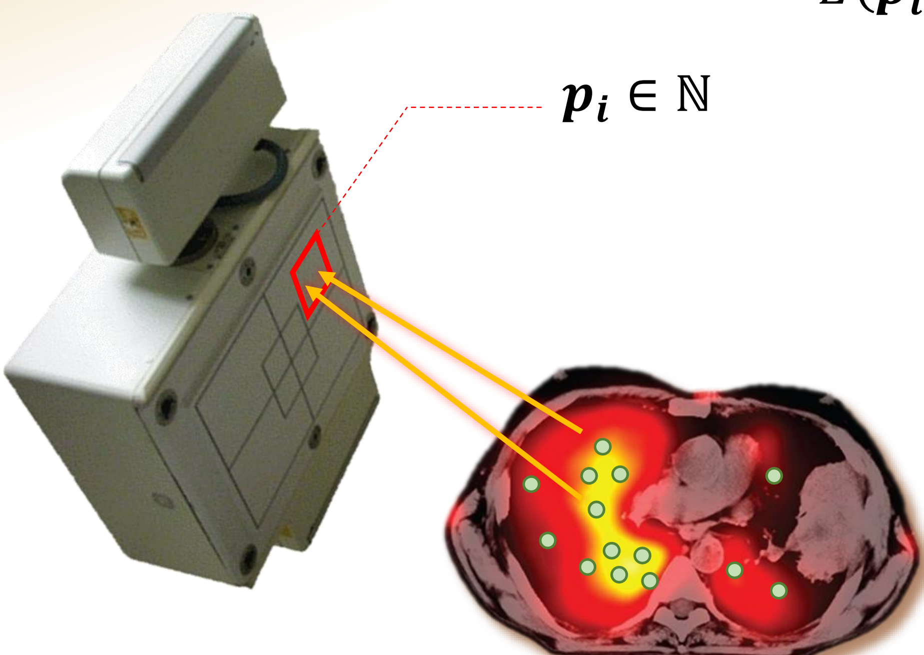
Reconstruction

- Modèle analytique — Limites

Bruit statistique

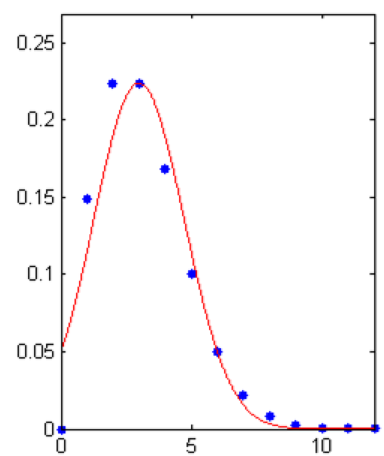
$$E(\boldsymbol{p_i}) = \boldsymbol{\kappa} \int_{\boldsymbol{\Gamma_i}} \boldsymbol{f}$$

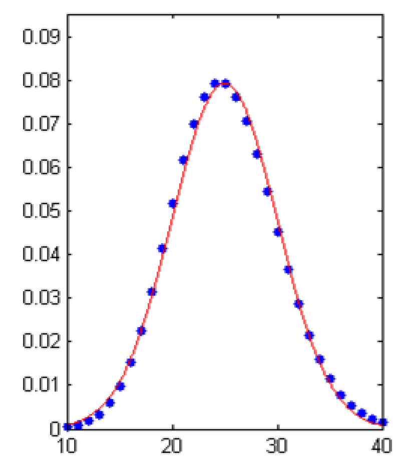
$\boldsymbol{p_i} \in \mathbb{N}$



$$Var(\boldsymbol{p_i}) = E(\boldsymbol{p_i})$$

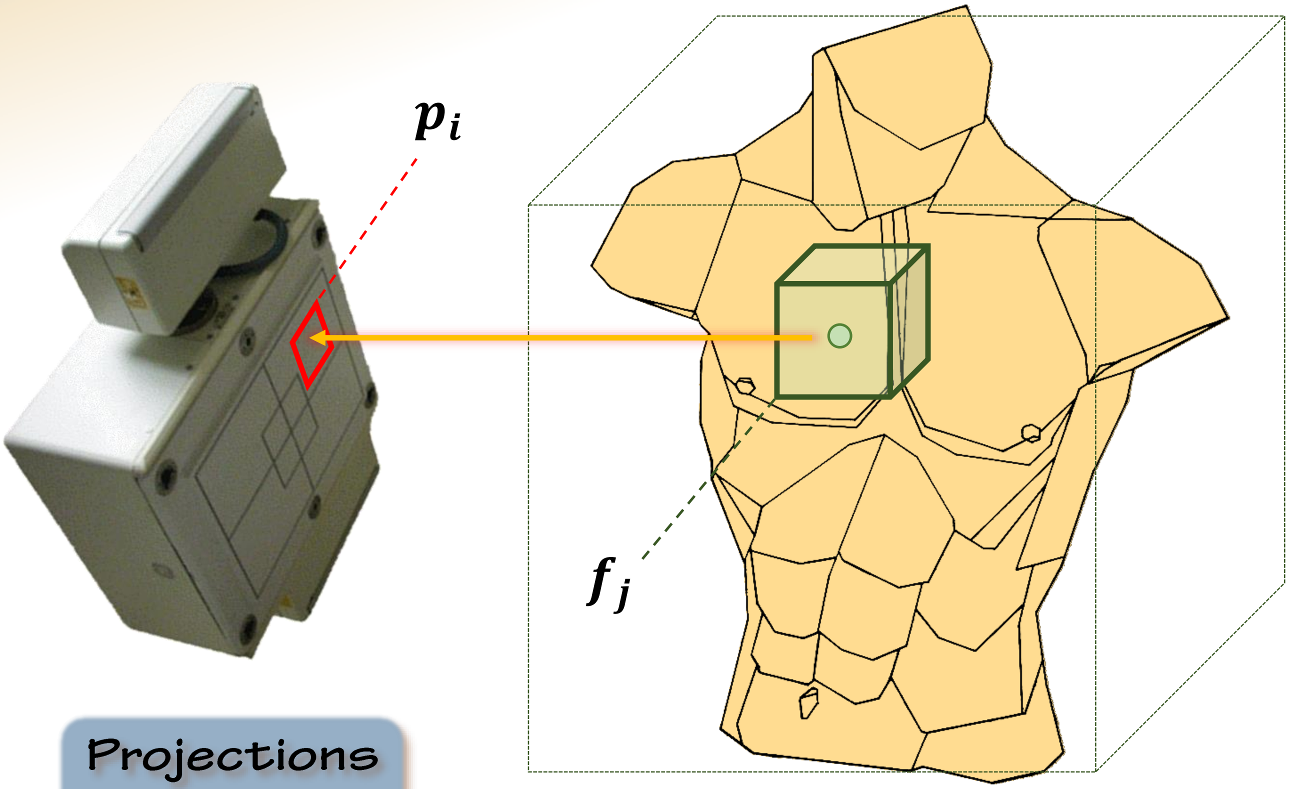
$$SNR = \frac{E(\boldsymbol{p_i})}{\sigma(\boldsymbol{p_i})} \approx \sqrt{\boldsymbol{p_i}}$$





Reconstruction

- Modèle algébrique

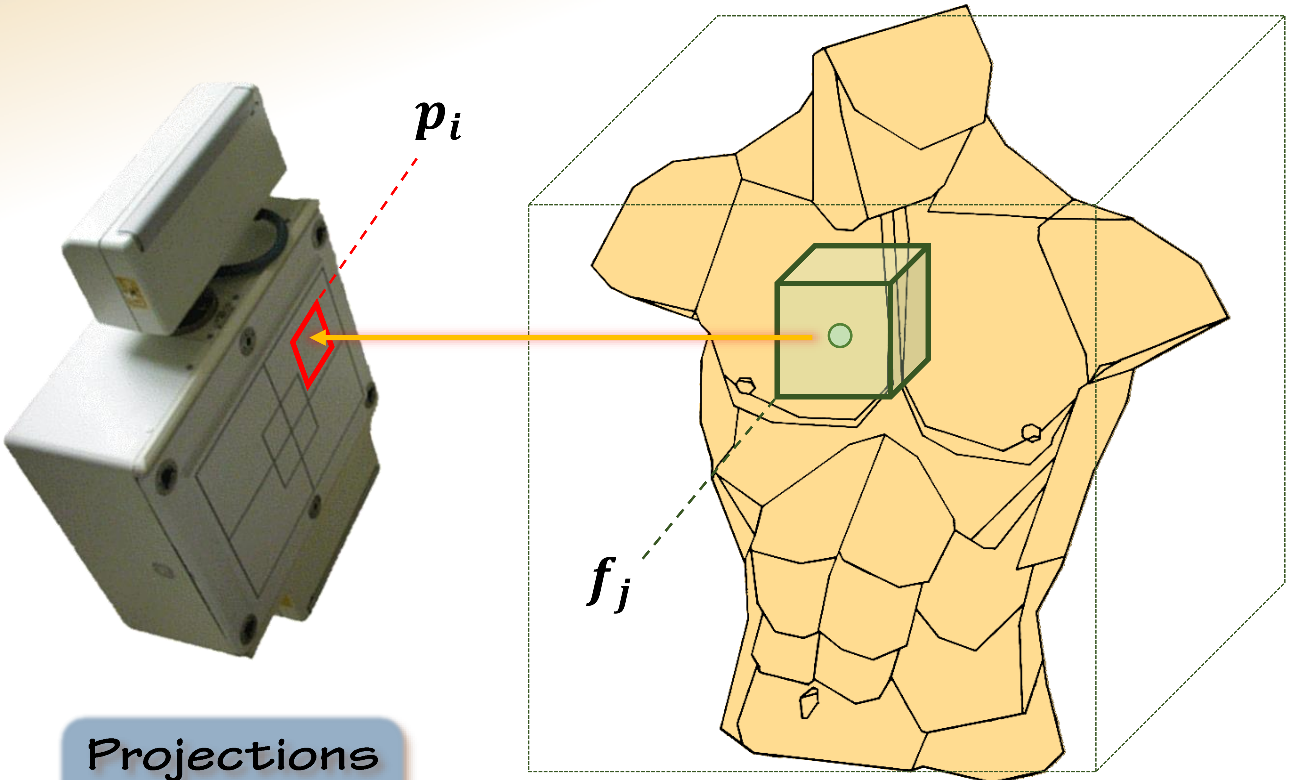


Projections
$(i = 1 \dots \mathrm{N})$

Voxels / pixels $(j = 1 \dots \mathrm{M})$

Reconstruction

- Modèle algébrique



Projections
$(i = 1 \ldots \mathrm{N})$

Voxels / pixels $(j = 1 \ldots \mathrm{M})$

Matrice système

$$\mathbf{R} \in \mathbb{R}^{\mathbf{N \times M}}$$

$$\mathbf{R}_{ij} = \wp(j \to i)$$

-> Géométrie

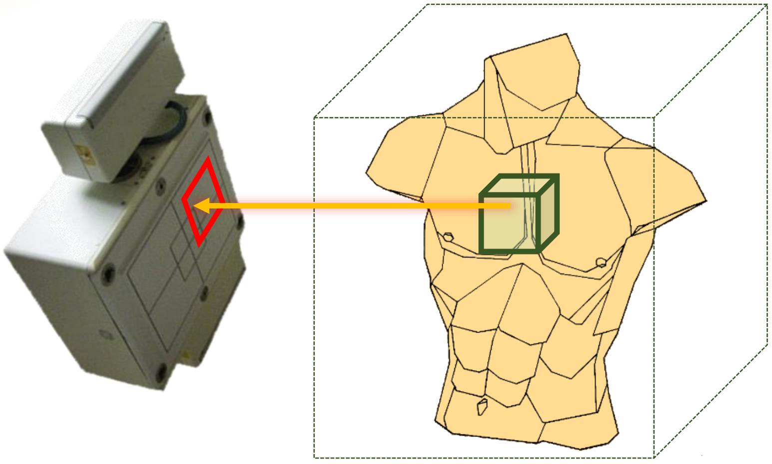
-> Atténuation

-> Réponse du détecteur

Reconstruction

- Modèle algébrique

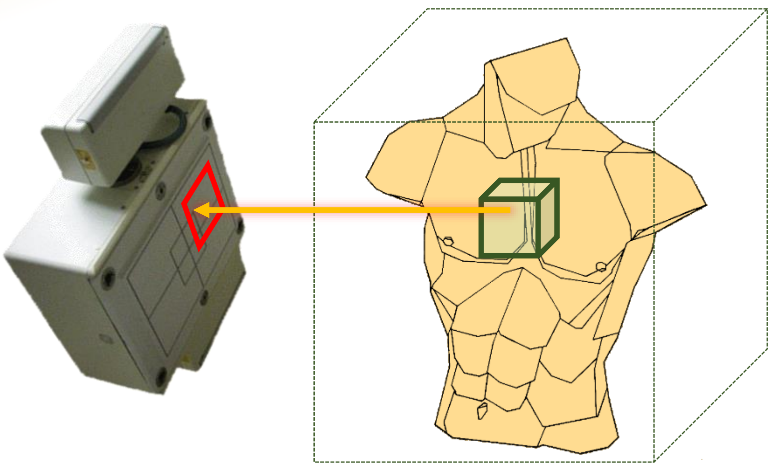
$$\boxed{p = \mathbf{R}f + s + n}$$

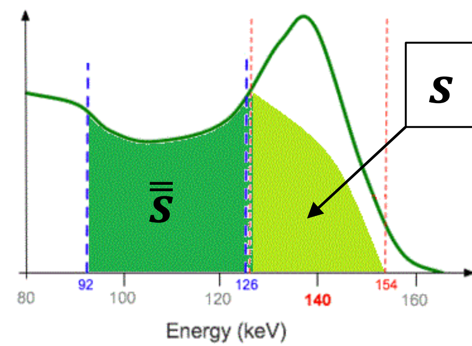


Reconstruction

- Modèle algébrique

$$\boxed{p = \mathrm{R}f + s + n}$$





$$\boxed{s = \kappa\,\bar{\bar{s}}}$$

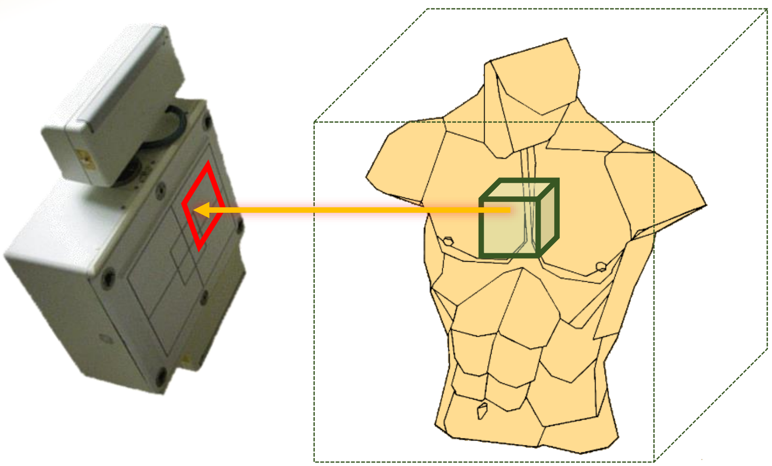
s

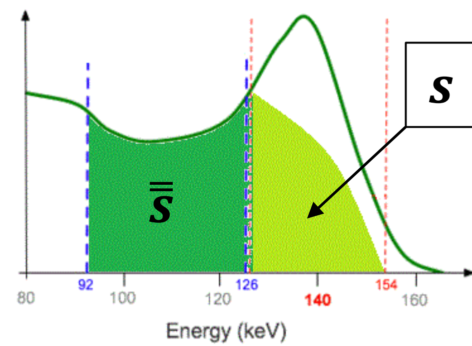
$$s \approx p * h$$

Reconstruction

- Modèle algébrique

$$\boxed{p = \mathrm{R}f + s + n}$$





$$\boxed{s = \kappa\, \bar{\bar{s}}}$$

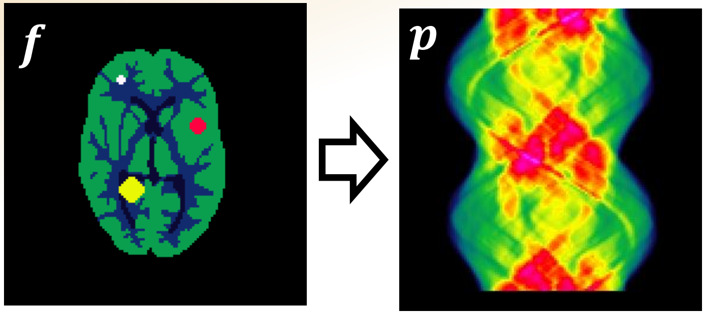
$$s$$

$$s \approx p * h$$

$$E(n) = 0 \;\; ; V(n) \approx p$$

Reconstruction

- Modèle algébrique

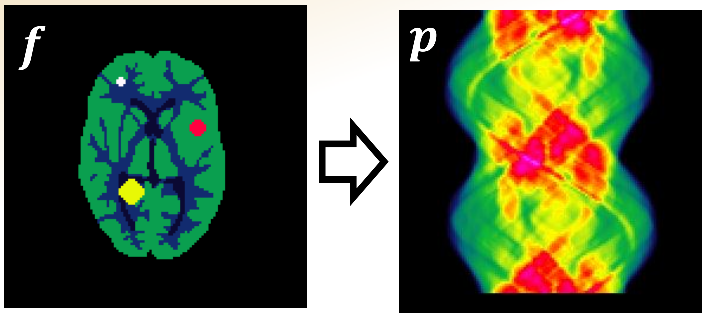


objet

$$\boldsymbol{p} = \mathbf{R}\boldsymbol{f} + \boldsymbol{n}$$

Reconstruction

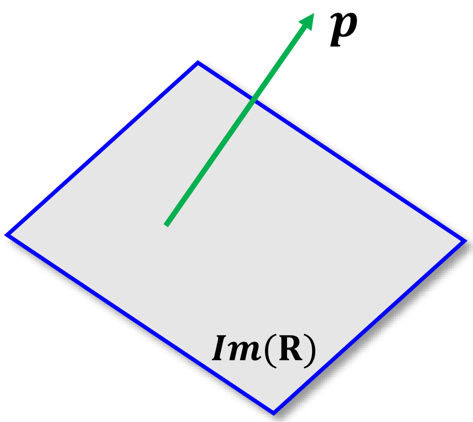
- Modèle algébrique



objet

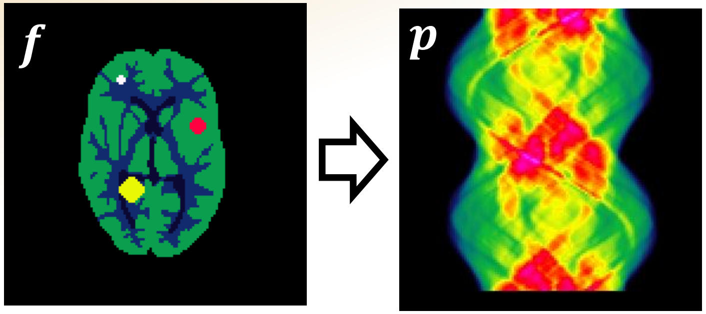
$$\boldsymbol{p} = \mathbf{R}\boldsymbol{f} + \boldsymbol{n}$$

$$\boldsymbol{p} \notin \boldsymbol{Im}(\mathbf{R}) \qquad \nexists \bar{\boldsymbol{f}} \; : \; \mathbf{R}\bar{\boldsymbol{f}} = \boldsymbol{p}$$



