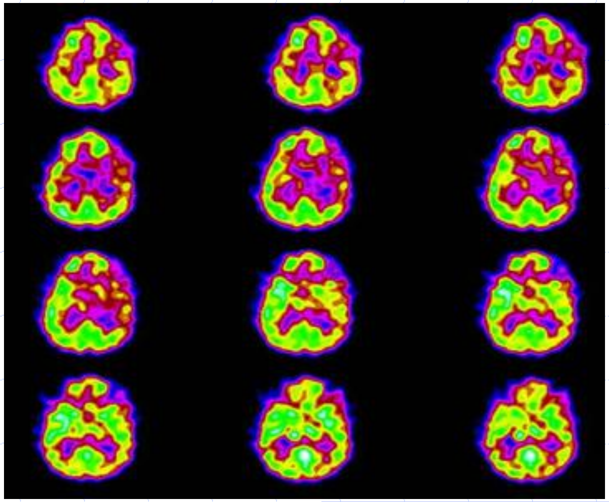
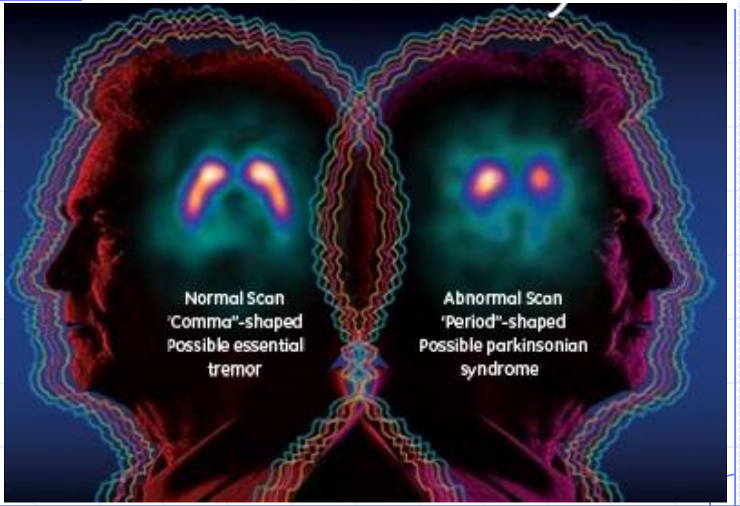
FORMATION TIC (Phymed, STIC)

COMPLEMENTS EN TOMOGRAPHIE MEDICALE





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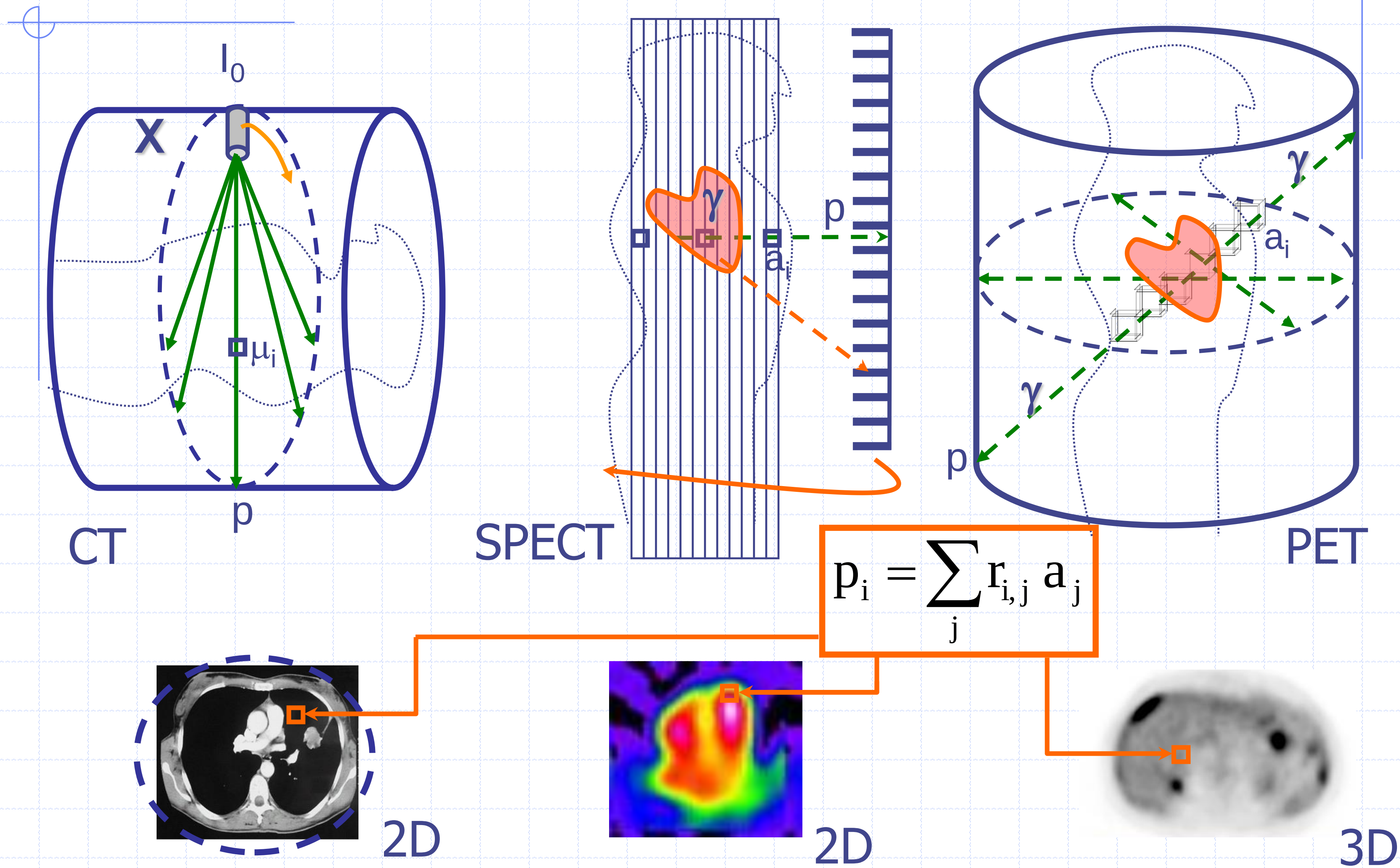
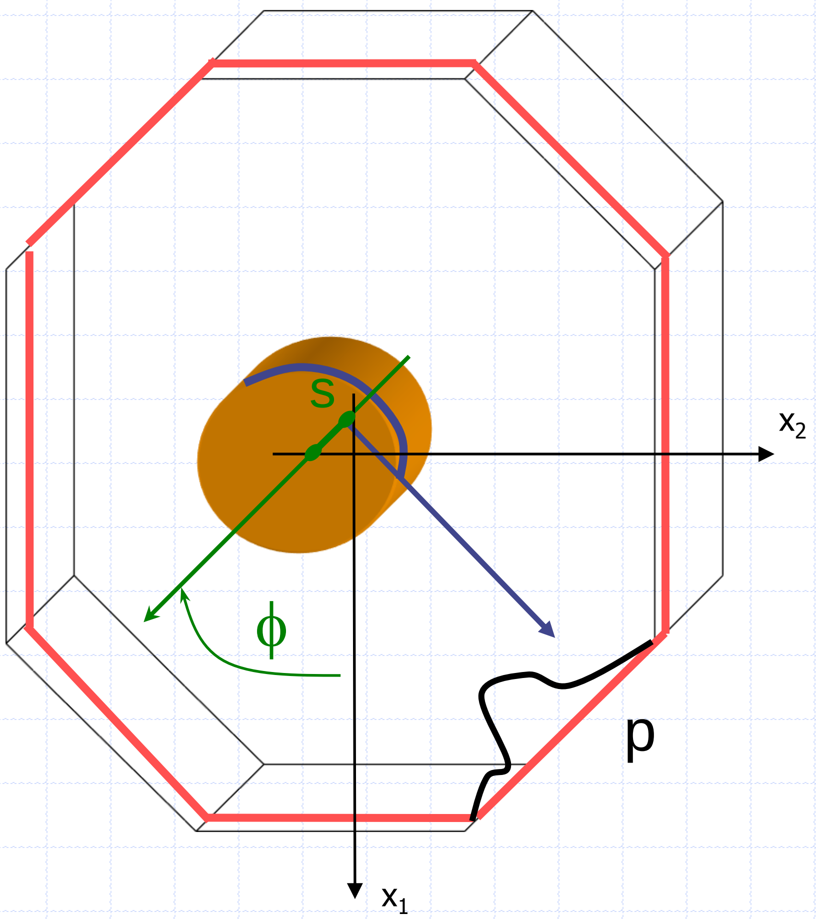
http://scinti.etud.univ-montp1.fr

