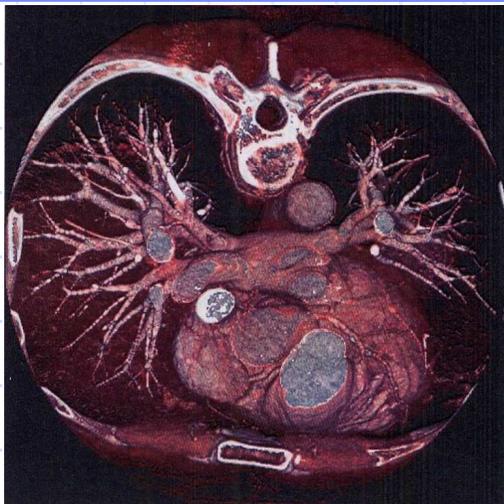


ECOLE CENTRALE DE MARSEILLE

TOMOGRAPHIE MEDICALE

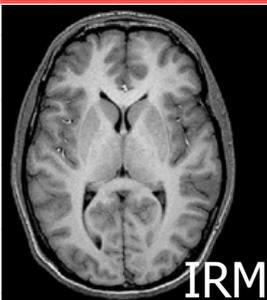
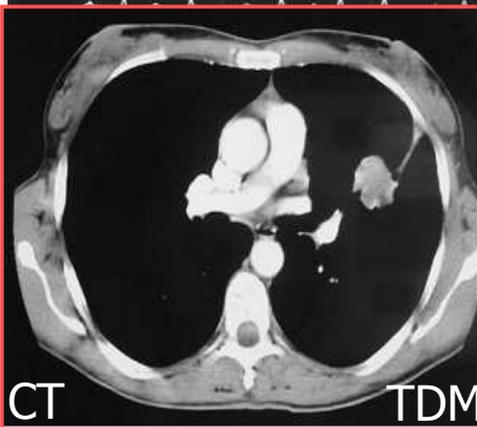


Denis Mariano-Goulart

Faculté de médecine & CHRU de Montpellier

<http://scinti.etud.univ-montp1.fr>

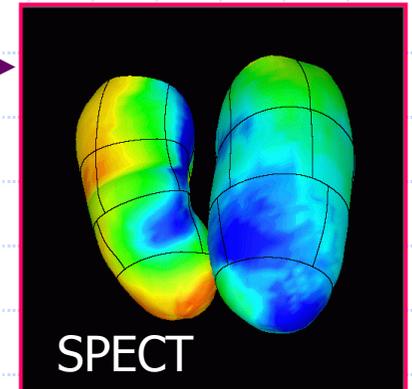
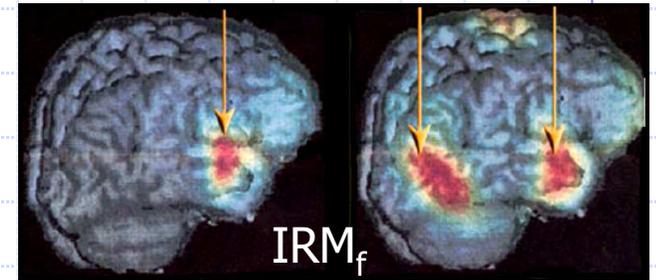
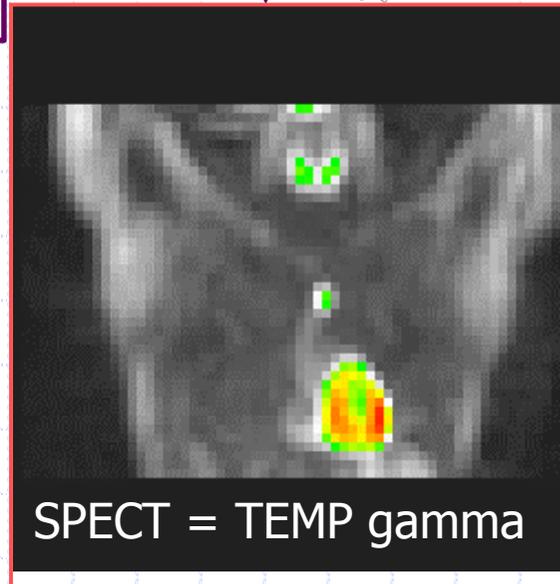
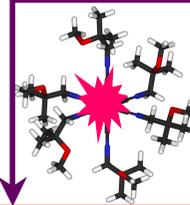
Imagerie médicale



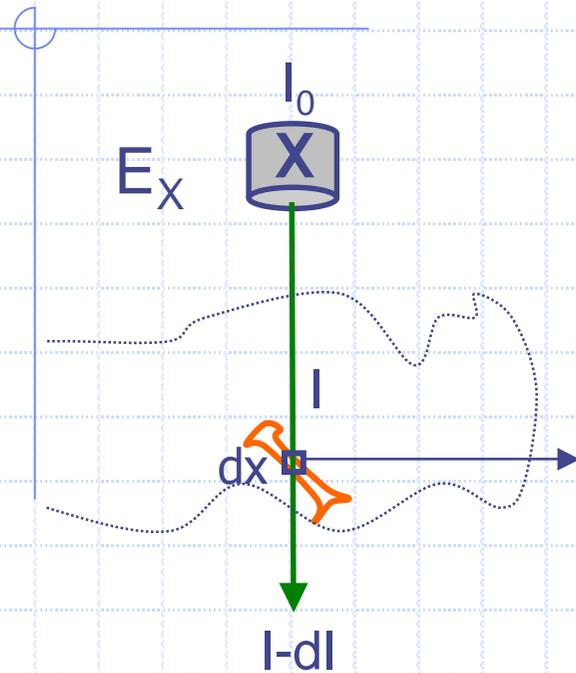
ANATOMIQUE

METABOLIQUE

FONCTIONNELLE

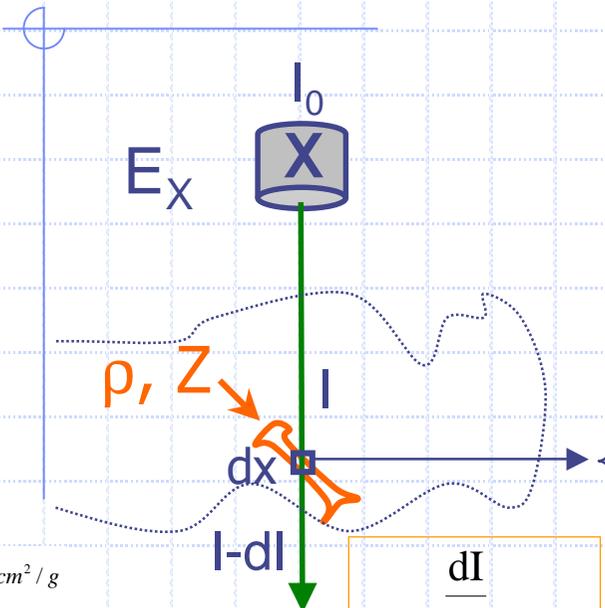


Scanner X = Computed Tomography

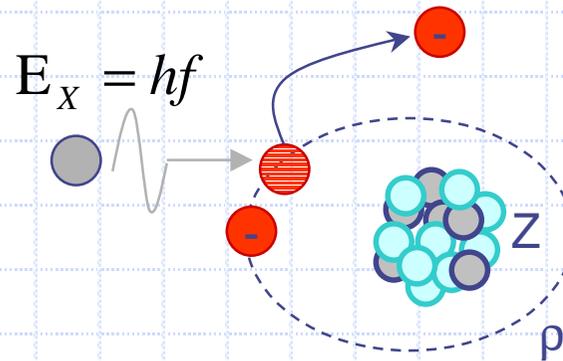
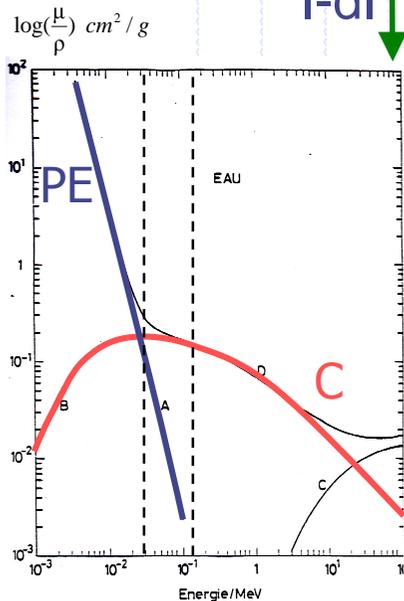


$$\mu = -\frac{dI}{I dx} \quad \text{probabilité d'interaction/cm}$$

Scanner X = Computed Tomography



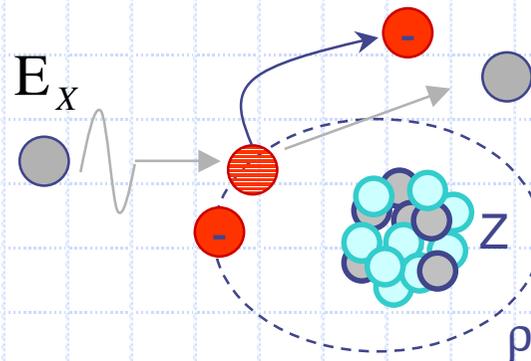
$$\mu = -\frac{dI}{dx} \text{ cm}^{-1}$$



Atténuation photo-électrique

$$\mu_{PE} \propto \rho \cdot \frac{Z^3}{E_X^3}$$

surtout si $10 \text{ keV} < E_X < 50 \text{ keV}$



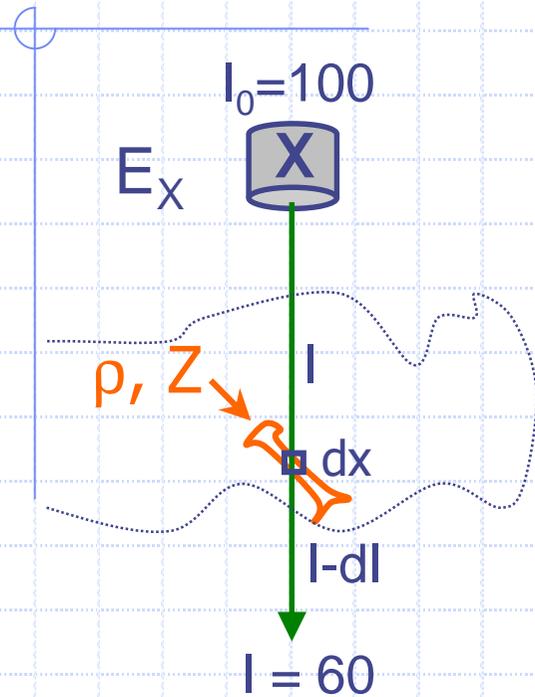
$$E'_x < E_x$$

Atténuation Compton

$$\mu_C \propto \rho$$

surtout si $E_X > 50 \text{ keV}$

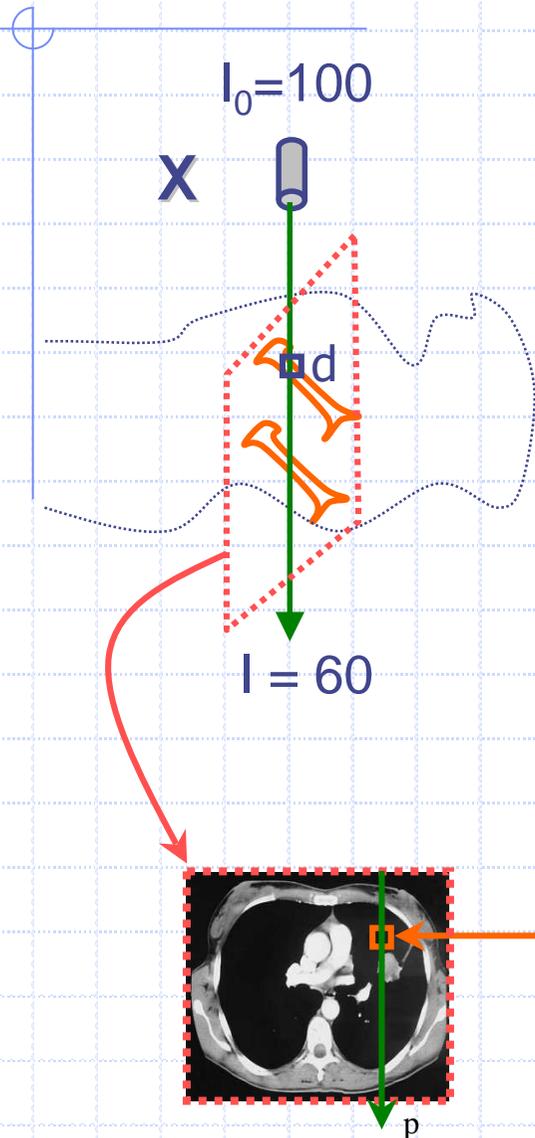
Scanner X = Computed Tomography



$$\mu = -\frac{dI}{I \cdot dx} \propto \rho$$

$$\frac{dI}{I} = -\mu \cdot dx \Rightarrow I = I_0 e^{-\mu \cdot x}$$

Scanner X = Computed Tomography



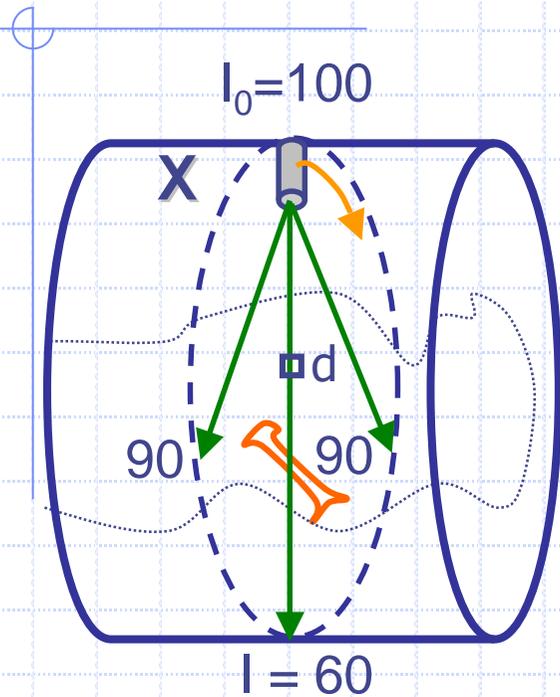
$$\mu = -\frac{dI}{I dx} \propto \rho$$

$$\frac{dI}{I} = -\mu \cdot dx \Rightarrow I = I_0 e^{-\mu \cdot x}$$

$$I = I_0 e^{-d \sum \mu_i} \Rightarrow p = \sum \mu_i = \frac{1}{d} \ln \frac{I_0}{I}$$

$$p = \mu_1 + \mu_2 + \dots + \mu_n \quad ? \quad \text{mesure}$$

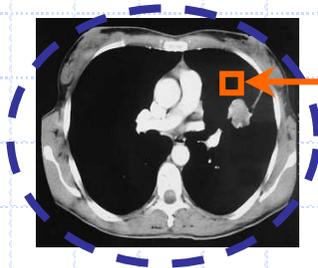
Scanner X = Computed Tomography



$$\mu = -\frac{dI}{I dx} \propto \rho$$

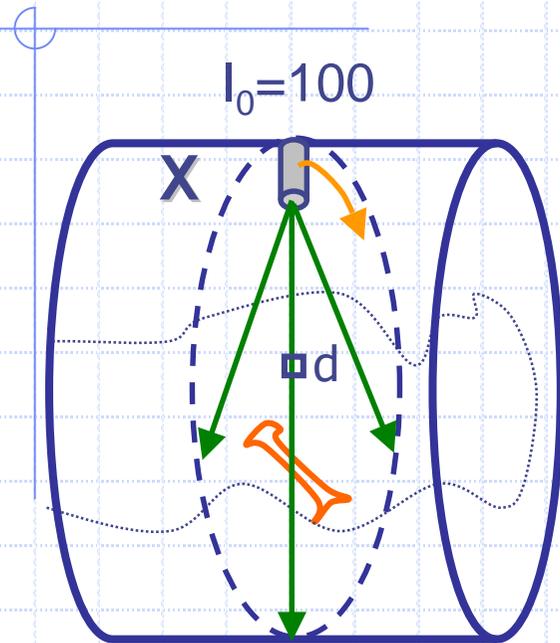
$$\frac{dI}{I} = -\mu \cdot dx \Rightarrow I = I_0 e^{-\mu \cdot x}$$

$$I = I_0 e^{-d \sum \mu_i} \Rightarrow p = \sum \mu_i = \frac{1}{d} \ln \frac{I_0}{I}$$

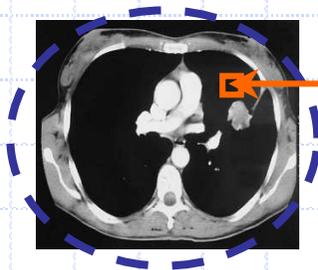


$$\begin{cases} p = \mu_1 + \mu_2 + \dots + \mu_n ? \\ \dots \\ p' = \mu'_1 + \mu'_2 + \dots + \mu'_n \end{cases} \quad \text{mesures}$$