Reconstruction

- Modèle algébrique

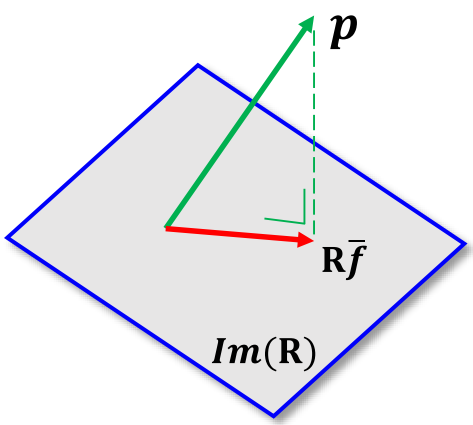


objet

$$\boldsymbol{p} = \mathrm{R}\boldsymbol{f} + \boldsymbol{n}$$

$$\boldsymbol{p} \notin \boldsymbol{Im}(\mathrm{R}) \qquad \nexists \bar{\boldsymbol{f}} \ : \ \mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{p}$$

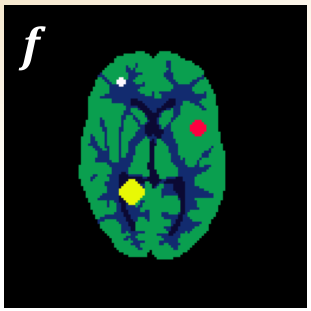
$$\bar{\boldsymbol{f}} \ : \mathrm{R}\bar{\boldsymbol{f}} = \mathrm{P}_{\boldsymbol{Im}(\mathrm{R})}(\boldsymbol{p})$$



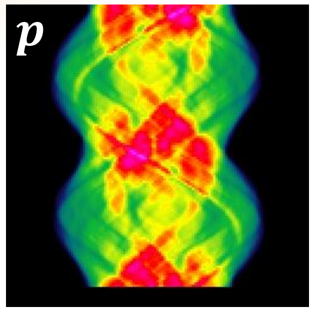
$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in \Omega}{argmin} \ \{\|\mathrm{R}\boldsymbol{f} - \boldsymbol{p}\|\}$$

Reconstruction

- Modèle algébrique



objet

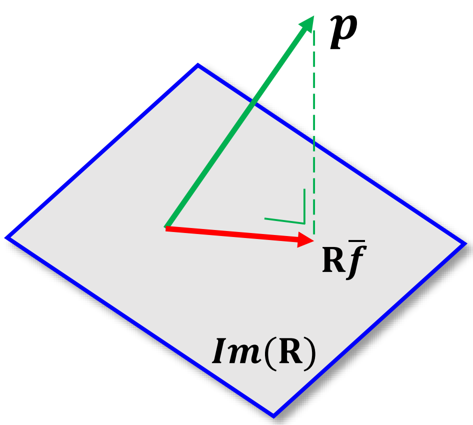


$$\boldsymbol{p} = \mathrm{R}\boldsymbol{f} + \boldsymbol{n}$$

$$\boldsymbol{p} \notin \boldsymbol{Im}(\mathrm{R}) \qquad \nexists \bar{\boldsymbol{f}} \ : \ \mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{p}$$

$$\bar{\boldsymbol{f}} \ : \mathrm{R}\bar{\boldsymbol{f}} = \mathrm{P}_{\boldsymbol{Im}(\mathrm{R})}(\boldsymbol{p})$$

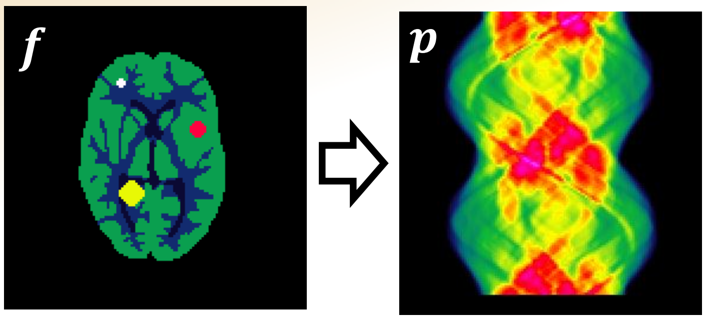
$$\forall \boldsymbol{w} \in \boldsymbol{Im}(\boldsymbol{R}) : \ \boldsymbol{w}^{\mathrm{T}}\mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{p}$$



$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\Omega}{argmin} \ \{\|\mathrm{R}\boldsymbol{f} - \boldsymbol{p}\|\}$$

Reconstruction

- Modèle algébrique



objet

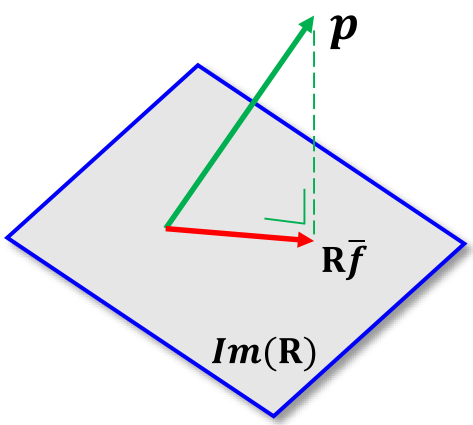
$$p = \mathrm{R}f + n$$

$$p \notin Im(\mathrm{R}) \qquad \nexists \bar{f} \;:\; \mathrm{R}\bar{f} = p$$

$$\bar{f} \;:\; \mathrm{R}\bar{f} = \mathrm{P}_{Im(\mathrm{R})}(p)$$

$$\forall w \in Im(R) \;:\; w^{\mathrm{T}}\mathrm{R}\bar{f} = w^{\mathrm{T}}p$$

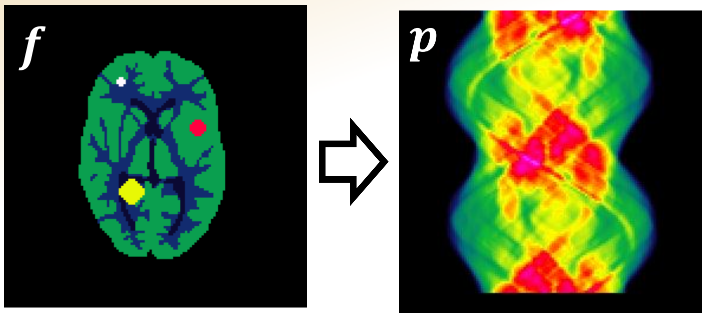
$$\forall g \in \Omega \;:\; (\mathrm{R}g)^{\mathrm{T}}\mathrm{R}\bar{f} = (\mathrm{R}g)^{\mathrm{T}}p$$



$$\bar{f} = \underset{f \in \Omega}{argmin}\,\{\|\mathrm{R}f - p\|\}$$

Reconstruction

- Modèle algébrique



objet

$$\boldsymbol{p} = \mathrm{R}\boldsymbol{f} + \boldsymbol{n}$$

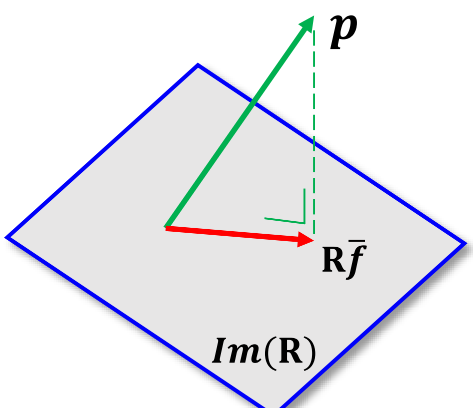
$$\boldsymbol{p} \notin \boldsymbol{Im}(\mathrm{R}) \qquad \nexists \bar{\boldsymbol{f}} \;:\; \mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{p}$$

$$\bar{\boldsymbol{f}} \;:\mathrm{R}\bar{\boldsymbol{f}} = \mathrm{P}_{\boldsymbol{Im}(\mathrm{R})}(\boldsymbol{p})$$

$$\forall \boldsymbol{w} \in \boldsymbol{Im}(\boldsymbol{R}) :\; \boldsymbol{w}^{\mathrm{T}}\mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{p}$$

$$\forall \boldsymbol{g} \in \boldsymbol{\Omega} :\; (\mathrm{R}\boldsymbol{g})^{\mathrm{T}}\mathrm{R}\bar{\boldsymbol{f}} = (\mathrm{R}\boldsymbol{g})^{\mathrm{T}}\boldsymbol{p}$$

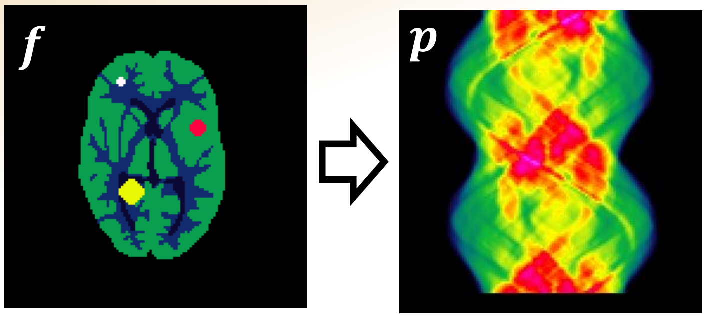
$$\forall \boldsymbol{g} \in \boldsymbol{\Omega} :\; \boldsymbol{g}^{\mathrm{T}}\mathrm{R}^{*}\mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{g}^{\mathrm{T}}\mathrm{R}^{*}\boldsymbol{p}$$



$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\Omega}{argmin}\; \{\|\mathrm{R}\boldsymbol{f} - \boldsymbol{p}\|\}$$

Reconstruction

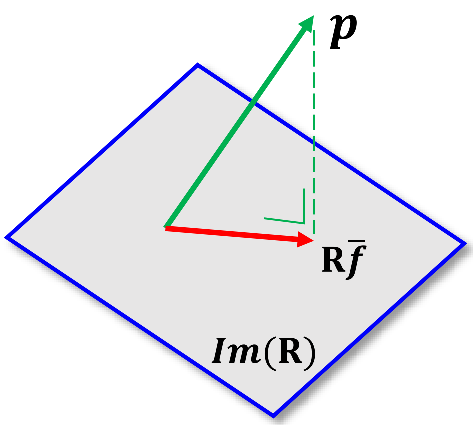
- Modèle algébrique



objet

$$\boldsymbol{p} = \mathrm{R}\boldsymbol{f} + \boldsymbol{n}$$

$$\boldsymbol{p} \notin \boldsymbol{Im}(\mathrm{R}) \qquad \nexists \bar{\boldsymbol{f}} \;:\; \mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{p}$$



$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\Omega}{argmin}\, \{\|\mathrm{R}\boldsymbol{f} - \boldsymbol{p}\|\}$$

$$\bar{\boldsymbol{f}} \;:\mathrm{R}\bar{\boldsymbol{f}} = \mathrm{P}_{\boldsymbol{Im}(\mathrm{R})}(\boldsymbol{p})$$

$$\forall \boldsymbol{w} \in \boldsymbol{Im}(\boldsymbol{R}) :\; \boldsymbol{w}^{\mathrm{T}}\mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{p}$$

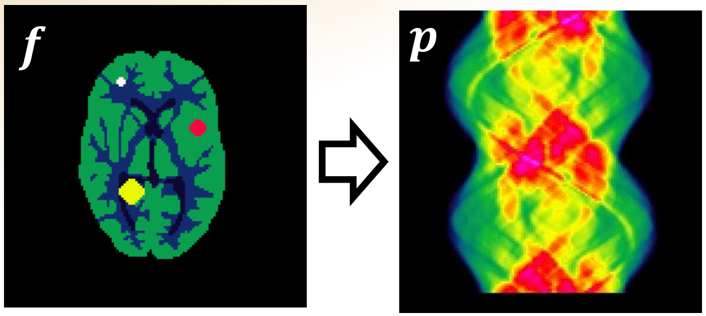
$$\forall \boldsymbol{g} \in \boldsymbol{\Omega} :\; (\mathrm{R}\boldsymbol{g})^{\mathrm{T}}\mathrm{R}\bar{\boldsymbol{f}} = (\mathrm{R}\boldsymbol{g})^{\mathrm{T}}\boldsymbol{p}$$

$$\forall \boldsymbol{g} \in \boldsymbol{\Omega} :\; \boldsymbol{g}^{\mathrm{T}}\mathrm{R}^{*}\mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{g}^{\mathrm{T}}\mathrm{R}^{*}\boldsymbol{p}$$

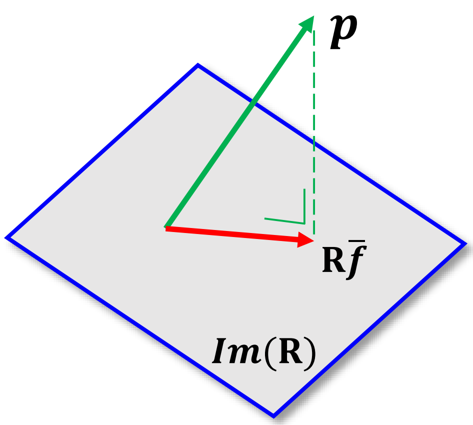
$$\mathrm{R}^{*}\mathrm{R}\bar{\boldsymbol{f}} = \mathrm{R}^{*}\boldsymbol{p}$$

Reconstruction

- Modèle algébrique



objet



$$\bar{f} = \underset{f \in \Omega}{argmin} \{\|\mathrm{R}f - p\|\}$$

$$p = \mathrm{R}f + n$$

$$p \notin Im(\mathrm{R}) \qquad \nexists \bar{f} \ : \ \mathrm{R}\bar{f} = p$$

$$\bar{f} \ : \mathrm{R}\bar{f} = \mathrm{P}_{Im(\mathrm{R})}(p)$$

$$\forall w \in Im(R) : \ w^{\mathrm{T}}\mathrm{R}\bar{f} = w^{\mathrm{T}}p$$

$$\forall g \in \Omega : \ (\mathrm{R}g)^{\mathrm{T}}\mathrm{R}\bar{f} = (\mathrm{R}g)^{\mathrm{T}}p$$

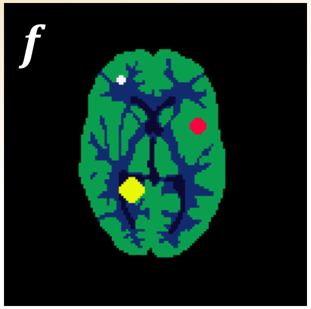
$$\forall g \in \Omega : \ g^{\mathrm{T}}\mathrm{R}^*\mathrm{R}\bar{f} = g^{\mathrm{T}}\mathrm{R}^*p$$

$$\mathrm{R}^*\mathrm{R}\bar{f} = \mathrm{R}^*p$$

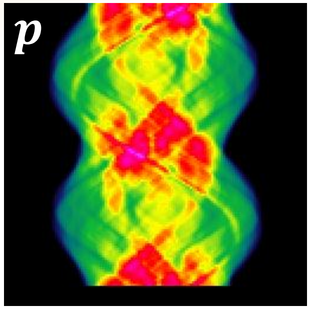
$$\bar{f} = (\mathrm{R}^*\mathrm{R})^{-1}\mathrm{R}^*p$$

Reconstruction

- Modèle algébrique

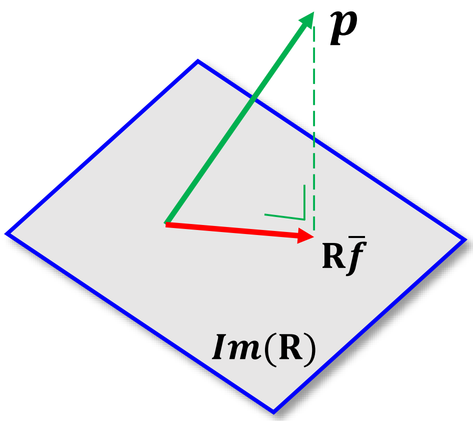


objet



$$\boldsymbol{p} = \mathrm{R}\boldsymbol{f} + \boldsymbol{s} + \boldsymbol{n}$$

$$\boldsymbol{p} \notin \boldsymbol{Im}(\mathrm{R}) \qquad \nexists \bar{\boldsymbol{f}} \;:\; \mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{p}$$

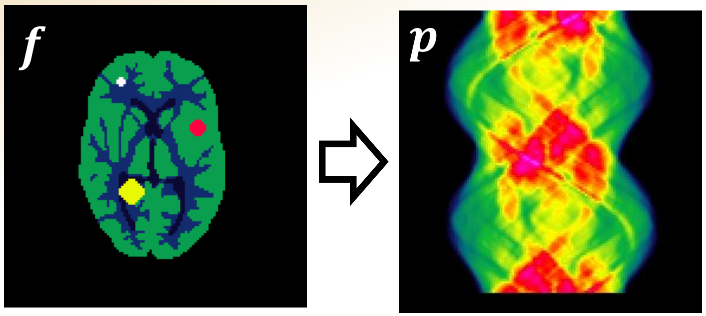


$$\bar{\boldsymbol{f}} = \underset{f \in \Omega}{argmin} \, \{\|\mathrm{R}\boldsymbol{f} - \boldsymbol{p}\|\}$$

$$\bar{\boldsymbol{f}} = (\mathrm{R}^*\mathrm{R})^{-1}\mathrm{R}^*\boldsymbol{p}$$

Reconstruction

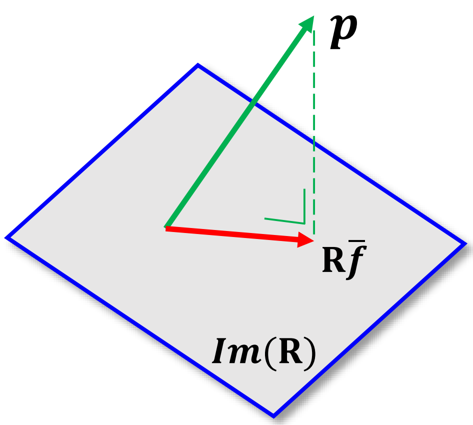
- Modèle algébrique



objet

$$\boldsymbol{p} = \mathbf{R}\boldsymbol{f} + \boldsymbol{s} + \boldsymbol{n}$$

$$\boldsymbol{p} \notin \boldsymbol{Im}(\mathbf{R}) \qquad \nexists \bar{\boldsymbol{f}} \;:\; \mathbf{R}\bar{\boldsymbol{f}} = \boldsymbol{p}$$



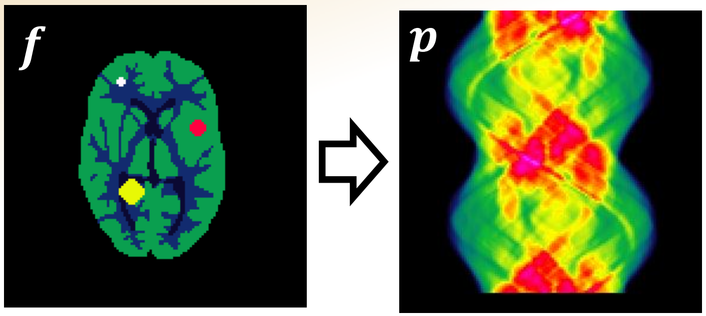
$$\bar{\boldsymbol{f}} = \underset{f \in \Omega}{argmin} \{\|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|\}$$

$$\bar{\boldsymbol{f}} = (\mathbf{R}^*\mathbf{R})^{-1}\mathbf{R}^*\boldsymbol{p}$$

$dim(\mathbf{R}^*\mathbf{R}) = \mathcal{O}(10^5)$
$\kappa(\mathbf{R}^*\mathbf{R}) \gg$

