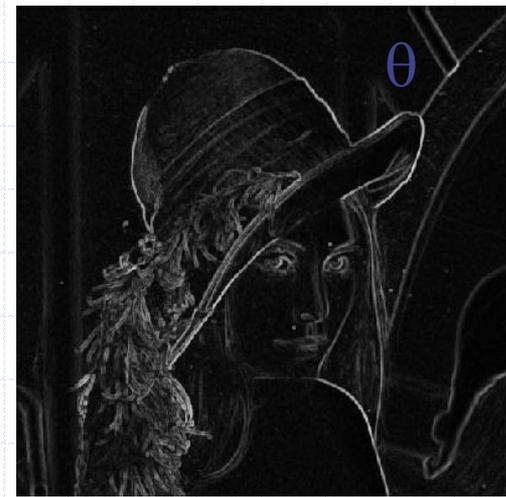
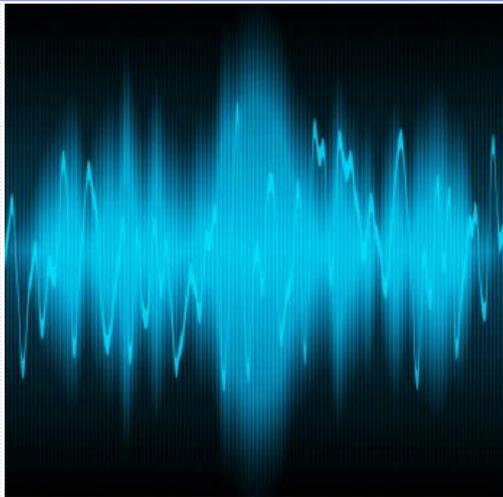


FORMATION TIC (Phymed)

TRAITEMENT DES IMAGES MEDICALES



Fayçal Ben Bouallègue - faybenb@hotmail.com

<http://scinti.etud.univ-montp1.fr>

Numérisation

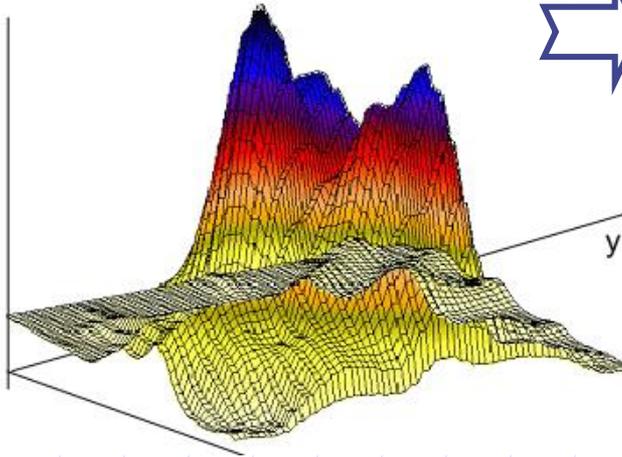
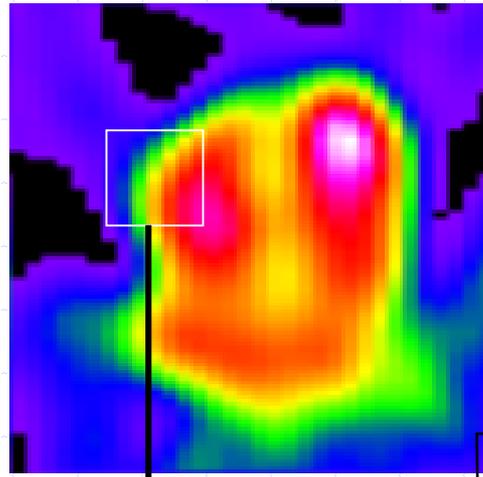
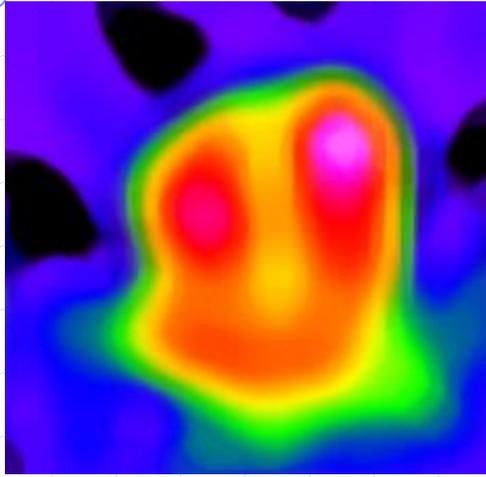
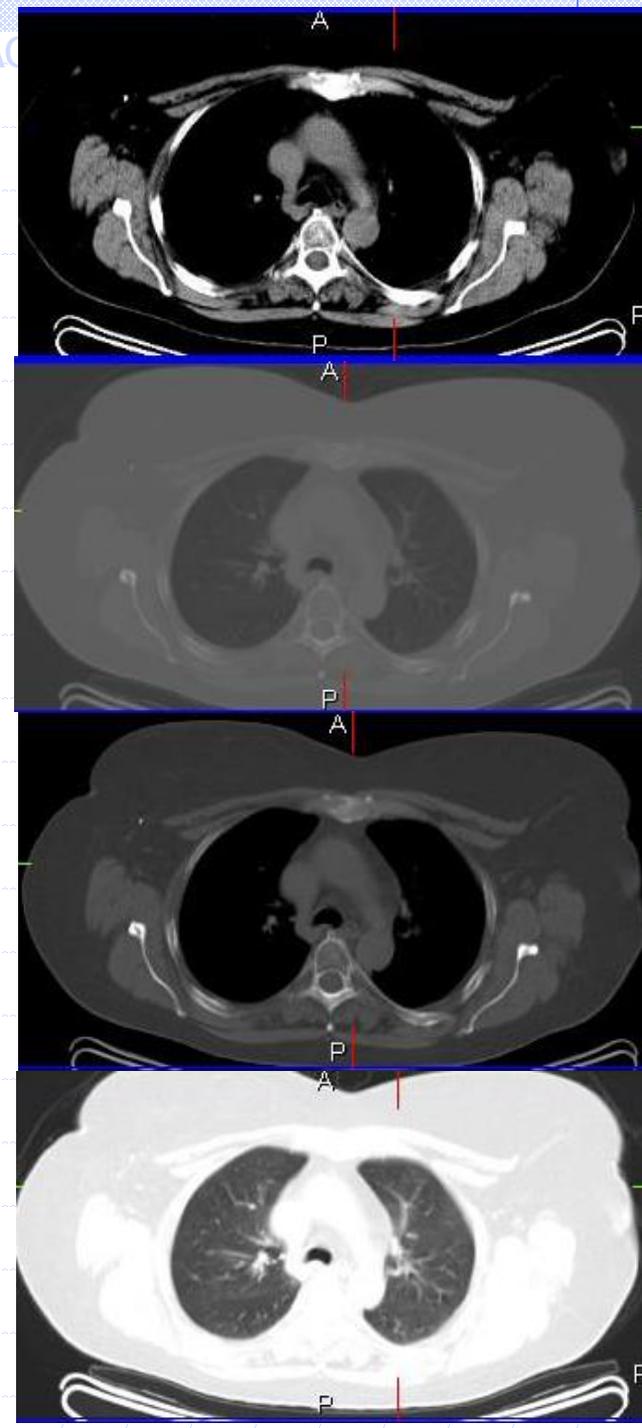