Scanner X = Computed Tomography

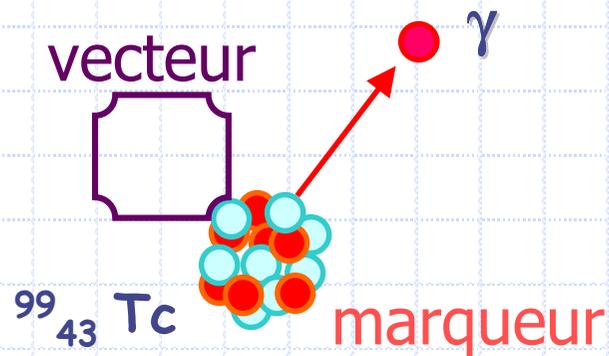
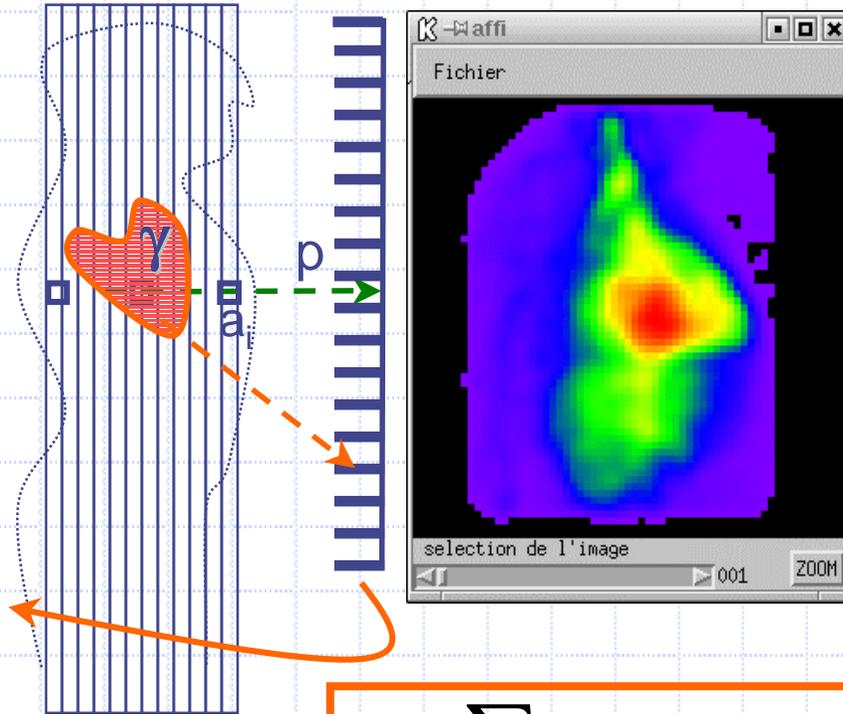


$$p_i = \sum_j r_{i,j} \mu_j, \quad i = 0 - 360^\circ$$

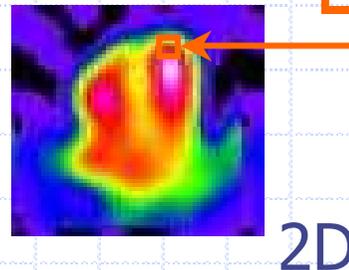


$r_{i,j}$ = contribution du pixel j à la projection i

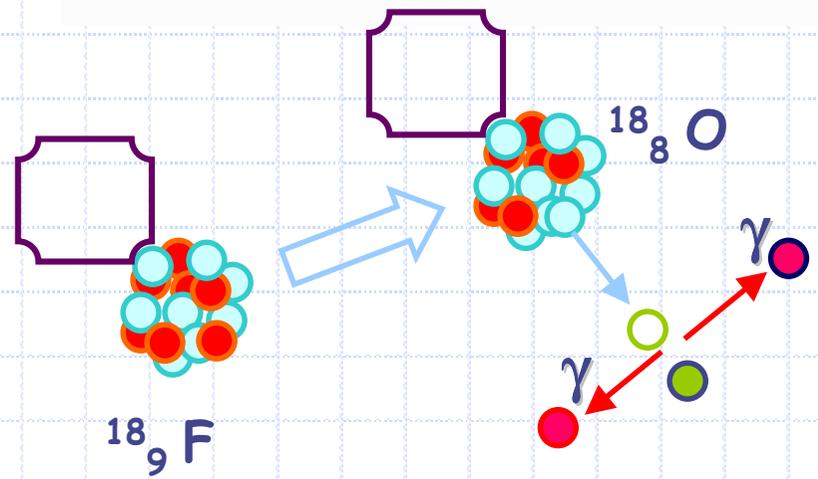
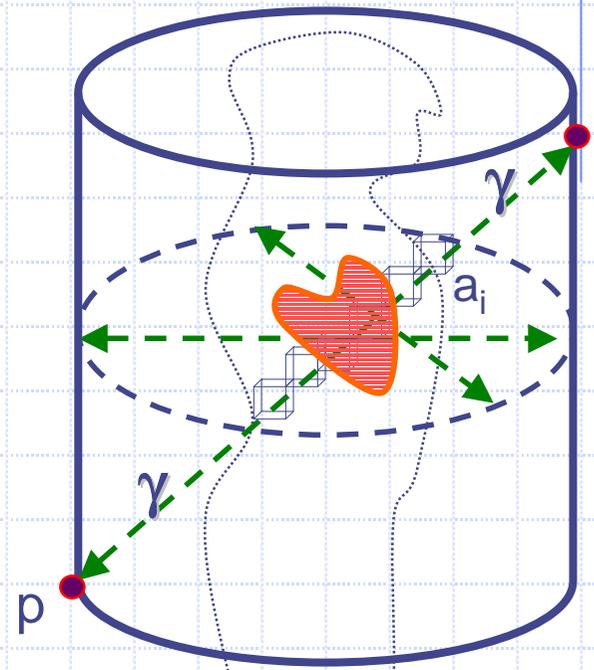
Single Photon Emission CT



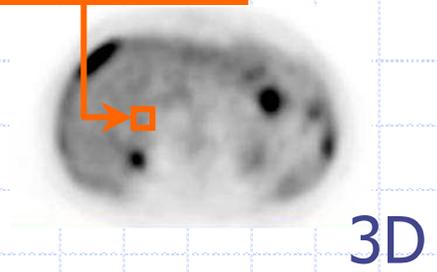
$$p_i = \sum_j r_{i,j} a_j, \quad i = 0-360^\circ$$



Tomographie par Emission de Positons

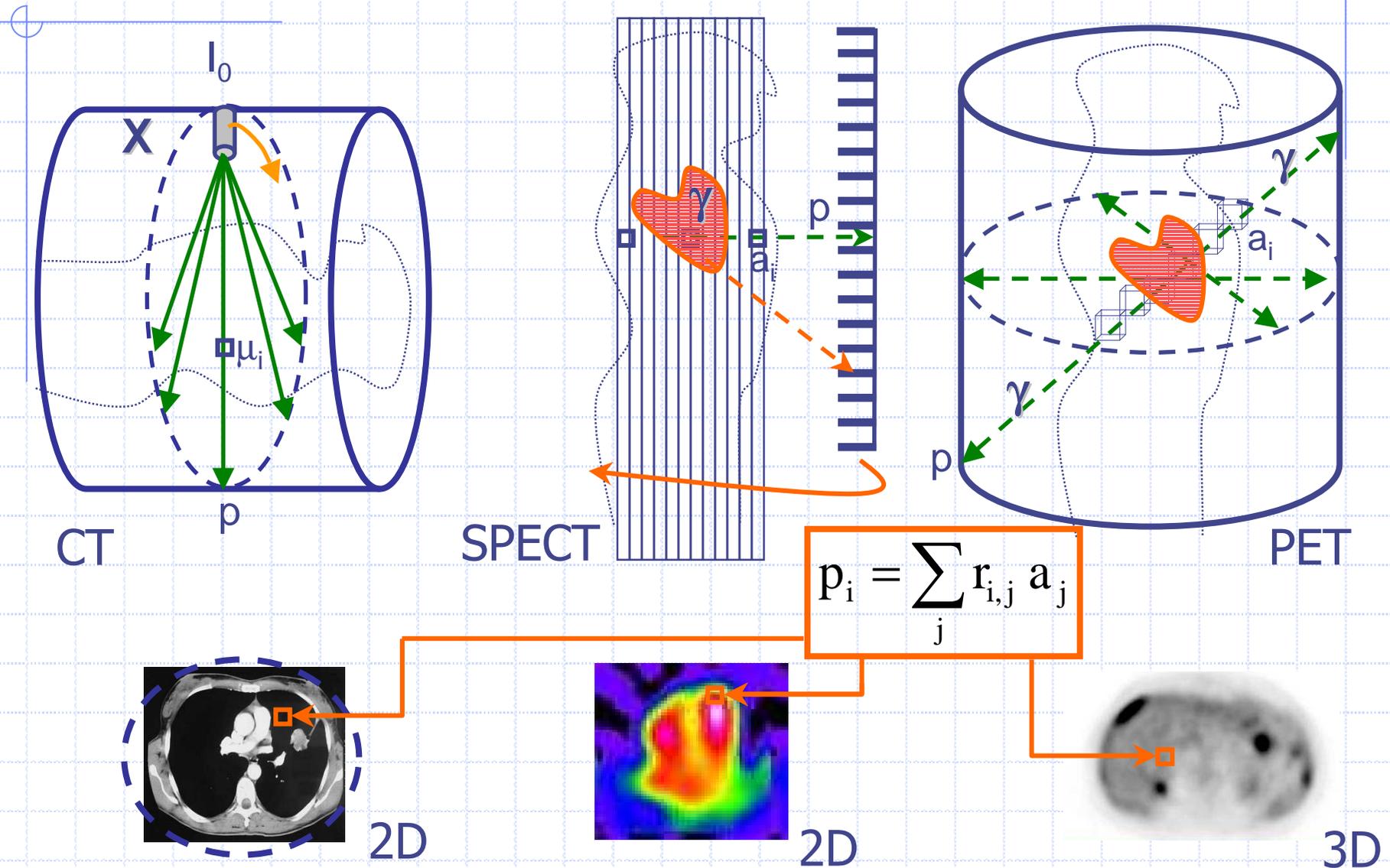


$$p_i = \sum_j r_{i,j} a_j$$

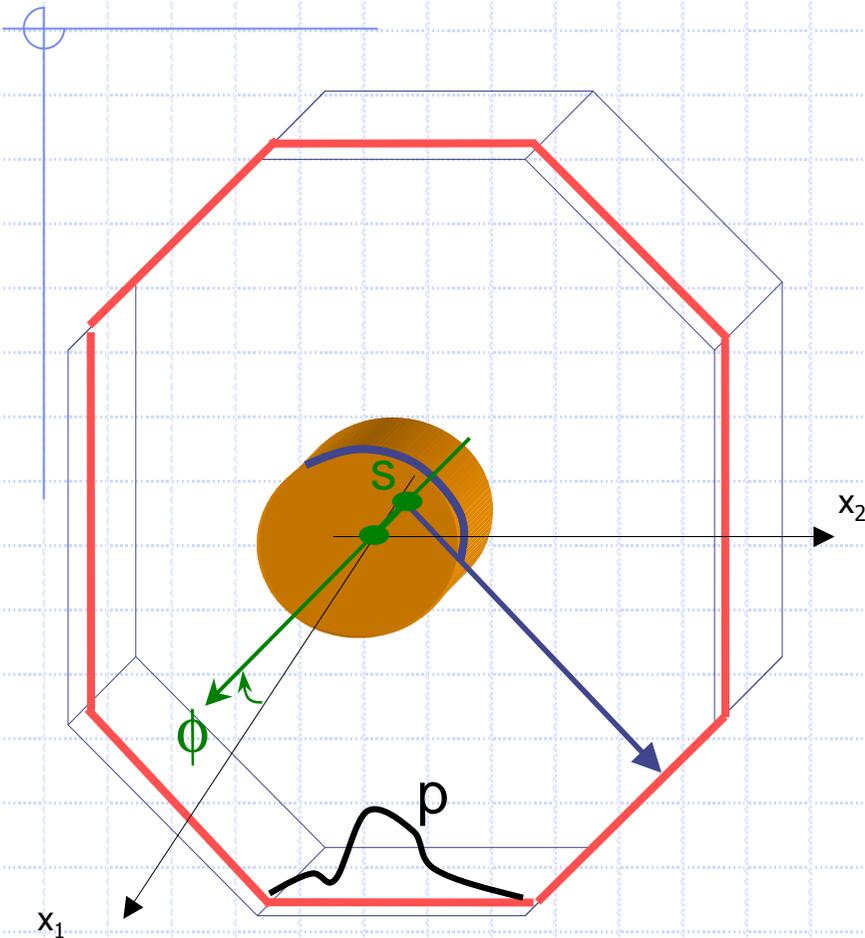


3D

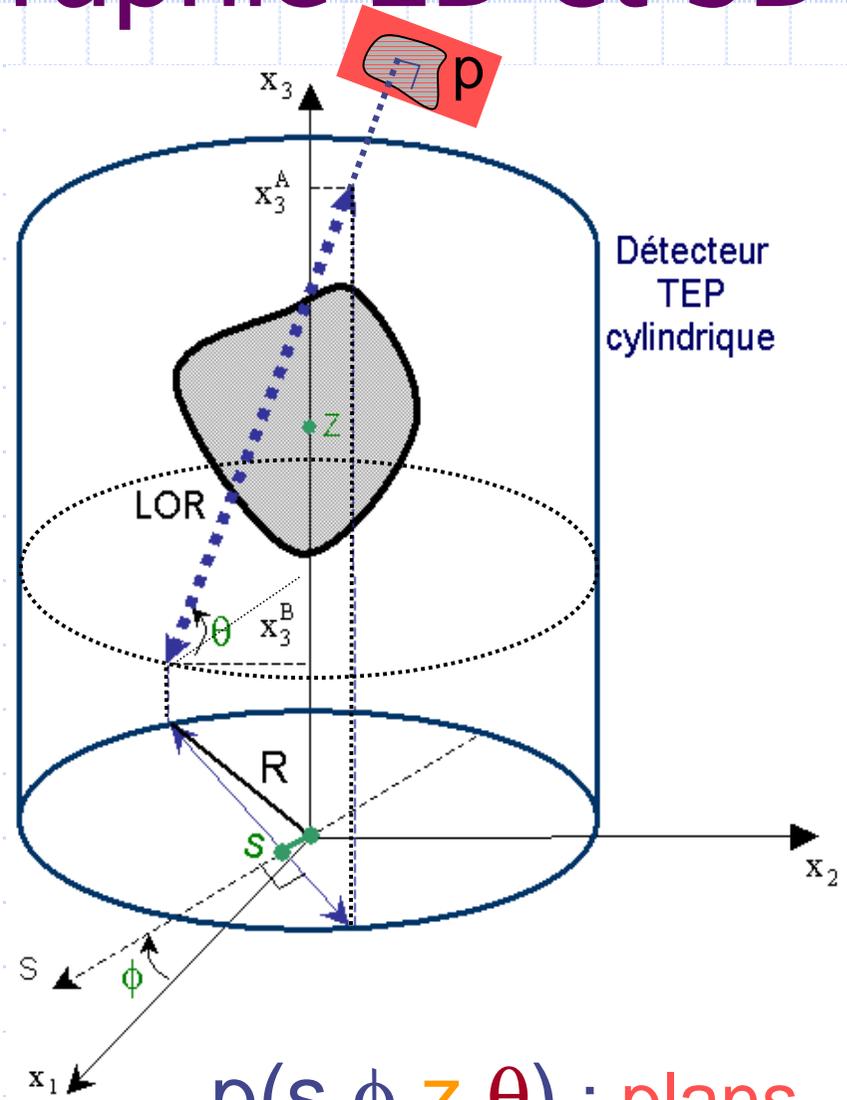
Tomographie: problème inverse linéaire



Codage en tomographie 2D et 3D

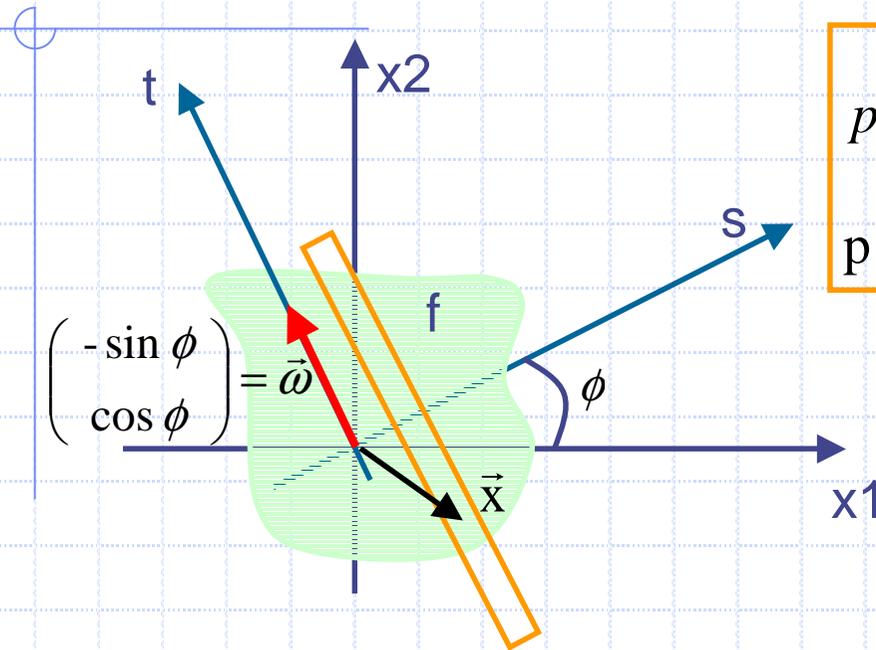


$p(s, \phi)$: lignes



$p(s, \phi, z, \theta)$: plans

Modélisation analytique

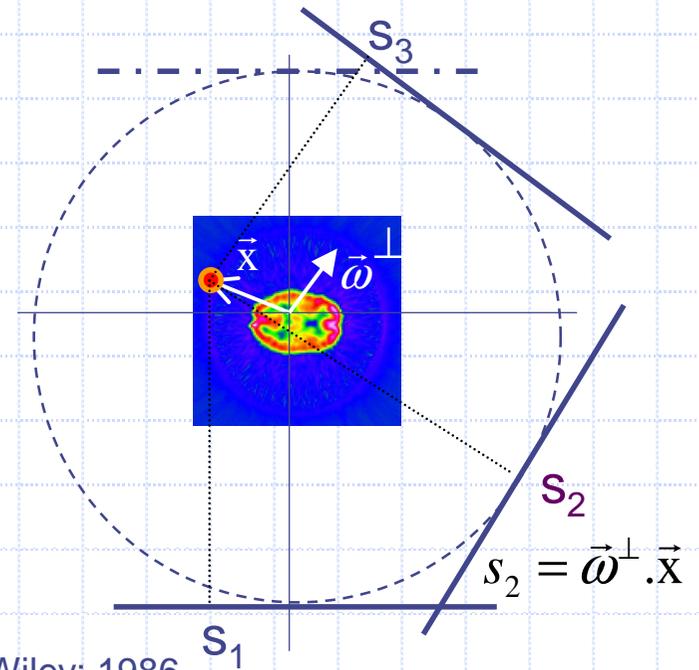


$$p(\vec{\omega}, s) = p_{\vec{\omega}}(s) = \int_t f(s \vec{\omega}^\perp + t \vec{\omega}) dt$$