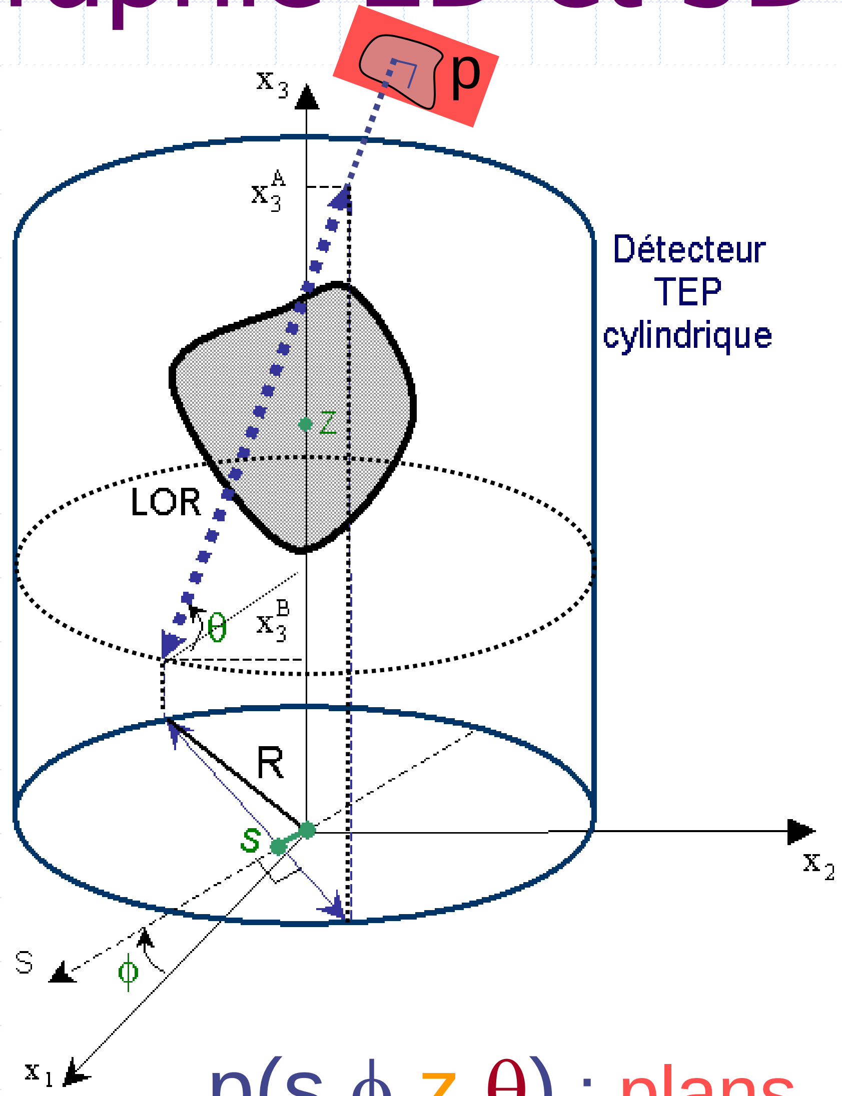
Tomographie: problème inverse linéaire



$$p_i = \sum_j r_{i,j}\, a_j$$

Codage en tomographie 2D et 3D

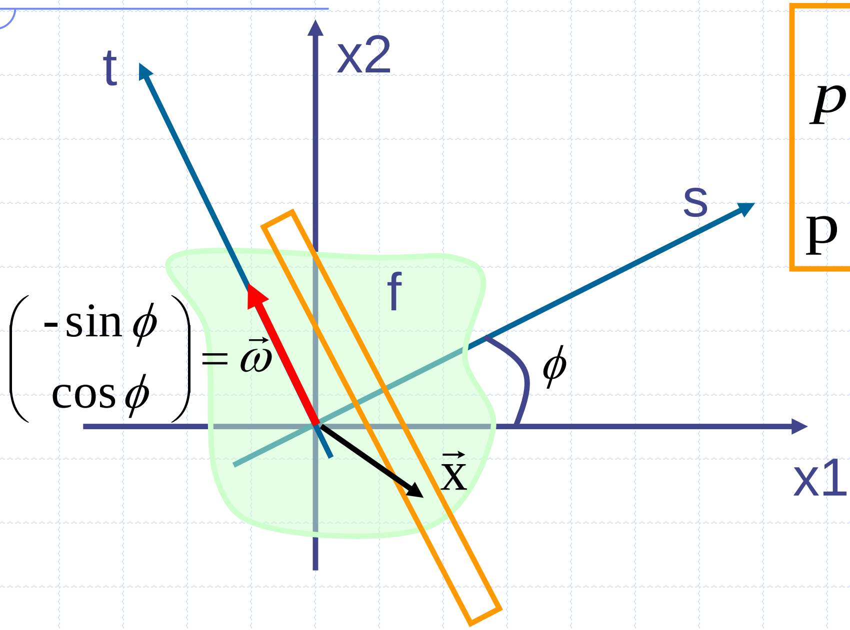




$p(s,\phi)$: lignes

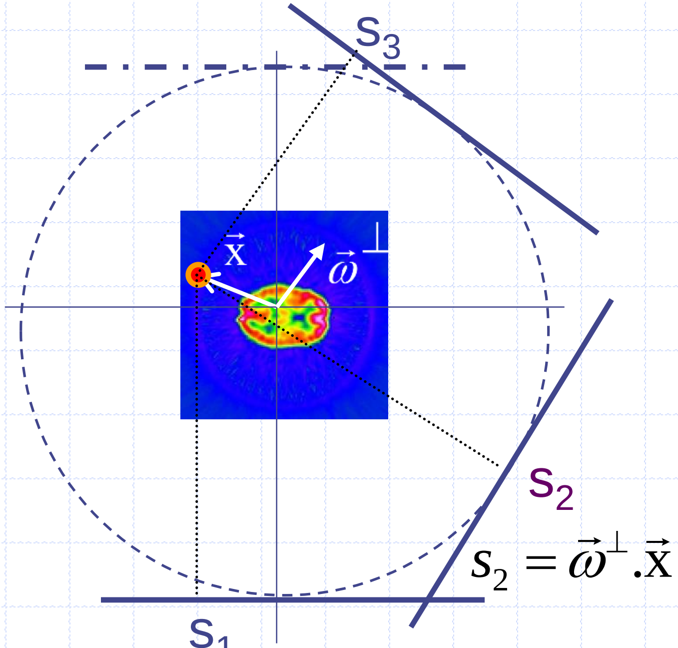
$p(s,\phi,z,\theta)$: plans

Modélisation analytique



$$p(\vec{\omega}, s) = p_{\vec{\omega}}(s) = \int_t f(s\vec{\omega}^{\perp} + t\vec{\omega})\,dt$$

$p = Rf$ transformée de Radon



$$(R^* p)(\vec{x}) = \int_{\phi=0}^{\pi} p\left(\vec{\omega}, \vec{\omega}^{\perp}.\vec{x}\right) d\phi$$

rétroprojection = épandage

Natterer F. The mathematics of computerized tomography. New York: Wiley; 1986.

Modélisation algébrique

f_1	f_2
f_3	f_4

$p_1 = r_{1,1} f_1 + r_{1,2} f_2$

$p_2 = r_{2,3} f_3 + r_{2,4} f_4$

$p_4 = r_{4,2} f_2 + r_{4,4} f_4$

$p_3 = r_{3,1} f_1 + r_{3,3} f_3$

$$\begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ r_{2,1} & r_{2,2} & r_{2,3} & r_{2,4} \\ r_{3,1} & r_{3,2} & r_{3,3} & r_{3,4} \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{pmatrix} . \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

$r_{i,j}$ = % du pixel j intersecté par la projection i

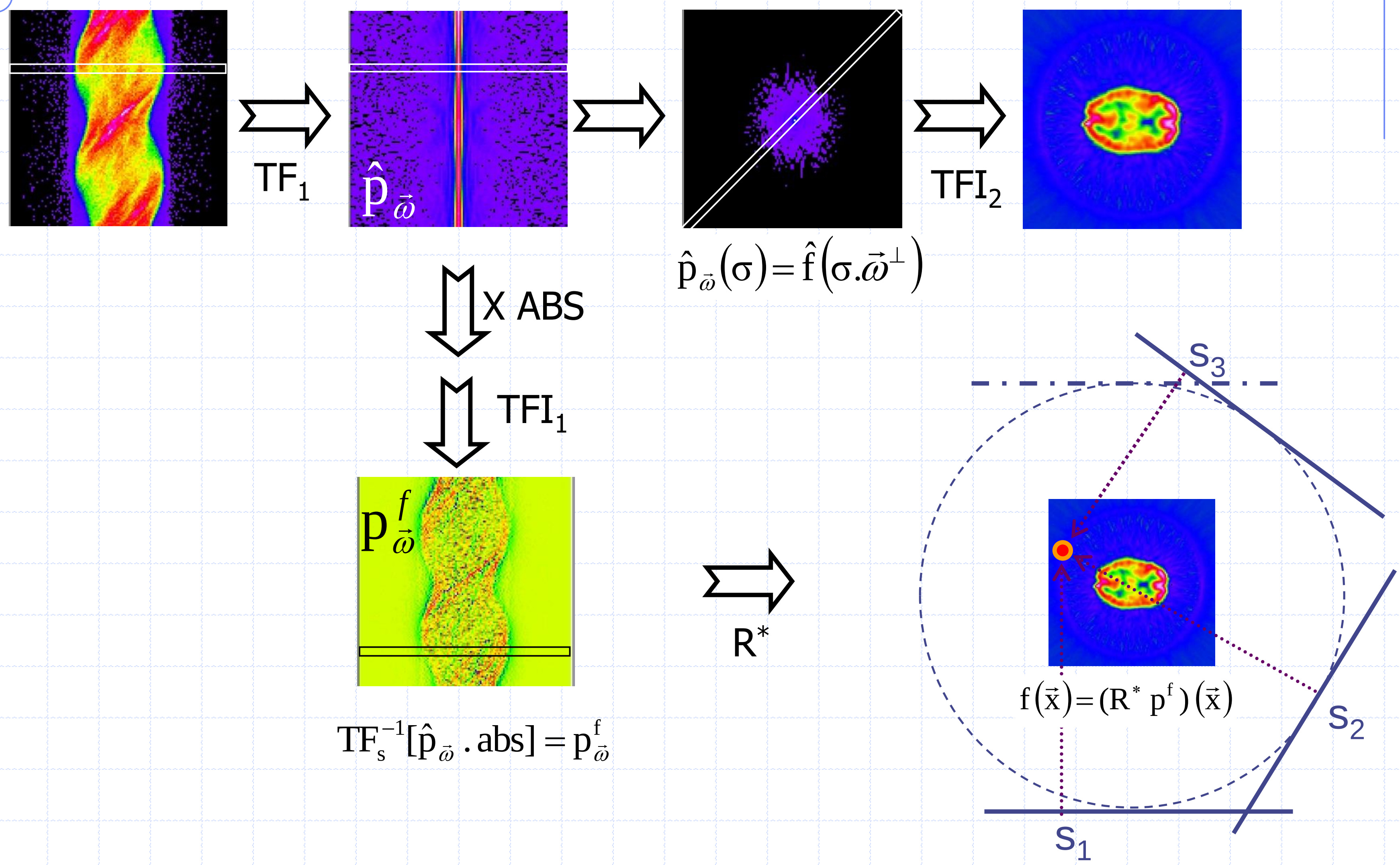
$$\boxed{R.\vec{f} = \vec{p}}$$

$$\begin{pmatrix} r_{1,1} & r_{2,1} & r_{3,1} & r_{4,1} \\ r_{1,2} & r_{2,2} & r_{3,2} & r_{4,2} \\ r_{1,3} & r_{2,3} & r_{3,3} & r_{4,3} \\ r_{1,4} & r_{2,4} & r_{3,4} & r_{4,4} \end{pmatrix} . \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