IMAGE ANALOGIQUE

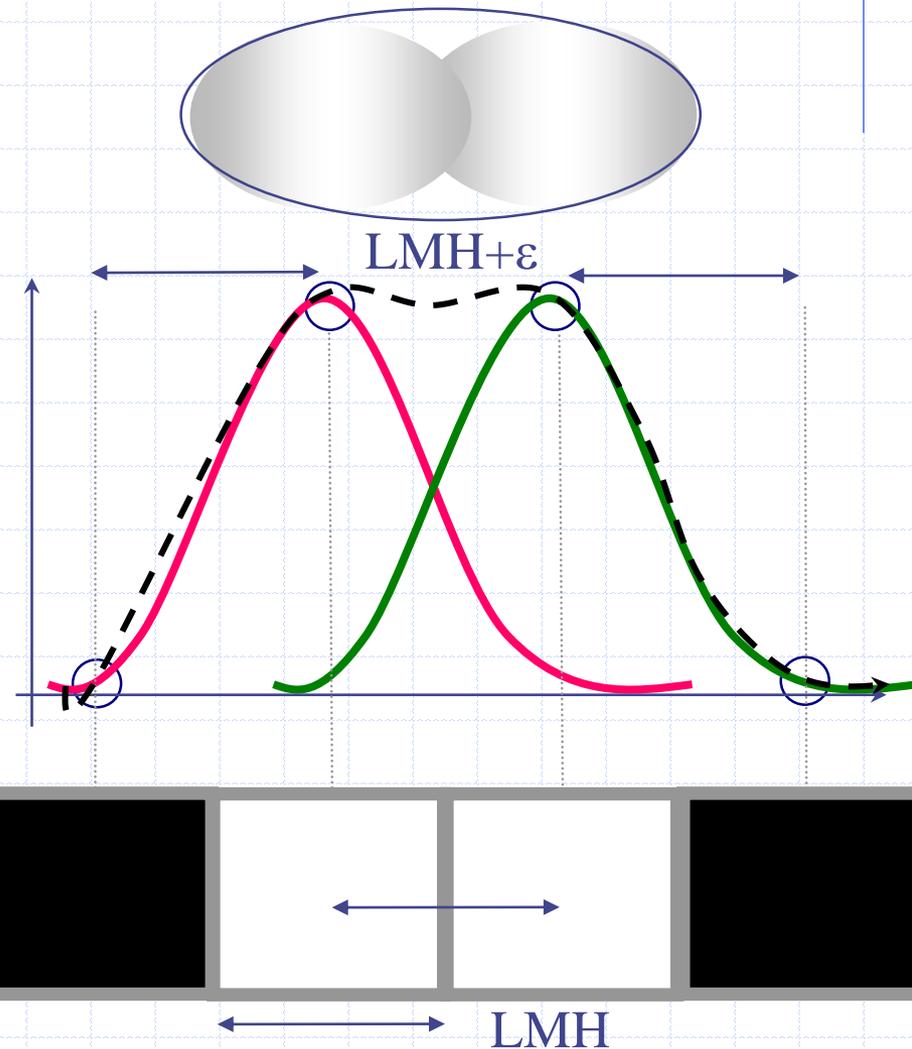
IMAGE NUMERIQUE



Approche intuitive

Si la taille du pixel est identique à la LMH, alors aucun contraste n'est produit pour des objets ponctuels distants d'un peu plus que la LMH:

Perte de résolution



Théorème de Shannon

ECHANTILLONNAGE
SANS PERTE DE
RESOLUTION



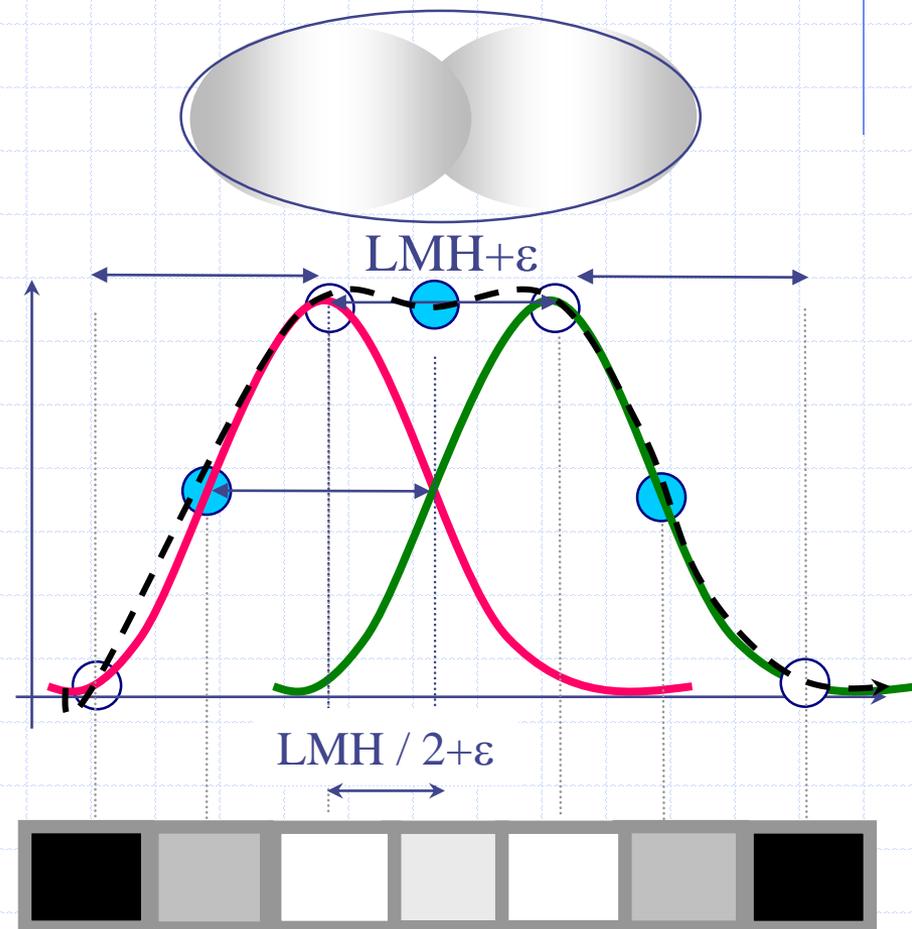
taille du pixel d

$$d \leq LMH/2$$

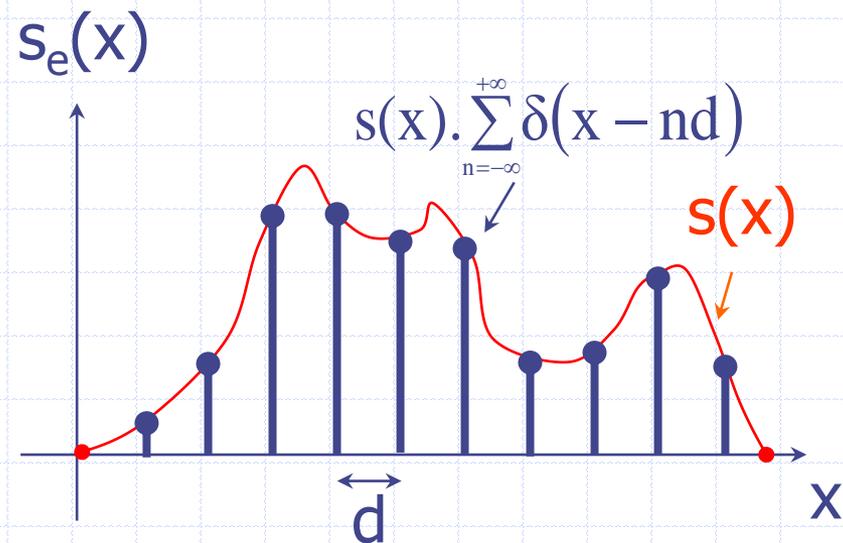
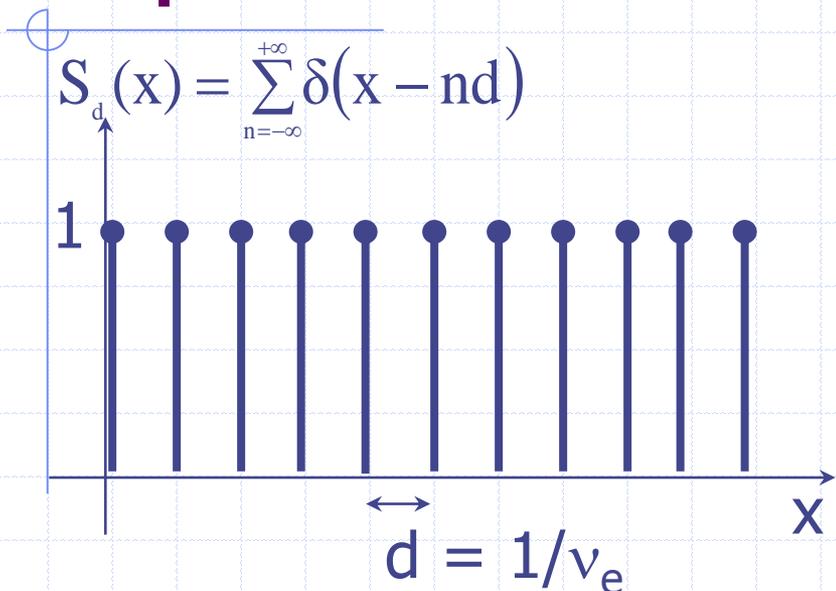
En pratique :

$$d = LMH/2$$

$$1/d = 2/LMH \Leftrightarrow f_e = 2 \cdot f_{\max}$$



Opérateur d'échantillonnage



$$s_e(x) = s(x) \cdot S_d(x) = \sum_{n=-\infty}^{+\infty} s(x) \cdot \delta(x - nd) = \sum_{n=-\infty}^{+\infty} s(n \cdot d) \cdot \delta(x - nd)$$

Nb :
$$S_d(x) = \sum_{n=-\infty}^{+\infty} \delta(x - n \cdot d) \xrightarrow{TF} \hat{S}_d(\nu) = \frac{1}{d} \sum_{n=-\infty}^{+\infty} \delta\left(\nu - \frac{n}{d}\right)$$

Théorème de Shannon

$$s_e(x) = s(x) \cdot \sum_{n=-\infty}^{+\infty} \delta(x - nd)$$

$$\hat{s}_e(v) = \frac{1}{d} \hat{s}(v) * \sum_{n=-\infty}^{+\infty} \delta\left(v - \frac{n}{d}\right)$$