Reconstruction

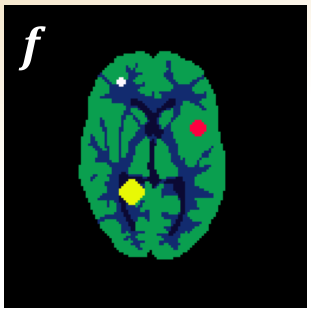
- Méthodes itératives



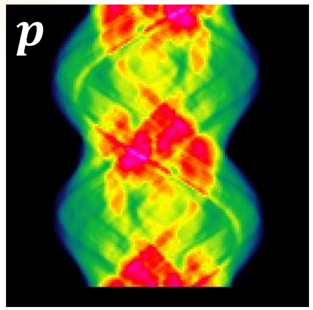
objet

Reconstruction

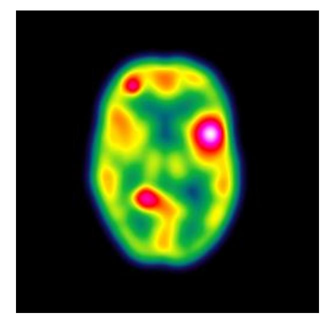
- Méthodes itératives



objet



estimation



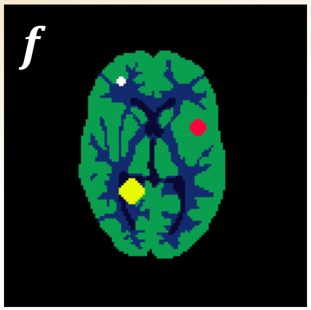
projection

$$\bar{p}^0 = \mathbf{R}\bar{f}^0(+s)$$

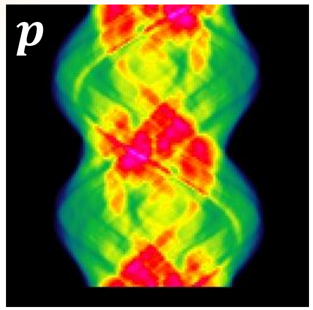
$\bar{f}^0$

Reconstruction

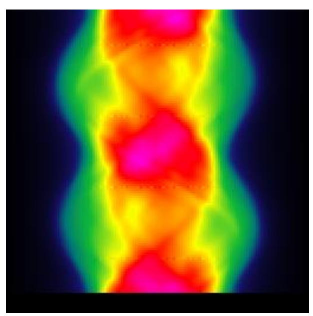
- Méthodes itératives



objet



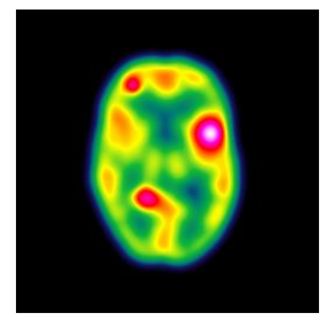
comparaison



$\overline{p}^0$

projection

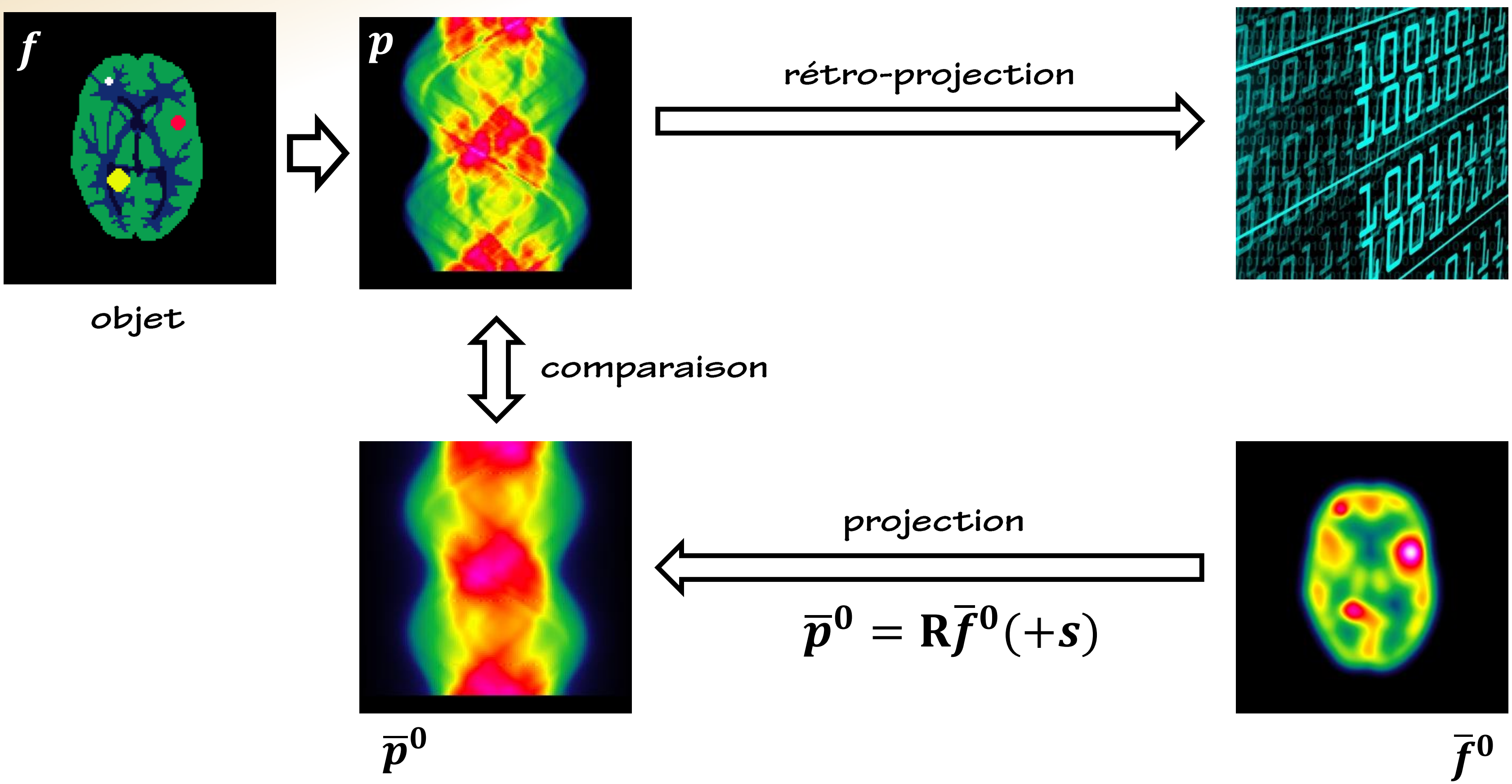
$$\overline{p}^0 = \mathbf{R}\overline{f}^0 (+s)$$



$\overline{f}^0$

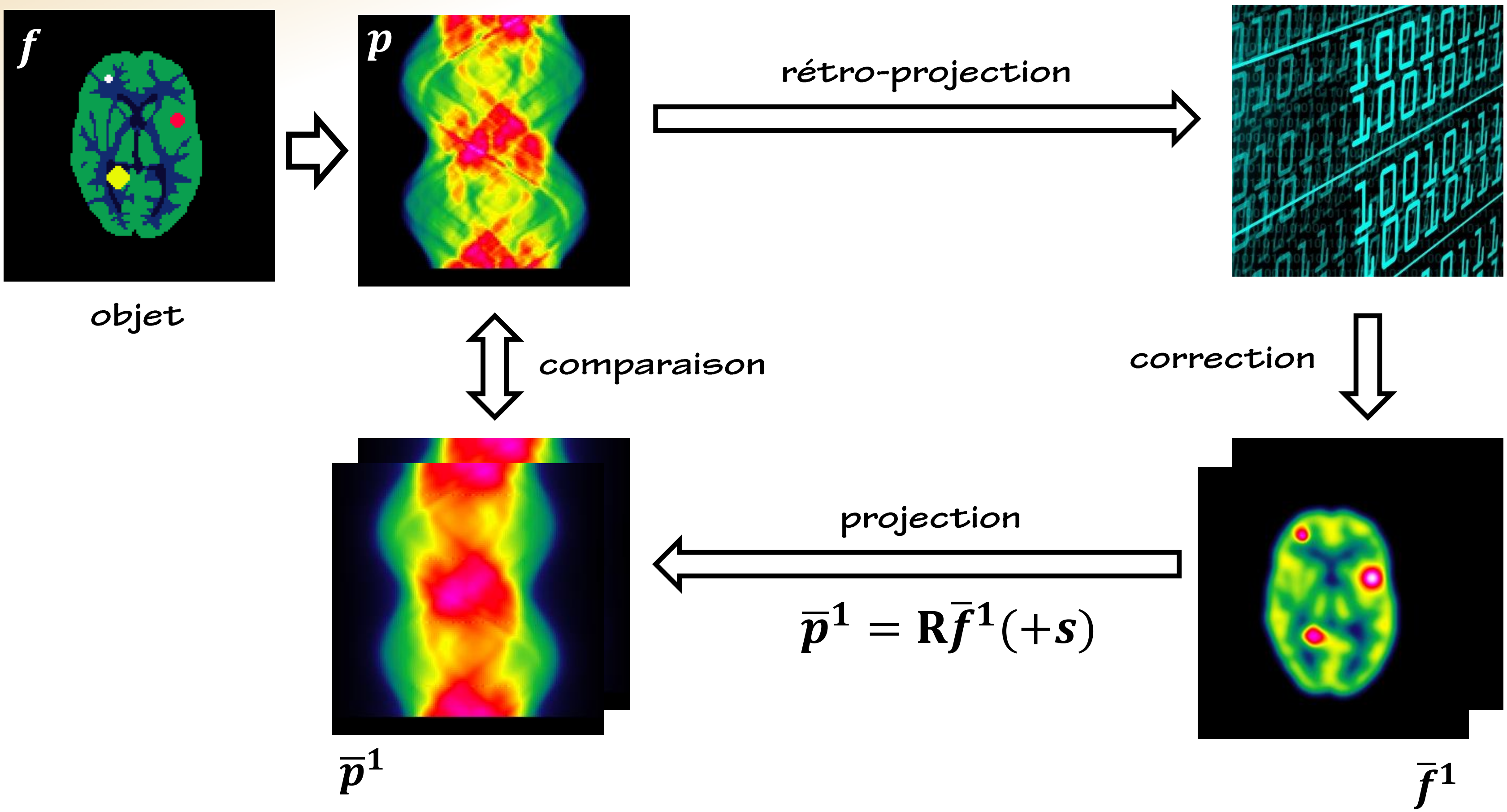
Reconstruction

- Méthodes itératives



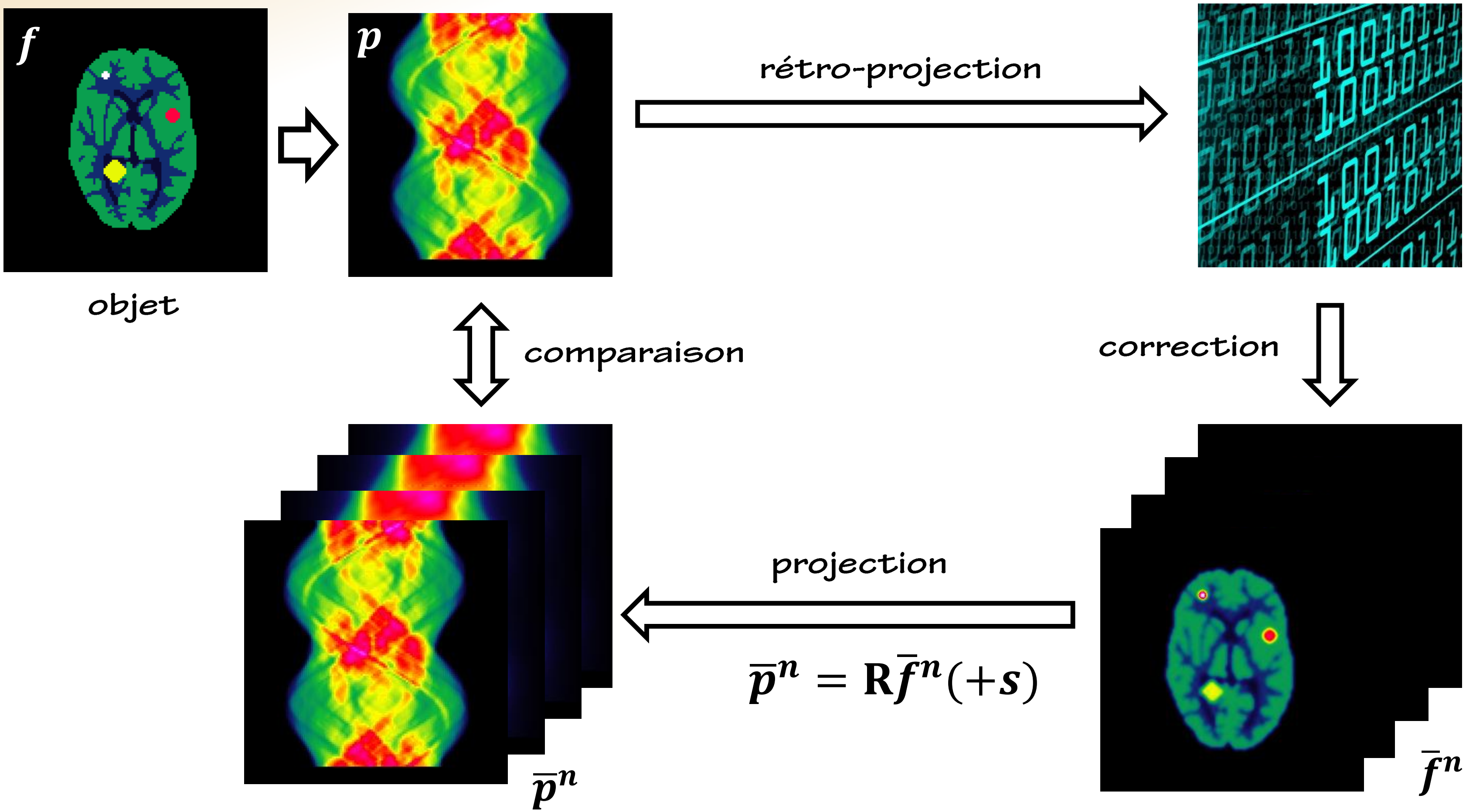
Reconstruction

- Méthodes itératives



Reconstruction

- Méthodes itératives



Reconstruction

- Méthodes itératives

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\boldsymbol{\Omega}}{argmin}\ \{J(\boldsymbol{f})\}$$

Méthode **LS**

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2$$

Reconstruction

- Méthodes itératives

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in \boldsymbol{\Omega}}{argmin} \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2$$

Méthode LS

$$\boxed{\bar{\boldsymbol{f}} = (\mathbf{R}^*\mathbf{R})^{-1}\,\mathbf{R}^*\,\boldsymbol{p}}$$

$dim(\mathbf{R}^*\mathbf{R}) = \sigma(10^5)$

$\kappa(\mathbf{R}^*\mathbf{R}) \gg$

Reconstruction

- Méthodes itératives

Méthode LS

$$\bar{f} = \underset{f \in \Omega}{argmin} \{J(f)\}$$

$$J(f) = \|\mathbf{R}f - p\|^2$$

$$\bar{f} = (\mathbf{R}^*\mathbf{R})^{-1}\,\mathbf{R}^*\,p$$

$$\boxed{\bar{f}^{n+1} = \bar{f}^{n} + \eta\,\mathbf{R}^*\left(p - \mathbf{R}\,\bar{f}^{n}\right)}$$

Reconstruction

- Méthodes itératives

Méthode LS

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\boldsymbol{\Omega}}{argmin}\ \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2$$

$$\bar{\boldsymbol{f}} = (\mathbf{R}^*\mathbf{R})^{-1}\,\mathbf{R}^*\,\boldsymbol{p}$$

$$\boxed{\bar{\boldsymbol{f}}^{n+1} = \bar{\boldsymbol{f}}^{n} + \boldsymbol{\eta}\,\mathbf{R}^*\left(\boldsymbol{p} - \mathbf{R}\,\bar{\boldsymbol{f}}^{n}\right)}$$

correction additive

Reconstruction

- Méthodes itératives

Méthode LS

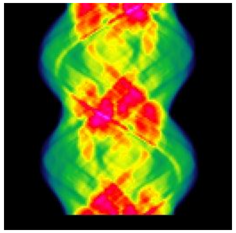
$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\boldsymbol{\Omega}}{argmin}\ \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2$$

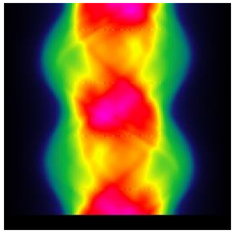
$$\bar{\boldsymbol{f}} = (\mathbf{R}^*\mathbf{R})^{-1}\,\mathbf{R}^*\,\boldsymbol{p}$$

$$\boxed{\bar{\boldsymbol{f}}^{n+1} = \bar{\boldsymbol{f}}^{n} + \boldsymbol{\eta}\,\mathbf{R}^*\left(\boldsymbol{p} - \mathbf{R}\,\bar{\boldsymbol{f}}^{n}\right)}$$

comparaison



$\Leftrightarrow$



Reconstruction

- Méthodes itératives

Méthode LS

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\boldsymbol{\Omega}}{argmin}\,\{J(\boldsymbol{f})\}$$

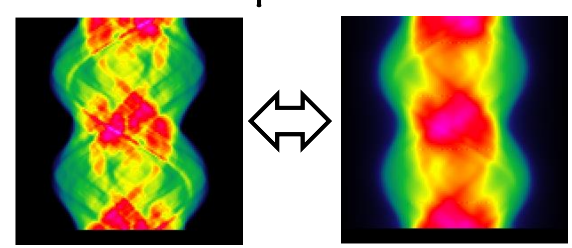
$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2$$

$$\bar{\boldsymbol{f}} = (\mathbf{R}^*\mathbf{R})^{-1}\,\mathbf{R}^*\,\boldsymbol{p}$$

$$\boxed{\bar{\boldsymbol{f}}^{n+1} = \bar{\boldsymbol{f}}^{n} + \boldsymbol{\eta}\,\mathbf{R}^*\left(\boldsymbol{p} - \mathbf{R}\,\bar{\boldsymbol{f}}^{n}\right)}$$

rétro-projection

comparaison



Reconstruction

- Méthodes itératives

Méthode **ML**

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in \boldsymbol{\Omega}}{argmin} \{ J(\boldsymbol{f}) \}$$

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{p}|\boldsymbol{f}) \}$$

Reconstruction

- Méthodes itératives

Méthode **ML**

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\boldsymbol{\Omega}}{argmin}\ \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = -log\{\ \wp(\boldsymbol{p}|\boldsymbol{f})\ \}$$

$$\wp(\boldsymbol{p}|\boldsymbol{f}) = \prod_i \frac{e^{-\breve{\boldsymbol{p}}_i}\ \breve{\boldsymbol{p}}_i^{\ \boldsymbol{p}_i}}{\boldsymbol{p}_i\ !}$$

$$\breve{\boldsymbol{p}}_i = (\mathbf{R}\boldsymbol{f})_i$$

Reconstruction

- Méthodes itératives

Méthode ML

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\boldsymbol{\Omega}}{argmin}\ \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = -log\{\, \wp(\boldsymbol{p}|\boldsymbol{f})\, \}$$

$$\wp(\boldsymbol{p}|\boldsymbol{f}) = \prod_i \frac{e^{-\breve{\boldsymbol{p}}_i}\ \breve{\boldsymbol{p}}_i^{\,\boldsymbol{p}_i}}{\boldsymbol{p}_i\,!}$$

$$\breve{\boldsymbol{p}}_i = (\mathbf{R}\boldsymbol{f})_i$$

$$J(\boldsymbol{f}) = \sum_i \{\mathbf{R}\boldsymbol{f}_i - \boldsymbol{p}_i\, log(\mathbf{R}\boldsymbol{f}_i)\,\}$$

Reconstruction

- Méthodes itératives

Méthode ML

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in \boldsymbol{\Omega}}{argmin} \{ J(\boldsymbol{f}) \}$$