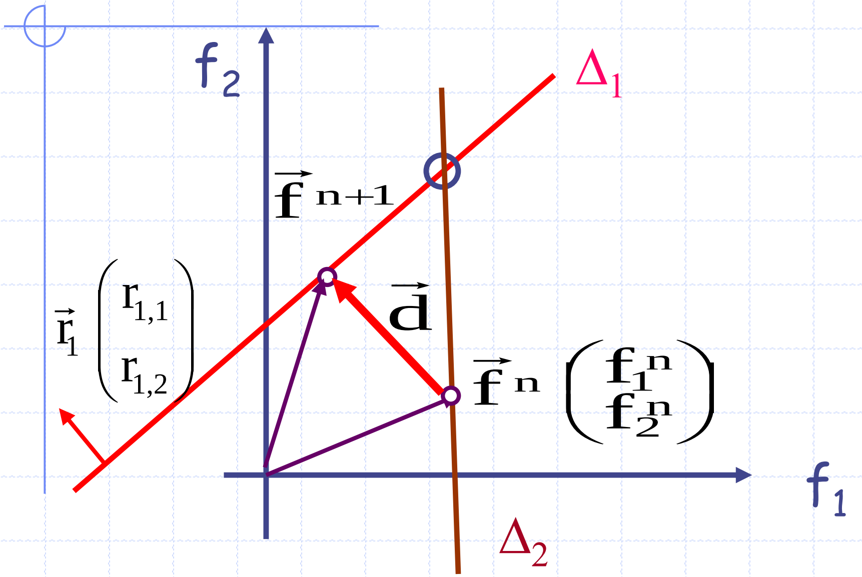
$$\boxed{{}^{t}R.\vec{p} = \vec{b}}$$

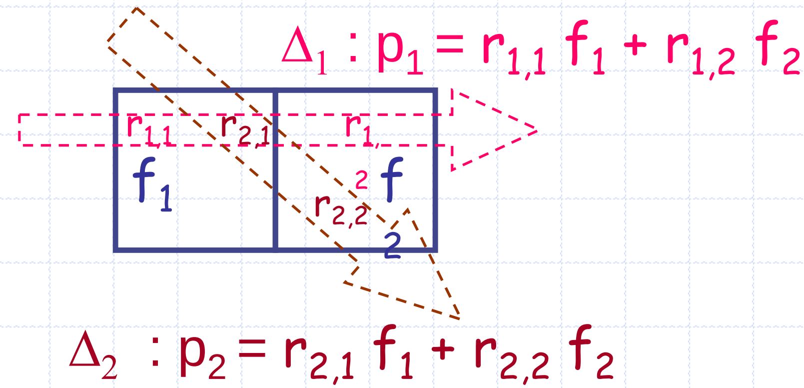
$b_1 = r_{1.1}\, p_1 + r_{3,1}\, p_3$	$b_2 = r_{1.2}\, p_1 + r_{4,2}\, p_4$	$\leftarrow p_1$
$b_3 = r_{2.3}\, p_2 + r_{3,3}\, p_3$	$b_4 = r_{2,4}\, p_2 + r_{4,4}\, p_4$	$\leftarrow p_2$
$\uparrow p_3$	$\uparrow p_4$	

Algorithmes analytiques



Algorithmes algébriques





S. Kaczmarz
1895-1940

ART $\boxed{\vec{f}^{\,n,j} = \vec{f}^{\,n,j\text{-}1} + R^* \left(\vec{p}_j - R\vec{f}^{\,n}\Big|_j\right) \ldots \vec{f}^{\,n+1,0} = \vec{f}^{\,n,J}}$

SIRT $\boxed{\vec{f}^{\,n+1} = \vec{f}^{\,n} + R^* \left(\vec{p} - R\vec{f}^{\,n}\right)}$

Kaczmarz S. Angenährte Auflösung von Systemen linearer Gleichungen. *Bull Int Acad Pol Sci Lett A* 1937;35:355-7.

Moindres carrés

$$\bar{\mathrm{f}} = \arg\min_{\mathrm{f} \in \mathrm{C}} (J)$$

$$J = \left\| \mathrm{R}\vec{\mathrm{f}} - \vec{\mathrm{p}} \right\|^2 = \left(\mathrm{R}\vec{\mathrm{f}} - \vec{\mathrm{p}}\right)^{\mathrm{T}} \left(\mathrm{R}\vec{\mathrm{f}} - \vec{\mathrm{p}}\right)$$

$$\nabla J = 2\mathrm{R}^{\mathrm{T}}\left(\mathrm{R}\vec{\mathrm{f}} - \vec{\mathrm{p}}\right) = 0$$

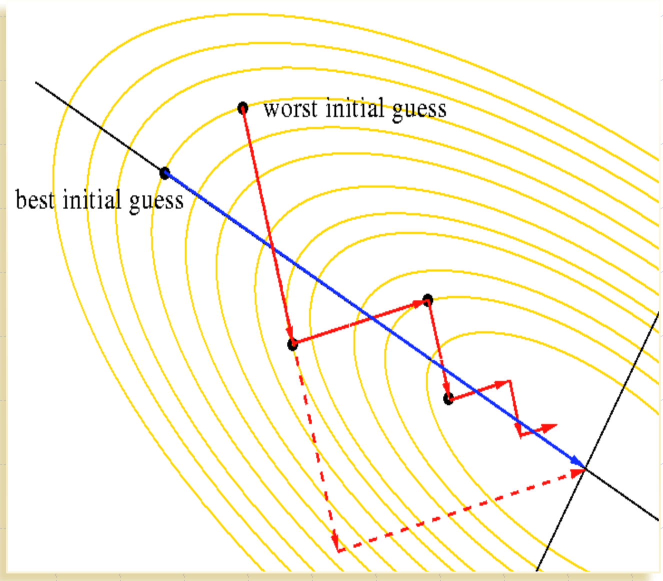
$$\mathrm{R}^{\mathrm{T}}\mathrm{R}\vec{\mathrm{f}} = \mathrm{R}^{\mathrm{T}}\vec{\mathrm{p}}$$

$$\boxed{\bar{\mathrm{f}} = (\mathrm{R}^{*}\mathrm{R})^{-1}\mathrm{R}^{*}\vec{\mathrm{p}}}$$

MAIS …
dim(R*R) ~ 10000
Et mal conditionnée

Steepest descent

$$\bar{f} = \underset{f \in C}{\arg\min} \left(\left\| R\vec{f} - \vec{p} \right\|^2 \right)$$



$$\bar{f}^{\,n+1} = \bar{f}^{\,n} - \gamma(\nabla J)^n = \bar{f}^{\,n} - \gamma R^T \left(R\bar{f}^{\,n} - \vec{p} \right)$$

$$J\left(\bar{f}^{\,n+1}\right) = \left\| R\bar{f}^{\,n} - \gamma R R^T \left(R\bar{f}^{\,n} - \vec{p} \right) - \vec{p} \right\|^2$$

$$\frac{1}{2} \frac{\partial J\left(\bar{f}^{\,n+1}\right)}{\partial \gamma} = -\left(R\bar{f}^{\,n} - \vec{p} \right)^T R R^T \left(R\bar{f}^{\,n} - \gamma R R^T \left(R\bar{f}^{\,n} - \vec{p} \right) - \vec{p} \right)$$

$$= -\left(R\bar{f}^{\,n} - \vec{p} \right)^T R R^T \left(R\bar{f}^{\,n} - \vec{p} \right) + \gamma \left(R\bar{f}^{\,n} - \vec{p} \right)^T R R^T R R^T \left(R\bar{f}^{\,n} - \vec{p} \right) = 0$$

$$\gamma = \frac{\left\| R^T \left(R\bar{f}^{\,n} - \vec{p} \right) \right\|^2}{\left\| R R^T \left(R\bar{f}^{\,n} - \vec{p} \right) \right\|^2}$$

$$\boxed{\bar{f}^{\,n+1} = \bar{f}^{\,n} + \frac{\left\| R^* \vec{\varepsilon} \right\|^2}{\left\| R R^* \vec{\varepsilon} \right\|^2} R^* \vec{\varepsilon} \; ; \; \vec{\varepsilon} = \vec{p} - R\bar{f}^{\,n}}$$

Gradient Conjugué

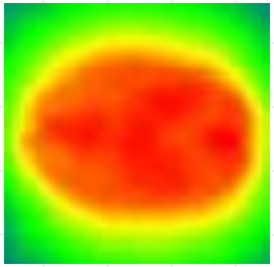
$$\bar{f} = \arg\min_{f \in C} \left\| R\vec{f} - \vec{p} \right\|^2$$

Directions de descente conjuguées :

$$(R\vec{d}^{\,m})^T R\vec{d}^{\,n} = 0 \ \text{ si } m \neq n$$

Initialisation :

$$\vec{d}^{\,0} = \vec{r}^{\,0} = R^*.\vec{p}$$

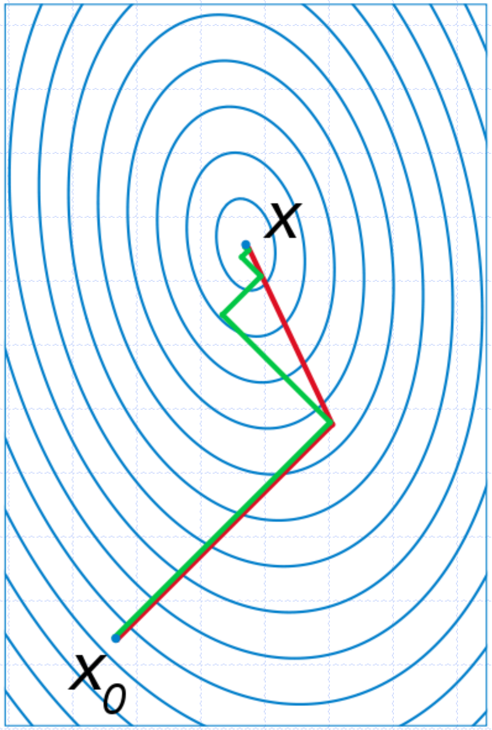


$$\omega^j = \frac{\left\|\vec{r}^{\,j}\right\|^2}{\vec{d}^{\,jT} R^* R \vec{d}^{\,j}}$$

$$\vec{r}^{\,j+1} = \vec{r}^{\,j} - \omega^j R^* R \vec{d}^{\,j}$$

$$\vec{d}^{\,j+1} = \vec{r}^{\,j+1} + \frac{\left\|\vec{r}^{\,j+1}\right\|^2}{\left\|\vec{r}^{\,j}\right\|^2} \vec{d}^{\,j}$$

$$\boxed{\vec{f}^{\,j+1} = \vec{f}^{\,j} + \omega^j \vec{d}^{\,j}}$$



Gilbert P Iterative methods for the three-dimensional reconstruction of an object from projections. J. Theor. Biol. 1972; 36:105-17

MLEM et OSEM

Bayes : $P(\vec{f}/\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})/P(\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})$

$$\tilde{\vec{f}} = \arg\min_{\vec{f}} \left[-\log P(\vec{p}/\vec{f}) - \cancel{\log P(\vec{f})} \right]$$

Adéquation aux données

$$P(\vec{p}/\vec{f}) = \prod_j \frac{e^{-\overline{p}_j}\ \overline{p}_j^{\ p_j}}{p_j!} \quad ; \quad \overline{p}_j = (Rf)_j$$

$$\tilde{\vec{f}} = \operatorname{argmin}\left[\sum_j \left(\overline{p}_j - p_j \log(\overline{p}_j)\right)\right]$$

ML-EM

$$\boxed{f_i^{n+1} = f_i^{n} \left(R^{*} \frac{p}{Rf^{n}} \right)_i}$$

Dempster A et al. Maximum likelihood from incomplete data via the EM algorithm. J R Stat Soc 1977;39:1-38. Hudson H et al. .Accelerated image reconstruction using ordered subsets of projection data. IEEE Trans Med Imaging 1994;13:601-9.