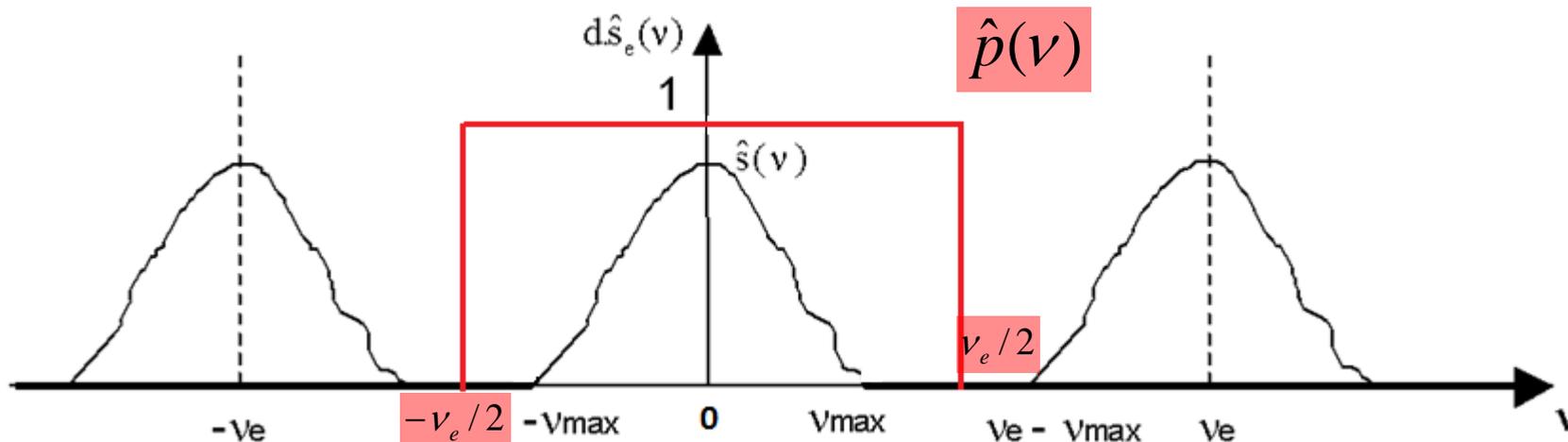
$$\hat{s}(v) * \delta\left(v - \frac{n}{d}\right) = \int_{\mathbb{R}} \hat{s}(v') \cdot \delta\left(v - \frac{n}{d} - v'\right) \cdot dv' = \hat{s}\left(v - \frac{n}{d}\right)$$

$$\hat{s}_e(v) = \frac{1}{d} \sum_{n=-\infty}^{+\infty} \hat{s}\left(v - \frac{n}{d}\right)$$

$$\hat{s}_e(v) = \frac{1}{d} \sum_{n=-\infty}^{+\infty} \hat{s}(v - n \cdot v_e)$$

Théorème de Shannon

$$d \cdot \hat{s}_e(v) = \sum_{n=-\infty}^{+\infty} \hat{s}(v - n \cdot v_e)$$

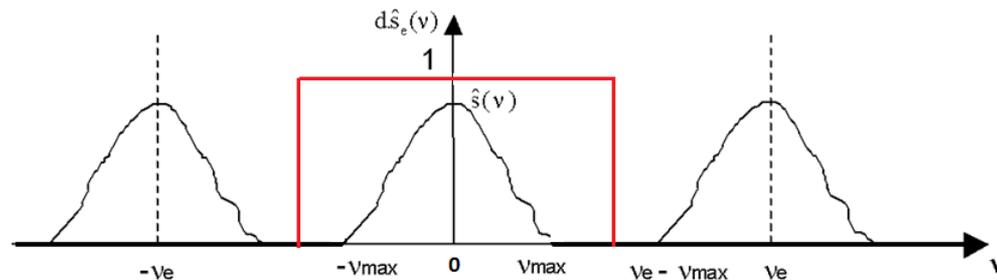


$$v_e > 2 \cdot v_{\max} \Rightarrow d \cdot \hat{s}_e(v) \cdot \hat{p}(v) = \hat{s}(v)$$

Théorème de Shannon

$$\hat{s}(v) = d \cdot \hat{s}_e(v) \cdot \hat{p}(v)$$

TF-1

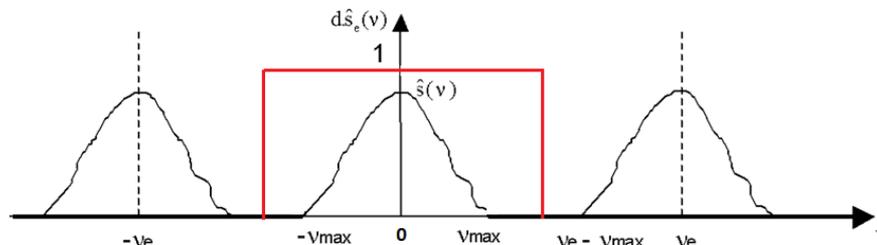


$$d \cdot p(x) * \sum_{n=-\infty}^{\infty} s(nd) \cdot \delta(x - nd) = d \cdot \sum_{n=-\infty}^{\infty} s(nd) \cdot p(x - nd)$$

$$= d \cdot s(x) \otimes_d p(x)$$

$$= s_e(x) \otimes_d \text{sinc}\left(\frac{\pi x}{d}\right) = s(x)$$

Théorème de Shannon



$$v_e = 2 \cdot v_{\max} \Rightarrow s(x) = s_e(x) \otimes_d \operatorname{sinc}\left(\frac{\pi x}{d}\right)$$

L'échantillonnage :

$$s_e(x) = s(x) \cdot S_d(x) = \sum_{n=-\infty}^{+\infty} s(n \cdot d) \cdot \delta(x - nd) \quad \text{avec } d = LMH/2$$