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{p}|\boldsymbol{f}) \}$$

$$J(\boldsymbol{f}) = \sum_i \{ \mathbf{R}\boldsymbol{f}_i - \boldsymbol{p}_i \, log(\mathbf{R}\boldsymbol{f}_i) \}$$

$$\bar{\boldsymbol{f}}^{n+1} = \bar{\boldsymbol{f}}^{n} \times \left(\mathbf{R}^* \frac{\boldsymbol{p}}{\mathbf{R}\,\bar{\boldsymbol{f}}^{n}} \right)$$

Reconstruction

- Méthodes itératives

Méthode ML

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\boldsymbol{\Omega}}{argmin}\ \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = -log\{\ \wp(\boldsymbol{p}|\boldsymbol{f})\ \}$$

$$J(\boldsymbol{f}) = \sum_i \{\mathbf{R}\boldsymbol{f}_i - \boldsymbol{p}_i\ log(\mathbf{R}\boldsymbol{f}_i)\ \}$$

$$\bar{\boldsymbol{f}}^{n+1} = \bar{\boldsymbol{f}}^{n} \times \left(\mathbf{R}^* \frac{\boldsymbol{p}}{\mathbf{R}\,\bar{\boldsymbol{f}}^{n}}\right)$$

correction multiplicative

Reconstruction

- Méthodes itératives

Méthode ML

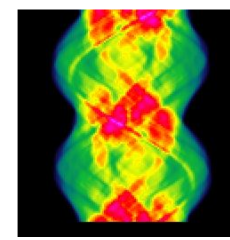
$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in \boldsymbol{\Omega}}{argmin} \, \{ J(\boldsymbol{f}) \}$$

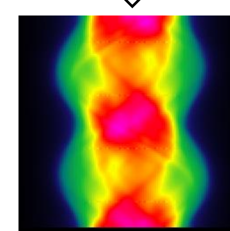
$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{p}|\boldsymbol{f}) \}$$

$$J(\boldsymbol{f}) = \sum_i \{ \mathbf{R}\boldsymbol{f}_i - \boldsymbol{p}_i \, log(\mathbf{R}\boldsymbol{f}_i) \}$$

$$\bar{\boldsymbol{f}}^{n+1} = \bar{\boldsymbol{f}}^{n} \times \left(\mathbf{R}^* \frac{\boldsymbol{p}}{\mathbf{R}\,\bar{\boldsymbol{f}}^{n}} \right)$$

comparaison





Reconstruction

- Méthodes itératives

Méthode ML

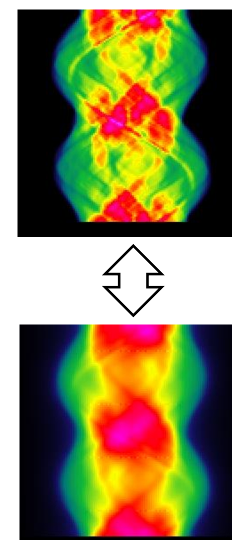
$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\boldsymbol{\Omega}}{argmin}\ \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = -log\{\,\wp(\boldsymbol{p}|\boldsymbol{f})\,\}$$

$$J(\boldsymbol{f}) = \sum_i \{\mathbf{R}\boldsymbol{f}_i - \boldsymbol{p}_i\, log(\mathbf{R}\boldsymbol{f}_i)\,\}$$

$$\bar{\boldsymbol{f}}^{n+1} = \bar{\boldsymbol{f}}^{n} \times \left(\boxed{\mathbf{R}^*}\,\frac{\boldsymbol{p}}{\mathbf{R}\,\bar{\boldsymbol{f}}^{n}}\right)$$

rétro-projection comparaison



Régularisation

$$\boxed{\boldsymbol{p} = \mathbf{R}\boldsymbol{f}}$$

R est mal conditionné : $\kappa(\mathbf{R}) \gg$

Régularisation

$$\boxed{\boldsymbol{p} = \mathbf{R}\boldsymbol{f}}$$

$\mathbf{R}$ est mal conditionné : $\kappa(\mathbf{R}) \gg$

$$\frac{\|\Delta \boldsymbol{f}\|}{\|\boldsymbol{f}\|} \leq \kappa(\mathbf{R}) \boxed{\frac{\|\Delta \boldsymbol{p}\|}{\|\boldsymbol{p}\|}} \approx o\left(\frac{1}{\sqrt{\boldsymbol{p}}}\right)$$

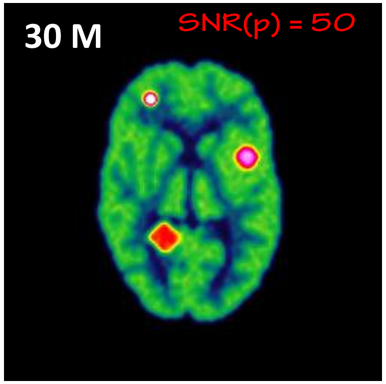
Régularisation

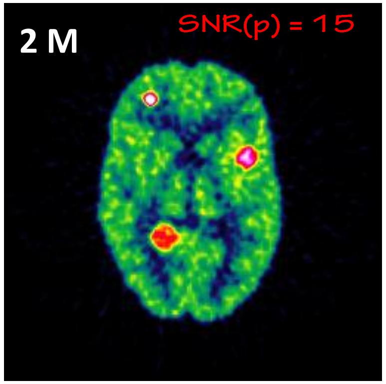
$$\boxed{p = \mathbf{R} f}$$

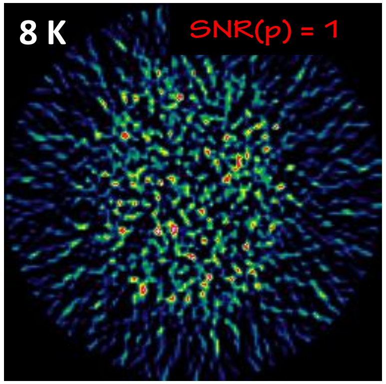
R est mal conditionné : $\kappa(\mathbf{R}) \gg$

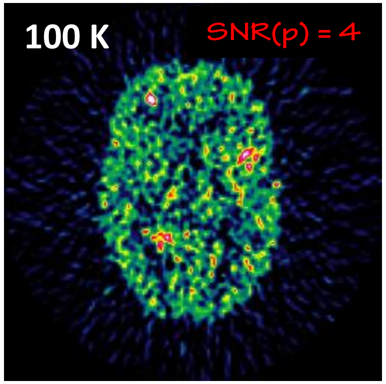
$$\frac{\|\Delta f\|}{\|f\|} \leq \kappa(\mathbf{R}) \boxed{\frac{\|\Delta p\|}{\|p\|}} \approx \sigma\left(\frac{1}{\sqrt{p}}\right)$$

$\Rightarrow$ Sensibilité au bruit





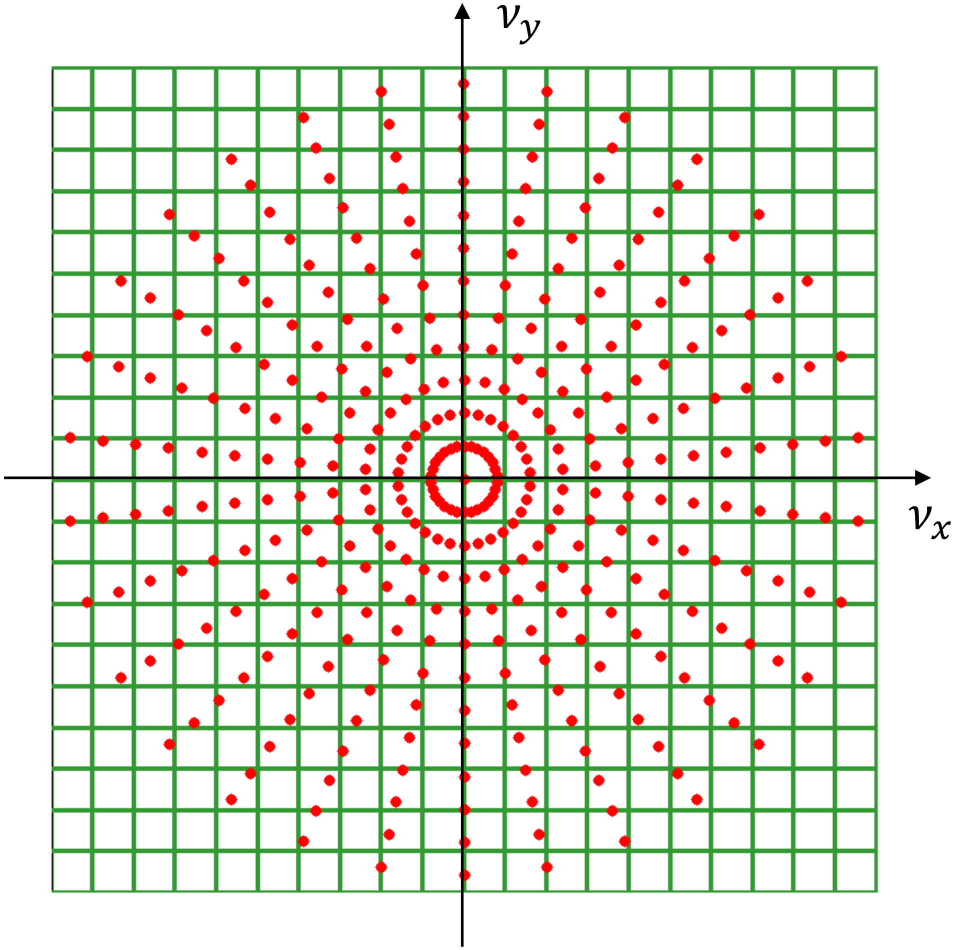




Régularisation

- Reconstruction analytique

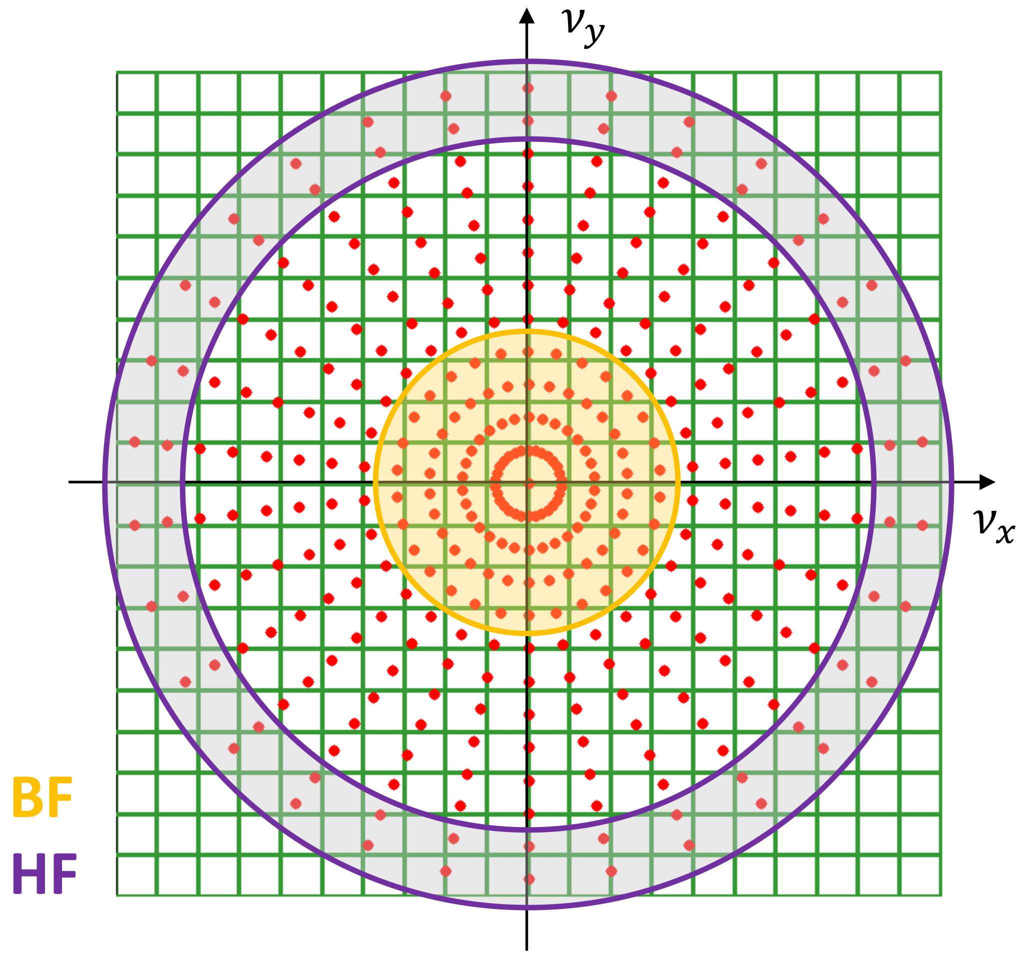
$$\boxed{\hat{f}(\nu\cos\theta, \nu\sin\theta)} = \boxed{\hat{p}_\theta(\nu)}$$



Régularisation

- Reconstruction analytique

$$\boxed{\hat{f}(\nu\cos\theta, \nu\sin\theta)} = \boxed{\hat{p}_\theta(\nu)}$$

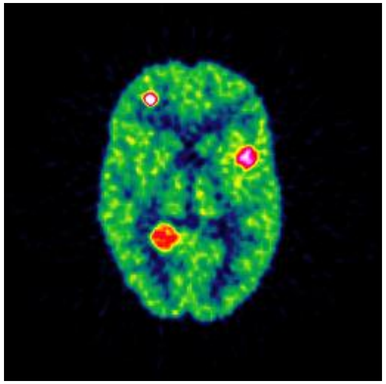


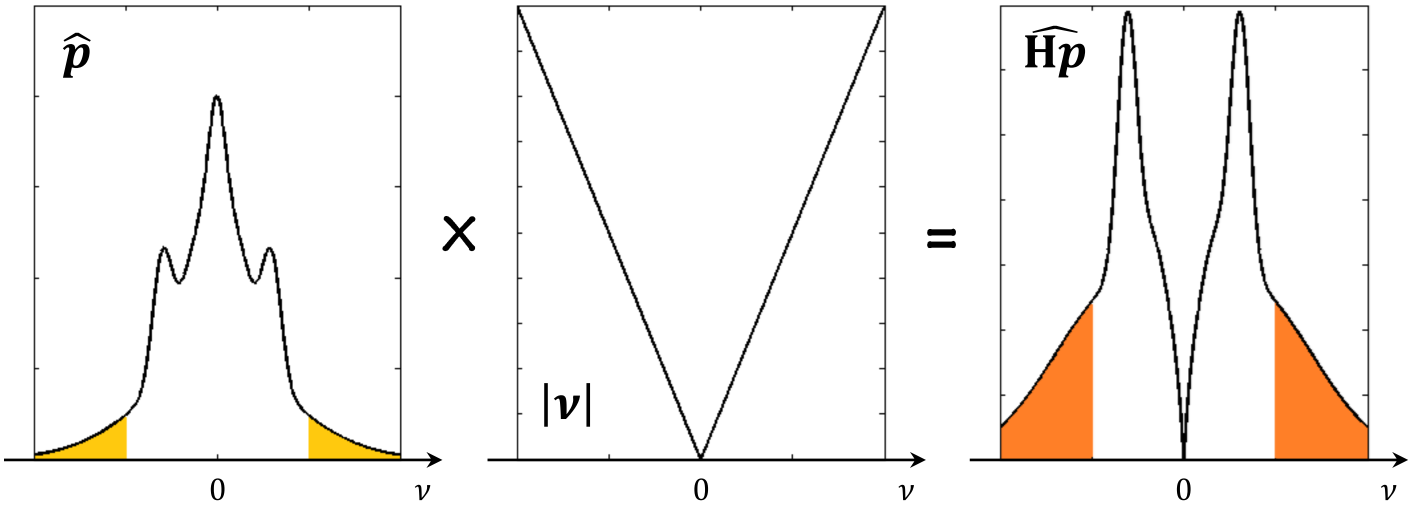
Régularisation

- Reconstruction analytique

$$f = \mathbf{R}^* \, \mathbf{H}p$$

$$\widehat{\mathbf{H}p} = |\nu| \, \hat{p}$$



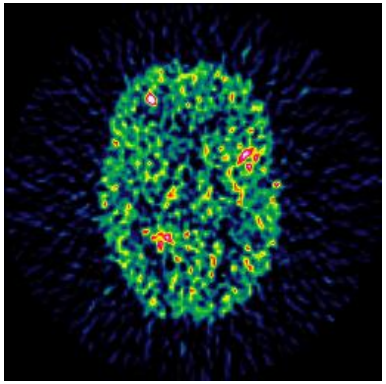


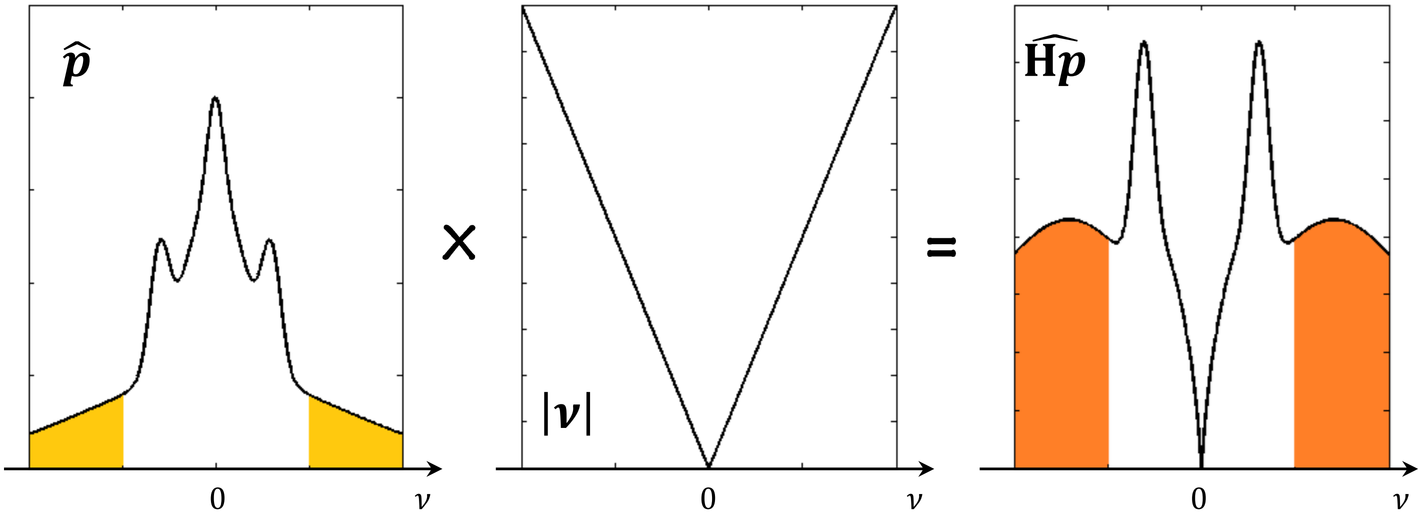
Régularisation

- Reconstruction analytique

$$f = \mathrm{R}^* \, \mathrm{H}p$$

$$\widehat{\mathrm{H}p} = |\nu| \, \hat{p}$$

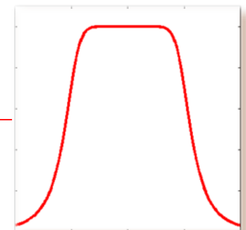


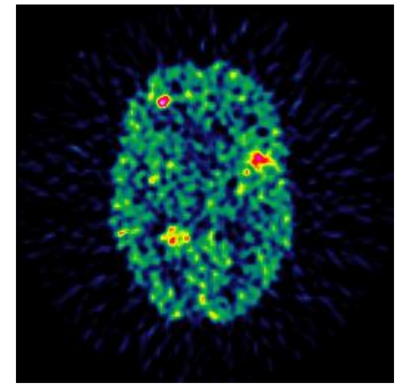


Régularisation

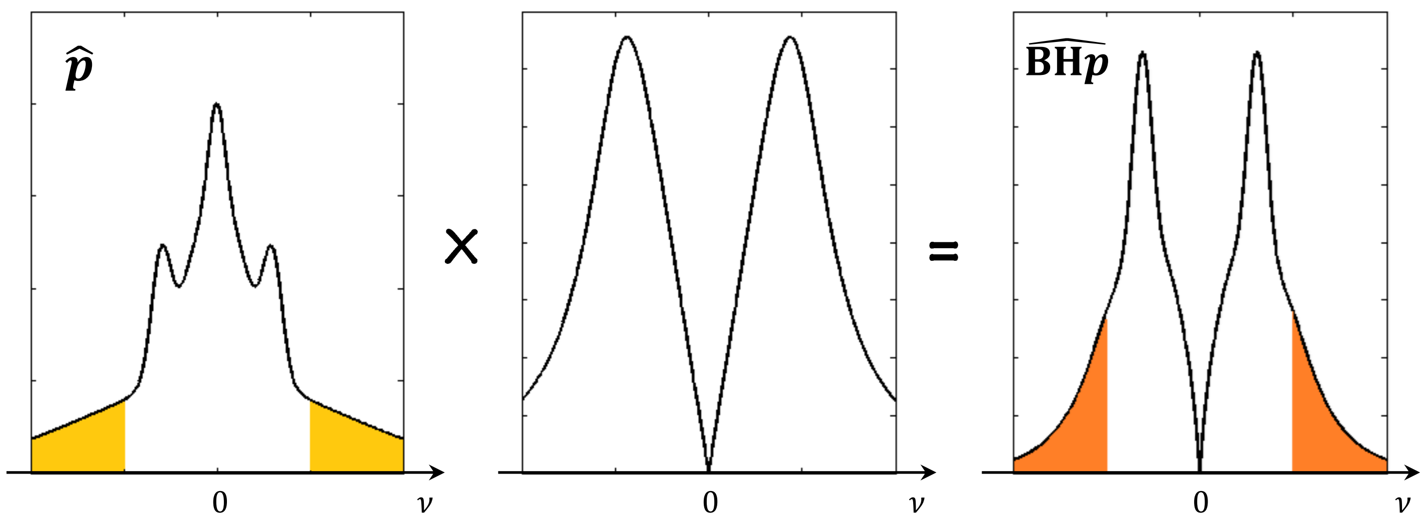
- Reconstruction analytique

$$\boldsymbol{f} = \mathrm{R}^* \, \mathbf{BH}\boldsymbol{p} \qquad \widehat{\mathbf{BH}\boldsymbol{p}} = \frac{|\boldsymbol{\nu}| \, \widehat{\boldsymbol{p}}}{\boxed{\sqrt{1 + \left(\frac{\boldsymbol{\nu}}{\boldsymbol{\nu_c}}\right)^{\beta}}}}$$