MLEM et OSEM

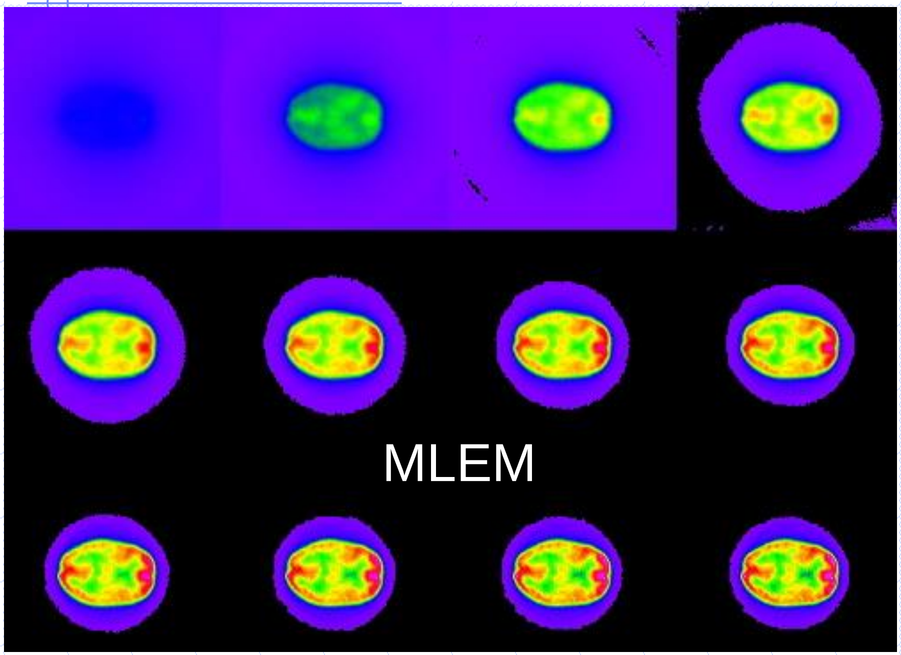
Avantages :

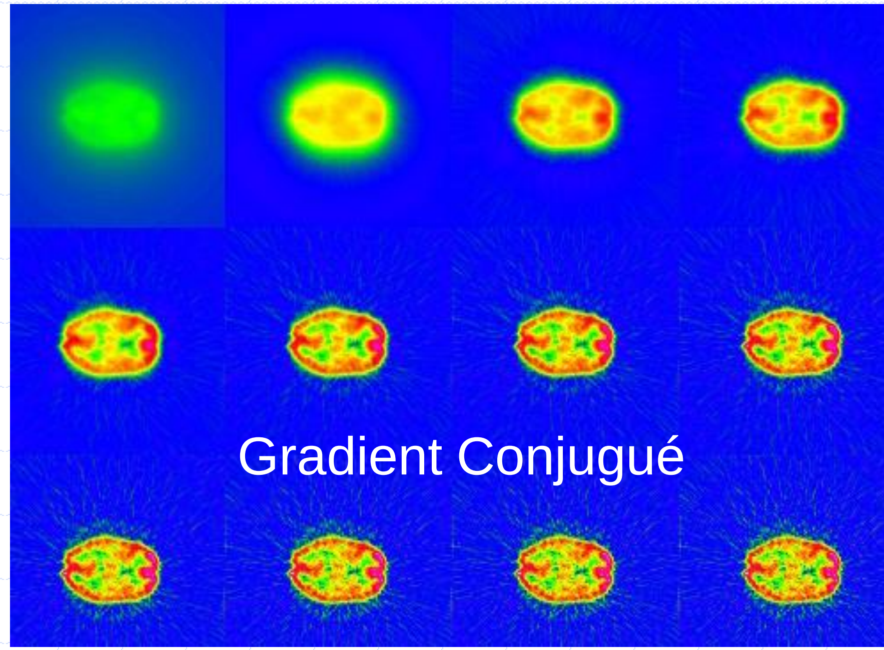
- Méthodes statistiques $\log P(\vec{p}/\vec{f})$
- Contrainte de non-négativité $\vec{f} \geq 0$
- Normalisation naturelle $\left\|\vec{f}\right\| = cste$

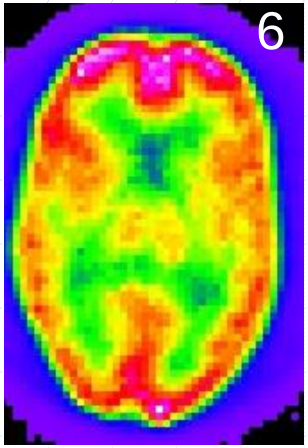
ML-EM $\boxed{f_i^{n+1} = f_i^n \left(R^* \frac{p}{Rf^n} \right)_i}$

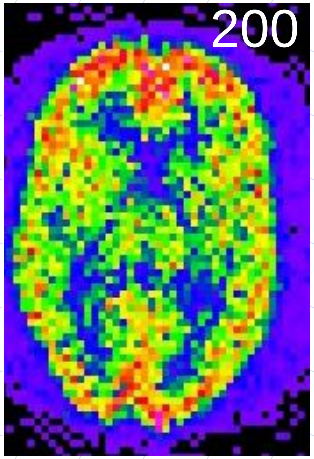
Dempster A et al. Maximum likelihood from incomplete data via the EM algorithm. J R Stat Soc 1977;39:1-38. Hudson H et al. .Accelerated image reconstruction using ordered subsets of projection data. IEEE Trans Med Imaging 1994;13:601-9.

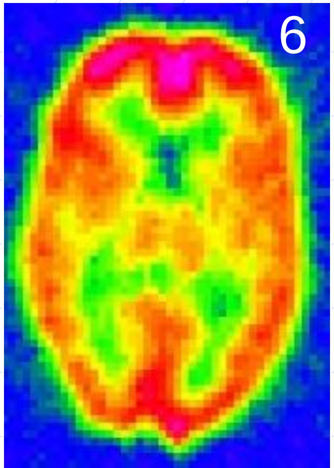
Exemples

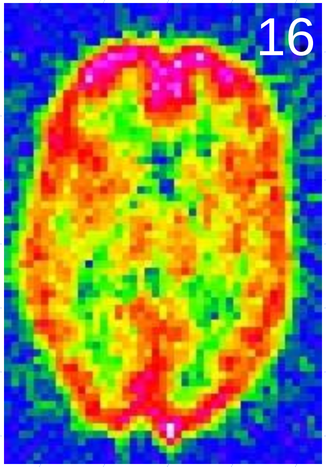




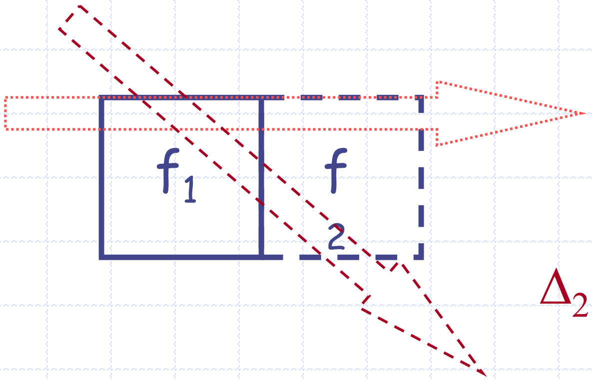






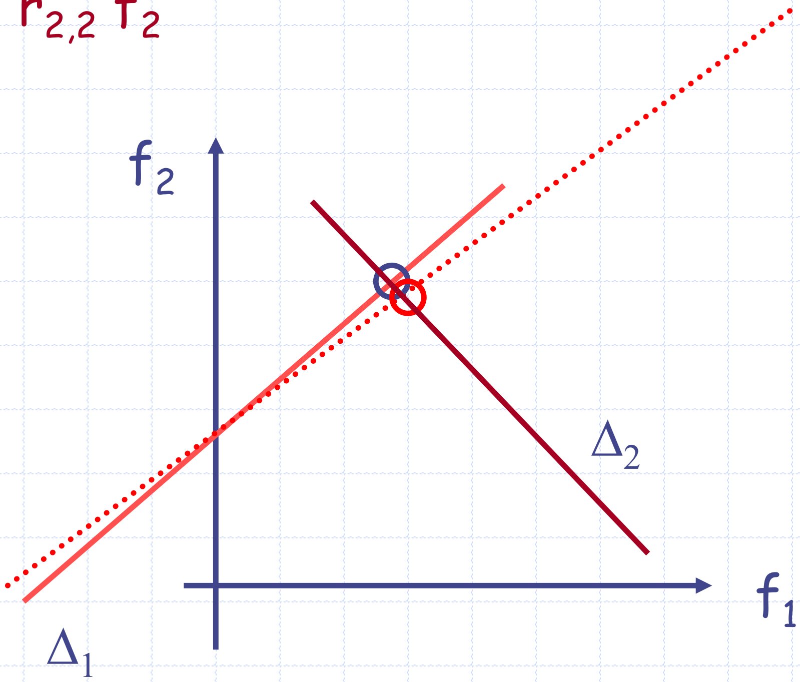


Régularisation … approche intuitive



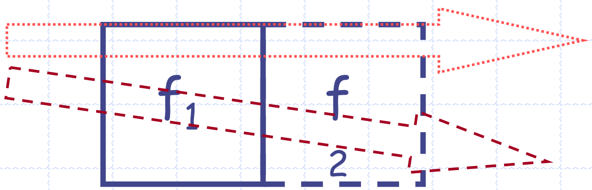
$$\Delta_1 : p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2$$