se fait sans perte d'information

$v_e = 2 \cdot v_{\max} = 2/LMH$ est appelé fréquence de Nyquist

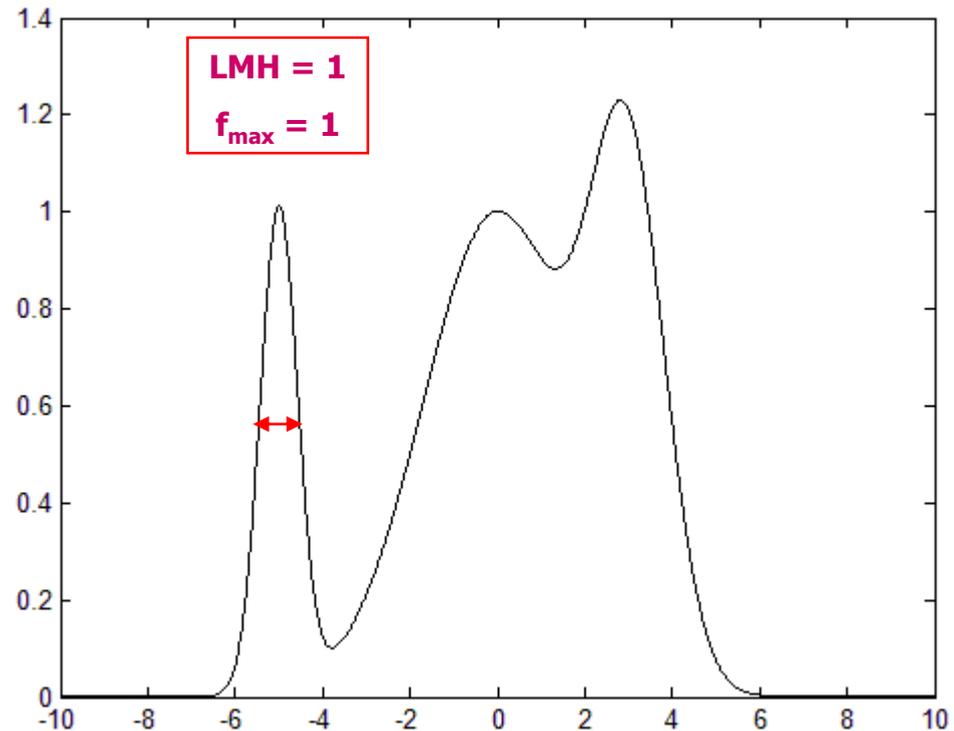
Théorème de Shannon

$$d = LMH/2$$

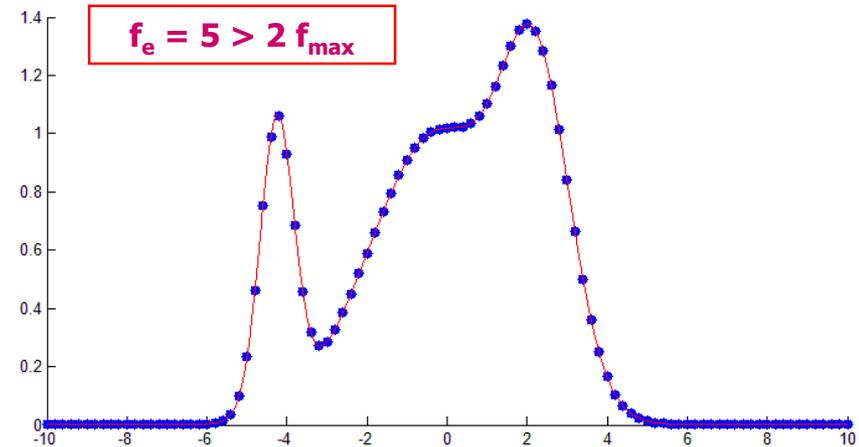
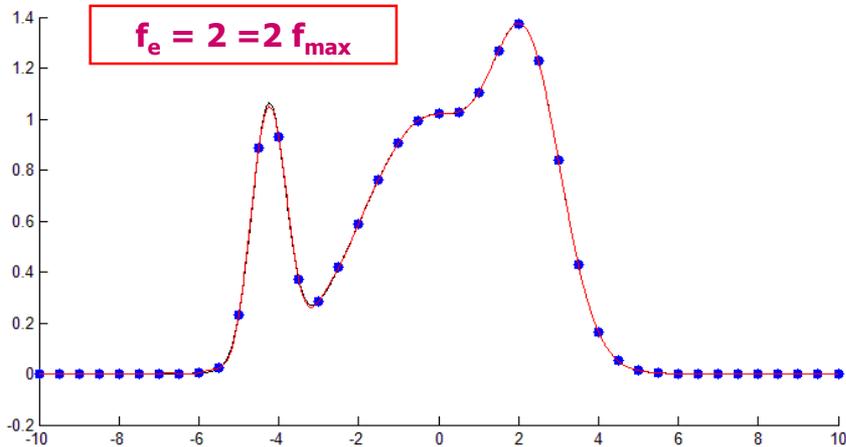
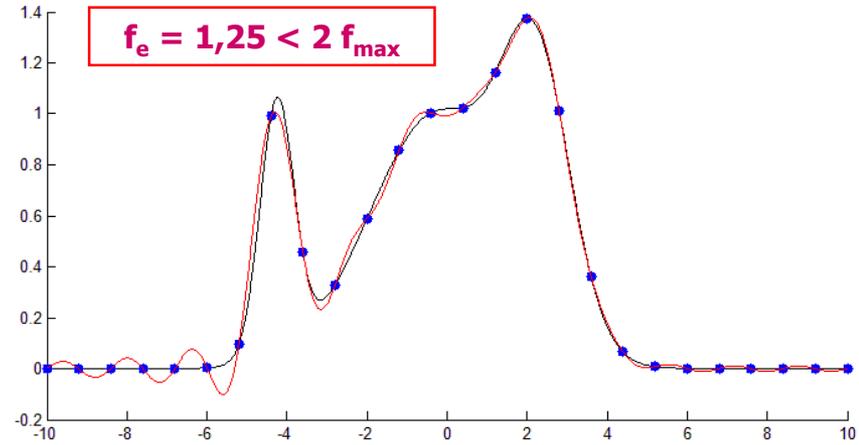
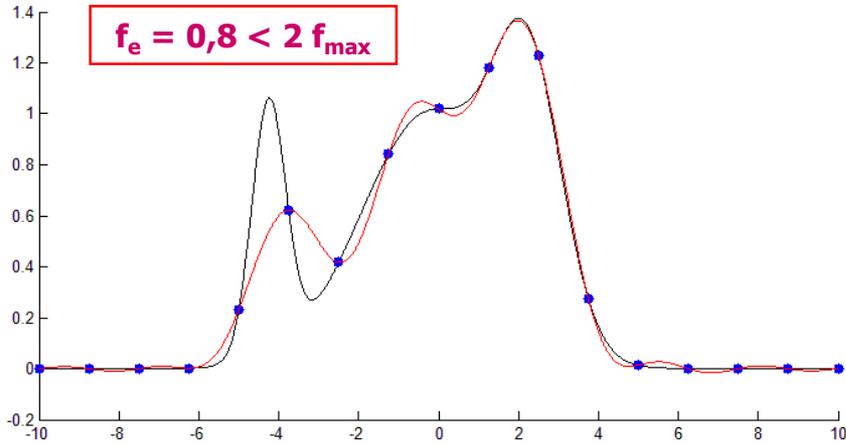
$$1/d = 2/LMH$$

$$f_e = 2 \cdot f_{\max}$$

$$s(x) \stackrel{?}{=} s_e(x) \otimes_d \text{sinc}\left(\frac{\pi x}{d}\right)$$



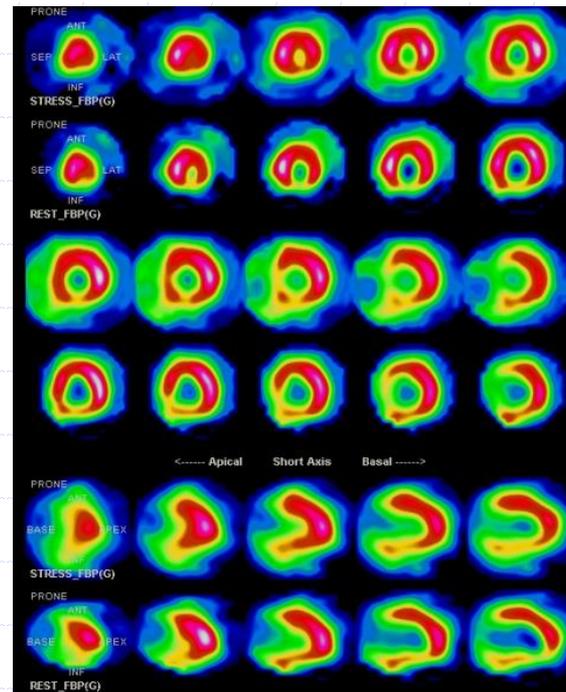
$$s(x) \stackrel{?}{=} s_e(x) \otimes_d \text{sinc}\left(\frac{\pi x}{d}\right)$$



Théorème de Shannon

*Exemple : Nombre de pixels
en scintigraphie myocardique*

*champ 50x50 cm
LMH = 16 mm*



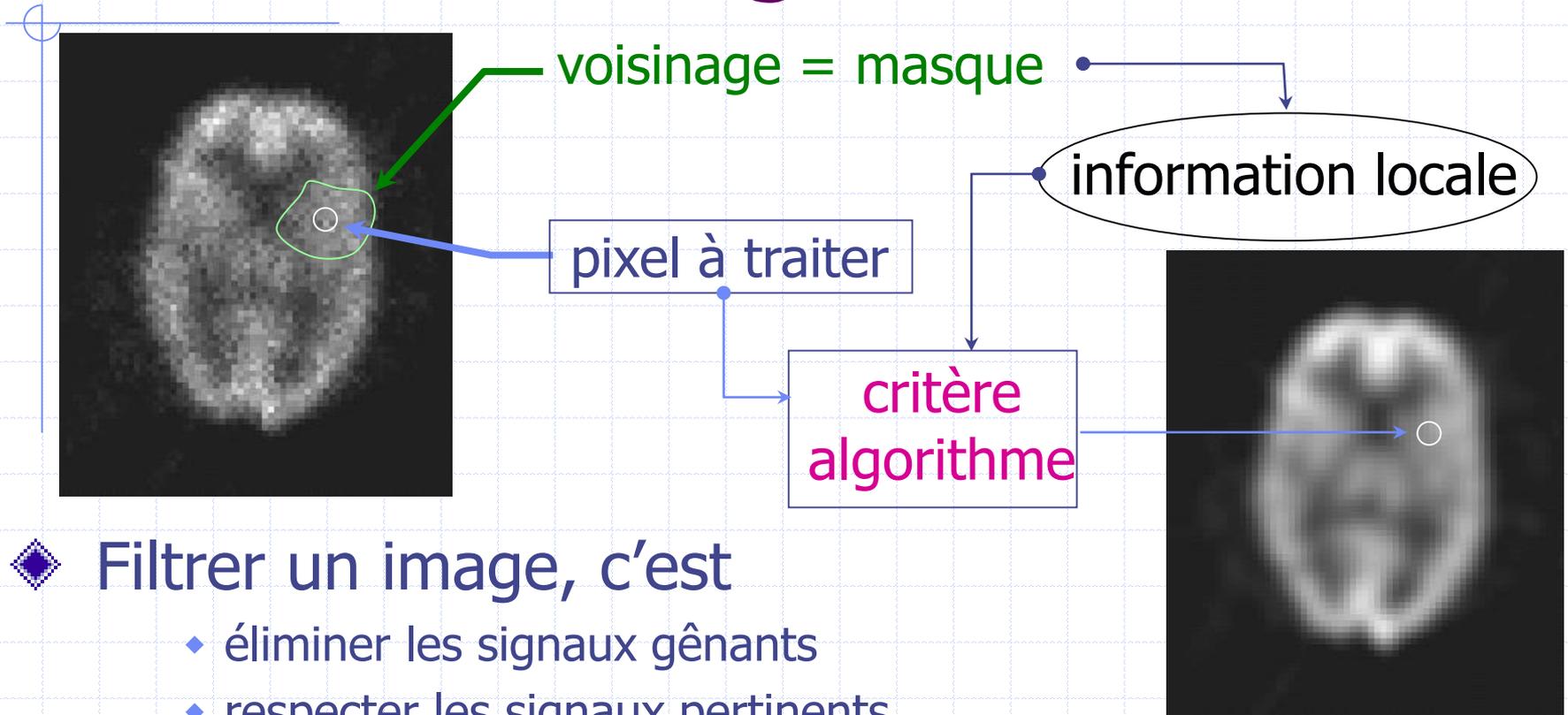
Réponse:

$$v_{\max} = 1/LMH = 1/16 = 0.0625 \text{ pixel/mm}$$

$$v_e = 2 \cdot v_{\max} = 2/16 = 0.125 \text{ pixel/mm}$$

Donc $0.125 \times 500 = 62.5$ i.e **64 pixels/côté**

Notion de filtrage



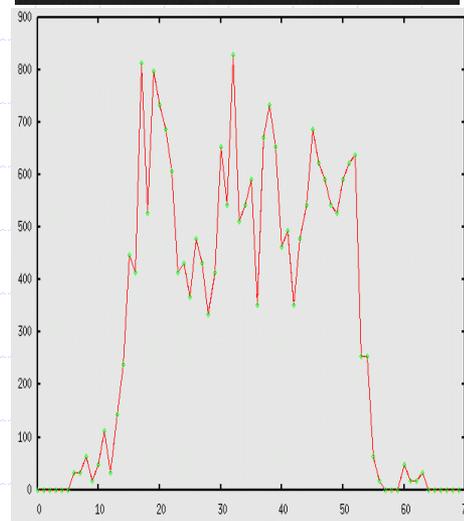
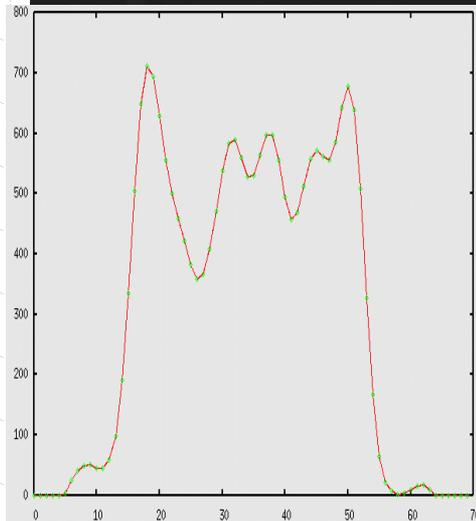
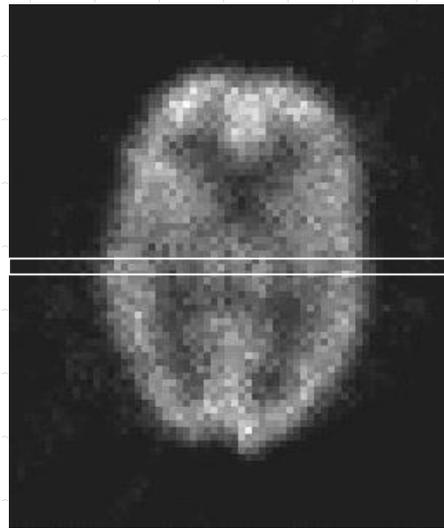
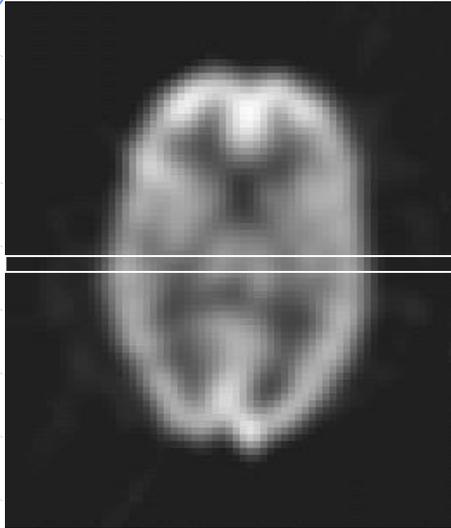
◆ Filtrer un image, c'est

- ◆ éliminer les signaux gênants
- ◆ respecter les signaux pertinents

◆ Il faut donc définir :

- ◆ un critère pour discriminer signaux parasites et pertinents
- ◆ un algorithme pour éliminer les signaux parasites

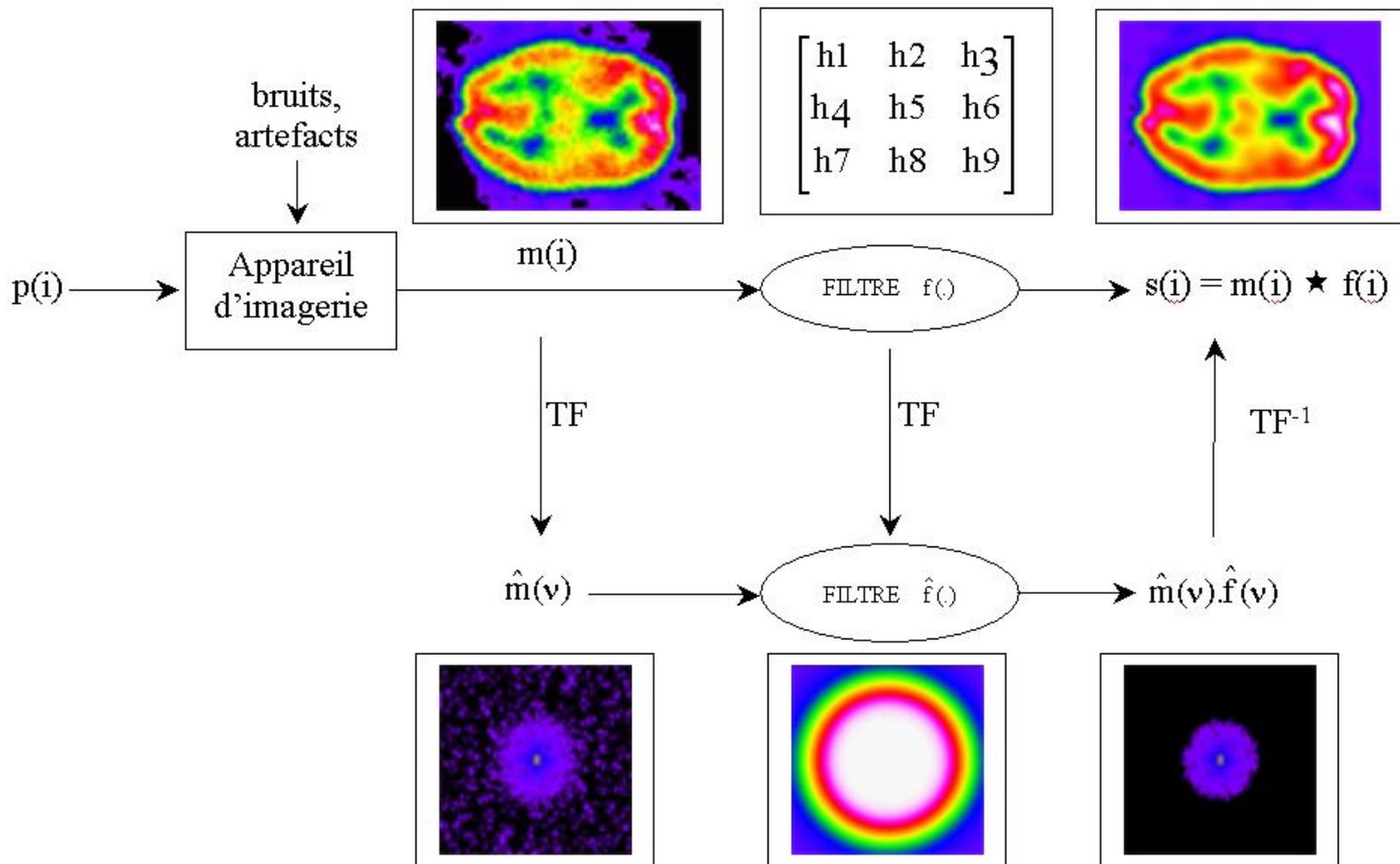
1° idée : critère fréquentiel



Ceci conduit à multiplier chaque harmonique par un facteur d'amplification variable, donc à appliquer un filtre linéaire et invariant dans le décalage (convolution).

Critère : fréquences
Algorithme: convolution ou multiplication dans l'espace des fréquences spatiales.