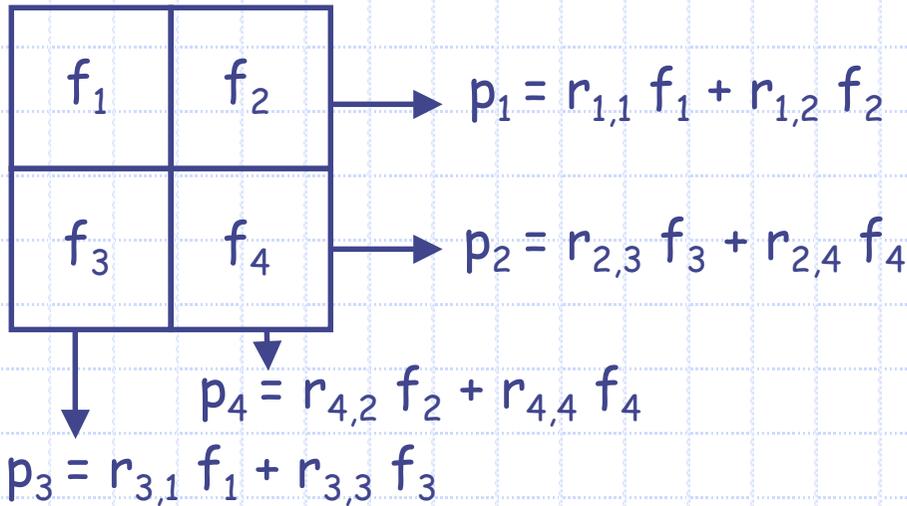
$p = Rf$ transformée de Radon

$$(R^* p)(\vec{x}) = \int_{\phi=0}^{\pi} p(\vec{\omega}, \vec{\omega}^\perp \cdot \vec{x}) d\phi$$

rétroprojection = épandage



Modélisation algébrique



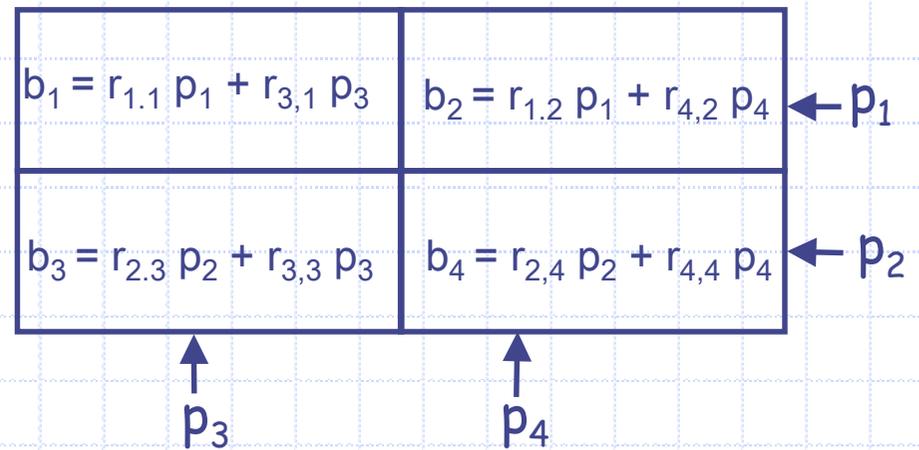
$$\begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ r_{2,1} & r_{2,2} & r_{2,3} & r_{2,4} \\ r_{3,1} & r_{3,2} & r_{3,3} & r_{3,4} \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{pmatrix} \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

$r_{i,j}$ = % du pixel j intersecté par la projection i

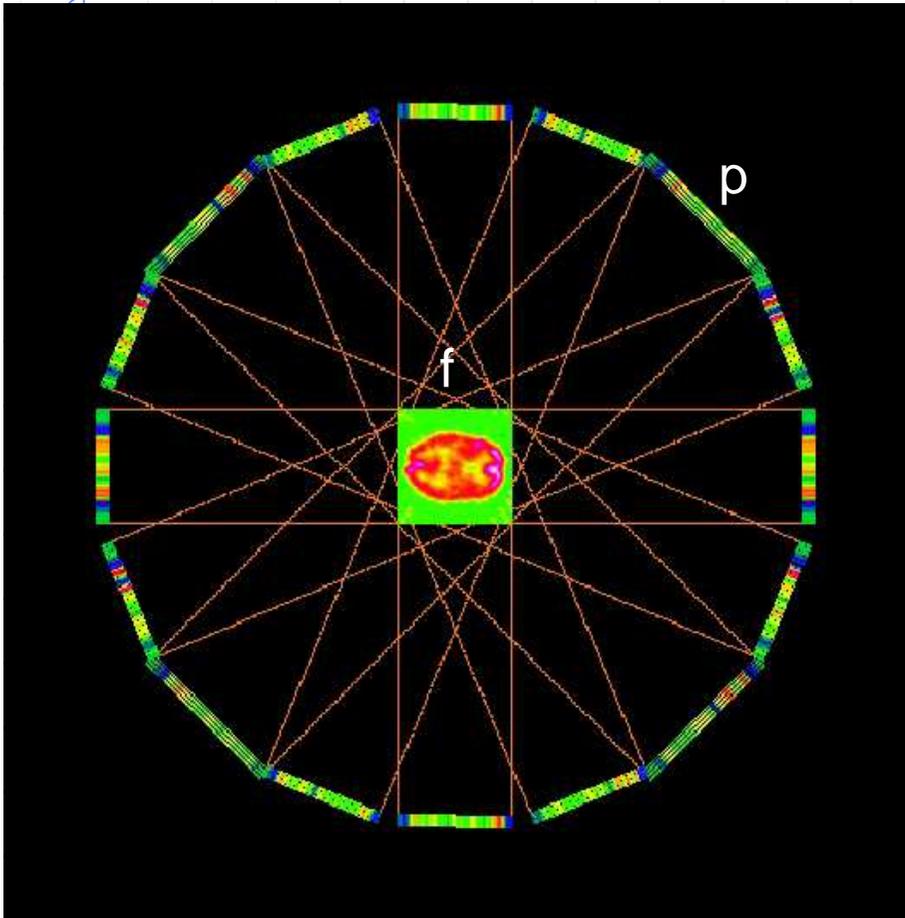
$$\mathbf{R} \cdot \vec{f} = \vec{p}$$

$$\begin{pmatrix} r_{1,1} & r_{2,1} & r_{3,1} & r_{4,1} \\ r_{1,2} & r_{2,2} & r_{3,2} & r_{4,2} \\ r_{1,3} & r_{2,3} & r_{3,3} & r_{4,3} \\ r_{1,4} & r_{2,4} & r_{3,4} & r_{4,4} \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

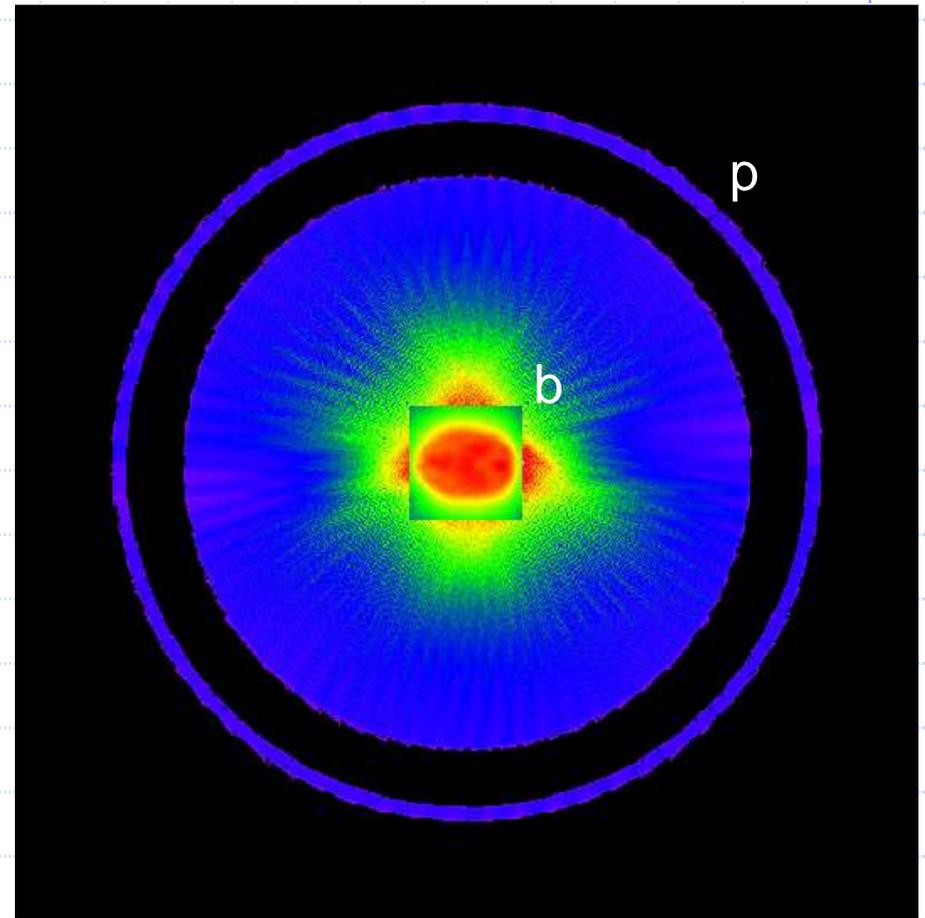
$${}^t\mathbf{R} \cdot \vec{p} = \vec{b}$$



Projection / Rétroprojection

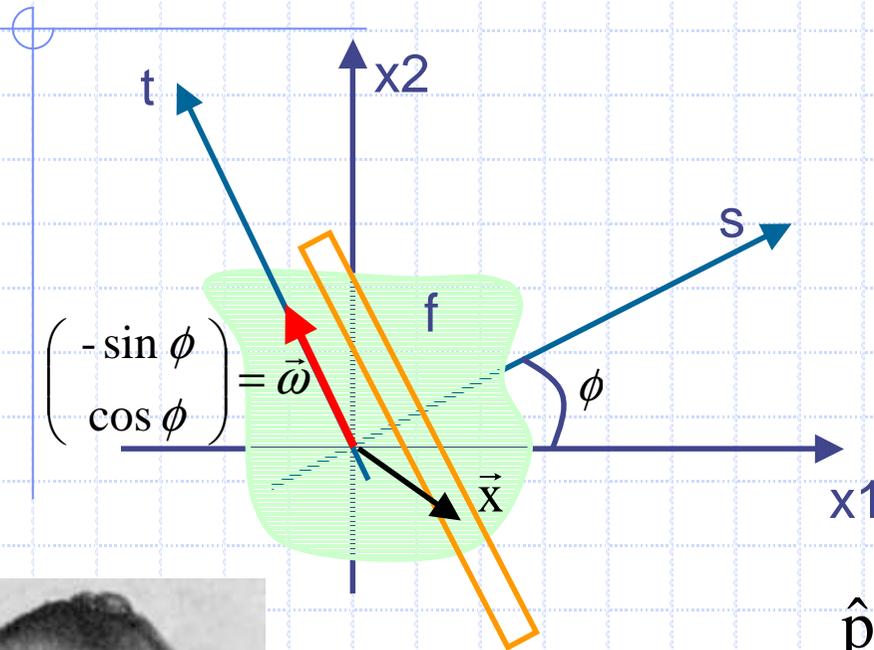


$$\mathbf{R} \cdot \vec{f} = \vec{p}$$



$${}^t\mathbf{R} \cdot \vec{p} = \vec{b}$$

Théorème de la projection



$$\begin{cases} p_{\vec{\omega}}(s) = \int_t f(s\vec{\omega}^\perp + t\vec{\omega}) dt \\ \hat{p}_{\vec{\omega}}(\sigma) = \int_s p_{\vec{\omega}}(s) \cdot e^{-i \cdot s \cdot \sigma} ds \end{cases}$$

$$\hat{p}_{\vec{\omega}}(\sigma) = \int_s \int_t f(s\vec{\omega}^\perp + t\vec{\omega}) e^{-i \cdot s \cdot \sigma} dt ds$$

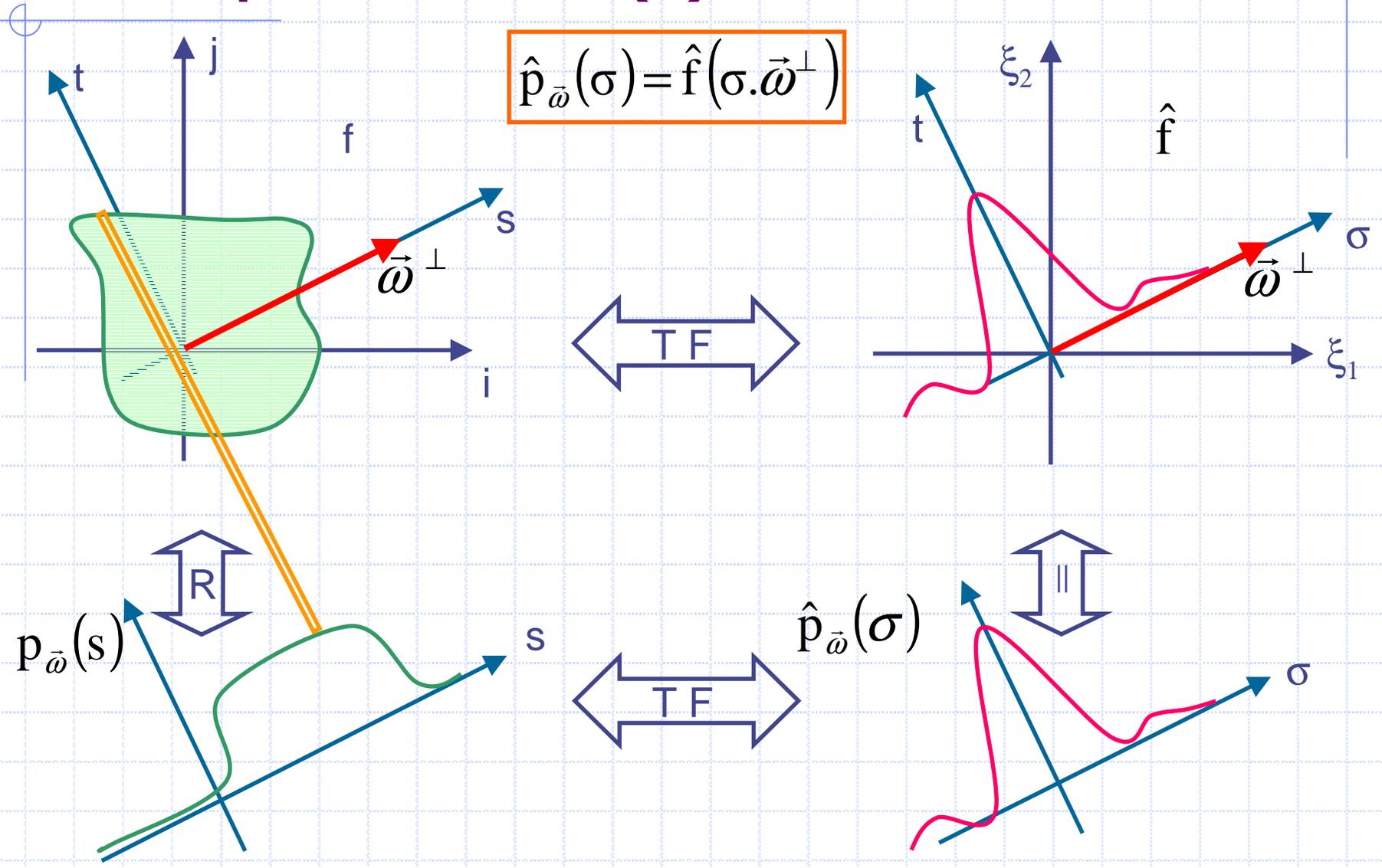
$$\hat{p}_{\vec{\omega}}(\sigma) = \iint f(\vec{x}) e^{-i \cdot \sigma \vec{x} \cdot \vec{\omega}^\perp} d\vec{x}$$

$$\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}(\sigma \cdot \cos \phi, \sigma \cdot \sin \phi) = \hat{f}(\sigma \cdot \vec{\omega}^\perp)$$

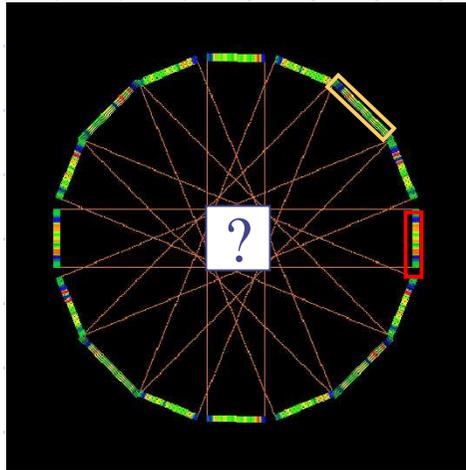


J. Radon
1887-1956

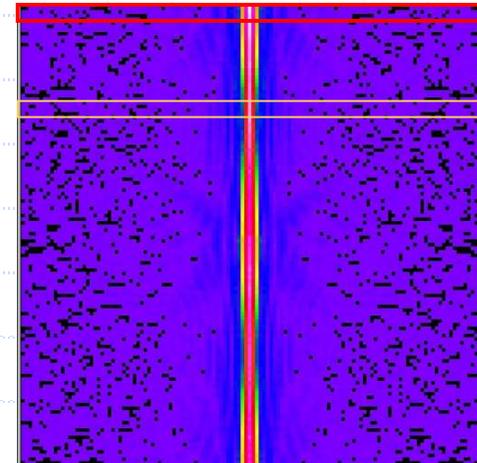
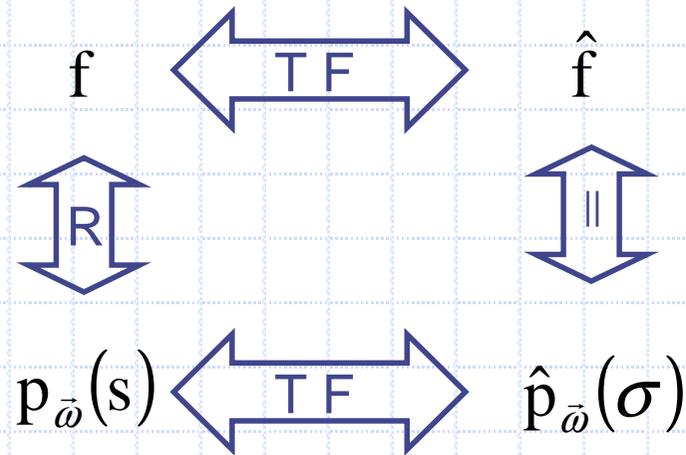
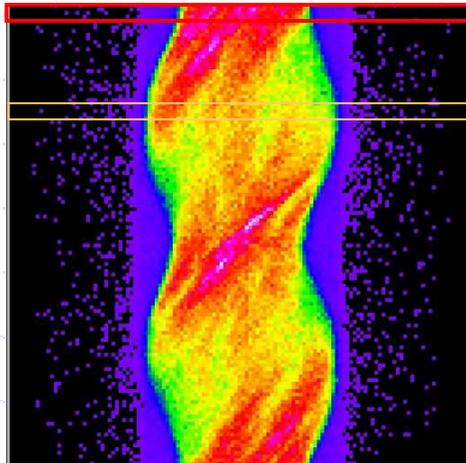
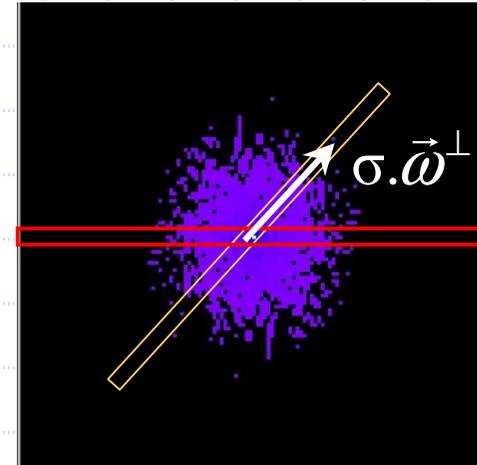
Interprétation (I)