$$\Delta_2 : p_2 = r_{2,1}\, f_1 + r_{2,2}\, f_2$$

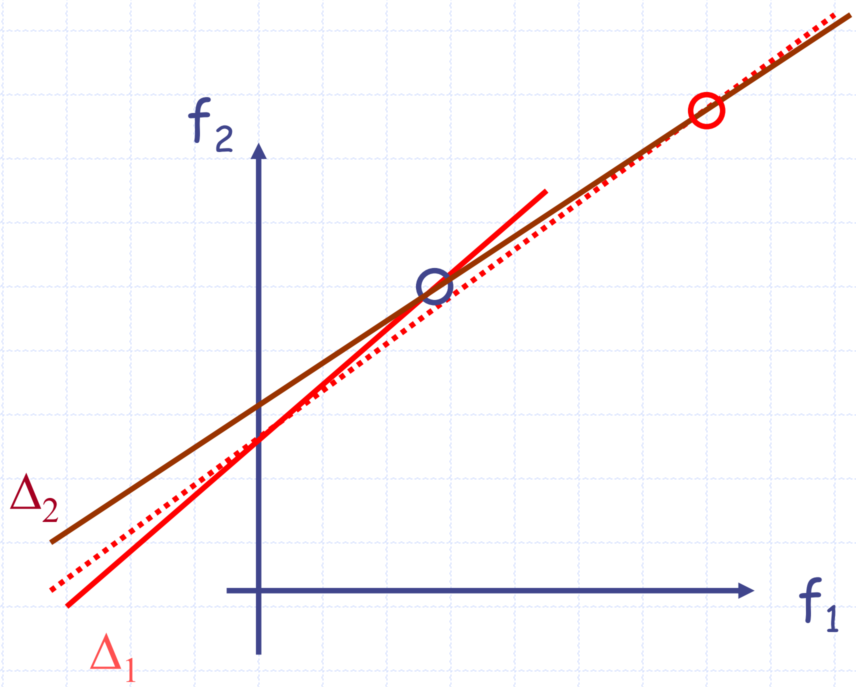


Régularisation … approche intuitive

$\Delta_1 : p_1 = r_{1,1}\, f_1 + r_{1,2}\, f_2$



$\Delta_2 : p_2 = r_{2,1}\, f_1 + r_{2,2}\, f_2$



$64^2 = 4\,096$

$128^2 = 16\,384$

$256^2 = 65\,536$

$512^2 = 262\,144$

Exemple

Matrice de Wilson

$$\text{Det} = 1$$

$$R.\begin{pmatrix}1\\1\\1\\1\end{pmatrix} = \begin{pmatrix}10 & 7 & 8 & 7\\ 7 & 5 & 6 & 5\\ 8 & 6 & 10 & 9\\ 7 & 5 & 9 & 10\end{pmatrix}\underbrace{\boxed{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}}}_{f} = \underbrace{\boxed{\begin{pmatrix}32\\23\\33\\31\end{pmatrix}}}_{p}$$

$$\begin{pmatrix}10 & 7 & 8 & 7\\ 7 & 5 & 6 & 5\\ 8 & 6 & 10 & 9\\ 7 & 5 & 9 & 10\end{pmatrix}\boxed{\begin{pmatrix}9{,}2\\ -12{,}6\\ 4{,}5\\ -1{,}1\end{pmatrix}} = \begin{pmatrix}32{,}\boxed{1}\\ 22{,}\boxed{9}\\ 33{,}\boxed{1}\\ 30{,}\boxed{9}\end{pmatrix} \qquad \begin{pmatrix}10 & 7 & 8 & 7\\ 7 & 5 & 6 & 5\\ 8 & 6 & 10 & 9\\ 7 & 5 & 9 & 10\end{pmatrix}\boxed{\begin{pmatrix}-7{,}2\\ 14{,}6\\ -2{,}5\\ 3{,}1\end{pmatrix}} = \begin{pmatrix}31{,}\boxed{9}\\ 23{,}\boxed{1}\\ 32{,}\boxed{9}\\ 31{,}\boxed{1}\end{pmatrix}$$

$$\kappa(R) = \|R\|.\|R^{-1}\| = \frac{\mu_{\max}}{\mu_{\min}}$$

$$\text{où } \mu = \sqrt{\text{valeurs propres de } R^{t}R}$$

$$\text{Sp}(R) \approx \{0{,}01;\ 0{,}84;\ 3{,}86;\ 30{,}29\} \Rightarrow \kappa(R) \approx \frac{30{,}29}{0.01} \approx 3029$$

La matrice de Wilson est très mal conditionnée ($\kappa >> 1$)

Problème d'Hadamard bien posé ?

- En continu : $\hat{p}_{\vec{\omega}}(\sigma) = \hat{f}\left(\sigma.\vec{\omega}^{\perp}\right)$, R bijectif d'inverse continue (conditions d'Hadamard).

- En discret, les choses sont moins simples :
  - R surjectif ? $\Leftarrow \boxed{{}^{t}R.}R\vec{f} = A\vec{f} = \boxed{{}^{t}R.}\vec{p} = \vec{q} \Leftarrow \bar{f} = \underset{f \in C}{\arg\min} \left\|\vec{p} - R\vec{f}\right\|^2$
  - R injectif ? : choix parmi les solutions possibles
  - R^{-1} continue mais $||R^{-1}||$ grande : $\kappa(R) = \|R\|\,\|R^{-1}\| = \dfrac{\mu_{max}}{\mu_{min}} >> 1$

$$\frac{\left\|\delta\vec{f}\right\|}{\left\|\vec{f}\right\|} \le \kappa(R) \frac{\left\|\delta\vec{p}\right\|}{\left\|\vec{p}\right\|}$$

Régularisation

Remplacer : $\bar{f} = \arg\min_{f \in C} \left\| \vec{p} - R\vec{f} \right\|^2$ par

$$\bar{f} = \arg\min_{f \in C} \left\{ \left\| \vec{p} - R\vec{f} \right\|^2 + \alpha.\rho(\vec{f}) \right\}$$

Adéquation aux données
Surjectivité du problème inverse

Régularisation
injectivité

Exemples : $\rho(\vec{f}) \in \left\{ \left\| \vec{f} \right\|^2 \; ; \; \left\langle \vec{f} \middle| Q\vec{f} \right\rangle \; ; \; \sum_i f_i \ln(f_i) \; ; \; \dots \right\}$

Régularisation de Tikhonov

$$\boxed{\bar{f} = \arg\min_{f \in C} \left\{ \left\| \vec{p} - R\vec{f} \right\|^2 + \alpha . \left\| \vec{f} \right\|^2 \right\}}$$

Adéquation aux données
Surjectivité du problème inverse

Régularisation
injectivité

$$\vec{f} = \arg\min_{\vec{f}} \left\{ \left\| \vec{p} - R\vec{f} \right\|^2 + \alpha . \left\| \vec{f} \right\|^2 \right\} \Leftrightarrow \left(R^t R + \alpha I \right) f = R^t p$$

Solution directe : $\vec{f} = \left(R^t R + \alpha I \right)^{-1} R^t p$ … ou solution par descente

Régularisation MAP-EM

Bayes : $P(\vec{f}/\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})/P(\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})$

$$\boxed{\tilde{\vec{f}} = \arg\min_{\vec{f}} \left[-\log P(\vec{p}/\vec{f}) - \log P(\vec{f}) \right]}$$

Adéquation aux données

?

$$P(\vec{p}/\vec{f}) = \prod_j \frac{e^{-q_j} q_j^{\,p_j}}{p_j!} \quad ; \quad q_j = (Rf)_j$$

Green PJ. Bayesian reconstructions from emission tomography data using a modified EM algorithm. IEEE Trans Med Imaging 1990;9:84-93

Régularisation MAP-EM-OSL

Bayes : $P(\vec{f}/\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})/P(\vec{p}) = P(\vec{p}/\vec{f}).P(\vec{f})$

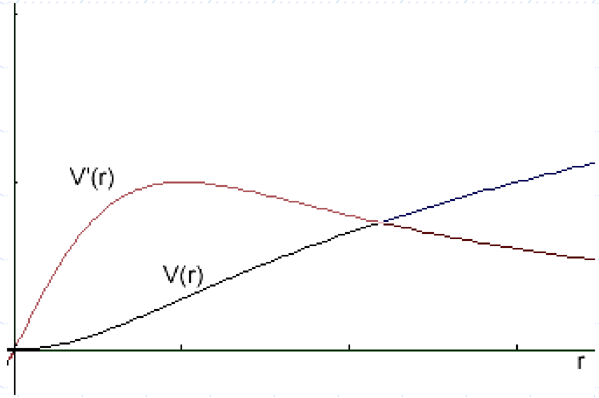
$$\boxed{\tilde{\vec{f}} = \arg\min_{\vec{f}}\left[-\log P(\vec{p}/\vec{f}) - \log P(\vec{f})\right]}$$

Adéquation aux données

$$P(\vec{f}) = \frac{1}{K} e^{-\beta.\sum_{i,j} w_{i,j}.V\left(f_i - f_j\right)}$$

Distribution de Gibbs

$$P(\vec{p}/\vec{f}) = \prod_j \frac{e^{-q_j} q_j^{\,p_j}}{p_j!} \quad ; \quad q_j = (Rf)_j$$



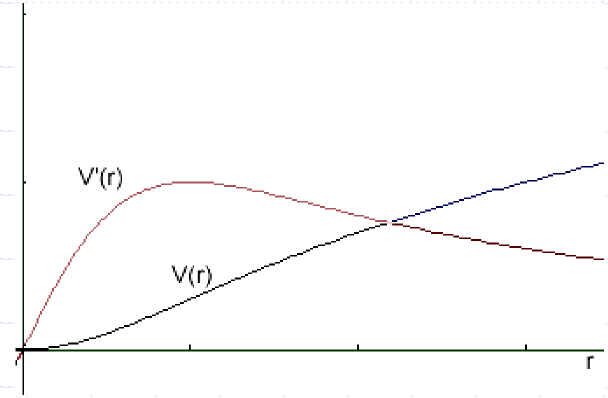
Green PJ. Bayesian reconstructions from emission tomography data using a modified EM algorithm. IEEE Trans Med Imaging 1990;9:84-93

Régularisation MAP-EM-OSL

$$\tilde{\vec{f}} = \arg\min_{\vec{f}} \left[-\log P\left(\vec{p}/\vec{f}\right) + \beta \underbrace{\sum_{i,j} w_{i,j} V(f_i - f_j)}_{U} \right]$$