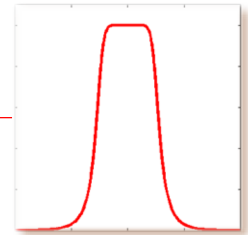
$\boldsymbol{\nu_c} = 0{,}5$

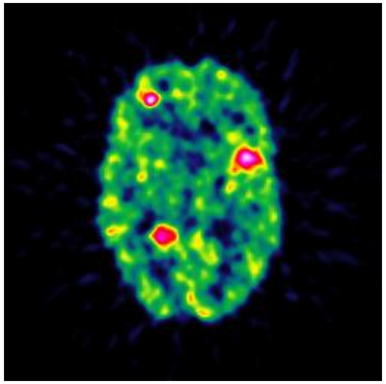


Régularisation

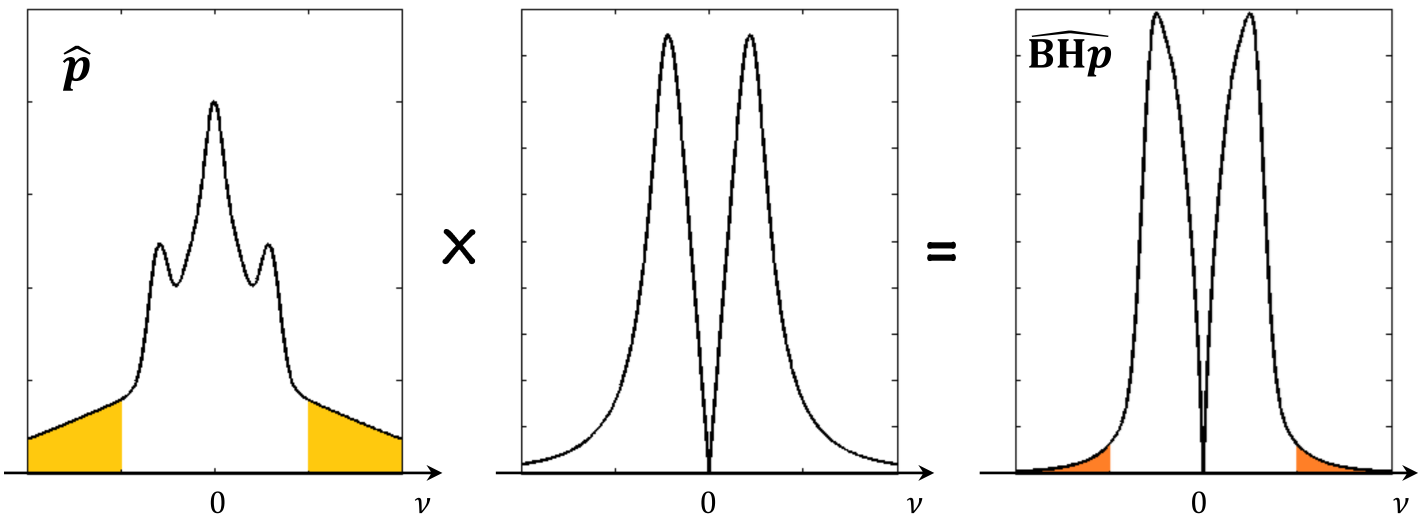
- Reconstruction analytique

$$f = \mathrm{R}^* \, \mathrm{BH}p \qquad \widehat{\mathrm{BH}p} = \frac{|\nu|\,\hat{p}}{\boxed{\sqrt{1+\left(\frac{\nu}{\nu_c}\right)^{\beta}}}}$$





$\nu_c = 0{,}25$

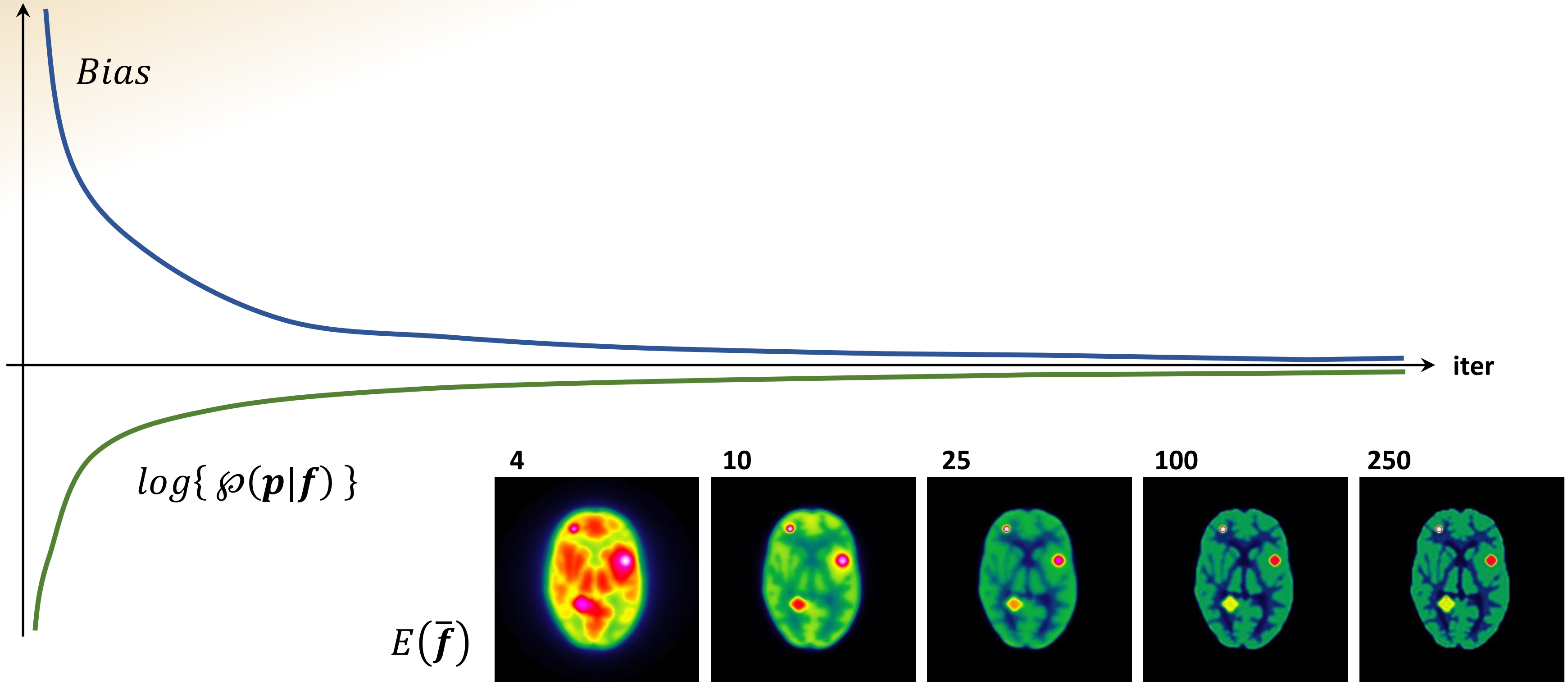


Régularisation

- Reconstruction itérative

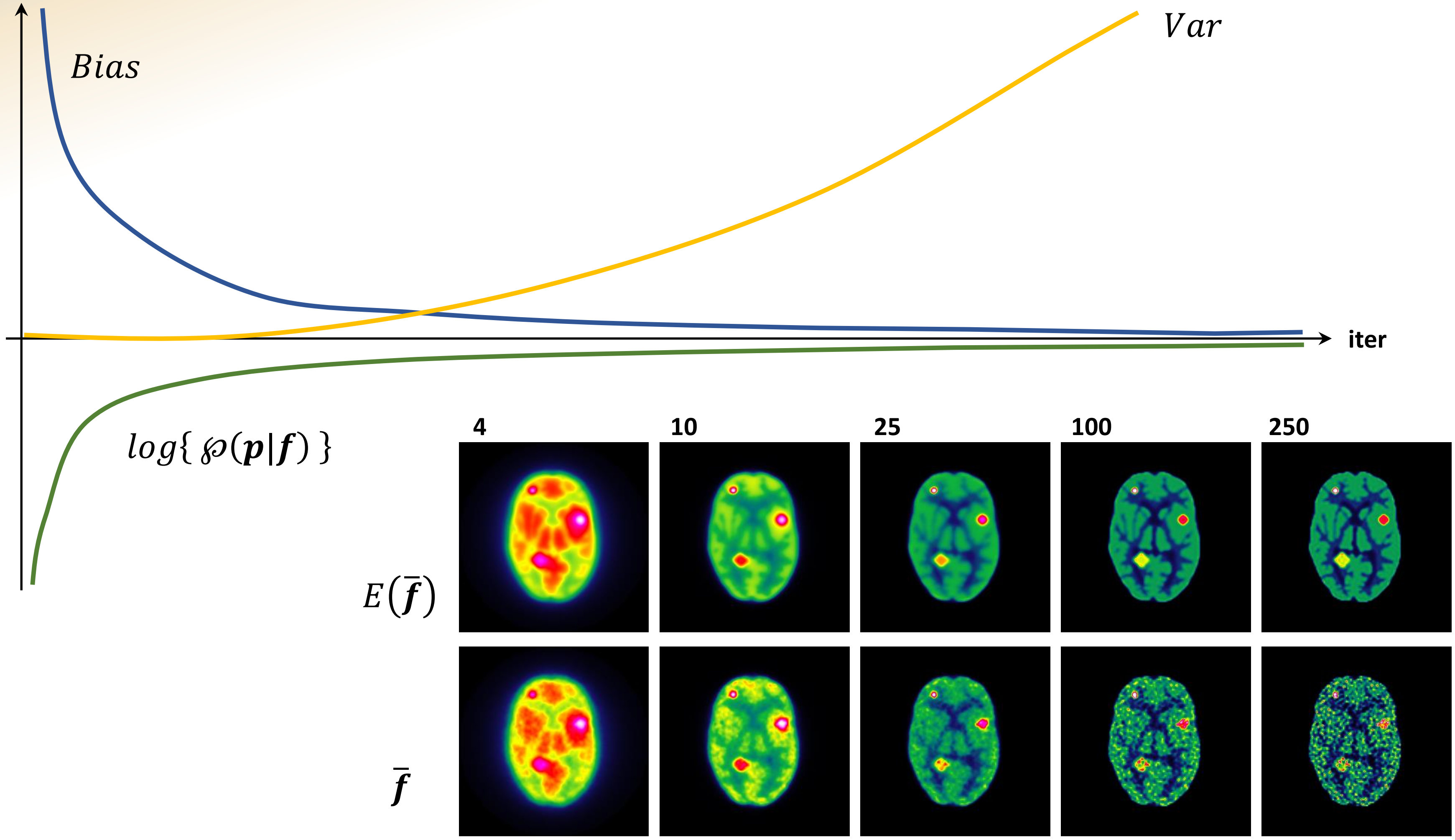
Régularisation

- Reconstruction itérative



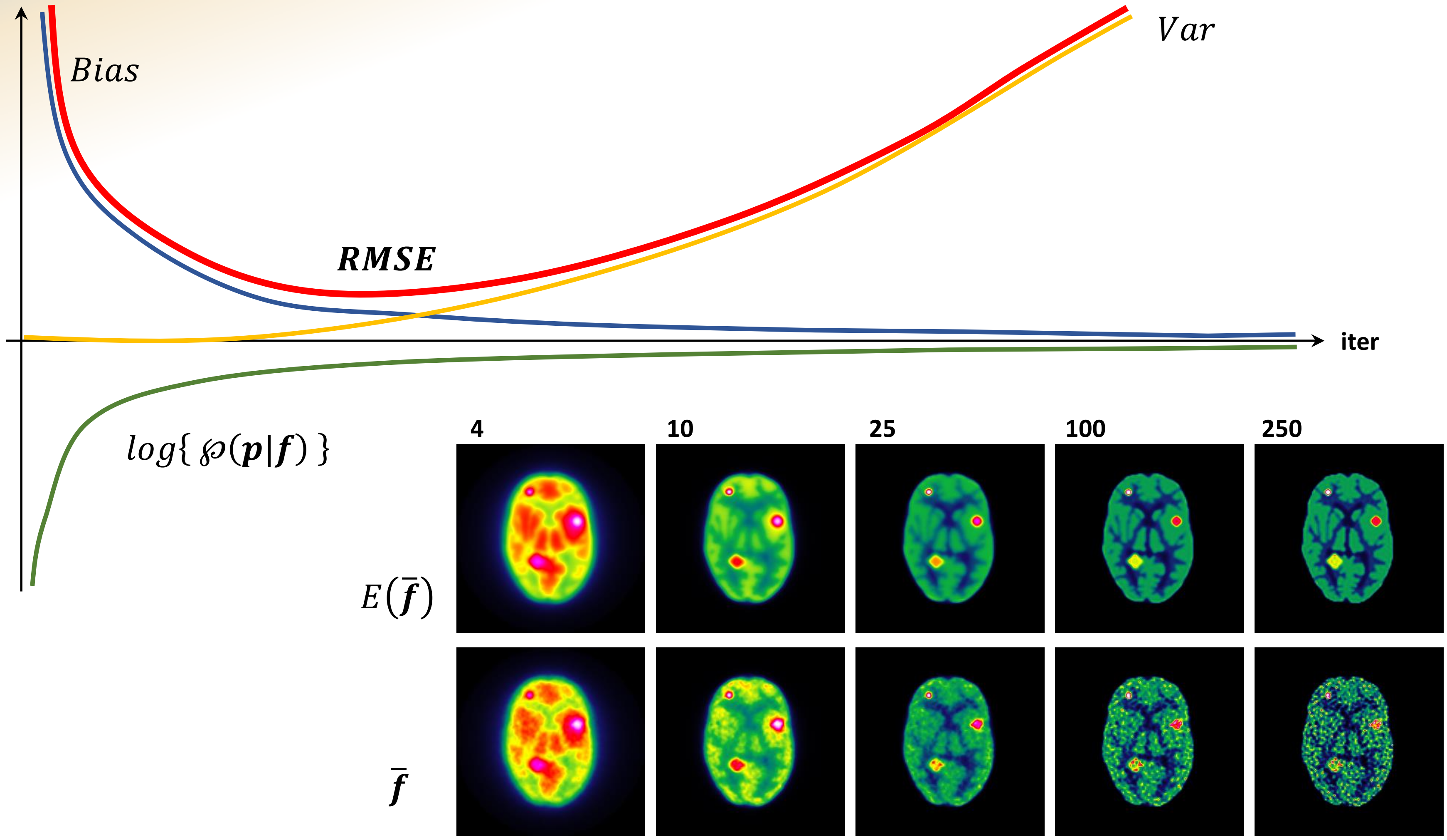
Régularisation

- Reconstruction itérative



Régularisation

- Reconstruction itérative



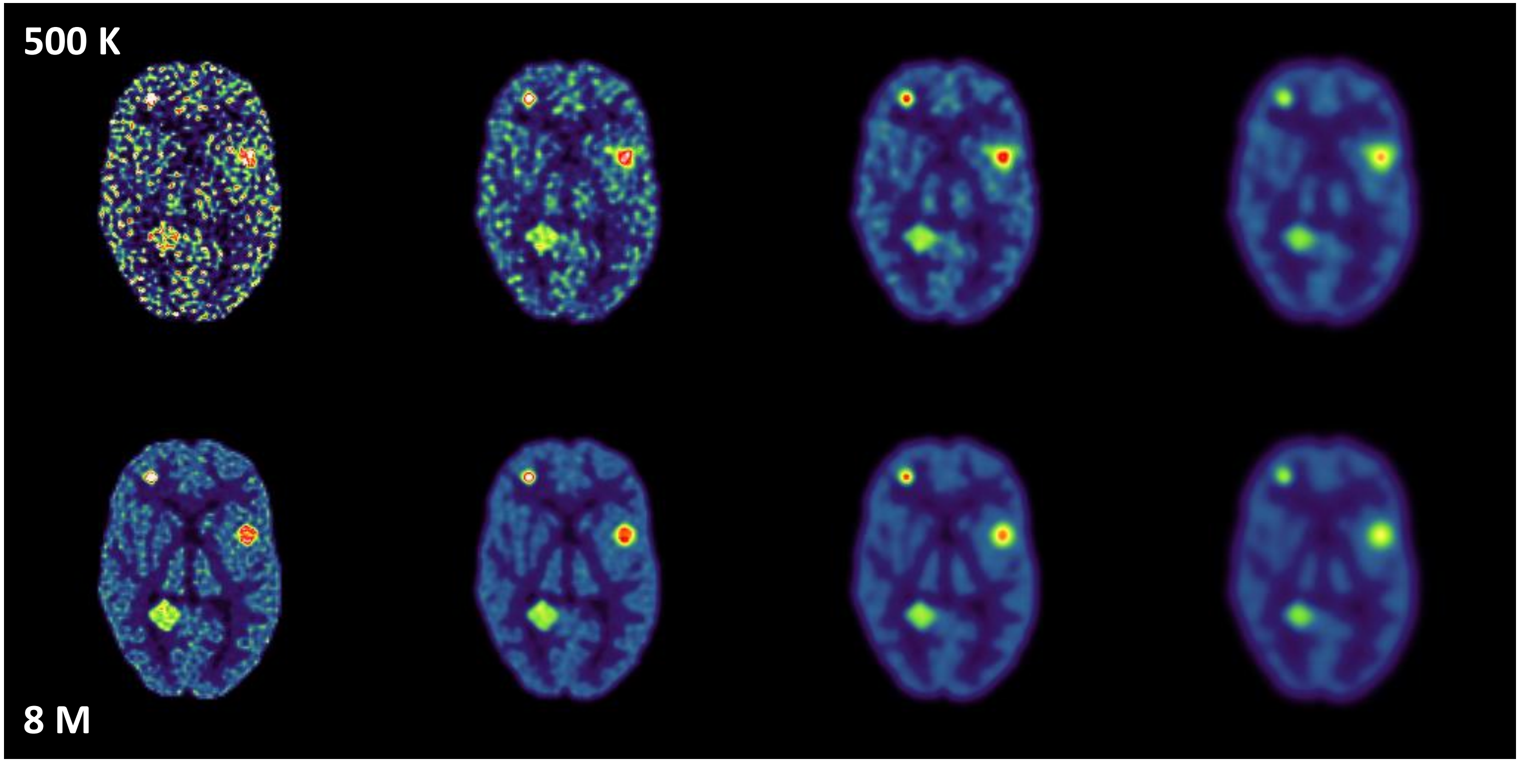
Régularisation

- Reconstruction itérative

Post-filtrage

$h * \bar{f}$

FWHM : 2 pix 3,5 pix 5 pix



Régularisation

- Reconstruction itérative

Tikhonov

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2 + \rho(\boldsymbol{f})$$

Régularisation

- Reconstruction itérative

Tikhonov

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \, \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \boxed{\|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2} + \rho(\boldsymbol{f})$$