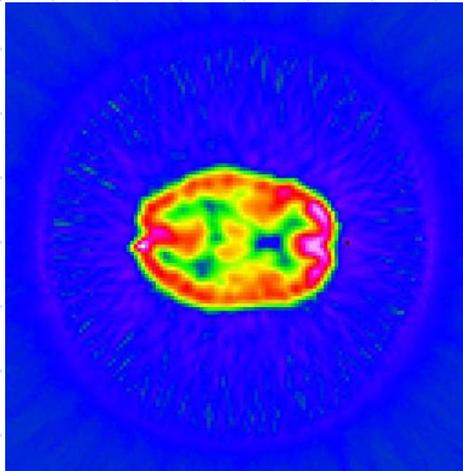
Interprétation (II)



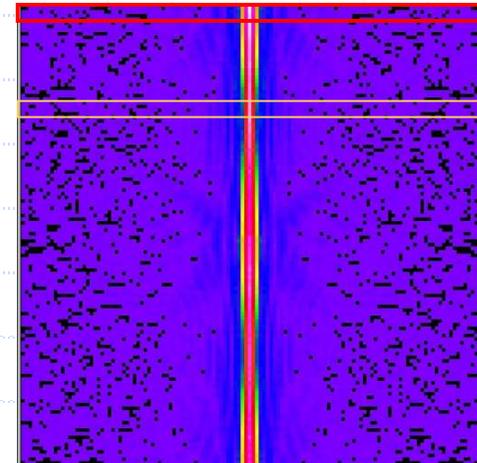
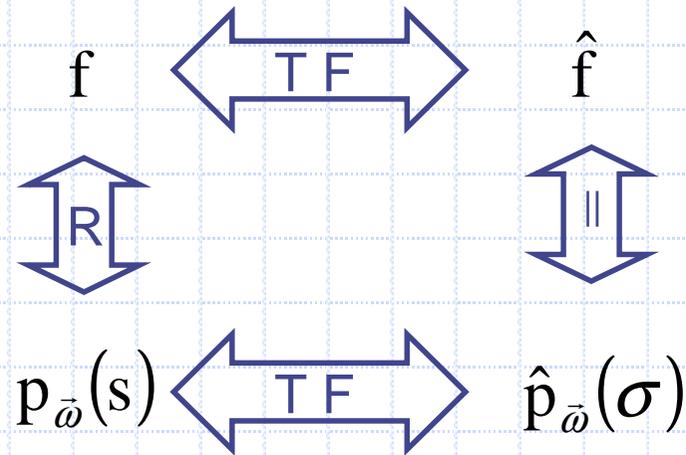
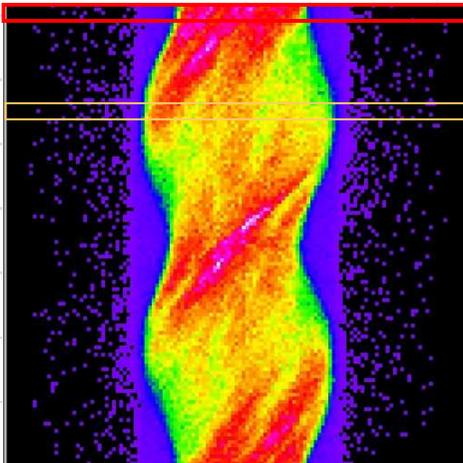
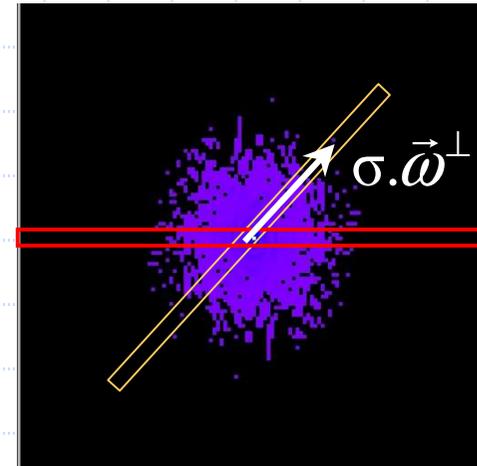
$$\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}(\sigma \cdot \vec{\omega}^\perp)$$



Interprétation (II)



$$\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}(\sigma \cdot \vec{\omega}^\perp)$$



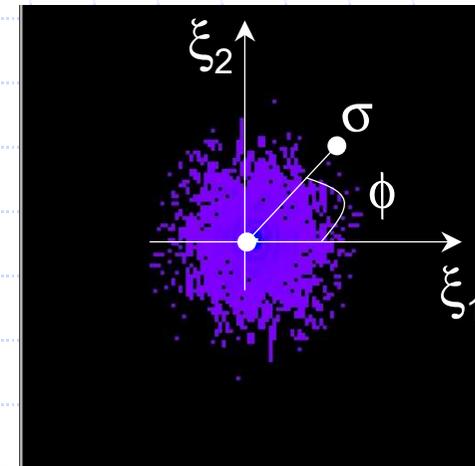
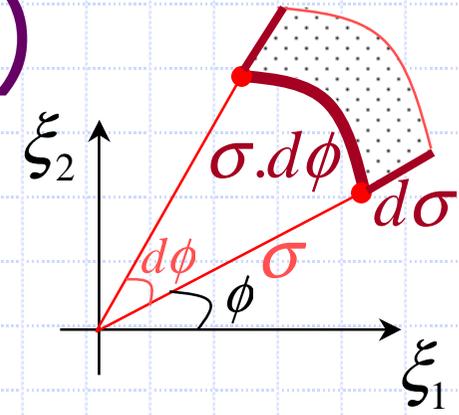
Rétroprojection filtrée (I)

$$f(\vec{x}) = \iint \widehat{f}(\vec{\xi}) e^{i\vec{x} \cdot \vec{\xi}} d\vec{\xi}$$

$$f(\vec{x}) = \int_{\phi=0}^{\pi} \int_{\sigma=-\infty}^{\sigma=+\infty} \widehat{f}(\sigma \vec{\omega}^{\perp}) e^{i\sigma \vec{\omega}^{\perp} \cdot \vec{x}} |\sigma| d\sigma d\phi$$

$$f(\vec{x}) = \int_{\phi=0}^{\pi} \int_{\sigma=-\infty}^{\sigma=+\infty} \widehat{p}_{\vec{\omega}}(\sigma) |\sigma| e^{i\sigma \vec{\omega}^{\perp} \cdot \vec{x}} d\sigma d\phi$$

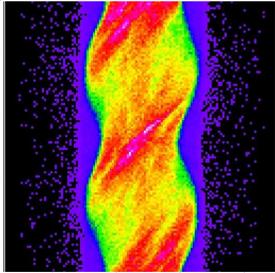
$$\underbrace{\text{TF}_s^{-1}[\widehat{p}_{\vec{\omega}} \cdot \text{abs}]}_{p_{\vec{\omega}}^f}(\vec{\omega}^{\perp} \cdot \vec{x})$$



J. Radon
1887-1956

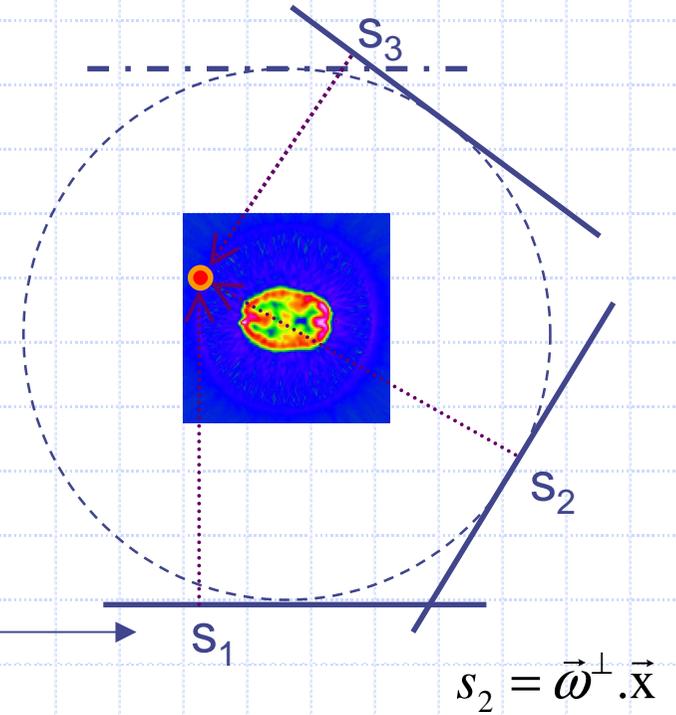
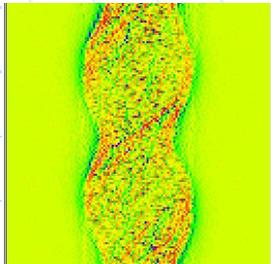
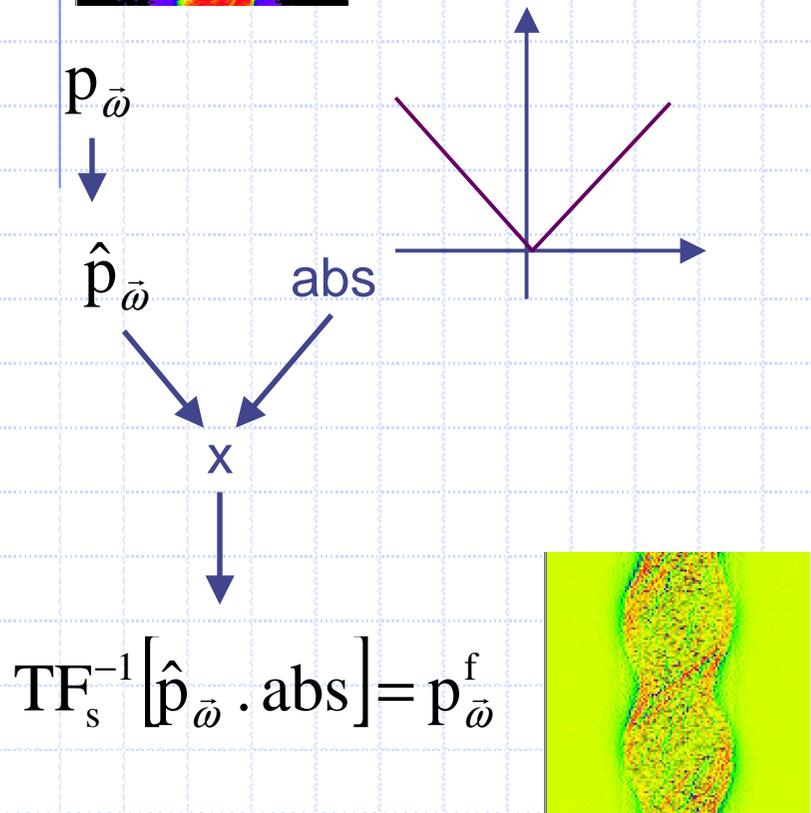
$$f(\vec{x}) = (\mathbf{R}^* p^f)(\vec{x})$$

Rétroprojection filtrée (II)

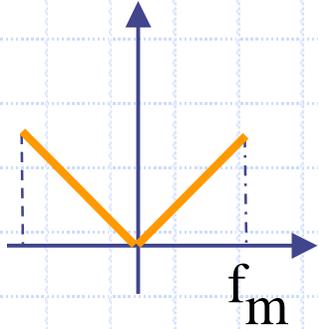


$$f(\vec{x}) = (\mathbf{R}^* p^f)(\vec{x})$$

Projections sur 180°



Rétroprojection filtrée (III)



$$f_m = \frac{1}{2 \cdot \Delta r} = \frac{f_e}{2}$$

$$RL(x) = \frac{f_m \sin(2\pi f_m x)}{\pi x} - \frac{1 - \cos(2\pi f_m x)}{2\pi^2 x^2}$$

$$RL(k \cdot \Delta r) = \begin{cases} \frac{1}{4(\Delta r)^2} & k = 0 \\ 0 & k \neq 0 \text{ pair} \\ -\frac{1}{(k\pi \Delta r)^2} & k \text{ impair} \end{cases}$$

$$p_{\bar{\omega}}^f = TF_s^{-1} [\hat{p}_{\bar{\omega}} \cdot \text{abs}]$$

$$p_{\bar{\omega}}^f = p_{\bar{\omega}} * RL$$

Exemple pour $\Delta r = \frac{1}{2}$, on obtient le filtre : $\left(0 \quad -\frac{1}{2,5} \quad 1 \quad -\frac{1}{2,5} \quad 0 \right)$

Rétroprojection filtrée (IV)

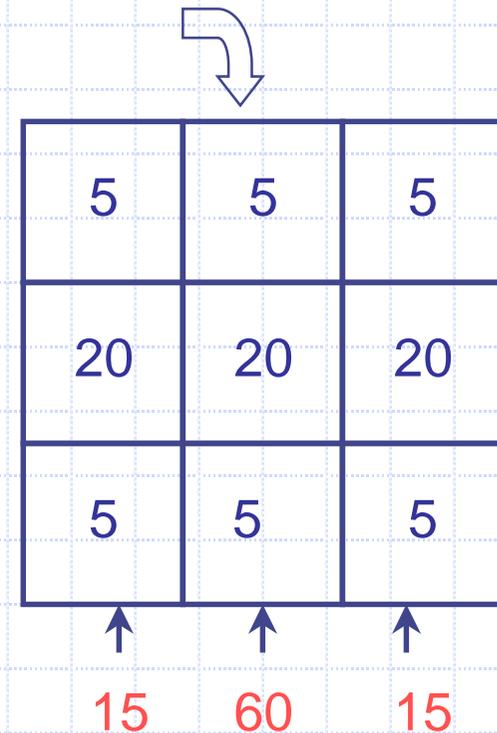
0	0	0	← 15 ← 45
0	0	0	← 60 ← 90
0	0	0	← 15 ← 45

45 90 45

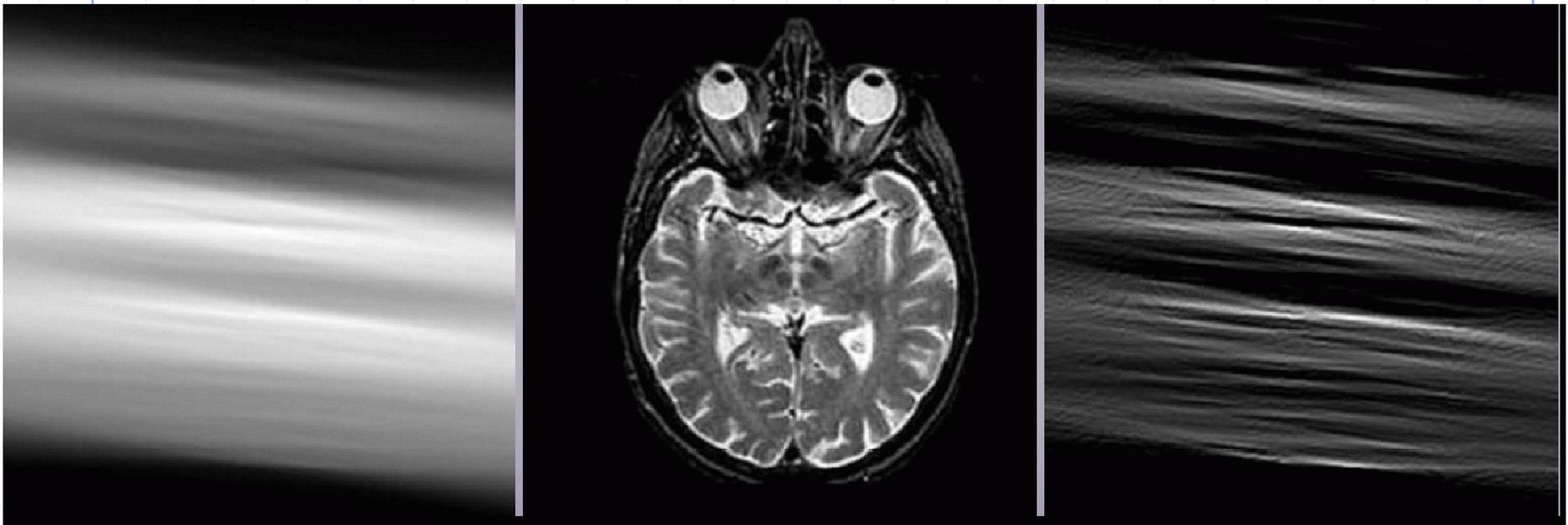
$$\text{Filtre} = \begin{pmatrix} -\frac{1}{3} & 1 & -\frac{1}{3} \end{pmatrix}$$

10	25	10
25	40	25
10	25	10

⇒



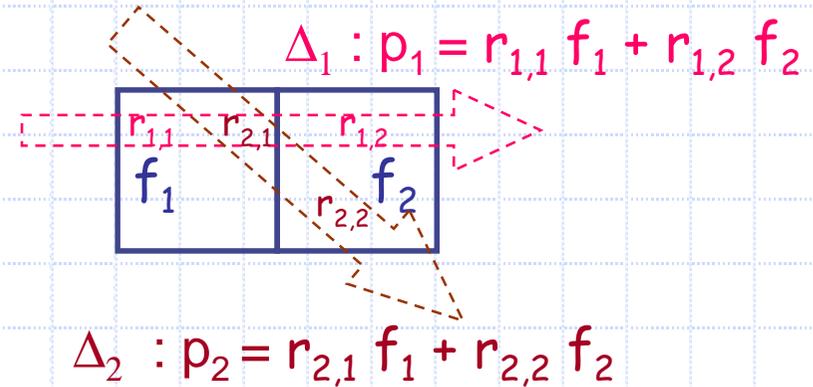
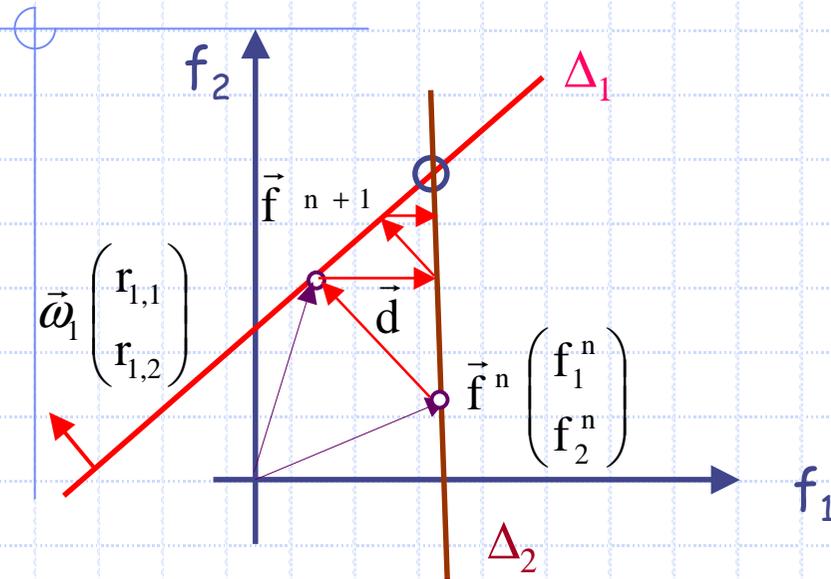
Rétroprojection filtrée (V)



$$R^* p$$

$$R^* TF_s^{-1} [\hat{p} . abs]$$

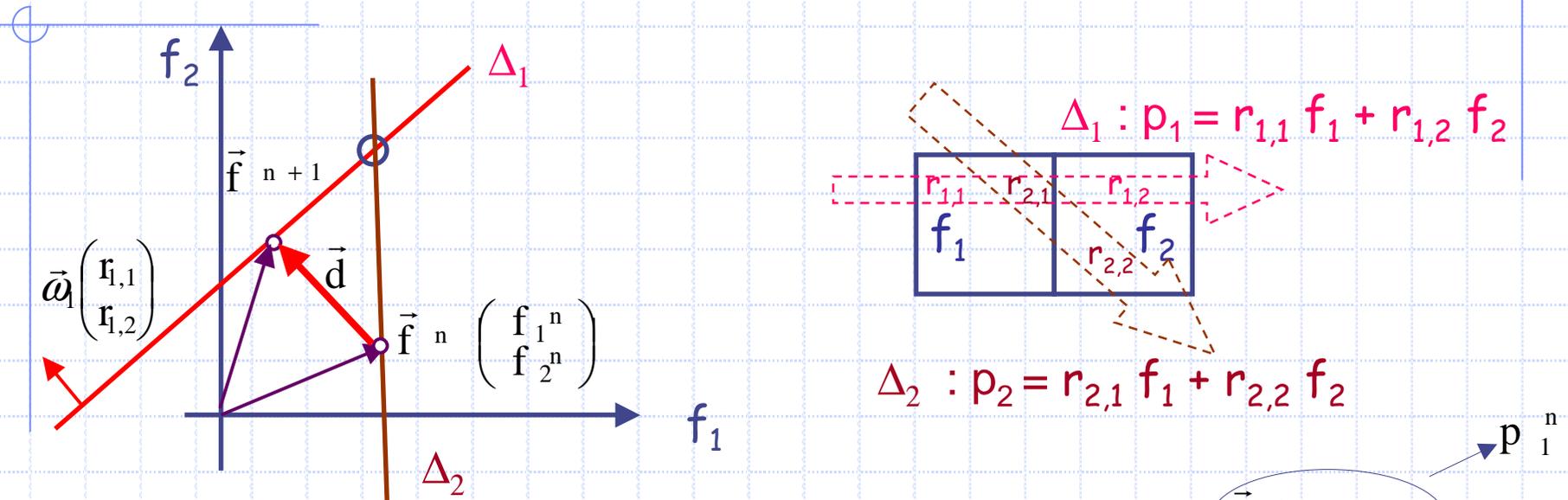
Algebraic Reconstruction Technique (I)



S. Kaczmarz
1895-1940

On construit une suite de coupes \vec{f}^n en projetant chaque itéré sur l'un puis l'autre hyperplan.