Filtrage linéaire d'image



Filtres passe-bas

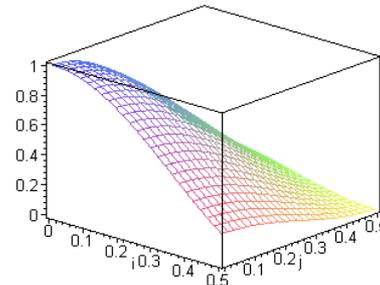
◆ Convolution: $s(i) = \sum_{k=-\infty}^{+\infty} f(k).m(i-k)$

◆ Remplace chaque valeur par une moyenne pondérée des valeurs voisines

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

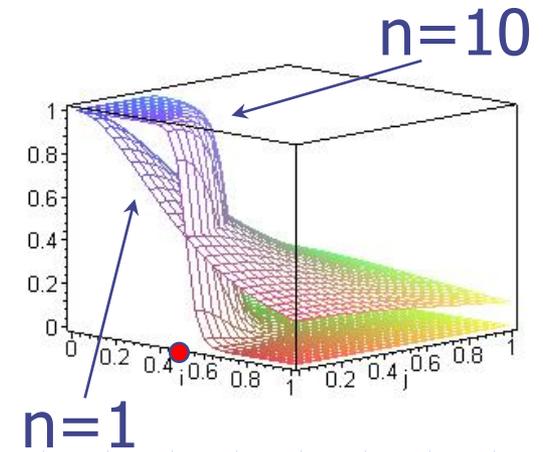
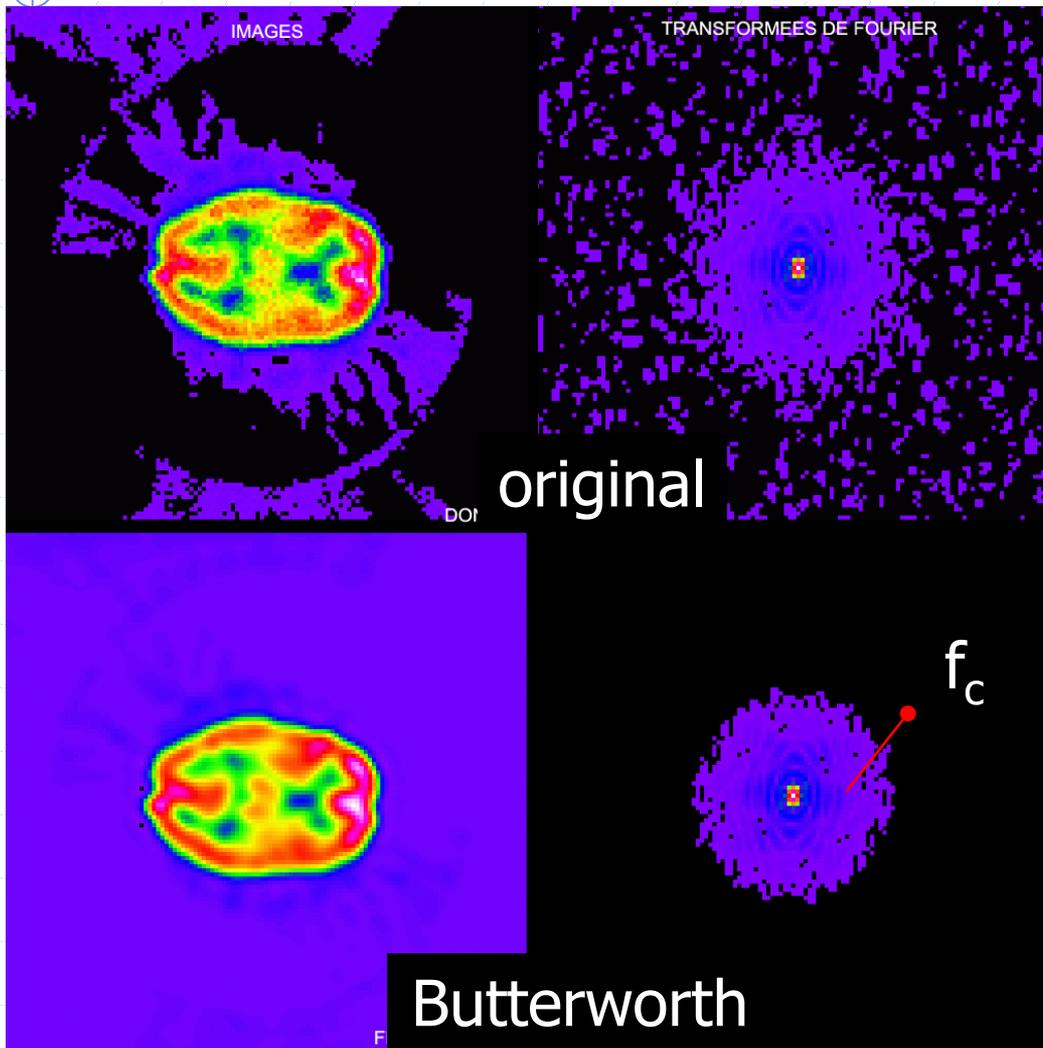
◆ Atténuation sélective de certaines fréquences

$$\hat{f}(v) = 0,5. \left[1 + \cos\left(\pi \frac{v}{v_e}\right) \right]$$



en 2D: $\sqrt{v_x^2 + v_y^2}$

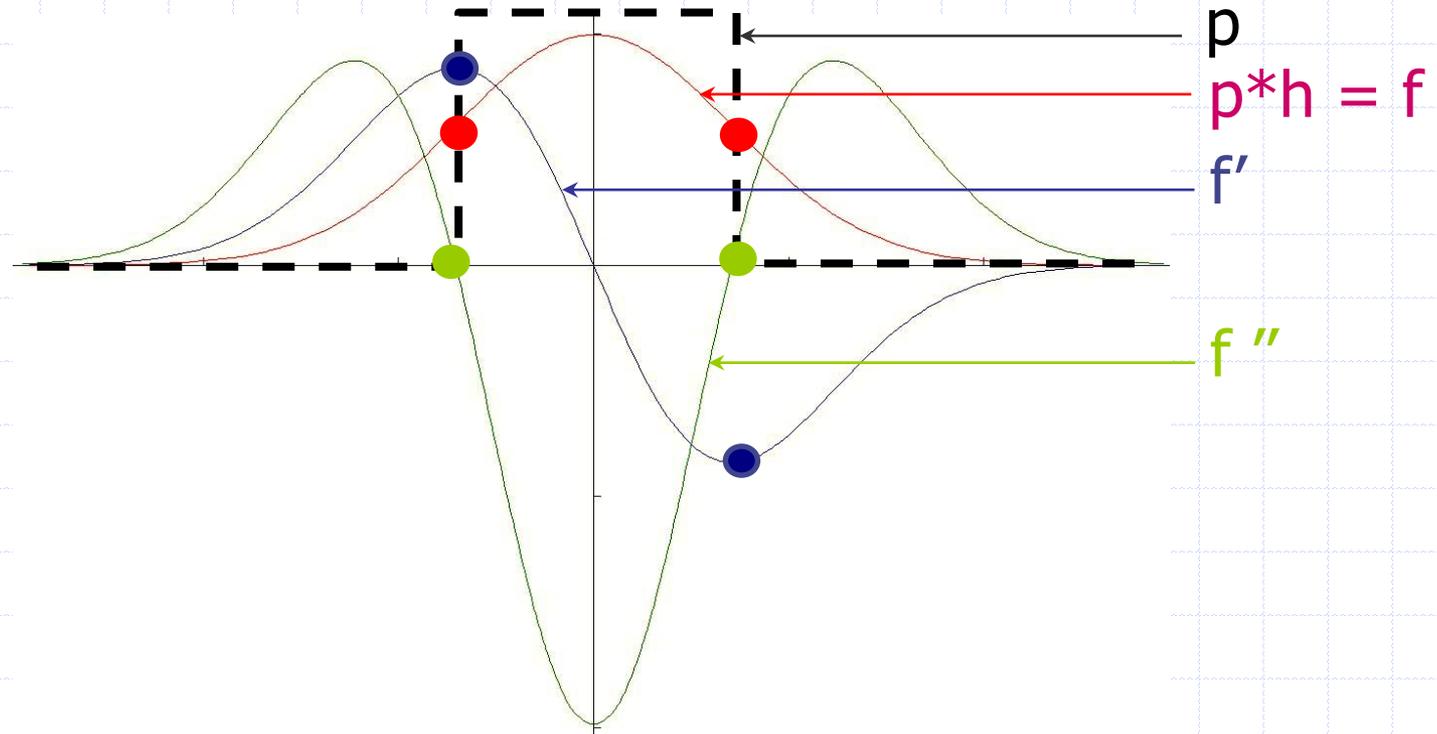
Exemple: filtres de Butterworth



$$\hat{m}(v_x, v_y) = \frac{1}{\sqrt{1 + \left(\frac{\sqrt{v_x^2 + v_y^2}}{f_c} \right)^{2n}}}$$

Filtre Passe-Bas

Exemple: segmentation



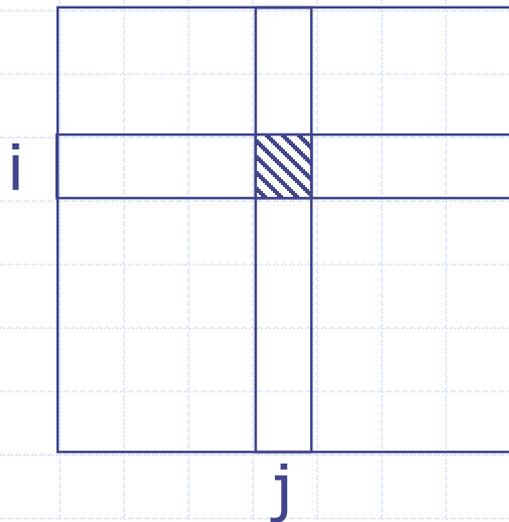
◆ Frontières :

- Extrema du gradient (f')
- Zéros du Laplacien (f'')

Filtres passe-haut: Gradients

$$g_h(i, j) = \frac{1}{2} [f(i+1, j) - f(i-1, j)] \longrightarrow [-1/2 \quad 0 \quad 1/2]$$