Adéquation
Surjectivité
↓ biais

Régularisation

- Reconstruction itérative

Tikhonov

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \{ J(\boldsymbol{f}) \}$$

$$J(\boldsymbol{f}) = \boxed{\|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2} + \boxed{\rho(\boldsymbol{f})}$$

Adéquation
Surjectivité
↓ biais

Régularisation
Injectivité
↓ variance

Régularisation

- Reconstruction itérative

Tikhonov

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \boxed{\|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2} + \boxed{\beta\|\boldsymbol{f}\|^2}$$

Régularisation

- Reconstruction itérative

Tikhonov

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \boxed{\|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2} + \boxed{\beta\|\boldsymbol{f}\|^2}$$

$$\bar{\boldsymbol{f}} = (\mathbf{R}^*\mathbf{R} + \beta\mathbf{I})^{-1}\, \mathbf{R}^*\, \boldsymbol{p}$$

$$\bar{\boldsymbol{f}}^{n+1} = (1-\beta)\bar{\boldsymbol{f}}^{n} + \boldsymbol{\eta}\,\mathbf{R}^*\left(\boldsymbol{p} - \mathbf{R}\,\bar{\boldsymbol{f}}^{n}\right)$$

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \, \{ J(\boldsymbol{f}) \}$$

Posterior

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{f}|\boldsymbol{p}) \}$$

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \{ J(\boldsymbol{f}) \}$$

Posterior

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{f}|\boldsymbol{p}) \}$$

$$\wp(\boldsymbol{f}|\boldsymbol{p}) = \frac{\wp(\boldsymbol{p}|\boldsymbol{f})\,\wp(\boldsymbol{f})}{\wp(\boldsymbol{p})}$$

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin}\ \{J(\boldsymbol{f})\}$$

Posterior

$$J(\boldsymbol{f}) = -log\{\wp(\boldsymbol{f}|\boldsymbol{p})\}$$

$$\wp(\boldsymbol{f}|\boldsymbol{p}) = \frac{\wp(\boldsymbol{p}|\boldsymbol{f})\,\wp(\boldsymbol{f})}{\wp(\boldsymbol{p})}$$

$$J(\boldsymbol{f}) = -log\{\wp(\boldsymbol{p}|\boldsymbol{f})\} - log\{\wp(\boldsymbol{f})\}$$

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin}\ \{J(\boldsymbol{f})\}$$

Posterior

$$J(\boldsymbol{f}) = -log\{\ \wp(\boldsymbol{f}|\boldsymbol{p})\ \}$$

Likelihood

$$J(\boldsymbol{f}) = -\boxed{log\{\ \wp(\boldsymbol{p}|\boldsymbol{f})\ \}} - log\{\ \wp(\boldsymbol{f})\ \}$$

Adéquation
Surjectivité
↓ biais

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \{ J(\boldsymbol{f}) \}$$

Posterior

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{f}|\boldsymbol{p}) \}$$

Likelihood **Prior**

$$J(\boldsymbol{f}) = -\boxed{log\{ \wp(\boldsymbol{p}|\boldsymbol{f}) \}} - \boxed{log\{ \wp(\boldsymbol{f}) \}}$$

Adéquation
Surjectivité
↓ biais

Régularisation
Injectivité
↓ variance

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin}\ \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = -log\{\overbrace{\wp(\boldsymbol{f}|\boldsymbol{p})}^{\text{Posterior}}\}$$

$$J(\boldsymbol{f}) = -\overbrace{\boxed{log\{\wp(\boldsymbol{p}|\boldsymbol{f})\}}}^{\text{Likelihood}} - \overbrace{\boxed{log\{\wp(\boldsymbol{f})\}}}^{\text{Prior}}$$

$$\wp(\boldsymbol{f}) = \kappa\, e^{-\beta \mathrm{U}}$$

Régularisation

- Reconstruction itérative

MAP

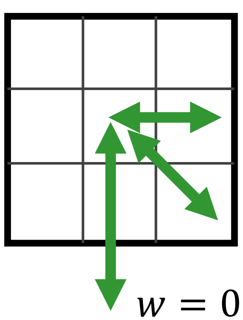
$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin}\ \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = -\overbrace{log\{\ \wp(\boldsymbol{f}|\boldsymbol{p})\ \}}^{\textit{Posterior}}$$

$$J(\boldsymbol{f}) = -\overbrace{\boxed{log\{\ \wp(\boldsymbol{p}|\boldsymbol{f})\ \}}}^{\textit{Likelihood}} - \overbrace{\boxed{log\{\ \wp(\boldsymbol{f})\ \}}}^{\textit{Prior}}$$

$$\mathrm{U} = \sum_{i,j} w_{ij} \left\| \boldsymbol{f}_i - \boldsymbol{f}_j \right\|^2$$

$$\wp(\boldsymbol{f}) = \kappa\, e^{-\beta \mathrm{U}}$$



$w_{1st} = 1$

$w_{2nd} = 1/\sqrt{2}$

$w = 0$

Régularisation

- Reconstruction itérative

MAP

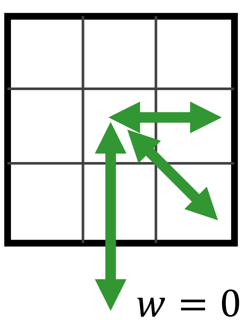
$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin}\ \{ J(\boldsymbol{f}) \}$$

$$J(\boldsymbol{f}) = \overbrace{-log\{\ \wp(\boldsymbol{f}|\boldsymbol{p})\ \}}^{\text{Posterior}}$$

$$J(\boldsymbol{f}) = -\overbrace{\boxed{log\{\ \wp(\boldsymbol{p}|\boldsymbol{f})\ \}}}^{\text{Likelihood}} - \overbrace{\boxed{log\{\ \wp(\boldsymbol{f})\ \}}}^{\text{Prior}}$$

$$\mathrm{U} = \sum_{i,j} w_{ij}\, \|\boldsymbol{f}_i - \boldsymbol{f}_j\|^2$$

$$\wp(\boldsymbol{f}) = \kappa\, e^{-\beta \mathrm{U}}$$



$w_{1st} = 1$

$w_{2nd} = 1/\sqrt{2}$

$w = 0$

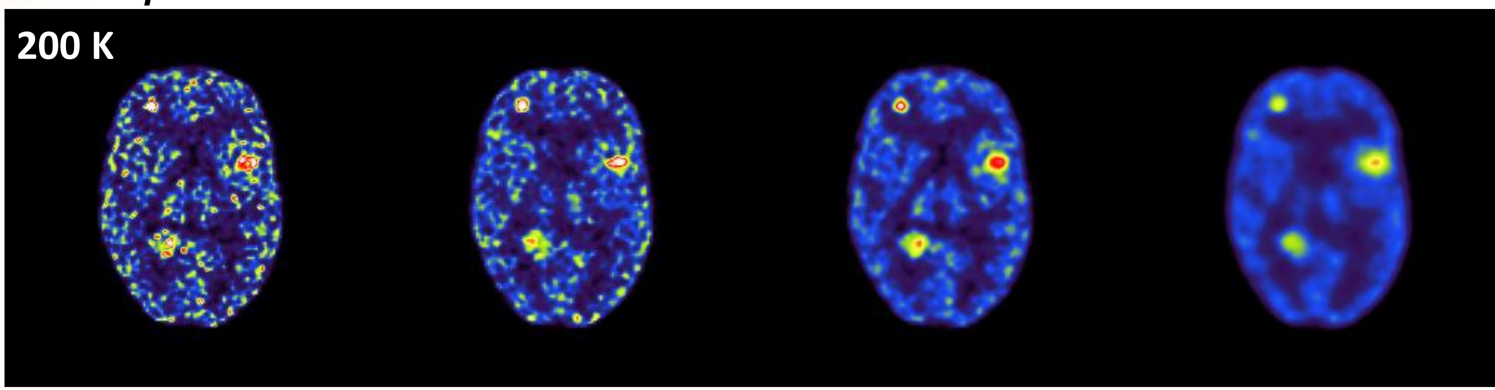
$$\bar{\boldsymbol{f}}^{n+1} = \frac{\bar{\boldsymbol{f}}^{n}}{1 + \beta\, \boldsymbol{\nabla}\mathrm{U}} \times \left(\mathbf{R}^{*} \frac{\boldsymbol{p}}{\mathbf{R}\, \bar{\boldsymbol{f}}^{n}} \right)$$

Régularisation

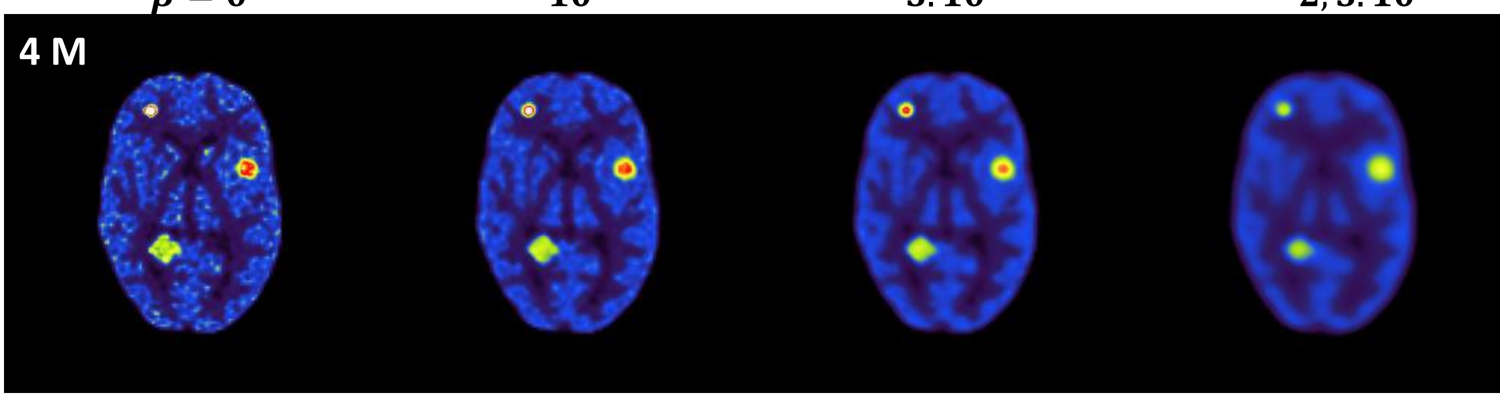
- Reconstruction itérative

MAP

$\beta = 0$ $\quad 2.10^{-4}$ $\quad 10^{-3}$ $\quad 5.10^{-3}$



$\beta = 0$ $\quad 10^{-5}$ $\quad 5.10^{-5}$ $\quad 2,5.10^{-4}$



Régularisation

- Reconstruction itérative

MAP

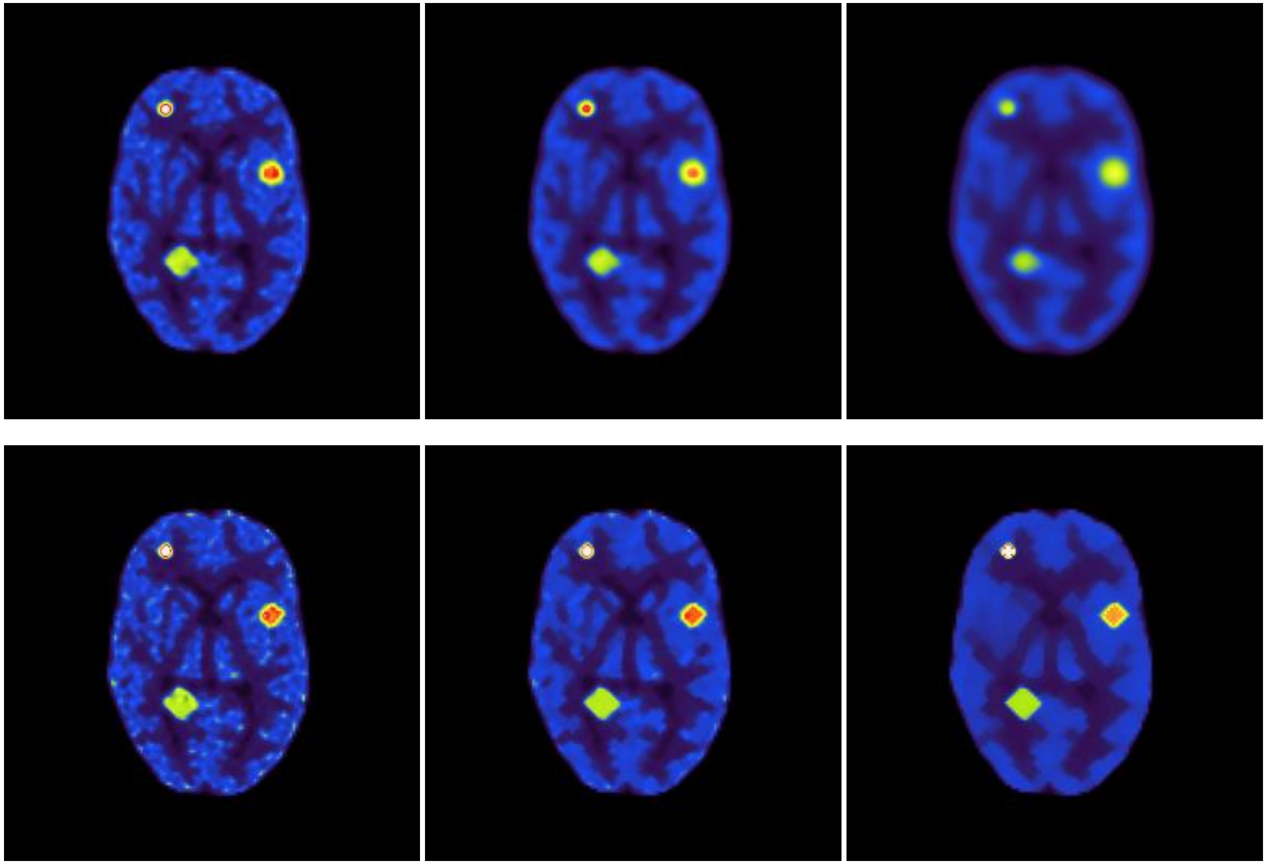
Quad. prior

$$U = \sum_{i,j} w_{ij} \|\boldsymbol{f}_i - \boldsymbol{f}_j\|^2$$

Median prior

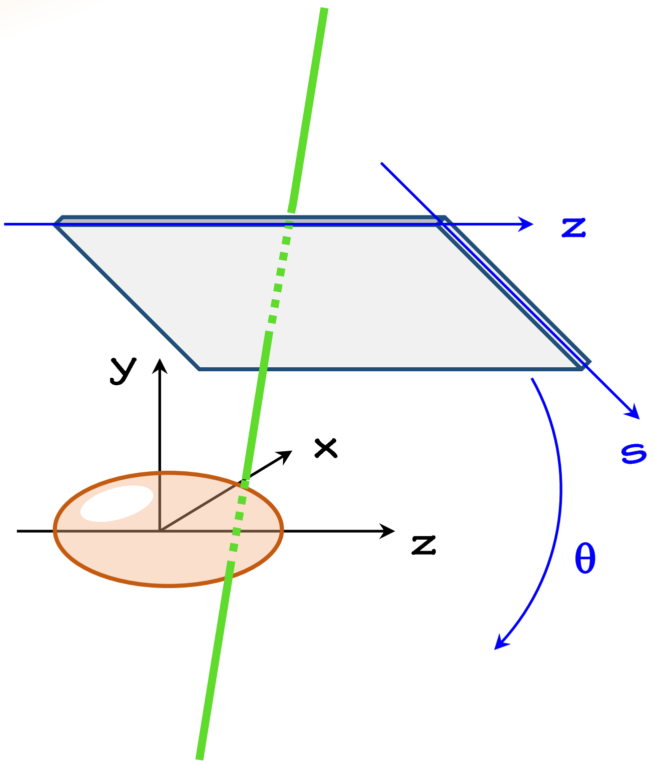
$$U = \sum_{i,j} w_{ij} |\boldsymbol{f}_i - \boldsymbol{f}_j|$$

« edge-preserving »