Algebraic Reconstruction Technique (II)



$$\vec{f}^{n+1} = \vec{f}^n + d \frac{\vec{\omega}_1}{\|\vec{\omega}_1\|} \quad d = \frac{p_1 - \vec{f}^n \cdot \vec{\omega}_1}{\|\vec{\omega}_1\|}$$

$$\vec{f}^{n+1} = \vec{f}^n + \frac{p_1 - p_1^n}{\|\vec{\omega}_1\|^2} \vec{\omega}_1$$

$$\vec{f}^{n+1} = \vec{f}^n + R^* (p_1 - p_1^n)$$



S. Kaczmarz
1895-1940

Algebraic Reconstruction Technique (III)

0	0	0
0	0	0
0	0	0

← 45 - 0 = 15 + 15 + 15

← 90 - 0 = 30 + 30 + 30

← 45 - 0

45 90 45

⇒

15	15	15
30	30	30
15	15	15

↓ ↓ ↓
 45 90 45
 - 60 60 60
 -15 30 -15

10	25	10
25	40	25
10	25	10

$$\vec{f}^{n+1} = \vec{f}^n + R^* (p_1 - p_1^n)$$

MLEM et OSEM

Bayes : $P(\vec{f} / \vec{p}) = P(\vec{p}/\vec{f}) \cdot P(\vec{f}) / P(\vec{p}) = P(\vec{p}/\vec{f}) \cdot P(\vec{f})$

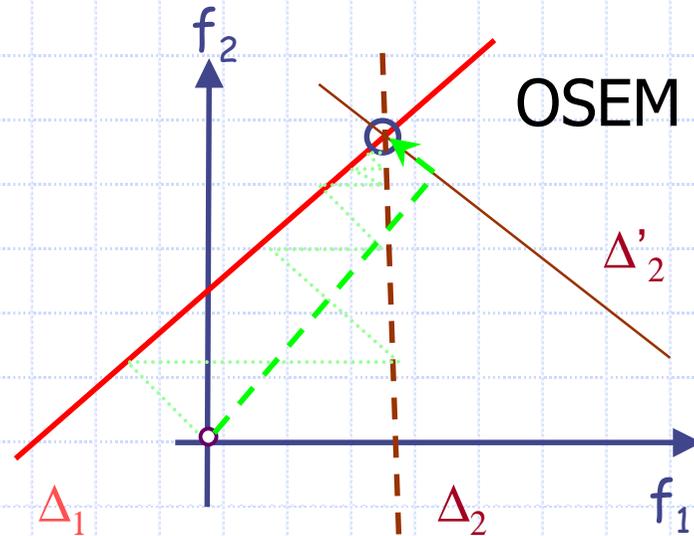
$$\vec{f} \approx = \arg \min_{\vec{f}} \left[-\log P(\vec{p}/\vec{f}) - \log P(\vec{f}) \right]$$

Adéquation aux données

$$\log \prod_i \frac{e^{-\tilde{p}_i} \tilde{p}_i^{p_i}}{p_i!}$$

$$f_i^{n+1} = f_i^n \cdot \frac{1}{\sum_{l'=1}^P r_{l',i}} \left(\sum_{l=1}^P r_{l,i} \frac{p_l}{\sum_{s=1}^N r_{l,s} f_s^n} \right)$$

$$R * \begin{bmatrix} p_1 \\ p_1^n \end{bmatrix}$$



Gradient Conjugué

$$\bar{\mathbf{f}} = \arg \min_{\mathbf{f} \in \mathbb{C}} \left\| \mathbf{R}\bar{\mathbf{f}} - \bar{\mathbf{p}} \right\|^2$$

Initialisation :

$$\bar{\mathbf{d}}^0 = \bar{\mathbf{r}}^0 = \mathbf{R}^* \cdot \bar{\mathbf{p}} \longrightarrow$$

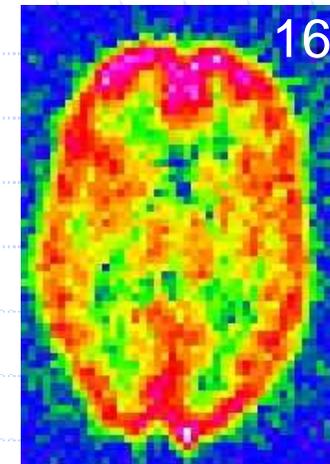
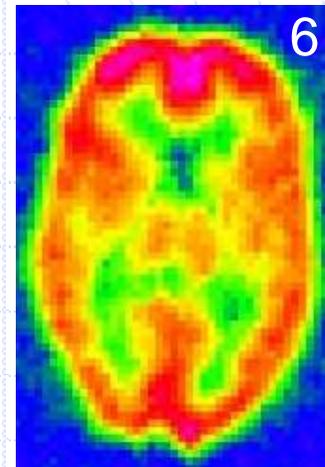
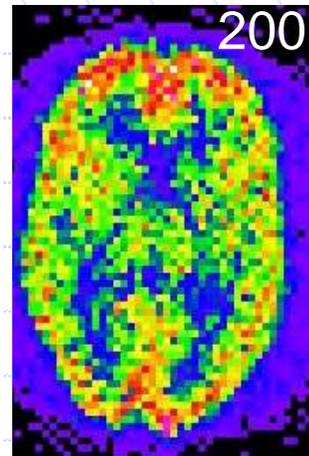
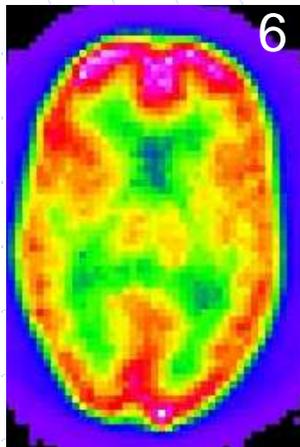
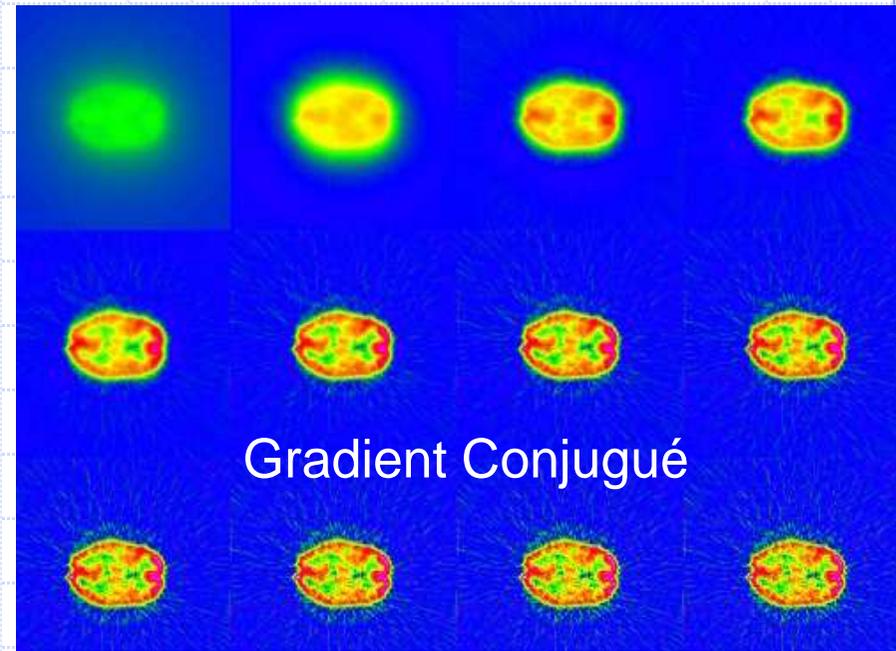
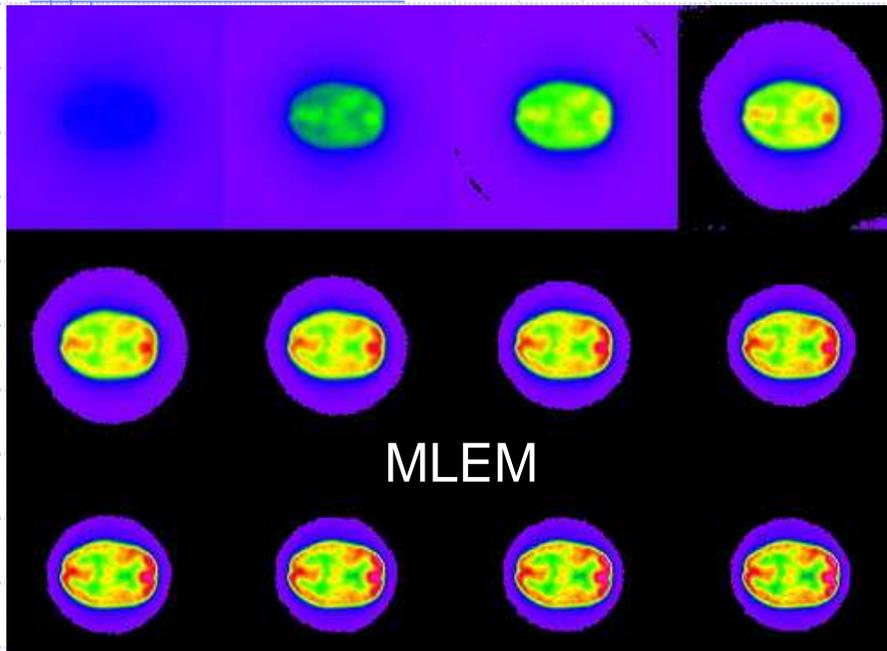
$$\omega^j = \frac{\|\bar{\mathbf{r}}^j\|^2}{\langle \bar{\mathbf{d}}^j | \mathbf{R}^* \cdot \mathbf{R} \cdot \bar{\mathbf{d}}^j \rangle}$$

$$\bar{\mathbf{r}}^{j+1} = \bar{\mathbf{r}}^j - \omega^j \cdot \mathbf{R}^* \cdot \mathbf{R} \cdot \bar{\mathbf{d}}^j$$

$$\bar{\mathbf{d}}^{j+1} = \bar{\mathbf{r}}^{j+1} + \frac{\|\bar{\mathbf{r}}^{j+1}\|^2}{\|\bar{\mathbf{r}}^j\|^2} \cdot \bar{\mathbf{d}}^j$$

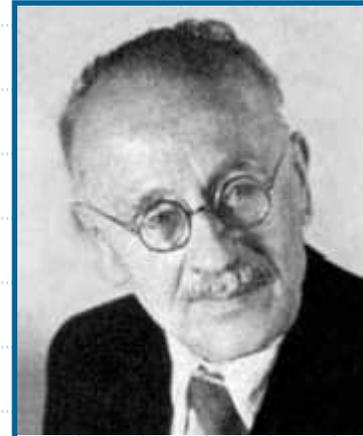
$$\bar{\mathbf{f}}^{j+1} = \bar{\mathbf{f}}^j + \omega^j \cdot \bar{\mathbf{d}}^j$$

Exemples





G. Hounsfield 1919-2004

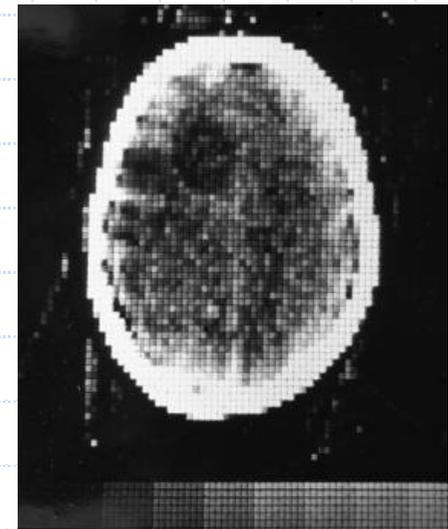
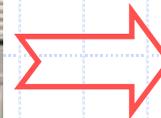


J. Radon
1887-1956

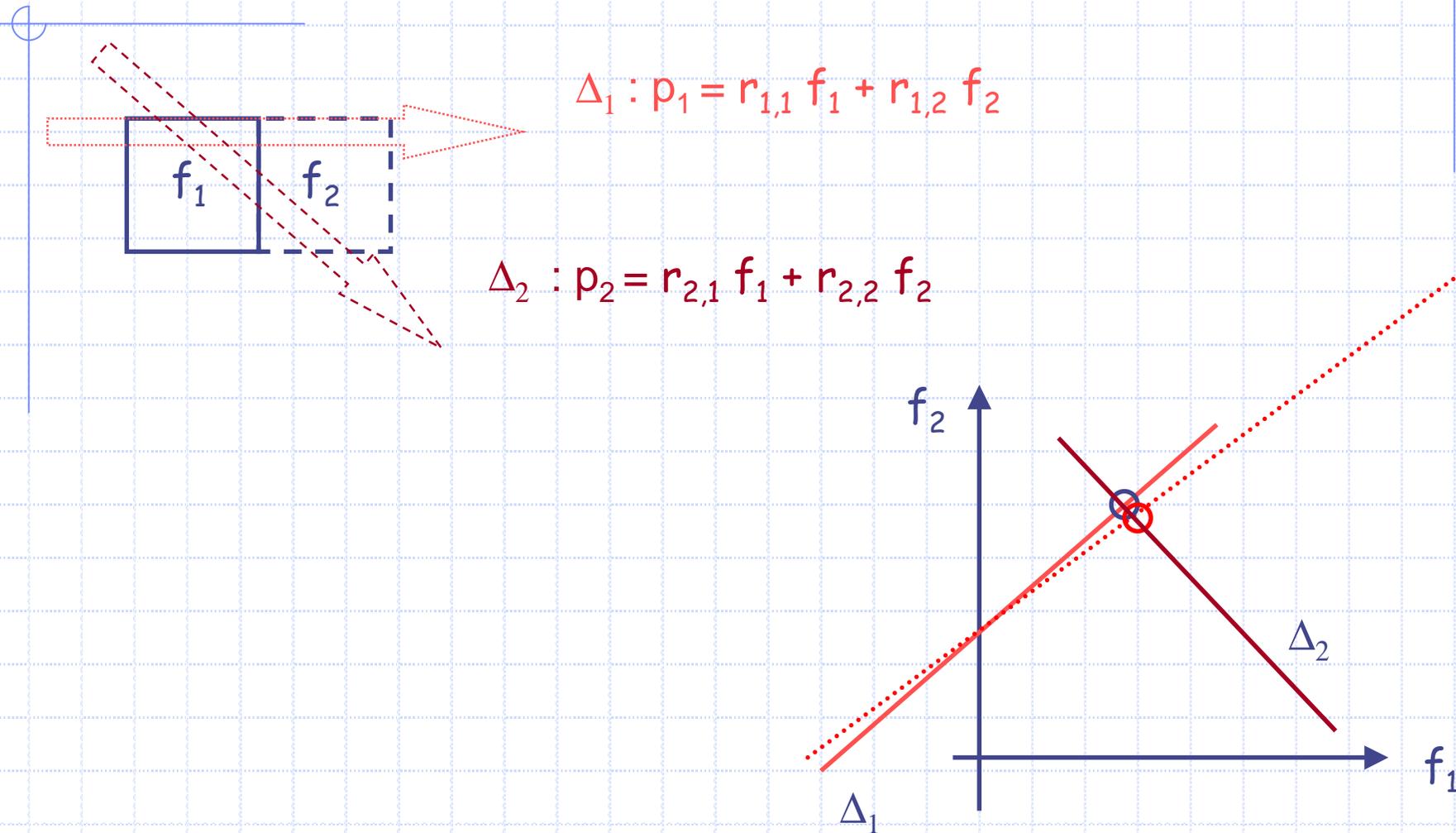


S. Kaczmarz
1895-1940

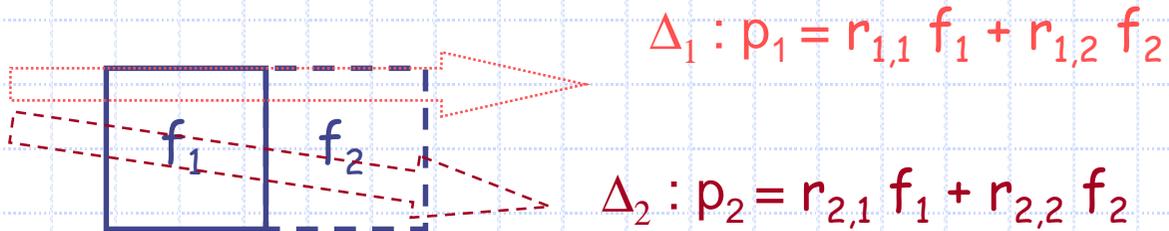
$\hat{p}_{\vec{\theta}}(\sigma) = \hat{f}(\sigma \cdot \vec{\theta})$