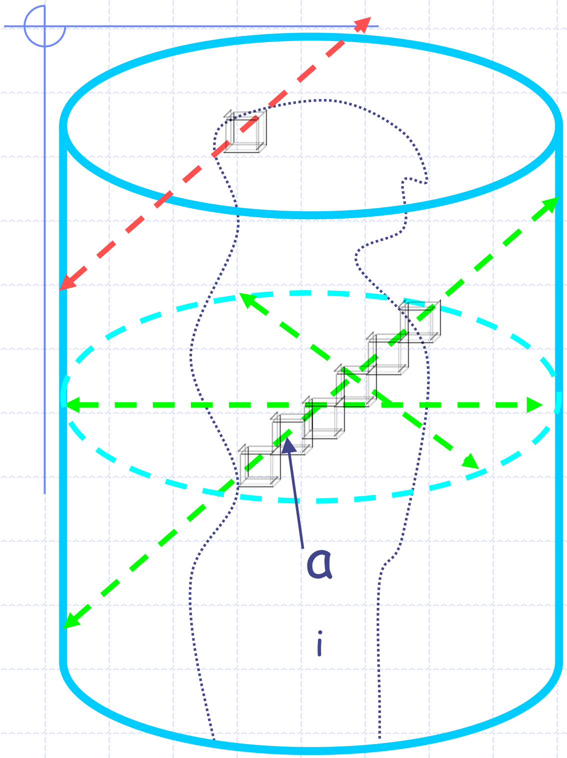
$$f_i^{n+1} = f_i^n \frac{1}{\sum_j R_{ij} + \beta . \partial_i U} \sum_j R_{ij} \frac{p_j}{q_j}$$

$$\partial_i U = \sum_{k \in V(i)} w_{i,k} . \frac{\partial V}{\partial r}(f_i - f_k)$$



Green PJ. Bayesian reconstructions from emission tomography data using a modified EM algorithm. IEEE Trans Med Imaging 1990;9:84-93

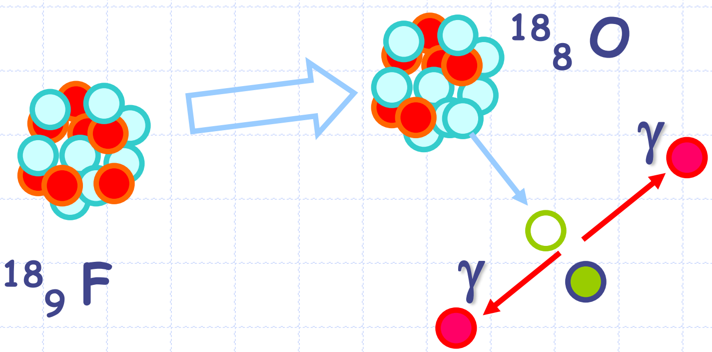
Tomographie en coïncidence 3D



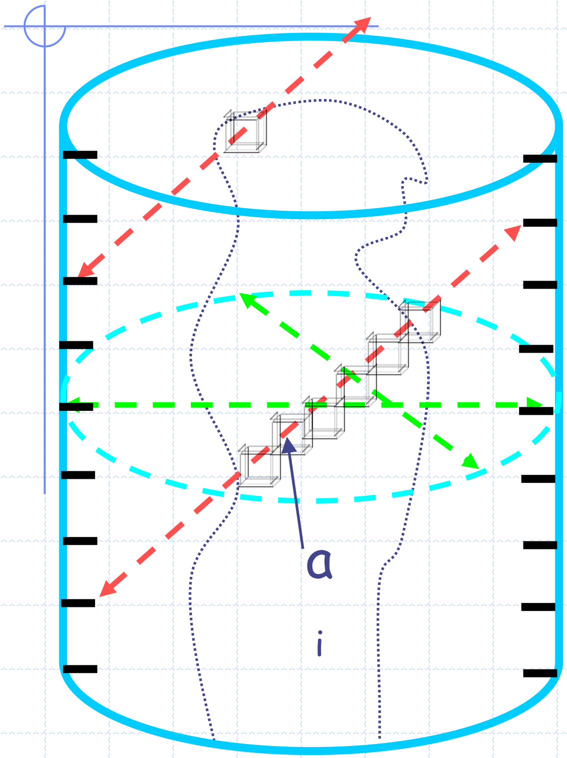
Exemple de la TEP

Projections 3D redondantes et incomplètes

- Recherche de $f(x,y,z)$ connaissant $p(s,\phi,z,\theta)$
- Certaines projections obliques ne sont pas enregistrées pour $\theta \neq 0$

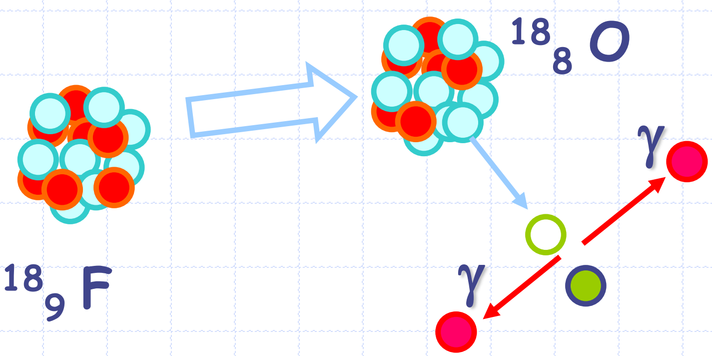


Tomographie en coïncidence 3D

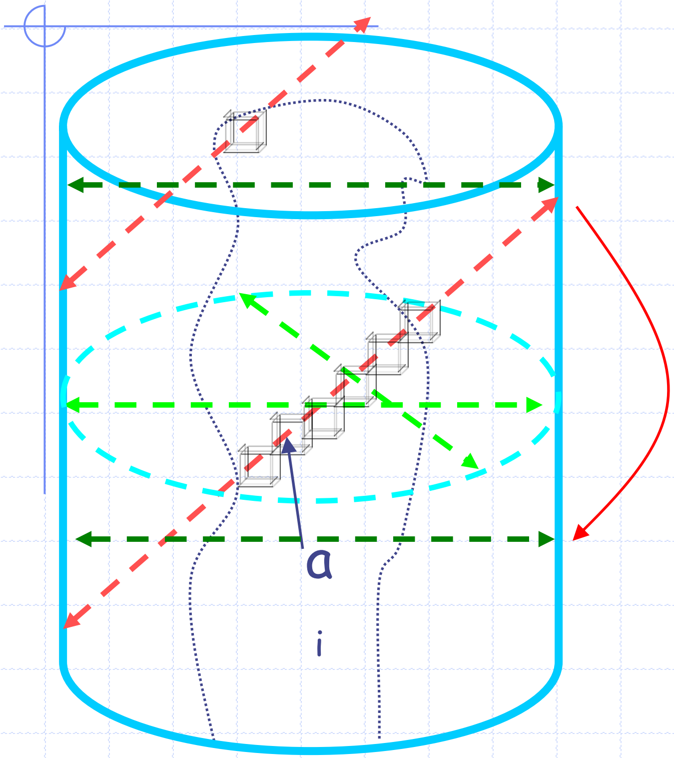


Projections 3D redondantes et incomplètes

- <u>**Reconstruction 2D de données 2D**</u>
  - Utilisation d'un collimateur
  - ↘ statistique de comptage, ↘ S/B



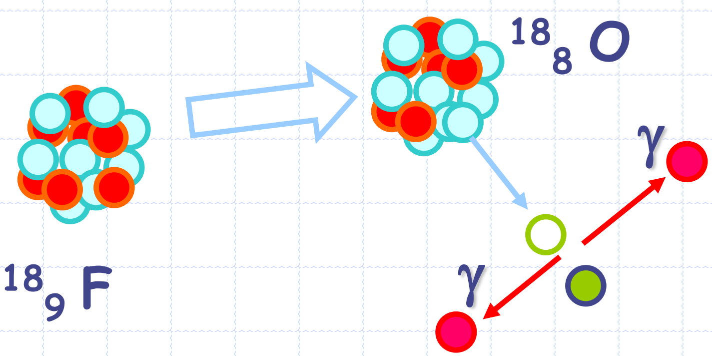
Tomographie en coïncidence 3D



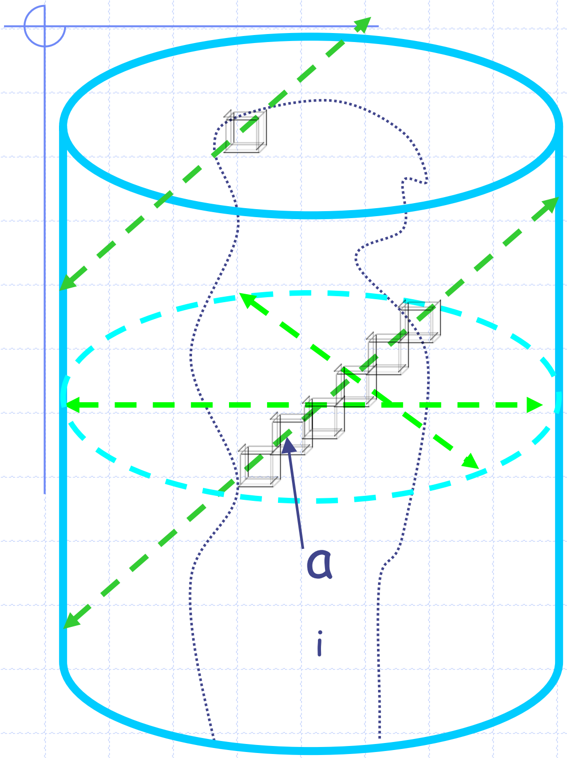
Projections 3D redondantes et incomplètes

- **<u>Reconstruction 2D de données 2D</u>**
  - Utilisation d'un collimateur
  - ↘ γ détectés N, ↘ S/B=N/√N= √N

- **<u>Réarrangement 2D de données 3D</u>**
  - Algorithmes de «rebinning »
  - S/B ↗ mais approximation



Tomographie en coïncidence 3D



Projections 3D redondantes et incomplètes

- <u>Reconstruction 2D de données 2D</u>
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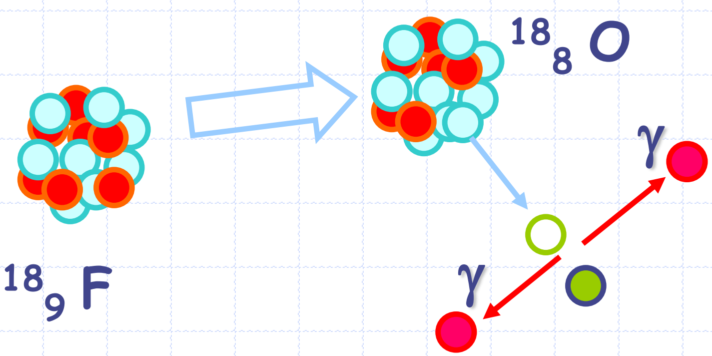
  - Algorithmes de «rebinning »
  - S/B ↗ mais approximation
- <u>Reconstruction 3D de données 3D</u>
  - Algorithmes algébriques 3D
  - RPF 3D si projections complètes
  - S/B ↗ mais temps de calcul ↗↗