TEP 3D

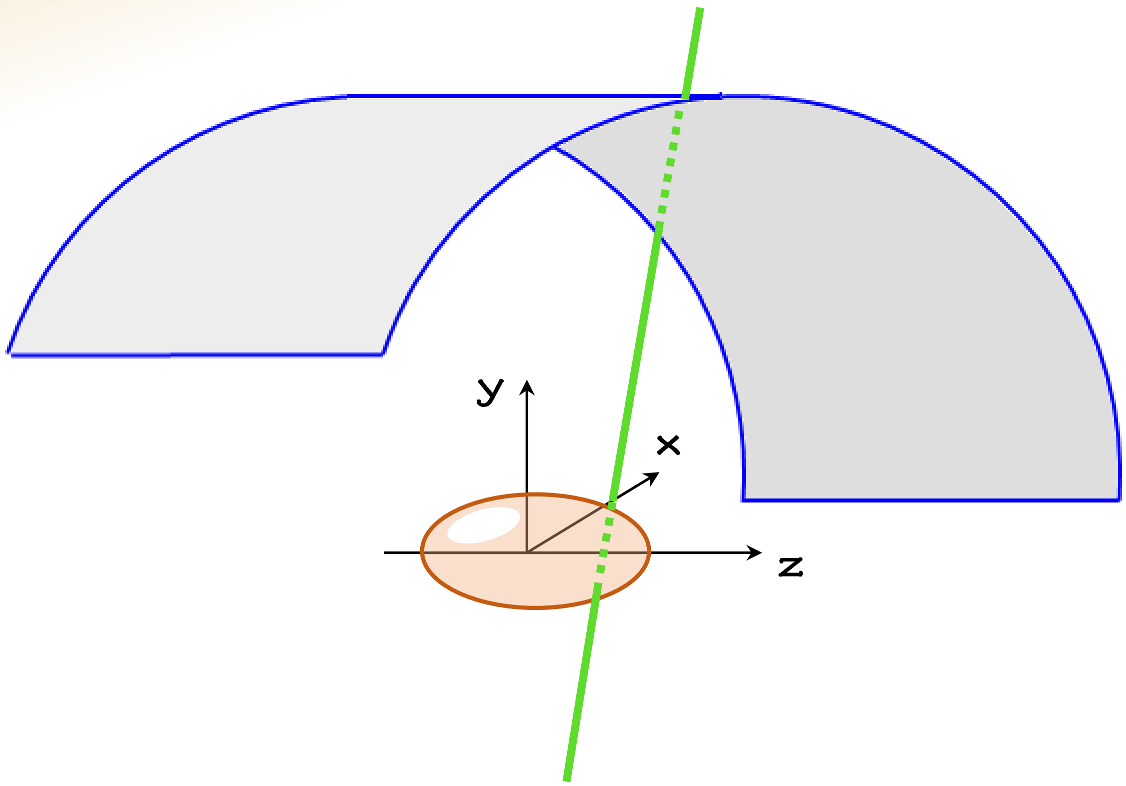
- Encodage TEMP



$p(z, \theta, s)$

TEP 3D

- Encodage TEP

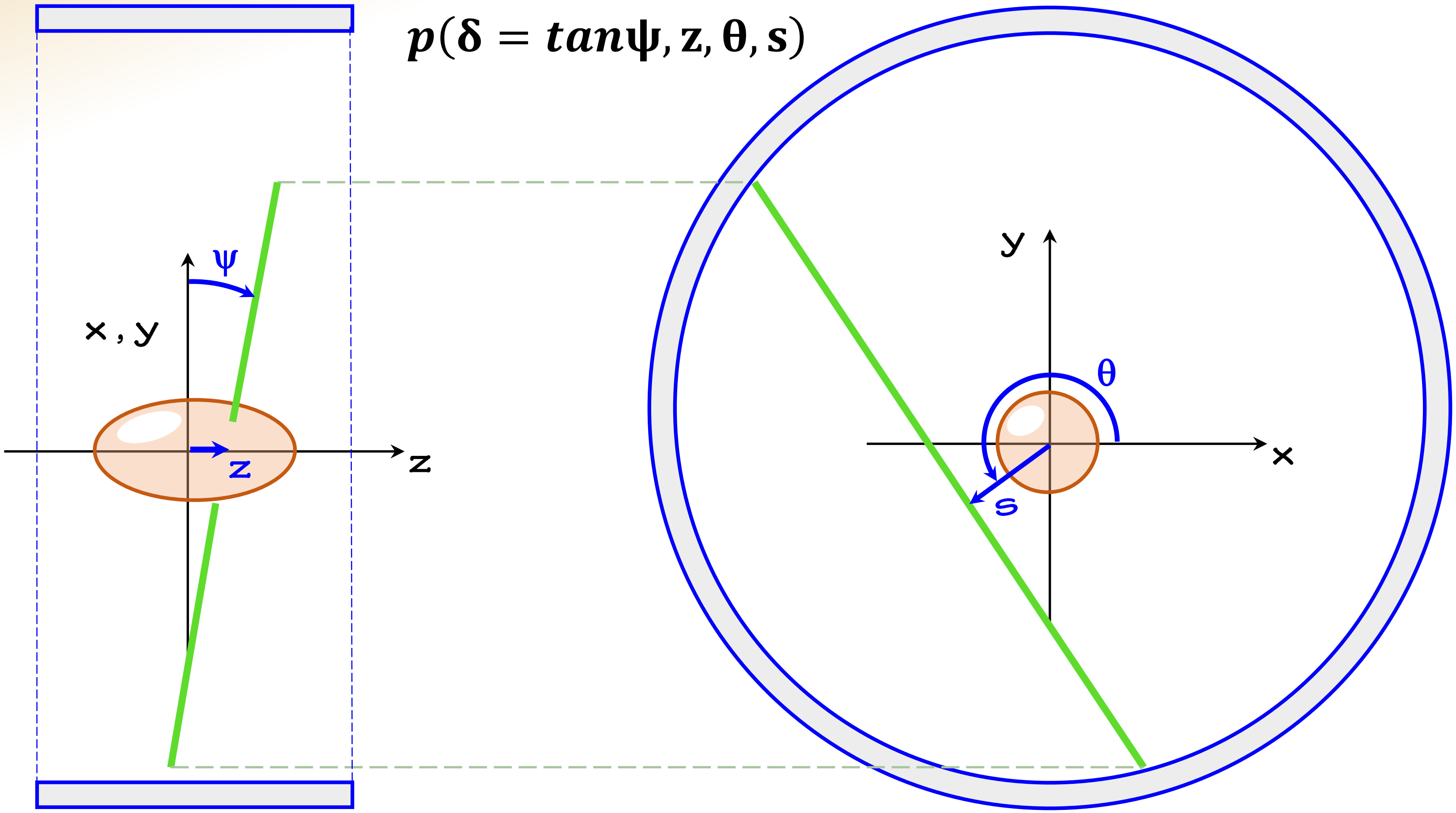


$p(\delta, z, \theta, s)$

TEP 3D

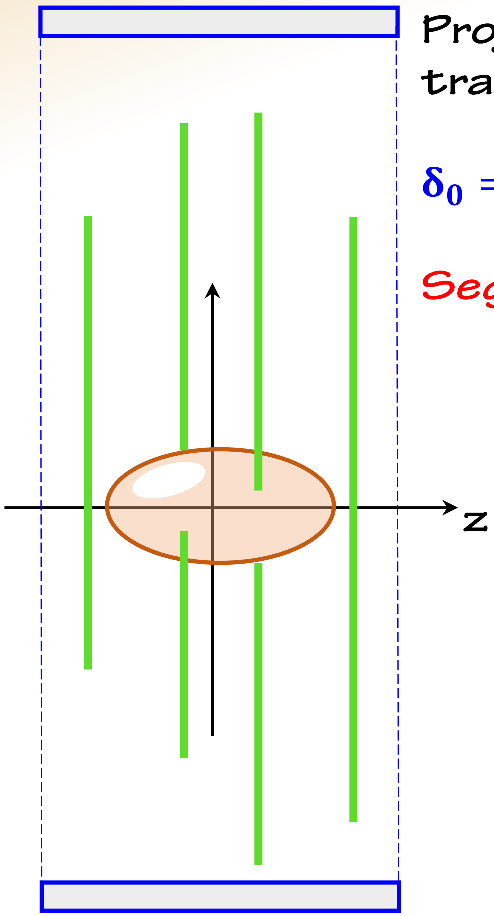
- Encodage TEP

$$p(\delta = tan\psi, z, \theta, s)$$



TEP 3D

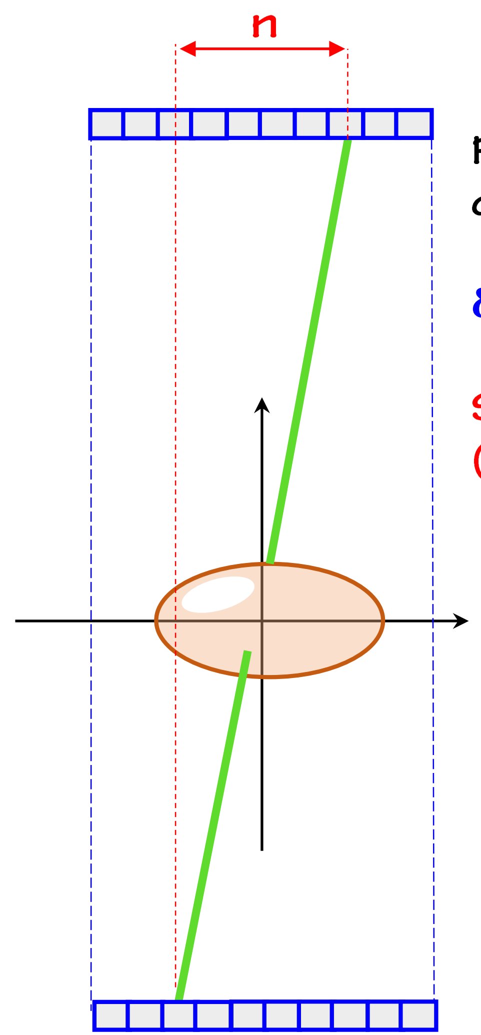
- Encodage TEP



Projections transverses :

$\delta_0 = 0$

Segment 0



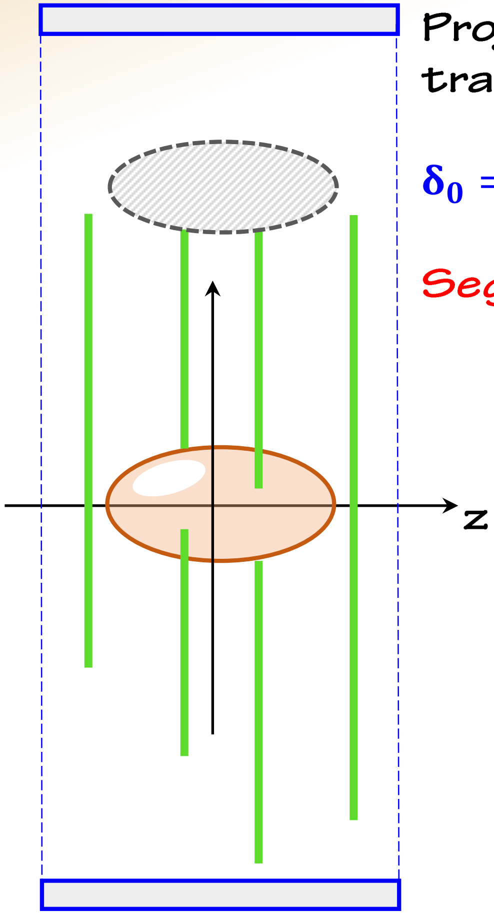
Projections obliques :

$\delta_n > 0$

Segment n (1…N)

TEP 3D

- Encodage TEP



Projections transverses :

$\delta_0 = 0$

Segment 0

COMPLETES

&

SUFFISANTES

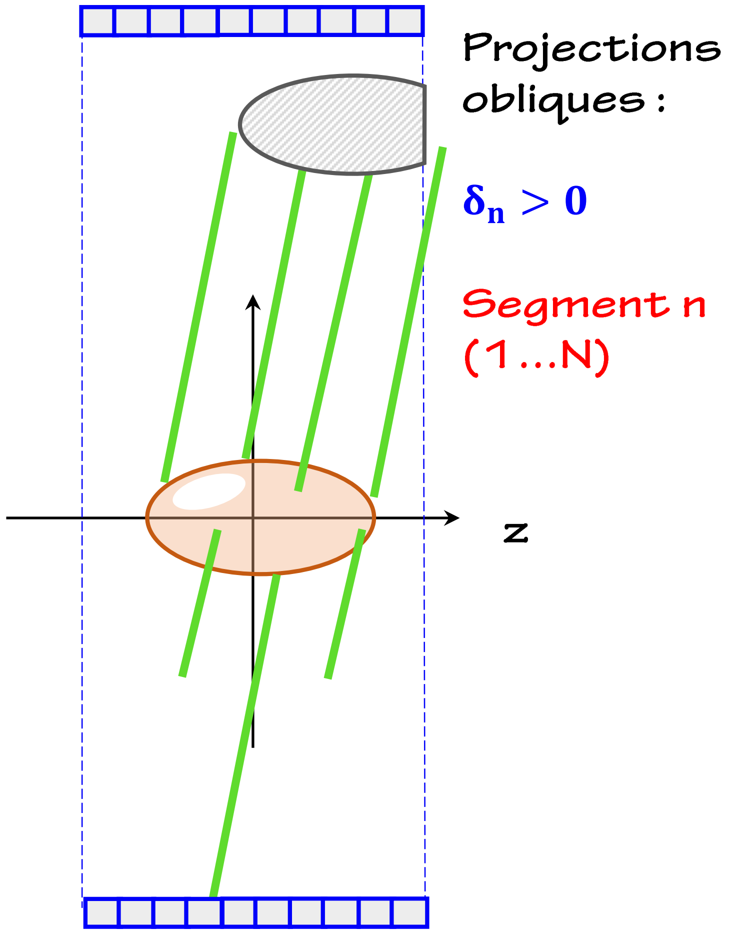
$$p(0, z, \theta, s) \rightarrow f(x, y, z)$$

TEP 3D

- Encodage TEP

TRONQUEES

$$p(\delta_{n>0}, z, \theta, s) \nrightarrow f(x, y, z)$$



TEP 3D

- Encodage TEP

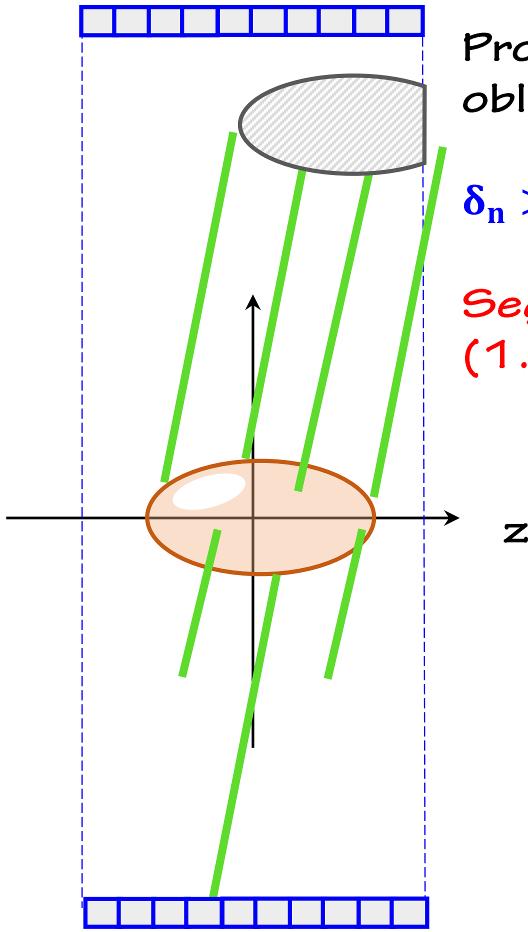
TRONQUEES

&

REDONDANTES

$$p(\delta_{n>0}, z, \theta, s) \nrightarrow f(x, y, z)$$

$$\left.\begin{array}{l} p(0, z, \theta, s) \\ p(\delta_1, z, \theta, s) \\ p(\delta_2, z, \theta, s) \\ p(\delta_3, z, \theta, s) \\ \vdots \end{array}\right\} \rightarrow f(x, y, z)$$



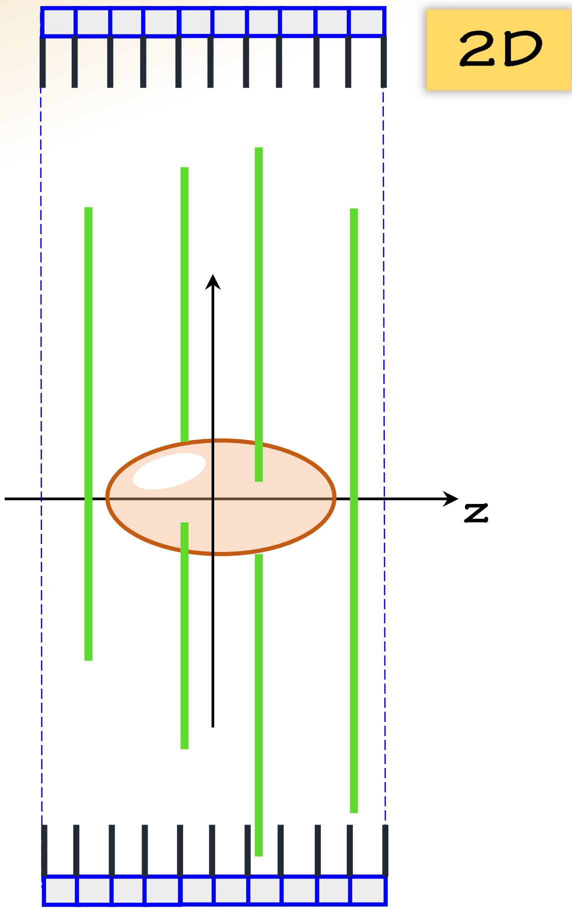
Projections obliques :

$\delta_n > 0$

Segment n (1…N)

TEP 3D

- Reconstruction



$$p(0, z, \theta, s) \rightarrow f(x, y, z)$$

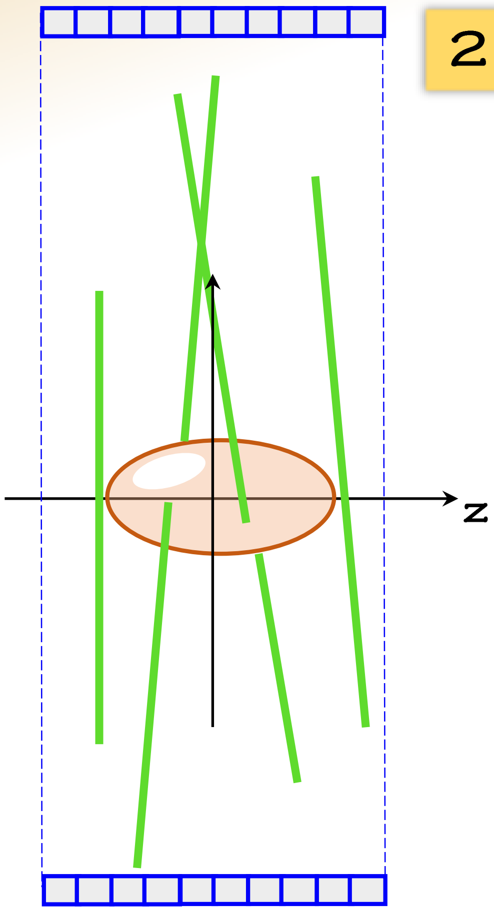
Collimation

Très rapide

SNR ↘

TEP 3D

- Reconstruction



2D ½

$$\underbrace{p(0,z,\theta,s)\,,p(\delta_1,z,\theta,s)\,,p(\delta_2,z,\theta,s)\,,\ldots}$$
$$\downarrow$$
$$p(0,z,\theta,s)$$
$$\downarrow$$
$$f(x,y,z)$$

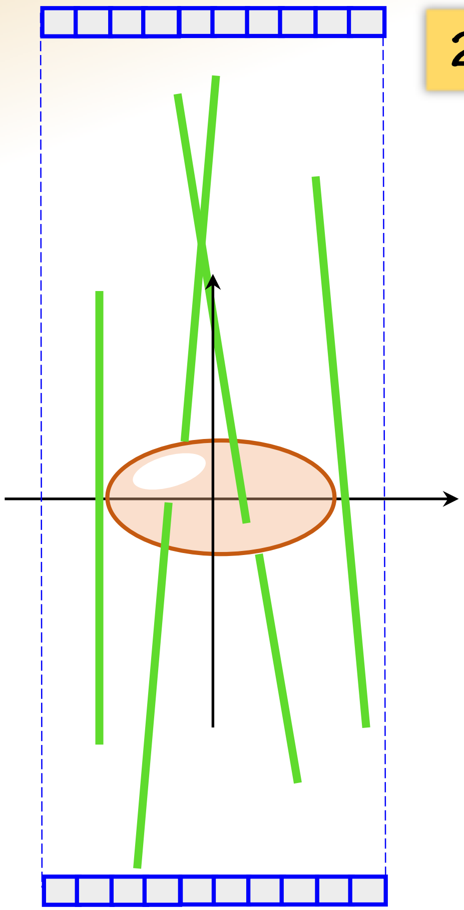
« Rebinning »