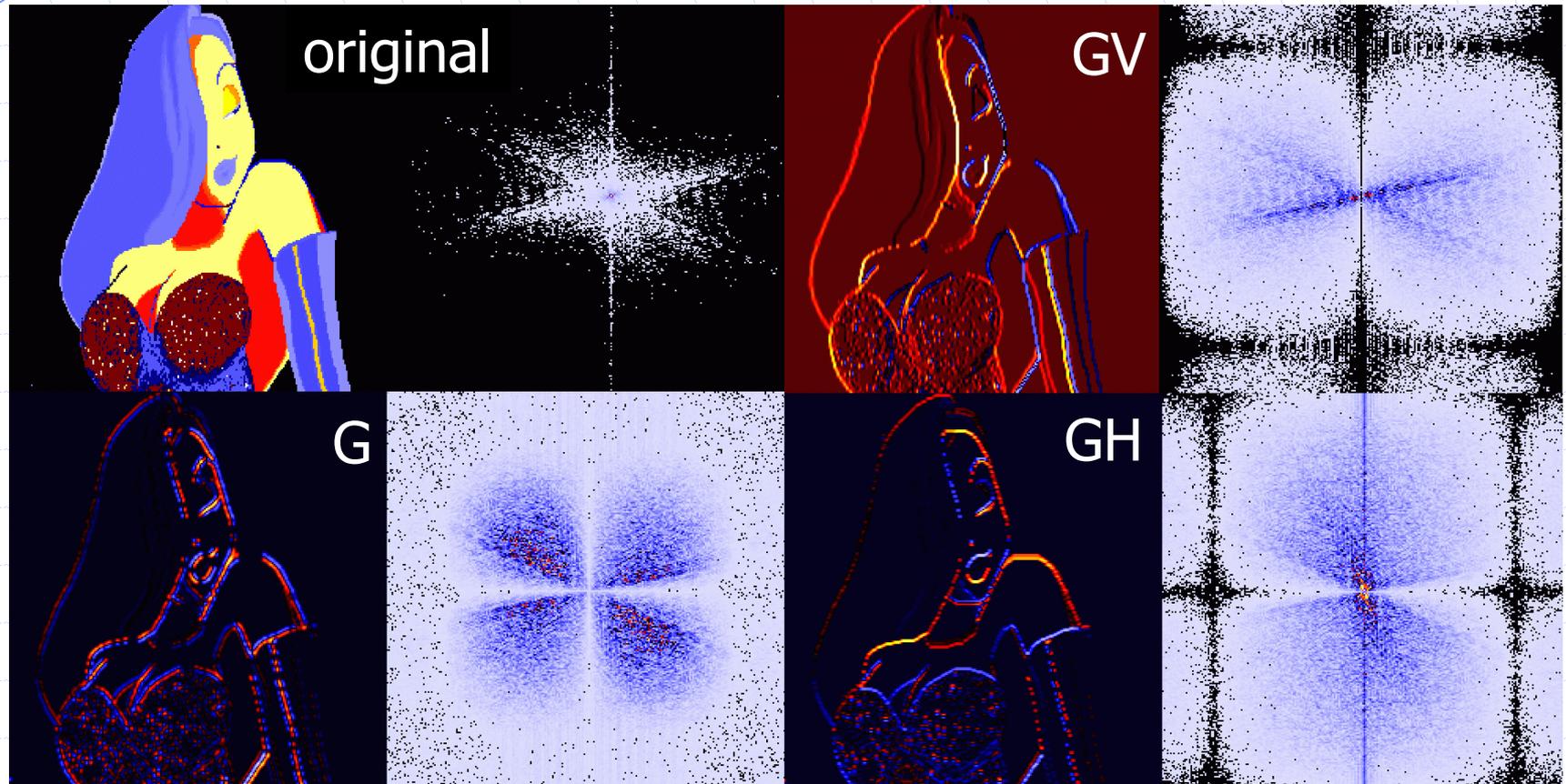
$$g_v(i, j) = \frac{1}{2} [f(i, j+1) - f(i, j-1)] \longrightarrow \begin{bmatrix} -1/2 \\ 0 \\ 1/2 \end{bmatrix}$$



Généralisation 2d:

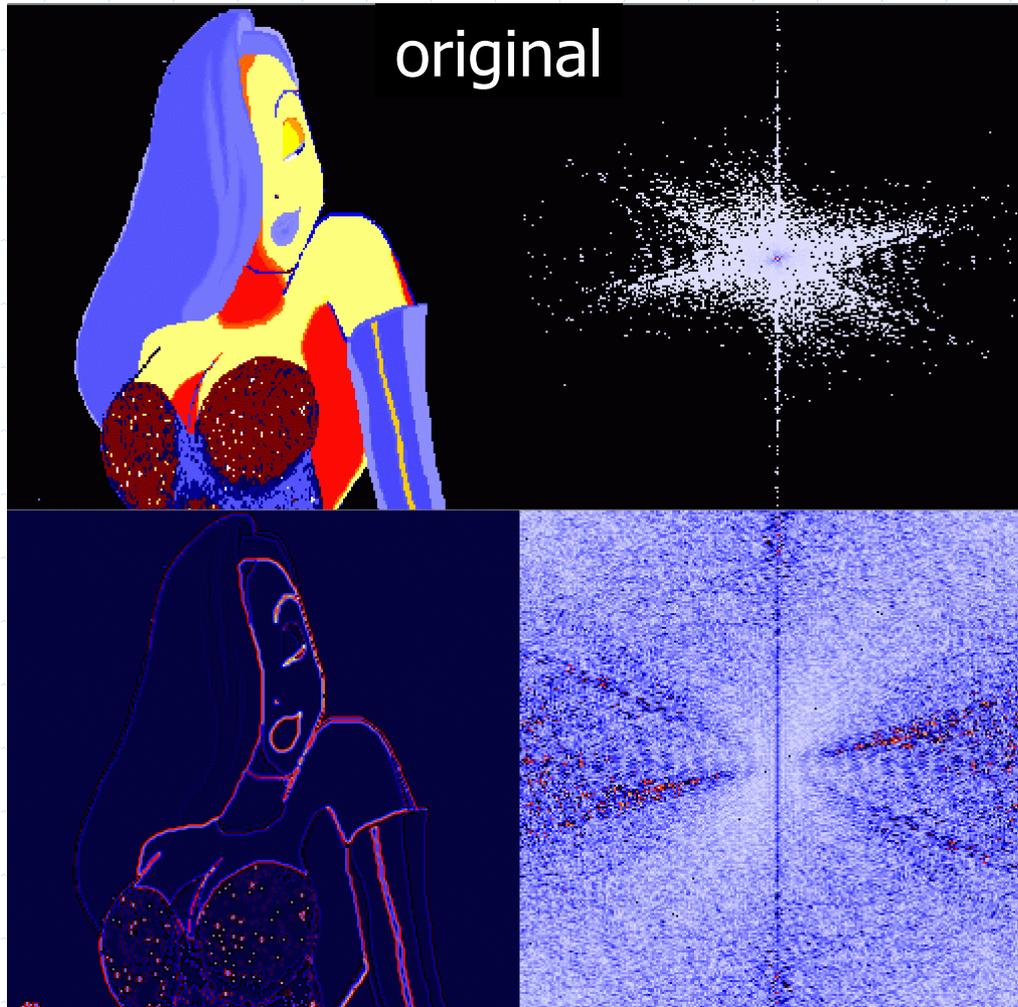
$$G_h = \begin{bmatrix} -1 & 0 & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \\ -1 & 0 & 1 \end{bmatrix} \quad G_v = \begin{bmatrix} -1 & -\sqrt{2} & -1 \\ 0 & 0 & 0 \\ 1 & \sqrt{2} & 1 \end{bmatrix}$$

Filtres passe-haut: Gradients



GH (GV) efface les frontières verticales (horizontales)