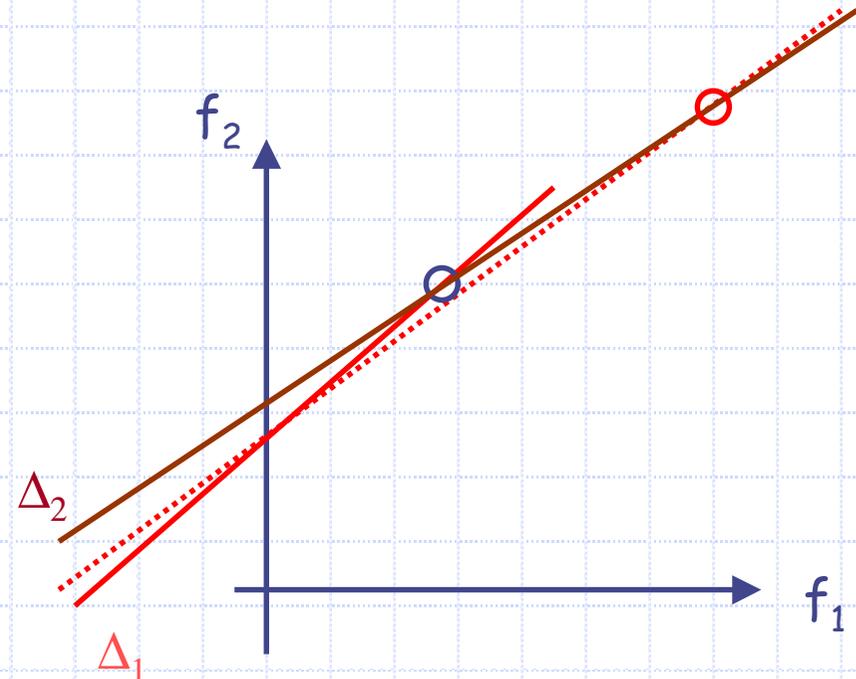
Approche intuitive (I)



Approche intuitive (II)



$64^2 = 4\ 096$
 $128^2 = 16\ 384$
 $256^2 = 65\ 536$
 $512^2 = 262\ 144$



Exemple

$$R \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 32 \\ 23 \\ 33 \\ 31 \end{pmatrix}$$

Matrice de Wilson

Det = 1

$$\begin{pmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{pmatrix} \begin{pmatrix} 9,2 \\ -12,6 \\ 4,5 \\ -1,1 \end{pmatrix} = \begin{pmatrix} 32,1 \\ 22,9 \\ 33,1 \\ 30,9 \end{pmatrix} \qquad \begin{pmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{pmatrix} \begin{pmatrix} -7,2 \\ 14,6 \\ -2,5 \\ 3,1 \end{pmatrix} = \begin{pmatrix} 31,9 \\ 23,1 \\ 32,9 \\ 31,1 \end{pmatrix}$$

$$\kappa(R) = \|R\| \cdot \|R^{-1}\| = \frac{\mu_{\max}}{\mu_{\min}}$$

où $\mu = \sqrt{\text{valeurs propres de } R^t R}$

$$\text{Sp}(R) \approx \{0,01; 0,84; 3,86; 30,29\} \Rightarrow \kappa(R) \approx \frac{30,29}{0,01} \approx 3029$$

La matrice de Wilson est très mal conditionnée ($\kappa \gg 1$)

Problème d'Hadamard bien posé ?

✓ En continu : $\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}(\sigma \cdot \vec{\omega}^\perp)$, R bijectif d'inverse continue (conditions d'Hadamard).

✓ En discret, les choses sont moins simples :

- R surjectif ? $\iff \overset{\square}{\mathbf{R}} \cdot \vec{f} = \mathbf{A} \vec{f} = \overset{\square}{\mathbf{R}} \cdot \vec{p} = \vec{q} \iff \vec{f} = \arg \min_{f \in C} \|\vec{p} - \mathbf{R} \vec{f}\|^2$
- R injectif ? : choix parmi les solutions possibles
- \mathbf{R}^{-1} continue mais $\|\mathbf{R}^{-1}\|$ grande : $\kappa(\mathbf{R}) = \|\mathbf{R}\| \|\mathbf{R}^{-1}\| = \frac{\mu_{\max}}{\mu_{\min}} \gg 1$

$$\frac{\|\delta \vec{f}\|}{\|\vec{f}\|} \leq \frac{\kappa(\mathbf{R})}{1 - \kappa(\mathbf{R}) \frac{\|\delta \mathbf{R}\|}{\|\mathbf{R}\|}} \left[\frac{\|\delta \vec{p}\|}{\|\vec{p}\|} + \frac{\|\delta \mathbf{R}\|}{\|\mathbf{R}\|} \right]$$

Estimation de $\kappa(R)$

◆ Gradient conjugué

◆ Approximation de Galerkin

$$\omega^j = \frac{\|\vec{r}^j\|^2}{\langle \vec{d}^j | R^* \cdot R \cdot \vec{d}^j \rangle} \quad , \quad \beta^j = \frac{\|\vec{r}^{j+1}\|^2}{\|\vec{r}^j\|^2}$$

$$\begin{pmatrix} \frac{1}{\omega^0} & -\frac{\sqrt{\beta^0}}{\omega^0} & 0 & 0 \\ -\frac{\sqrt{\beta^0}}{\omega^0} & \frac{1}{\omega^1} + \frac{\beta^0}{\omega^0} & \ddots & 0 \\ 0 & \ddots & \ddots & -\frac{\sqrt{\beta^{j-1}}}{\omega^{j-1}} \\ 0 & 0 & -\frac{\sqrt{\beta^{j-1}}}{\omega^{j-1}} & \frac{1}{\omega^j} + \frac{\beta^{j-1}}{\omega^{j-1}} \end{pmatrix}$$

- ◆ dont le spectre CV vers celui de R: estimation de $\kappa(R)$
- ◆ qui borne l'erreur sur la solution et le spectre

Régularisation de Tikhonov

Remplacer : $\bar{f} = \arg \min_{f \in C} \|\vec{p} - R\vec{f}\|^2$ par

$$\bar{f} = \arg \min_{f \in C} \left\{ \|\vec{p} - R\vec{f}\|^2 + \alpha \cdot \rho(\vec{f}) \right\}$$

Adéquation aux données
Surjectivité du problème inverse

Régularisation
injectivité

Exemple : régularisation de Tikhonov (cf. pseudo-inverse de Moore-Penrose)

$$\vec{f} = \arg \min_{\vec{f}} \left\{ \|\vec{p} - R\vec{f}\|^2 + \alpha \cdot \|\vec{f}\|^2 \right\} \Leftrightarrow (R^t R + \alpha I) f = R^t p$$

$$\vec{f} = (R^t R + \alpha I)^{-1} R^t p$$

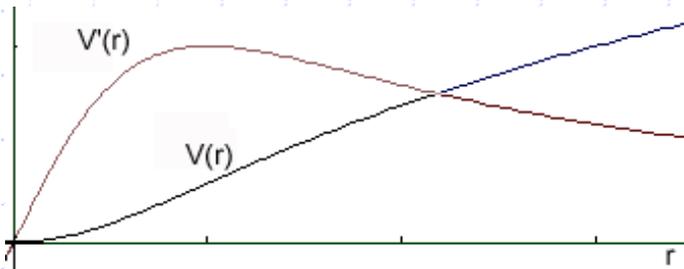
Régularisation MAP-EM-OSL

Bayes : $P(\vec{f} / \vec{p}) = P(\vec{p}/\vec{f}) \cdot P(\vec{f})/P(\vec{p}) = P(\vec{p}/\vec{f}) \cdot P(\vec{f})$

$$\vec{f} \approx \arg \min_{\vec{f}} \left[-\log P(\vec{p}/\vec{f}) - \log P(\vec{f}) \right]$$

Adéquation aux données

$$\log \prod_i \frac{e^{-\tilde{p}_i} \tilde{p}_i^{p_i}}{p_i!}$$



régularisation

$$-\beta \cdot \sum_{i,j} w_{i,j} \cdot V(f_i - f_j)$$

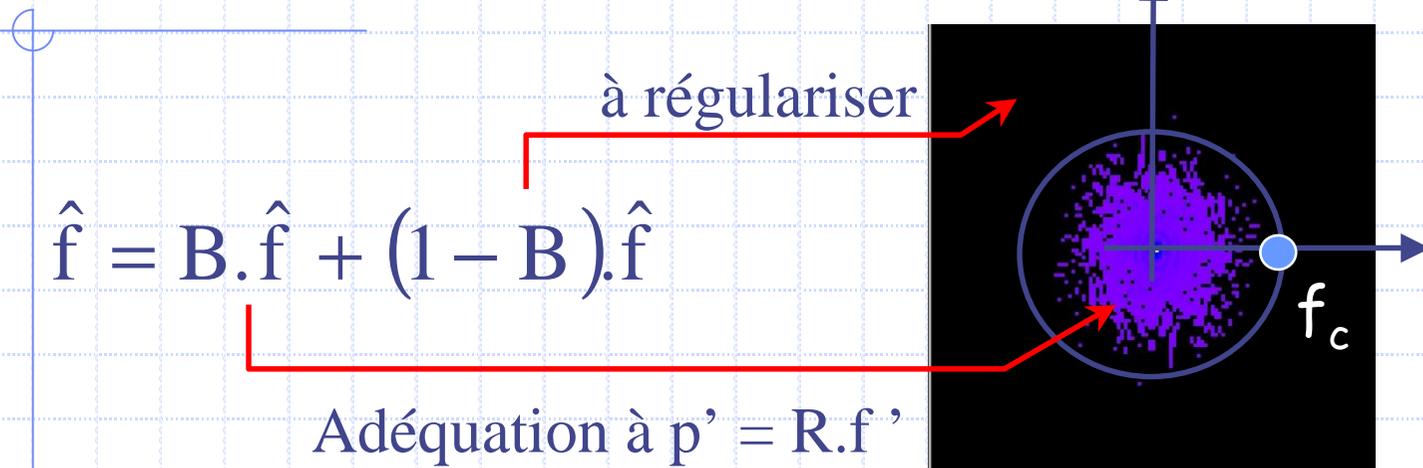
Gibbs : $P(\vec{f}) = \frac{1}{K} e$



$$f_i^{n+1} = f_i^n \cdot \frac{1}{\sum_{l=1}^P r_{l,i} + \beta \cdot \partial U} \cdot \sum_{l=1}^P r_{l,i} \frac{p_l}{\sum_{s=1}^N r_{l,s} f_s^n}$$

$$\partial U = \sum_{f_k \in V(f_i)} w_{i,k} \cdot \frac{\partial V}{\partial r} (f_i - f_k)$$

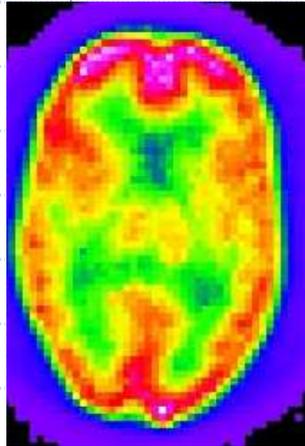
Régularisation de Fourier FRECT



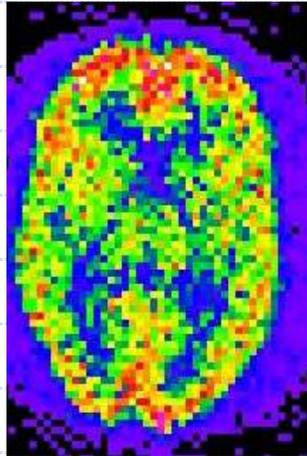
$$\hat{p}' = \hat{f}' = B.\hat{f} = B.\hat{p} \Rightarrow p' = TF_s^{-1} B.\hat{p}$$

$$E(f) = \left\| TF_s^{-1}(B.\hat{p}) - Rf \right\|^2 + \left\| (1 - B).\hat{f} \right\|^2 = \left\| \begin{pmatrix} R \\ (1 - B)U \end{pmatrix} \vec{f} - \begin{pmatrix} b * \vec{p} \\ 0 \end{pmatrix} \right\|^2$$

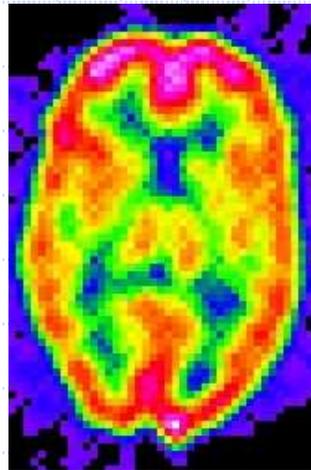
Comparaison MLEM-GC-FRECT



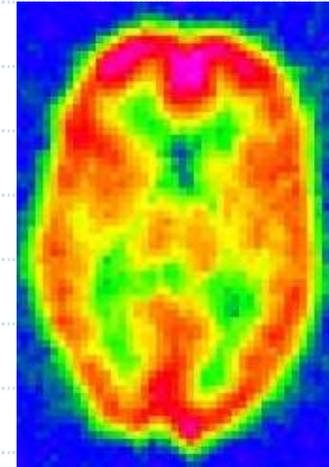
MLEM 6



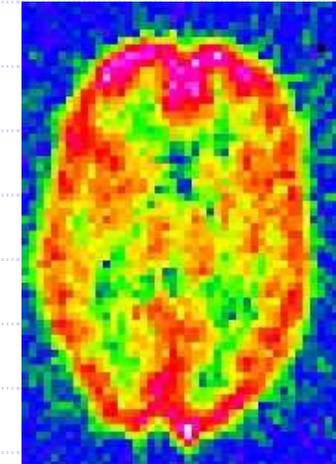
MLEM 200



FRECT 34 (CV)

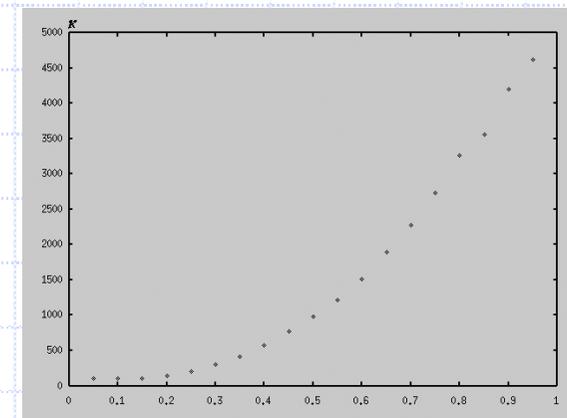


GC 6

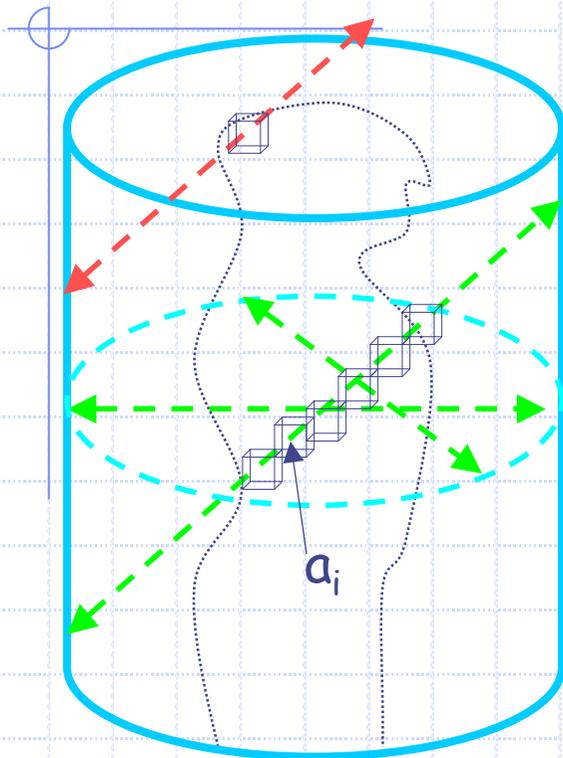


GC 16

$$E(f) = \left\| \text{TF}_s^{-1} (B \cdot \hat{p}) - Rf \right\|^2 + \left\| (1 - B) \cdot \hat{f} \right\|^2$$



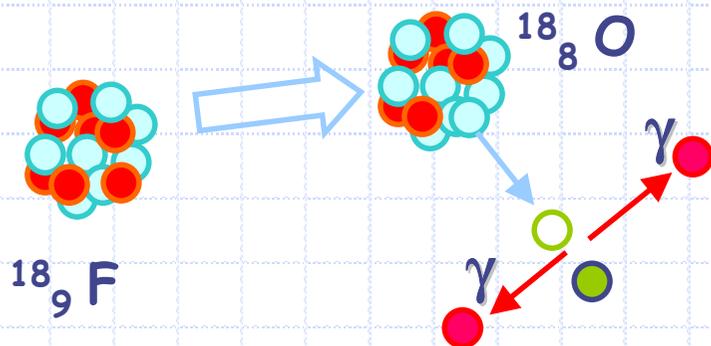
Tomographie en coïncidence 3D



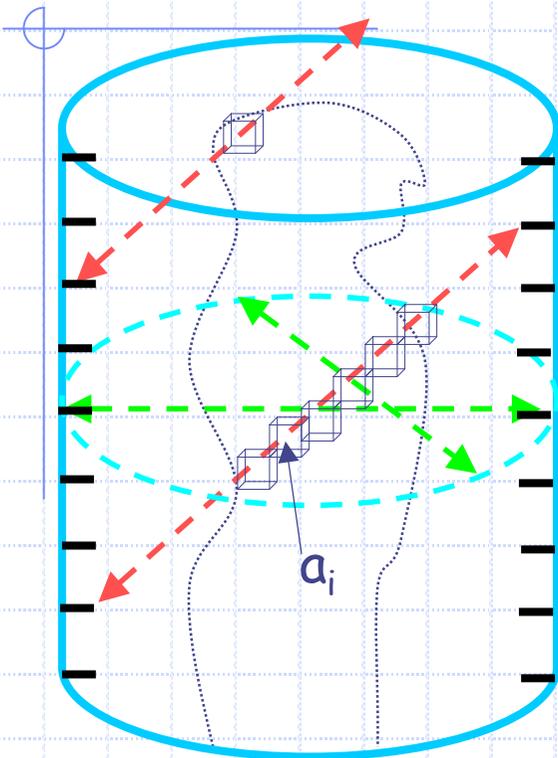
Exemple de la TEP

Projections 3D **redondantes** et **incomplètes**

- Recherche de $f(x,y,z)$ connaissant $p(s,\phi,z,\theta)$
- Certaines projections obliques ne sont pas enregistrées si $\theta \neq 0$