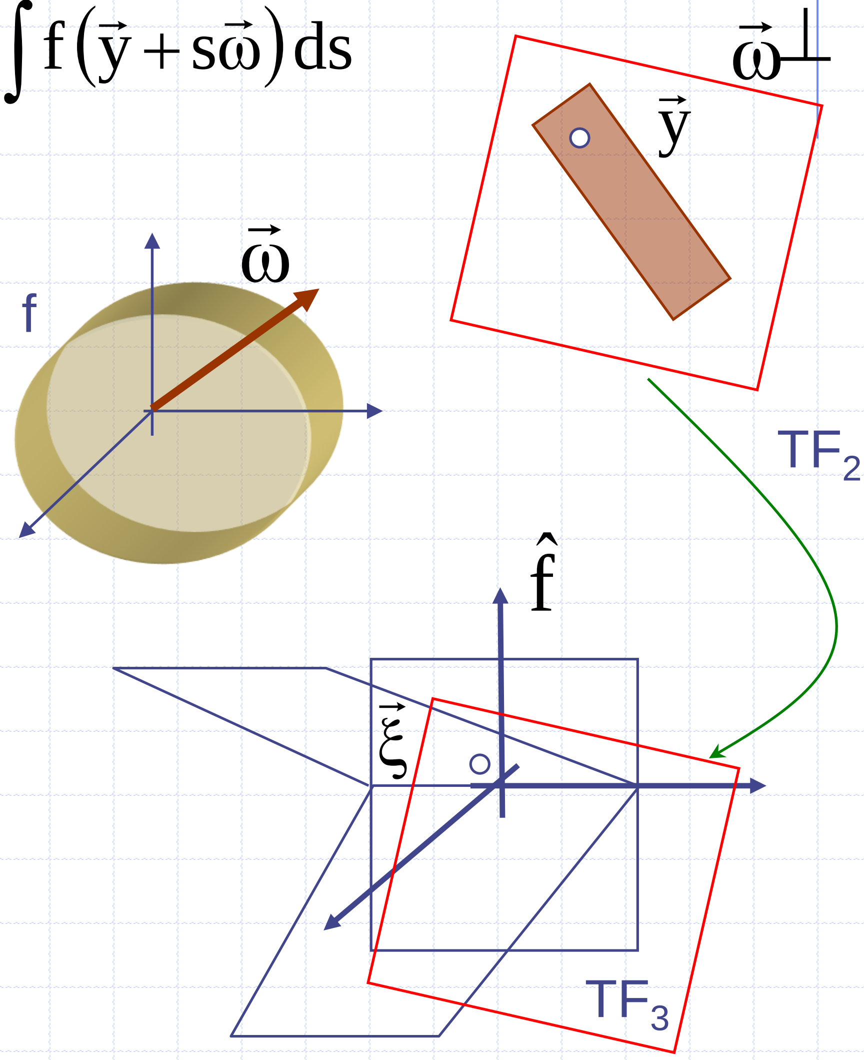
Un théorème de Radon 3D…

$$\forall \vec{\omega} \in S^2, \qquad \forall \vec{y} \in \vec{\omega}^{\perp}, p_{\vec{\omega}}(\vec{y}) = \int f(\vec{y} + s\vec{\omega})\, ds$$

$$\hat{p}_{\vec{\omega}}(\vec{\xi}) = \iint_{\vec{\omega}^{\perp}} \int f(\vec{y} + s\vec{\omega})\, e^{-2i\pi\, \vec{y}.\vec{\xi}}\, ds\, d\vec{y}$$

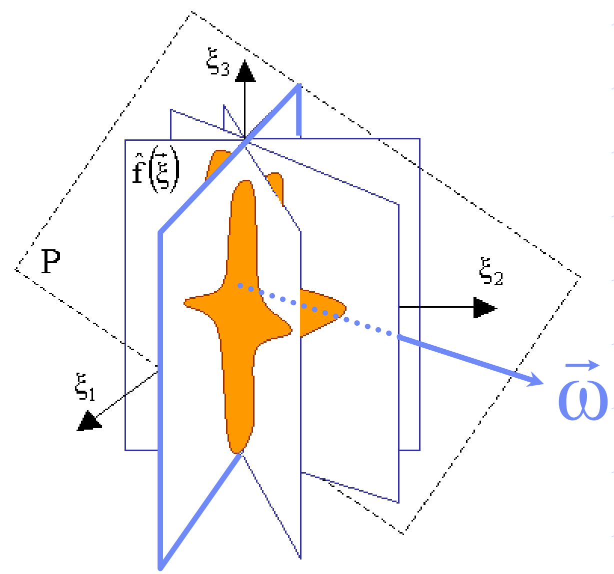
$$\hat{p}_{\vec{\omega}}(\vec{\xi}) = \iiint f(\vec{x})\, e^{-2i\pi\, \vec{x}.\vec{\xi}}\, d\vec{x} = \hat{f}(\vec{\xi})$$

$$\boxed{\forall \vec{\xi} \in \vec{\omega}^{\perp}, \hat{p}_{\vec{\omega}}(\vec{\xi}) = \hat{f}(\vec{\xi})}$$



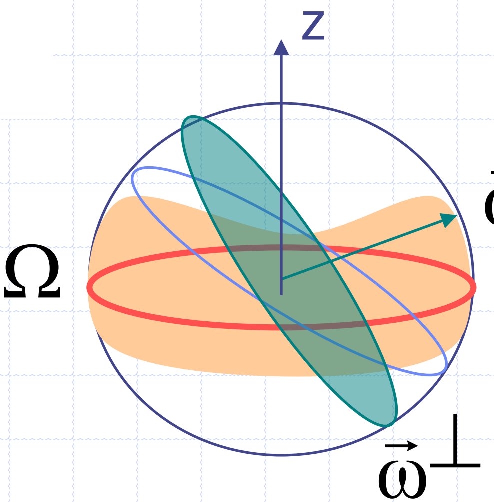
…

1- Condition d'Orlov :



Condition nécessaire à l'affectation
de toutes les fréquences spatiales de $\mathbb{R}^3$:

Ω contient au moins un cercle équatorial de S
i.e
Ω intersecte tout cercle équatorial de S

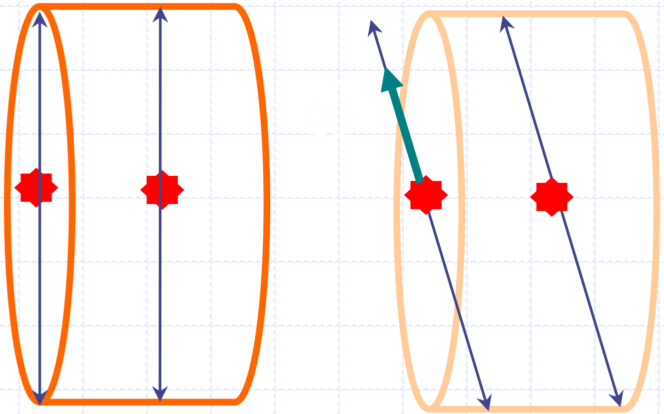


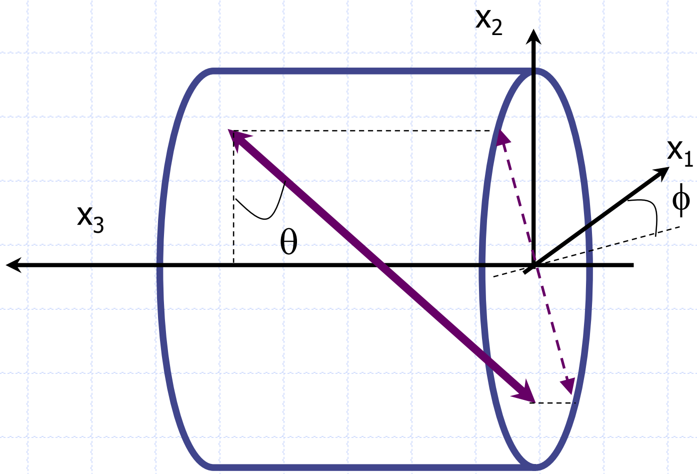
SS. Orlov. Sov.Phys. Crystallogr.,1976. Vol 20, 3:312-4 et 4:429-433

… plutôt difficile à appliquer !

1- Condition d'Orlov

2 - Projections non tronquées





3 – moyennant une interpolation 3D dans le domaine des fréquences

$$\hat{p}(\xi_1, \xi_2) = \hat{f}(\xi_1 \cos\theta \sin\phi - \xi_2 \sin\theta, \xi_1 \sin\theta \sin\phi + \xi_2 \cos\theta, -\xi_1 \cos\phi)$$

Solutions possibles

1– Condition d'Orlov
- Détecteur TEP cylindrique

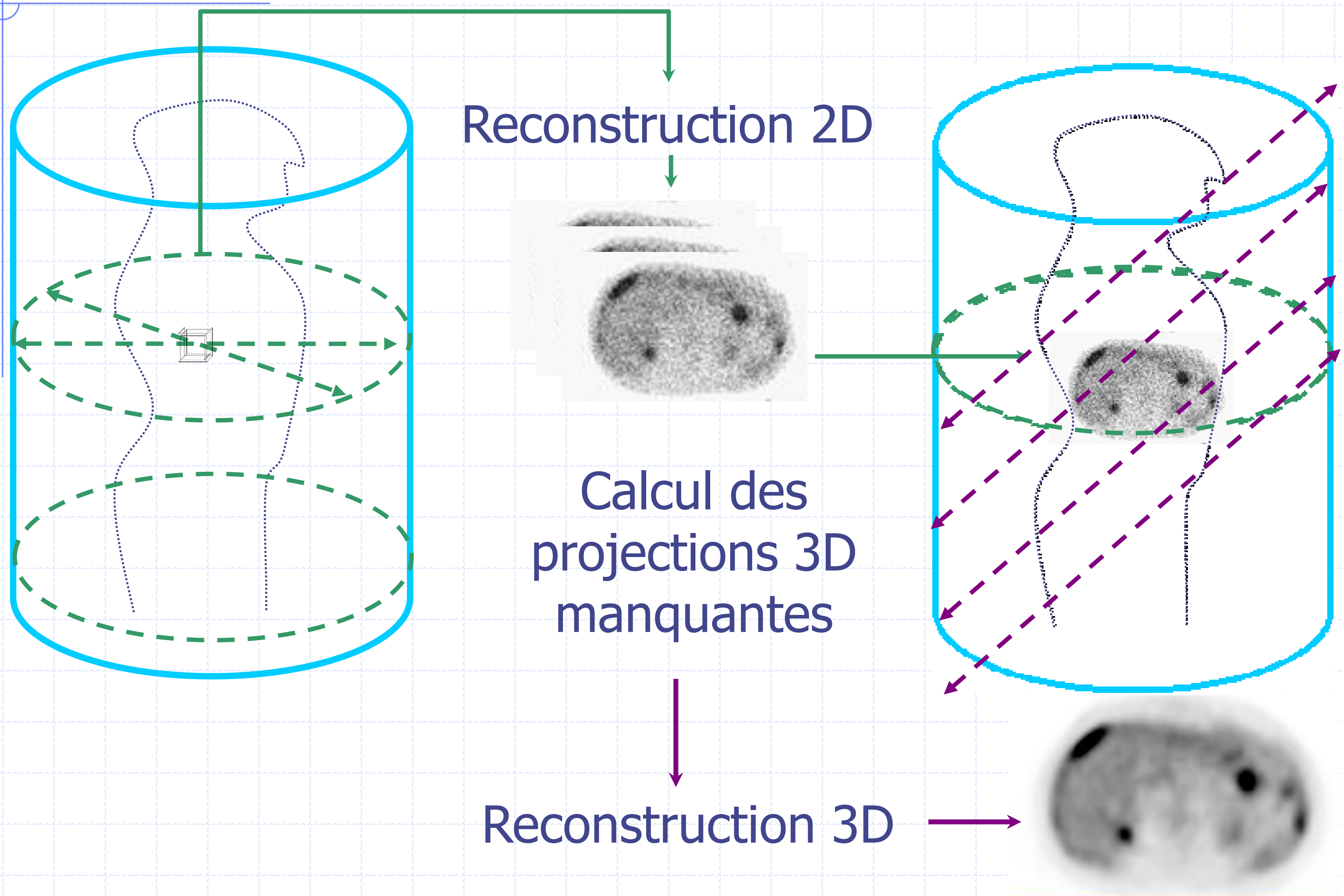
2 – Projections tronquées
- Estimées par reconstruction 2D puis projection ou <u>rebining</u>