Rapide

Approximatif

TEP 3D

- Reconstruction



2D ½

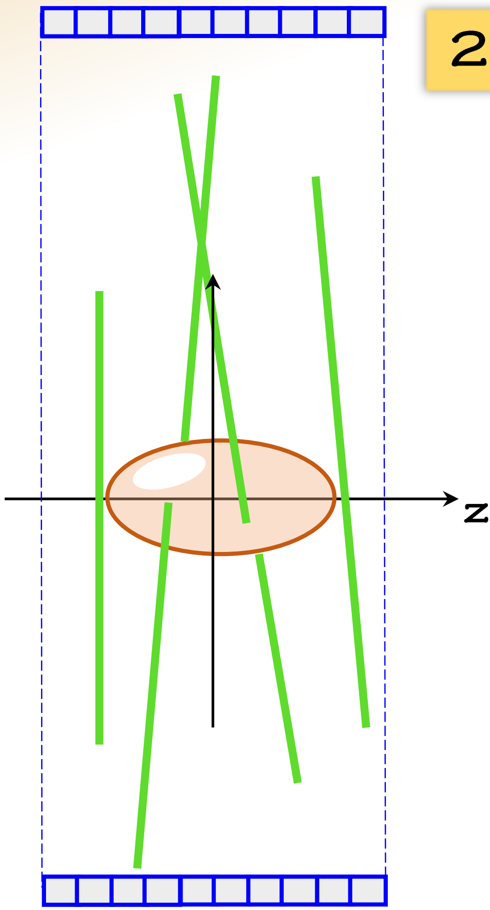
REBINNING EXACT

$$\hat{p}(0,\underbrace{\zeta,\mathrm{k},\omega}) = e^{-i\,\mathrm{k}\,atan\left(\frac{\delta\zeta}{\omega'}\right)}\;\hat{p}(\delta,\underbrace{\zeta,\mathrm{k},\omega'})$$

$$\omega' = \sqrt{\omega^2 - \delta^2\zeta^2}$$

TEP 3D

■ Reconstruction



2D ½

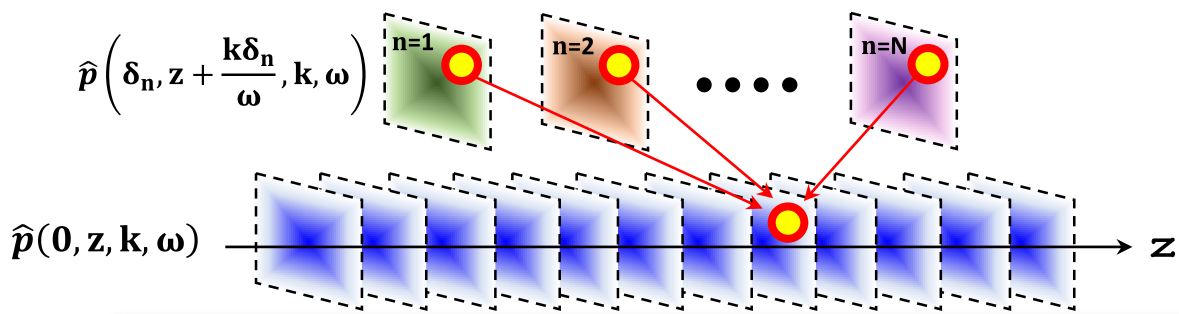
REBINNING EXACT

$$\hat{p}(0,\underline{\zeta,\mathrm{k},\omega}) = e^{-i\,\mathrm{k}\,atan\left(\frac{\delta\zeta}{\omega'}\right)}\ \hat{p}(\delta,\underline{\zeta,\mathrm{k},\omega'})$$

$$\omega' = \sqrt{\omega^2 - \delta^2\zeta^2}$$

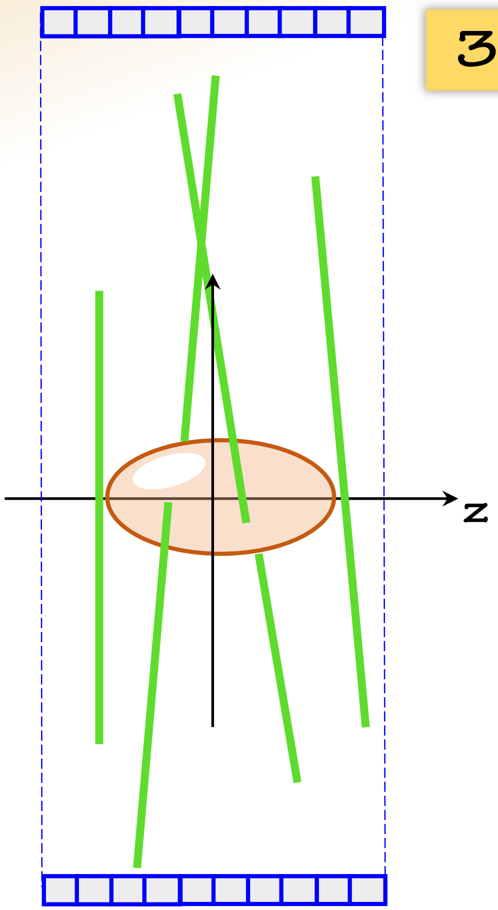
REBINNING APPROCHE

$$\hat{p}(0,\mathrm{z},\underline{\mathrm{k},\omega}) = \hat{p}\left(\delta, \mathrm{z} + \frac{\mathrm{k}\delta}{\omega}, \underline{\mathrm{k},\omega}\right)$$



TEP 3D

- Reconstruction



3D

$$\left.\begin{array}{c} p(0, z, \theta, s) \\ p(\delta_1, z, \theta, s) \\ p(\delta_2, z, \theta, s) \\ p(\delta_3, z, \theta, s) \\ \vdots \end{array}\right\} \rightarrow f(x, y, z)$$

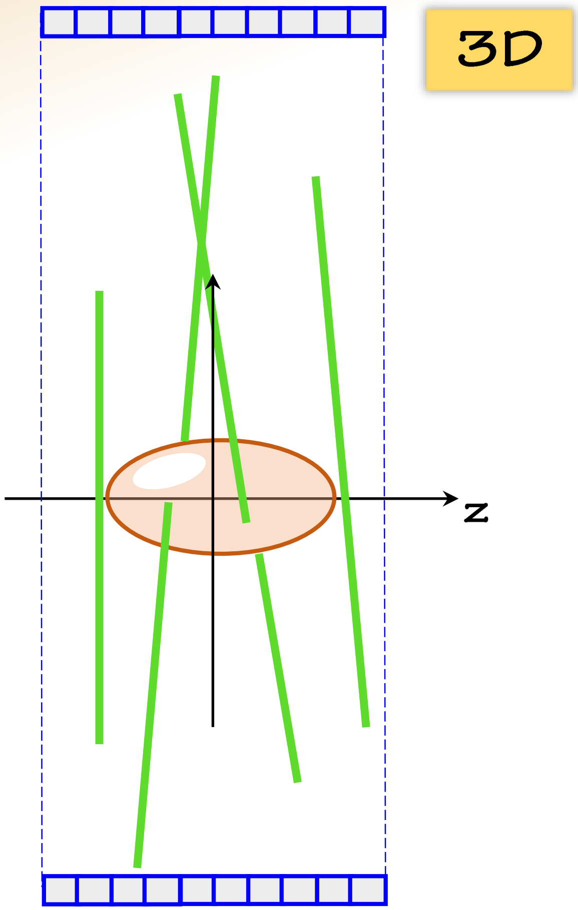
3D vrai

Reconstruction
analytique ou itérative

SNR ↗

TEP 3D

- Reconstruction



ALGEBRIQUE - ITERATIF

$$\boldsymbol{p}(\delta, z, \theta, s) = \mathbf{R_{3D}}\, \boldsymbol{f}\,(x, y, z)$$

$$\mathbf{R_{3D}} : \mathbb{R}^2 \times C_1 \times \{0 \ldots N\} \to \mathbb{R}^3$$

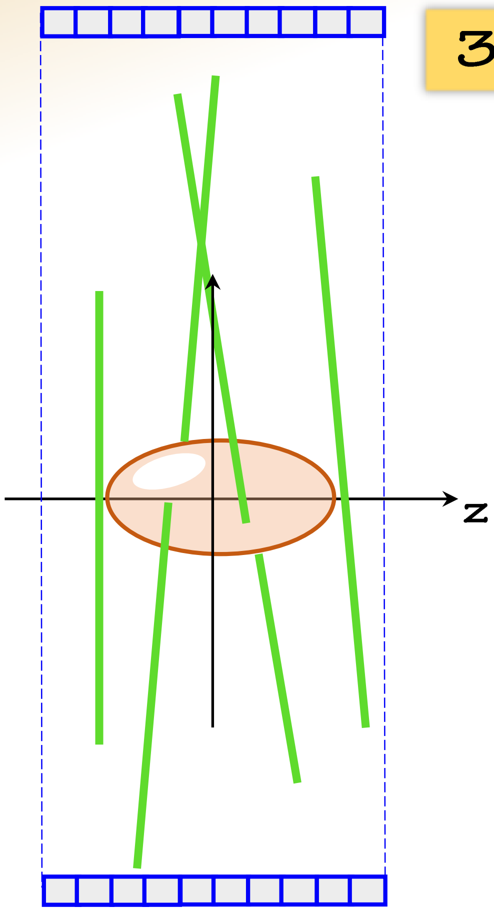
$$\boldsymbol{Dim}(\mathbf{R_{3D}}) \gg$$

Stockage ?

Temps de calcul ...

TEP 3D

- Reconstruction



3D

ANALYTIQUE : Fourier ou RPF

!! Complétion des données obliques !!

$$\boxed{p(0, z, \theta, s)} \rightarrow \tilde{f}\,(x, y, z)$$

$$\Big\downarrow \; \mathbf{R}\,\tilde{f}$$

$$\underbrace{\tilde{p}(\delta_{n>0}, z, \theta, s) \qquad \boxed{p(\delta_{n>0}, z, \theta, s)}}_{\text{Fusion}}$$

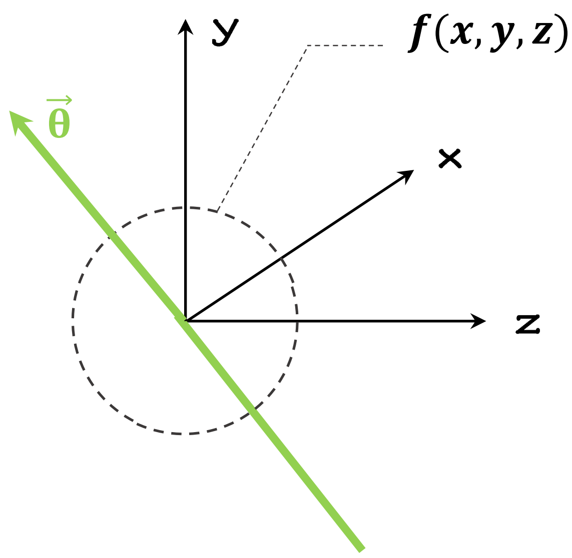
$$\left.\begin{array}{c} p(\delta_{n>0}, z, \theta, s) \\[2em] p(0, z, \theta, s) \end{array}\right\} \rightarrow f(x, y, z)$$

TEP 3D

- Reconstruction

3D

Synthèse de Fourier 3D

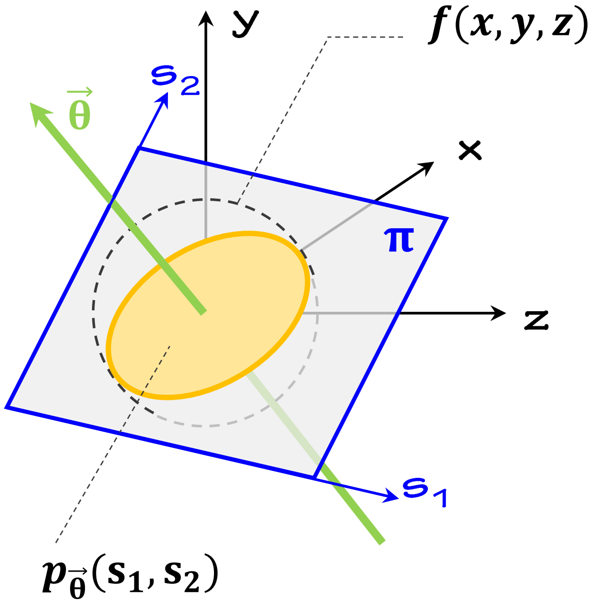


TEP 3D

- Reconstruction

3D

Synthèse de Fourier 3D

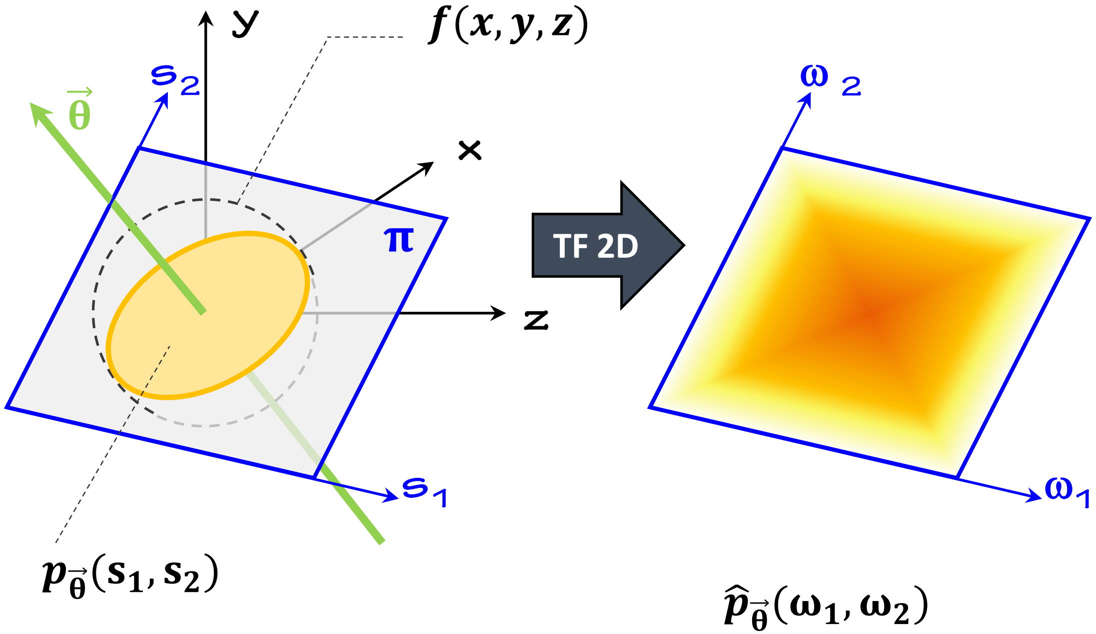


TEP 3D

- Reconstruction

3D

Synthèse de Fourier 3D



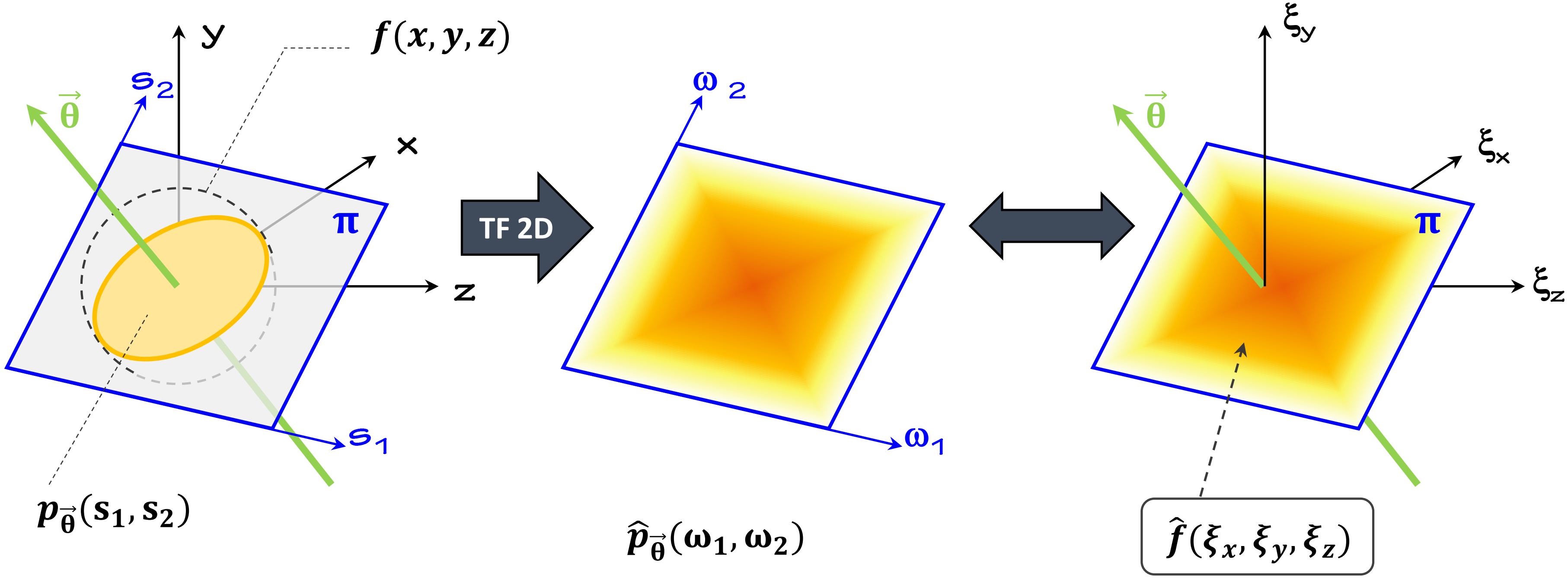
TEP 3D

- Reconstruction

3D

Synthèse de Fourier 3D

$$\hat{f}\left(\left[\xi_x, \xi_y, \xi_z\right] = R_{\vec{\theta}}[\omega_1, \omega_2]\right) = \hat{p}_{\vec{\theta}}(\omega_1, \omega_2)$$



TEP 3D

■ Reconstruction

3D

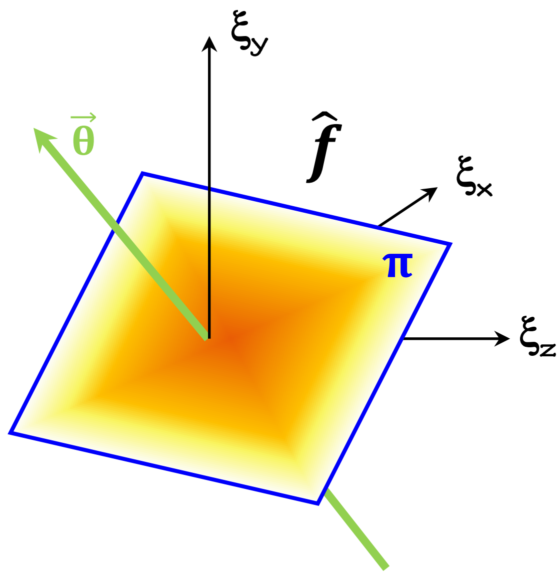
Synthèse de Fourier 3D

$$\hat{f}\left(\left[\xi_x, \xi_y, \xi_z\right] = R_{\vec{\theta}}[\omega_1, \omega_2]\right) = \hat{p}_{\vec{\theta}}(\omega_1, \omega_2)$$

Condition d'ORLOV

Nécessaire et suffisante pour que π décrive $\mathbb{R}^3$

$$\vec{\theta} \in \Omega \subset S^2$$
$$\exists\, C \in S^2 : C \subset \Omega$$
$$\forall\, C \in S^2 : \Omega \cap C \neq \emptyset$$



TEP 3D

■ Reconstruction

3D

Synthèse de Fourier 3D

$$\hat{f}\left(\left[\xi_x, \xi_y, \xi_z\right] = R_{\vec{\theta}}[\omega_1, \omega_2]\right) = \hat{p}_{\vec{\theta}}(\omega_1, \omega_2)$$

Condition d'ORLOV

Nécessaire et suffisante pour que π décrive $\mathbb{R}^3$

$$\vec{\theta} \in \Omega \subset S^2$$
$$\exists\, C \in S^2 : C \subset \Omega$$
$$\forall\, C \in S^2 : \Omega \cap C \neq \emptyset$$