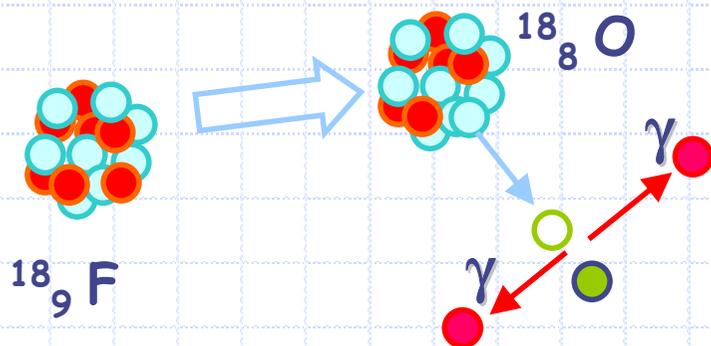


Tomographie en coïncidence 3D

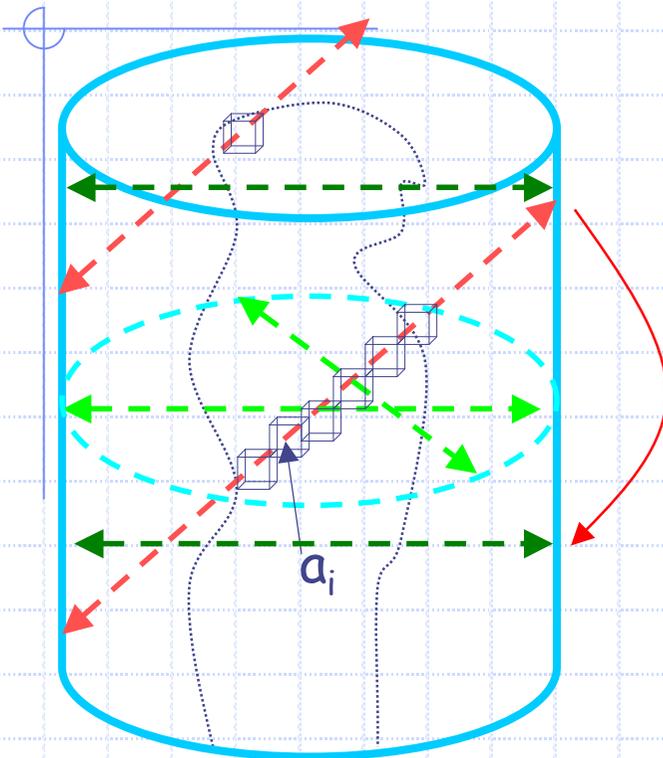


Projections 3D **redondantes** et **incomplètes**

- Reconstruction 2D de données 2D
 - Utilisation d'un collimateur
 - \searrow statistique de comptage, \searrow S/B

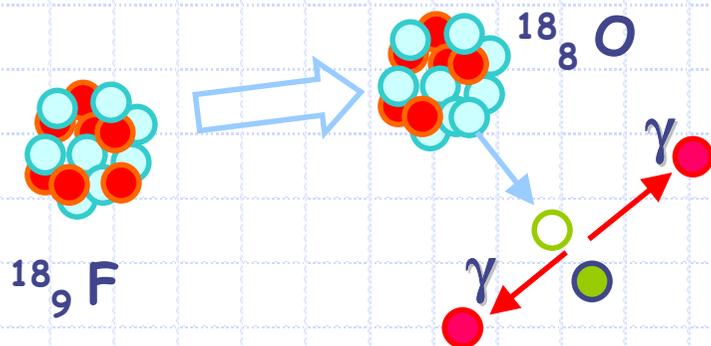


Tomographie en coïncidence 3D

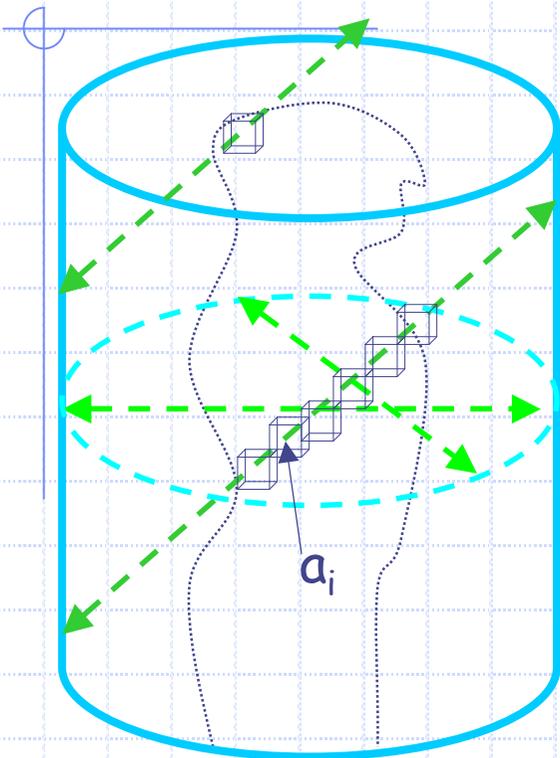


Projections 3D **redondantes** et **incomplètes**

- Reconstruction 2D de données 2D
 - Utilisation d'un collimateur
 - $\sphericalangle \gamma$ détectés N , $\sphericalangle S/B = N/\sqrt{N} = \sqrt{N}$
- Réarrangement 2D de données 3D
 - Algorithmes de «rebinning»
 - $S/B \nearrow$ mais approximation

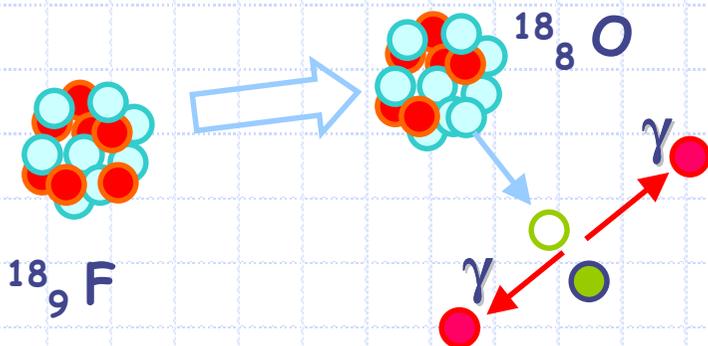


Tomographie en coïncidence 3D



Projections 3D **redondantes** et **incomplètes**

- Reconstruction 2D de données 2D
 - Utilisation d'un collimateur
 - $\sphericalangle \gamma$ détectés N , $\sphericalangle S/B = N/\sqrt{N} = \sqrt{N}$
- Réarrangement 2D de données 3D
 - Algorithmes de «rebinning»
 - $S/B \nearrow$ mais approximation
- Reconstruction 3D de données 3D
 - Algorithmes algébriques 3D
 - RPF 3D si projections complètes
 - $S/B \nearrow$ mais temps de calcul $\nearrow \nearrow$



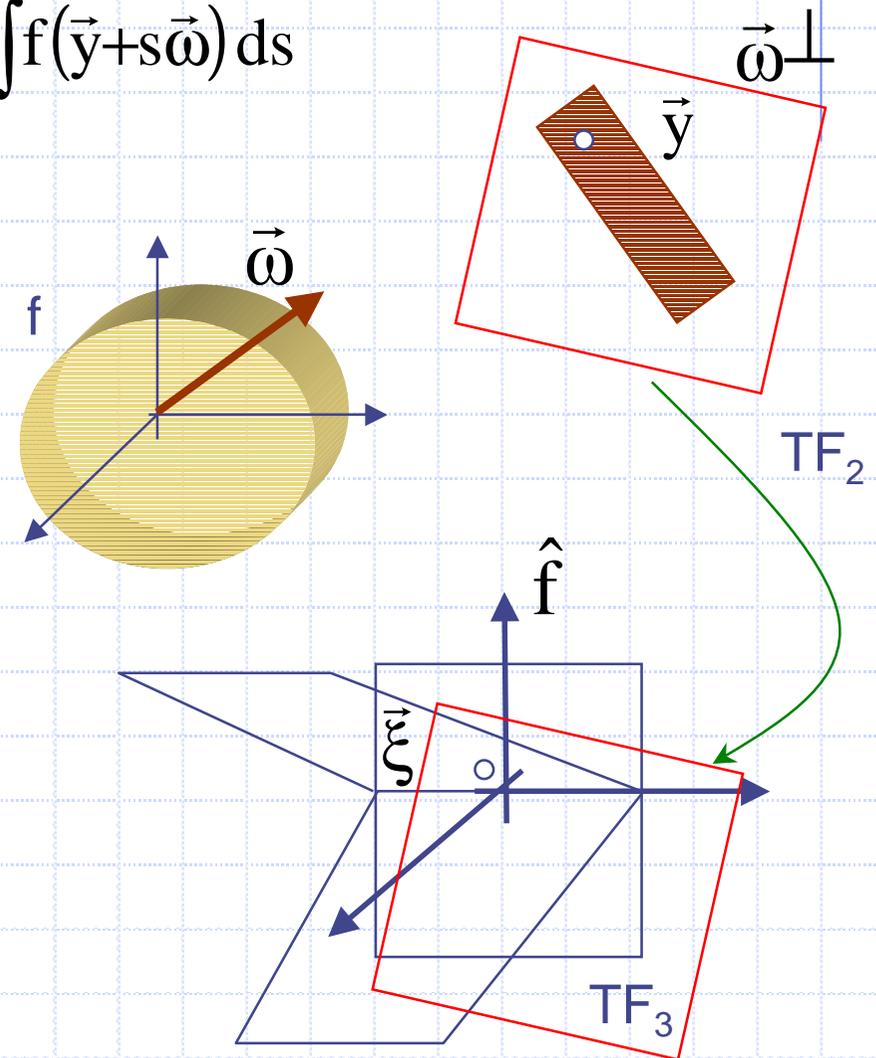
Un théorème de Radon 3D...

$$\forall \vec{\omega} \in S, \quad \forall \vec{y} \in \vec{\omega}^\perp, p_{\vec{\omega}}(\vec{y}) = \int f(\vec{y} + s\vec{\omega}) ds$$

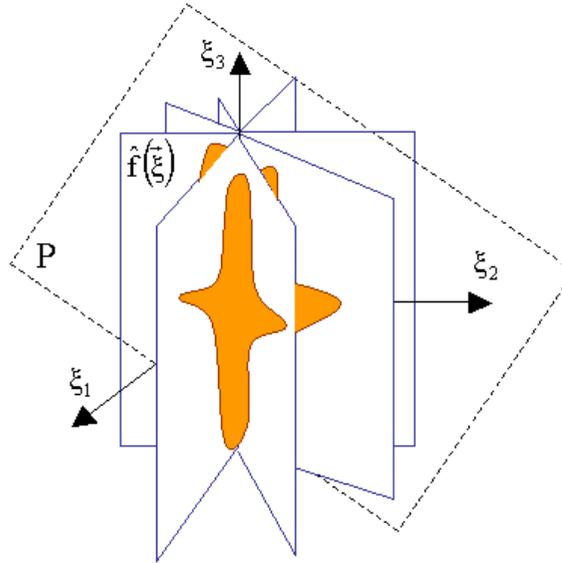
$$\hat{p}_{\vec{\omega}}(\vec{\xi}) = \iint_{\vec{\omega}^\perp} \int f(\vec{y} + s\vec{\omega}) e^{-2i\pi \vec{y} \cdot \vec{\xi}} ds d\vec{y}$$

$$\hat{p}_{\vec{\omega}}(\vec{\xi}) = \iiint f(\vec{x}) e^{-2i\pi \vec{x} \cdot \vec{\xi}} d\vec{x} = \hat{f}(\vec{\xi})$$

$$\forall \vec{\xi} \in \vec{\omega}^\perp, \hat{p}_{\vec{\omega}}(\vec{\xi}) = \hat{f}(\vec{\xi})$$



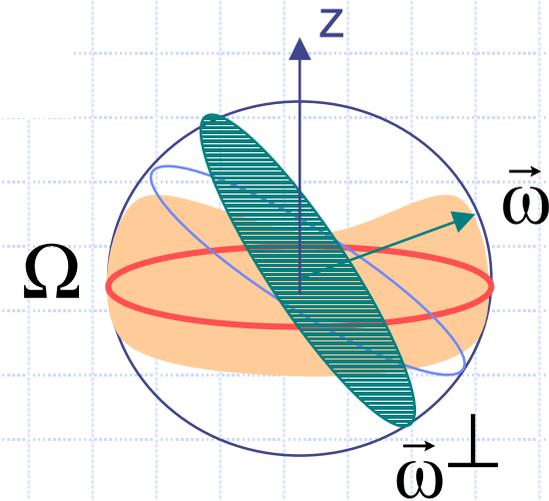
1- Condition d'Orlov :



Condition nécessaire à l'affectation de toutes les fréquences spatiales de \mathbb{R}^3 :

Ω contient au moins un cercle équatorial de S
i.e

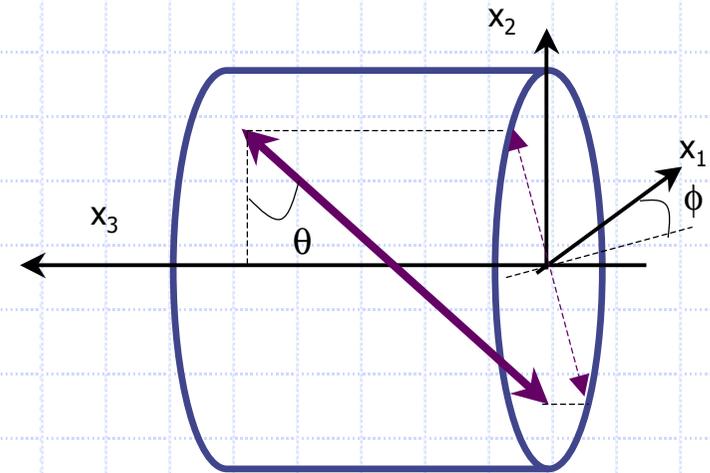
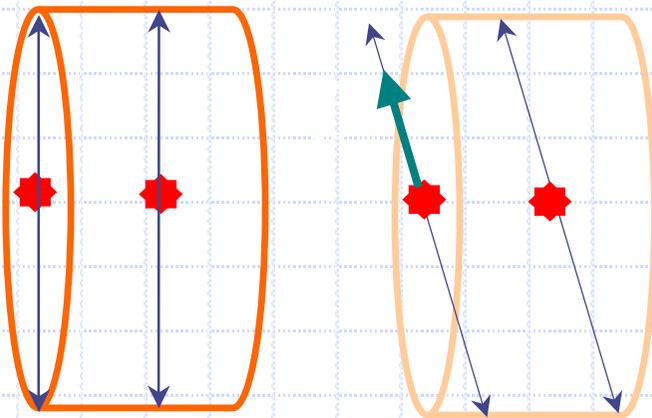
Ω intersecte tout cercle équatorial de S



... plutôt difficile à appliquer !

1- Condition d'Orlov

2 - Projections non tronquées



3 – moyennant une interpolation 3D dans le domaine des fréquences

$$\hat{p}(\xi_1, \xi_2) = \hat{f}(\xi_1 \cos \theta \sin \phi - \xi_2 \sin \theta, \xi_1 \sin \theta \sin \phi + \xi_2 \cos \theta, -\xi_1 \cos \phi)$$

Solutions possibles

1 – Condition d'Orlov

- Détecteur TEP cylindrique

2 – Projections tronquées

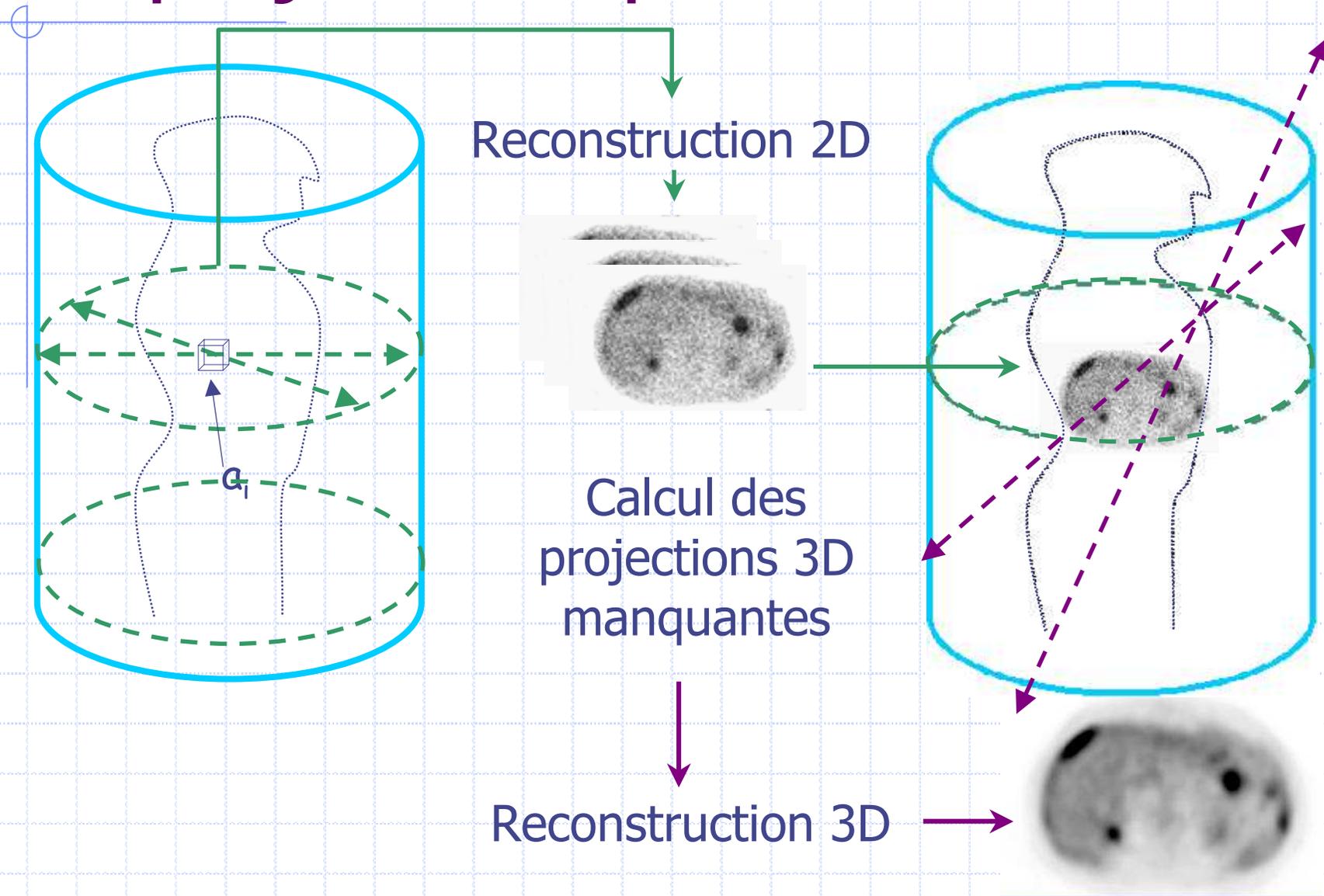
- Estimées par reconstruction 2D puis projection ou rebinning