3 –Interpolation 3D en fréquence
- Optimisation de l'interpolation (fonctions de Kaiser-Bessel)
- Utilisation d'une rétro-projection filtrée (filtre de Colsher)

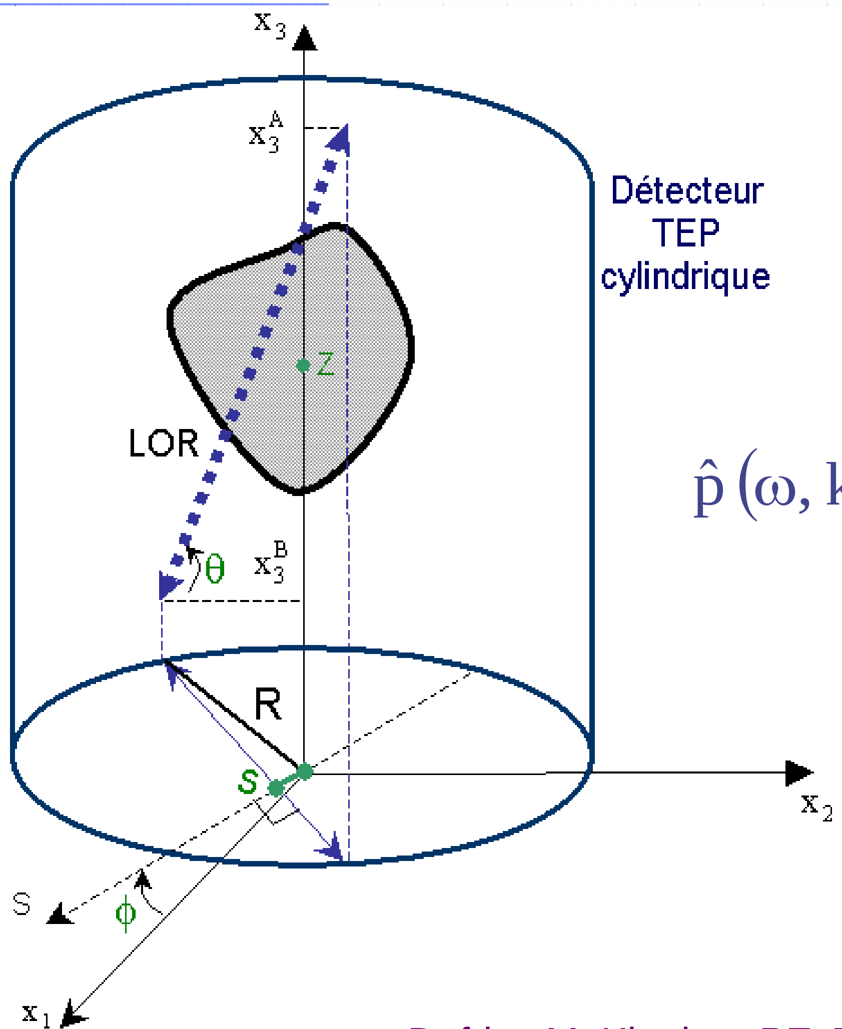
En « routine » : Utilisation d'algorithmes algébriques (OSEM 3D)
Reconstruction 2D après <u>rebining</u> des projections 3D

Fourier-based reconstruction for fully 3-DPET. Matej S, Kazantsev IG. IEEE Trans Med Imaging 2006;25:845-54.
Evaluation of a new gridding method for fully 3D direct Fourier PET reconstruction based on a two-plane geometry F Ben Bouallègue, J F Crouzet, D Mariano-Goulart. *Comput Med Imaging Graph.* 2008;32:580-589.
Colsher JG. Fully three-dimensional PET. Phys Med Biol 25(1), 103-115, 1980

Reprojection après RPF 2D



Ré-arrangement (rebining) exact



$$p\left(s,\varphi,z=\frac{x_3^A+x_3^B}{2},\delta=\mathrm{tg}\theta\right)$$

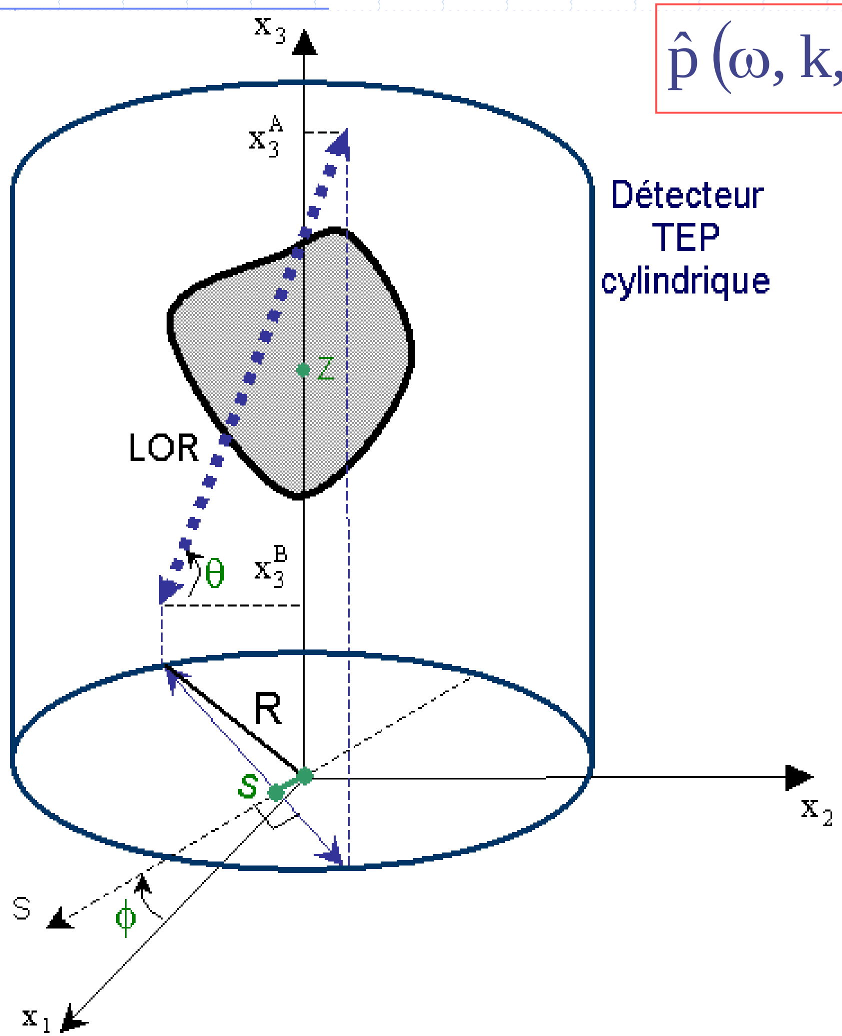
TF(s,φ) puis TF(z)
si invariance en Tz

$$\hat{p}\left(\omega,k,\zeta,\delta\right)=e^{-ik\arctan(\alpha)}\,\hat{p}\left(\omega\sqrt{1+\alpha^2},k,\zeta,0\right)$$

$$\alpha=\frac{\delta\zeta}{\omega}$$

Defrise M, Kinahan PE, Townsend DW, Michel C, Sibomana M, Newport DF. Exact and approximate rebinning algorithms for3-D PET data. IEEE Trans Med Imaging 1997;16:145-58.

Ré-arrangement approximatif



$$\boxed{\hat{p}(\omega, k, \zeta, \delta) = e^{-ik\arctan(\alpha)}\,\hat{p}\left(\omega\sqrt{1+\alpha^2}, k, \zeta, 0\right)}$$

DL à l'ordre 1 sur $\alpha = \dfrac{\delta\zeta}{\omega}$

$$\hat{p}(\omega, k, \zeta, \delta) \approx e^{-ik\alpha}\,\hat{p}(\omega, k, \zeta, 0)$$

$$\hat{p}(\omega, k, z, \delta) \approx \hat{p}\left(\omega, k, z\text{-}k\frac{\delta}{\omega}, 0\right)$$

SYNTHESE DE DONNEES 2D à S/B ↑ :

$$\boxed{\hat{p}(\omega, k, z, 0) \approx \hat{p}\left(\omega, k, z+k\frac{\delta}{\omega}, \delta\right)}$$

SYNTHESE DE DONNEES MANQUANTES :

$$\boxed{\hat{p}(\omega, k, z, \delta) \approx \hat{p}\left(\omega, k, z-k\frac{(\delta-\delta')}{\omega}, \delta'\right)}$$

Ben Bouallègue F, Crouzet JF, Comtat C, Fourcade M, Mohammadi B, Mariano-Goulart D. Exact & approximate Fourier rebinning algorithms for the solution of the data truncation problem in 3-DPET. IEEE Trans Med Imaging 2007;26:1001-9.

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2003. Springer.

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D. Mariano-Goulart
Encyclopédie Médico-chirurgicale,
35-105-A-10, 2009.