3 – Interpolation 3D en fréquence

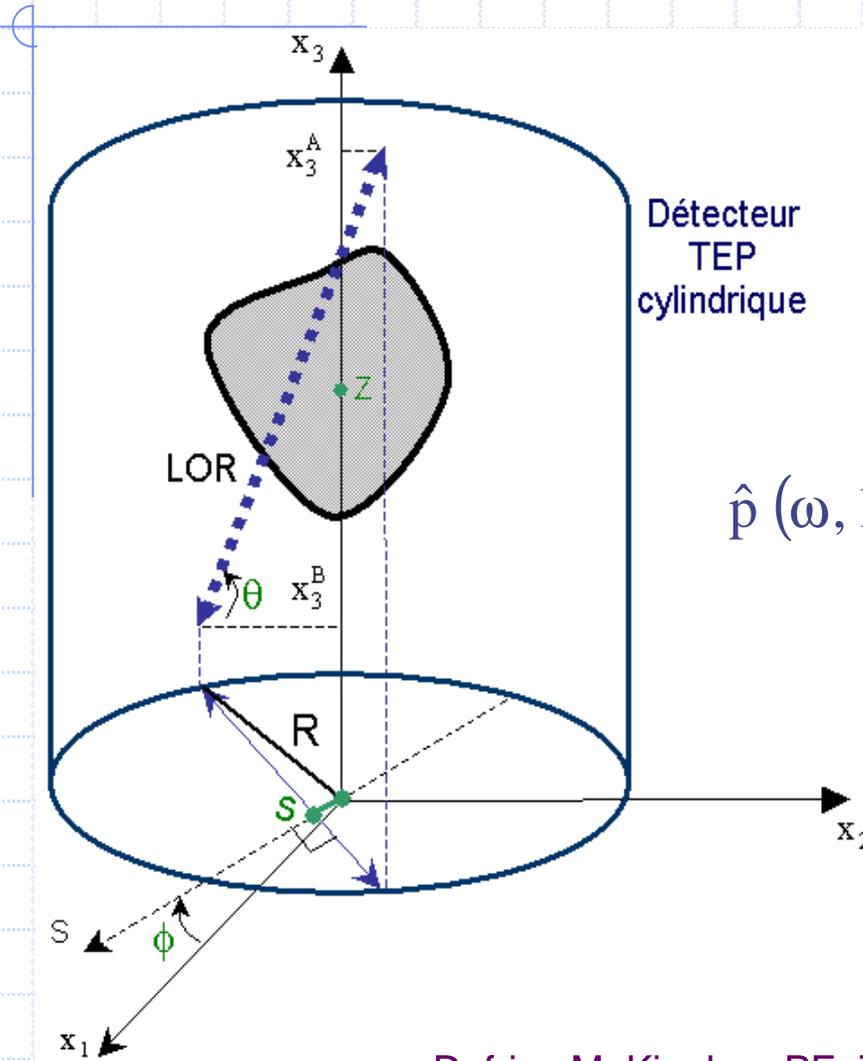
- Optimisation de l'interpolation (fonctions de Kaiser-Bessel)
- Utilisation d'une rétro-projection filtrée (filtre de Colsher)

En « routine » : Utilisation d'algorithmes algébriques (OSEM 3D)
Reconstruction 2D après rebinning des projections 3D

Reprojection après RPF 2D



Ré-arrangement (rebinning) exact



$$p \left(s, \phi, z = \frac{x_3^A + x_3^B}{2}, \delta = tg \theta \right)$$

TF(s,phi) puis TF(z)
si invariance en Tz

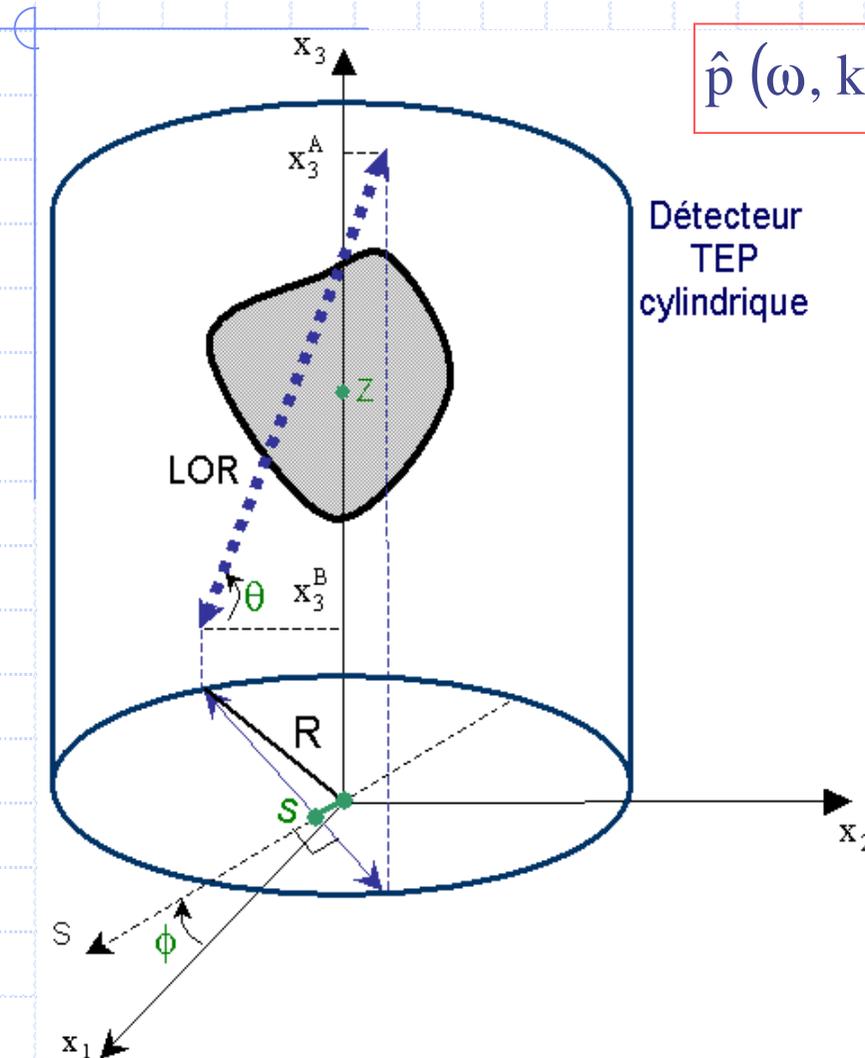
$$\hat{p}(\omega, k, \zeta, \delta) = e^{-ik \arctan(\alpha)} \hat{p}(\omega\sqrt{1+\alpha^2}, k, \zeta, 0)$$

$$\alpha = \frac{\delta\zeta}{\omega}$$

Defrise M, Kinahan PE, Townsend DW, Michel C, Sibomana M, Newport DF.

Exact and approximate rebinning algorithms for 3-D PET data. IEEE Trans Med Imaging 1997;16:145-58.

Ré-arrangement approximatif



$$\hat{p}(\omega, k, \zeta, \delta) = e^{-ik \arctan(\alpha)} \hat{p}(\omega \sqrt{1+\alpha^2}, k, \zeta, 0)$$

DL à l'ordre 1 sur $\alpha = \frac{\delta \zeta}{\omega}$

$$\hat{p}(\omega, k, \zeta, \delta) \approx e^{-ik\alpha} \hat{p}(\omega, k, \zeta, 0)$$

$$\hat{p}(\omega, k, z, \delta) \approx \hat{p}\left(\omega, k, z - k \frac{\delta}{\omega}, 0\right)$$

SYNTHESE DE DONNEES 2D à S/B \uparrow :

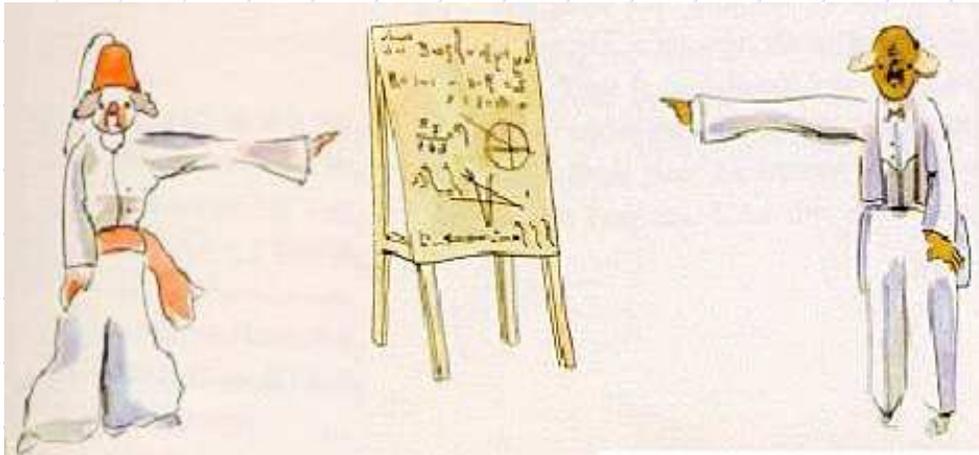
$$\hat{p}(\omega, k, z, 0) \approx \hat{p}\left(\omega, k, z + k \frac{\delta}{\omega}, \delta\right)$$

SYNTHESE DE DONNEES MANQUANTES:

$$\hat{p}(\omega, k, z, \delta) \approx \hat{p}\left(\omega, k, z - k \frac{(\delta - \delta')}{\omega}, \delta'\right)$$

Conclusions

- ◆ Résolutions de grands systèmes linéaires
 - ◆ Problématique fréquente en ingénierie numérique
 - ◆ Pour tout problème complexe : attention au conditionnement
- ◆ Intérêt d'une base fonctionnelle adaptée
 - ◆ Transformation de Fourier, analyse factorielle...
- ◆ La recherche en imagerie :
 - ◆ Très active, exemples sur notre propos :
 - Artefacts, quantification, dosimétrie...
 - Améliorations fréquentes disponibles en routine.
 - ◆ Nécessité de collaborations pluridisciplinaires
 - Compétences techniques pointues + ouverture
 - Pour choisir les directions de recherche, puis valider



Analyse numérique matricielle
appliquée à l'art de l'ingénieur.

P. Lascaux et R. Théodor. 2 tomes.
MASSON.

The Mathematics of
Computerized Tomography.

F. Natterer. 2001. SIAM.

Positron Emission Tomography.
Basic Sciences and Clinical Practice.

PE Valk, DL Bailey,
DW Townsend, MN Maisey.
2003. Springer.

Reconstruction tomographique en
imagerie médicale. D. Mariano-Goulart
Encyclopédie Médico-chirurgicale,
35-105-A-10, 2009.

Merci de votre attention...



Médecine et sciences de l'ingénieur.

Aspects complémentaires et
Passerelles



Plusieurs carrières

◆ Dans l'industrie :

- Nombreuses PME (sous-traitantes) : hard et soft
- SIEMENS, GE-MS, PHILLIPS :
 - ◆ Carrières de commerciaux
 - ◆ Principalement aux USA pour la R&D

◆ Dans les Hôpitaux :

- Ingénieurs biomédicaux
- Physiciens d'hôpitaux
- Hospitalo-Universitaires

Ingénieur Biomédical

◆ Rôles

- ◆ Elaboration des plans d'équipement
 - Besoins, chiffrage, contraintes...
 - Lancement des appel d'offres
 - suivi des achats (installation, maintenance...)
- ◆ Veille technologique

◆ Formation spécifique

- ◆ Ecoles d'ingénieurs généralistes
 - ± formation complémentaire spécialisée

Physicien d'hôpital

◆ Rôle

- ◆ « Qualité et sécurité dans l'utilisation des rayonnements ionisants »
- ◆ Tests d'acceptation et de conformité
- ◆ Assurance et contrôle de qualité
- ◆ Optimisation de la dosimétrie au patient en radiologie et en médecine nucléaire
- ◆ Simulation de balistique en radiothérapie

Physicien d'hôpital

◆ Formation spécifique (Arrêtés du 3/3/97 & 18/3/09)

- ◆ Diplôme de Qualification en Physique Radiologique et Médicale (**DQPRM**) : 1 an → N° agrément
- ◆ Concours d'admission (\approx 100 places) après :
 - M2, DI, thèse + formation aux RI
 - validation d'un M2 parmi
 - « Radiophysique et imageries médicales », Toulouse.
 - « IPSM », UJF Grenoble.
 - « Ingénierie pour la santé », UCL Lyon I.
 - « Physique médicale », Paris Sud.
 - « Physique électronique » Nantes.
 - Programme : Physique des RI, dosimétrie, imagerie

Hospitalo-Universitaire: MCU & PU-PH

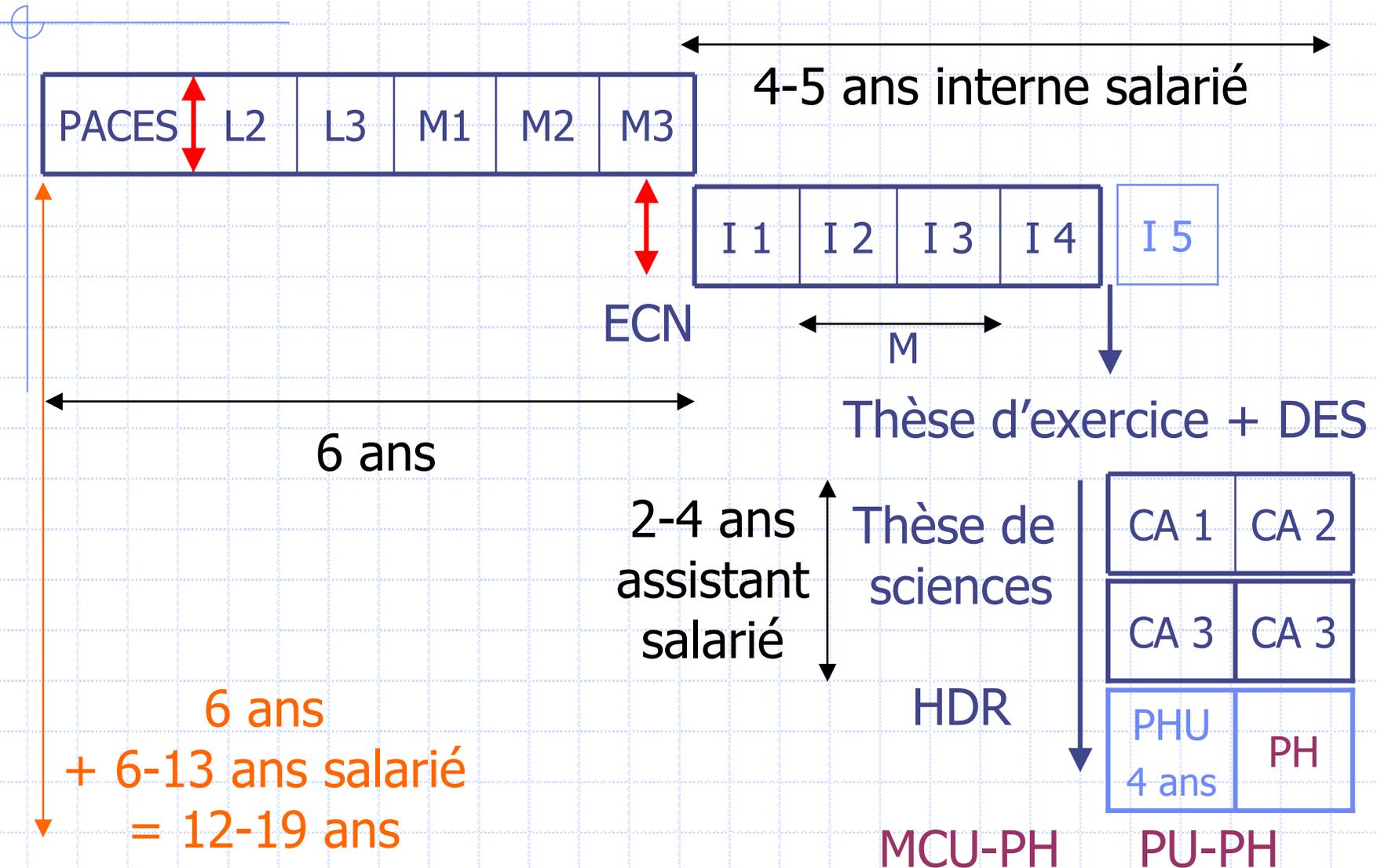
◆ Rôles :

- ◆ Soins en Centre Hospitalier Universitaire (ou CRLCC)
- ◆ Gestion médicale de départements hospitaliers
- ◆ Recherche
- ◆ Enseignement universitaire

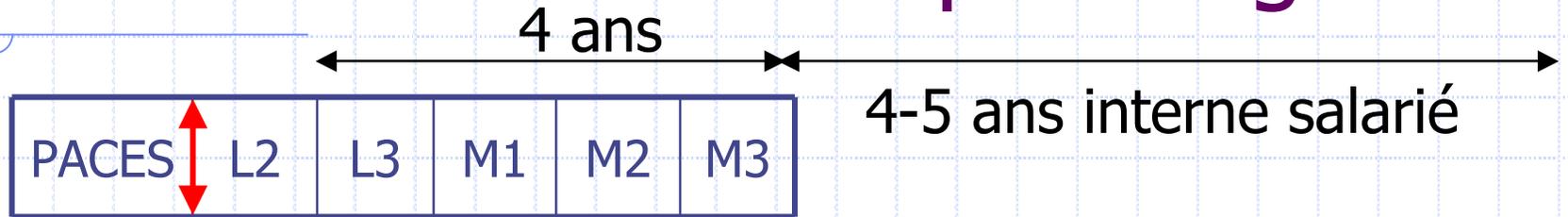
◆ Formation spécifique :

- ◆ Doctorat en médecine (sauf exception)
- ◆ Doctorat d'Université (sciences) ± HDR

Etudes de médecine



Etudes de médecine pour ingénieur



Thèse de sciences

Thèse d'exercice + DES

± 2 ans
assistant



MCU-PH

HDR

PU-PH

2 % du numerus clausus :

dossier, entretien

- Doctorat d'Université
- Doctorat d'État en santé
- Diplôme d'Ingénieur,

4 ans
+ 4-7 ans salarié
= 8-11 ans
(gain de 1 à 11 ans)

Ingénieur - HU : pourquoi ?

- ◆ Biophysique et médecine nucléaire
- ◆ Radiologie
- ◆ Epidémiologie – Statistiques
- ◆ Informatique médicale
- ◆ Mais aussi :
 - Cardiologie (ECG, écho, prothèses, PMK)
 - Neurologie (EEG, activations)
 - Néphrologie (dialyse)
 - Orthopédie (biomatériaux)
 - ...