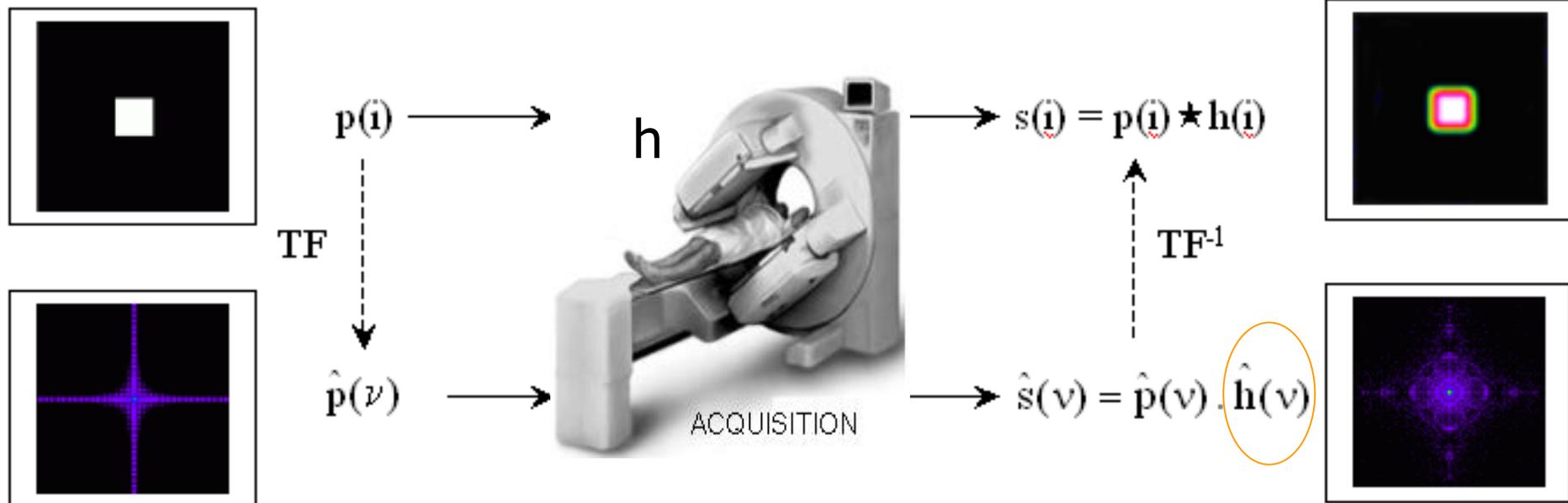
Filtres passe-haut: Laplacien



$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

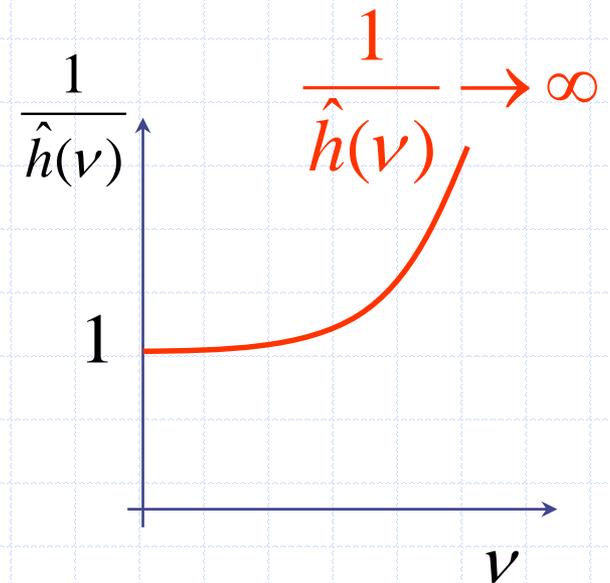
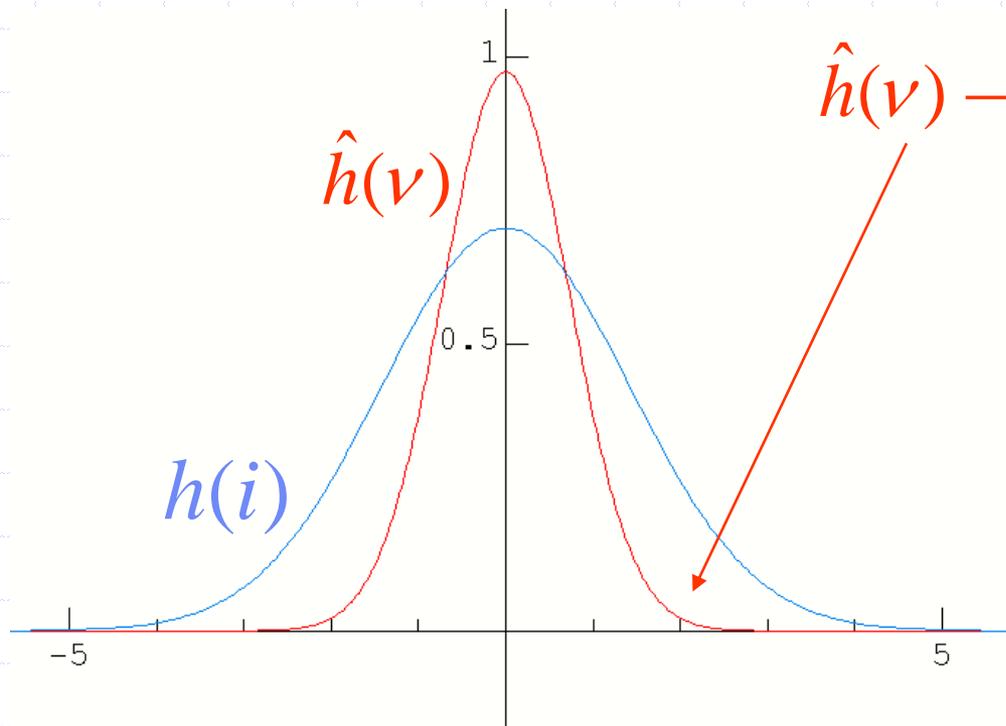
Déconvolution planaire

Dans une image de projection, la distance entre la source et le détecteur où se forme l'image est inconnue. On considère donc la réponse impulsionnelle comme invariante.

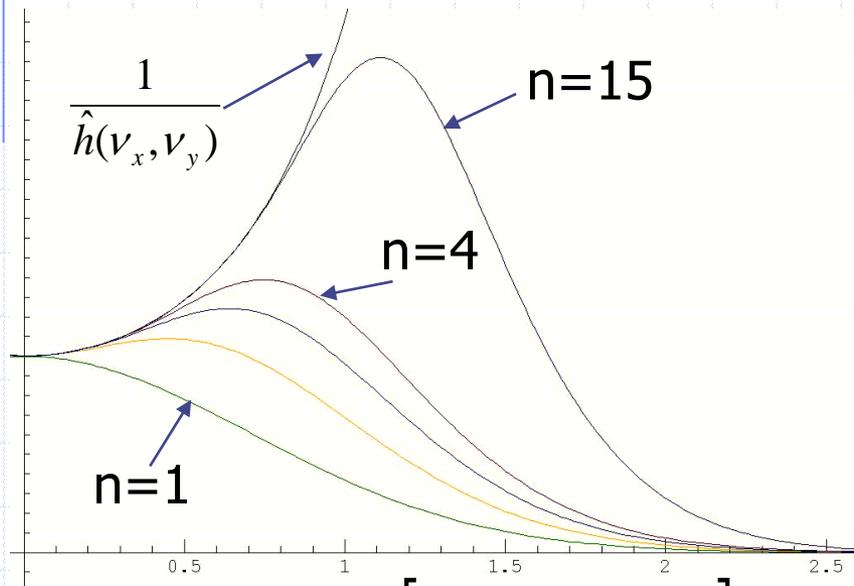
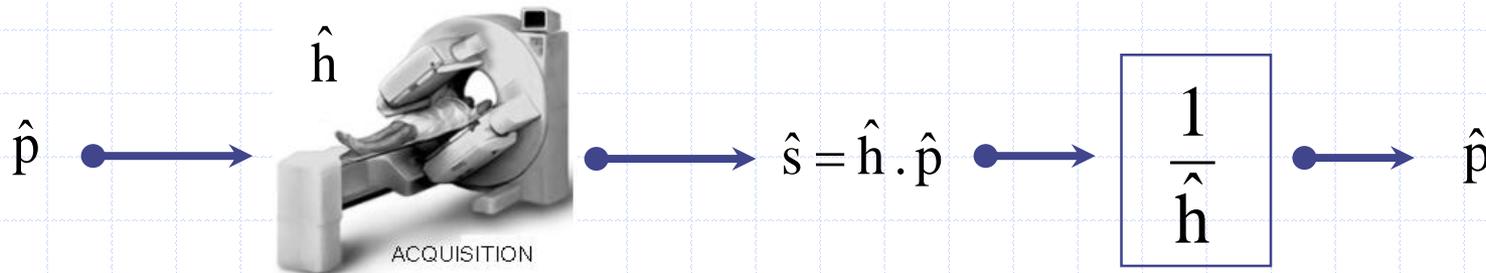


TF d'une gaussienne

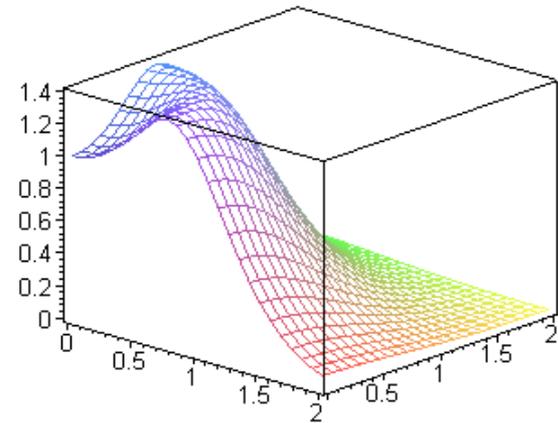
$$h(i) = \frac{1}{\sigma} e^{-\frac{i^2}{2\sigma^2}} \Leftrightarrow \hat{h}(v) = e^{-\frac{v^2}{2/\sigma^2}}$$



Filtre de déconvolution de Metz



$$\hat{m}(v_x, v_y) = \frac{1 - \left[1 - \hat{h}(v_x, v_y)^2 \right]^n}{\hat{h}(v_x, v_y)}$$



$n=0,834 \cdot \ln(C) - 7,774$

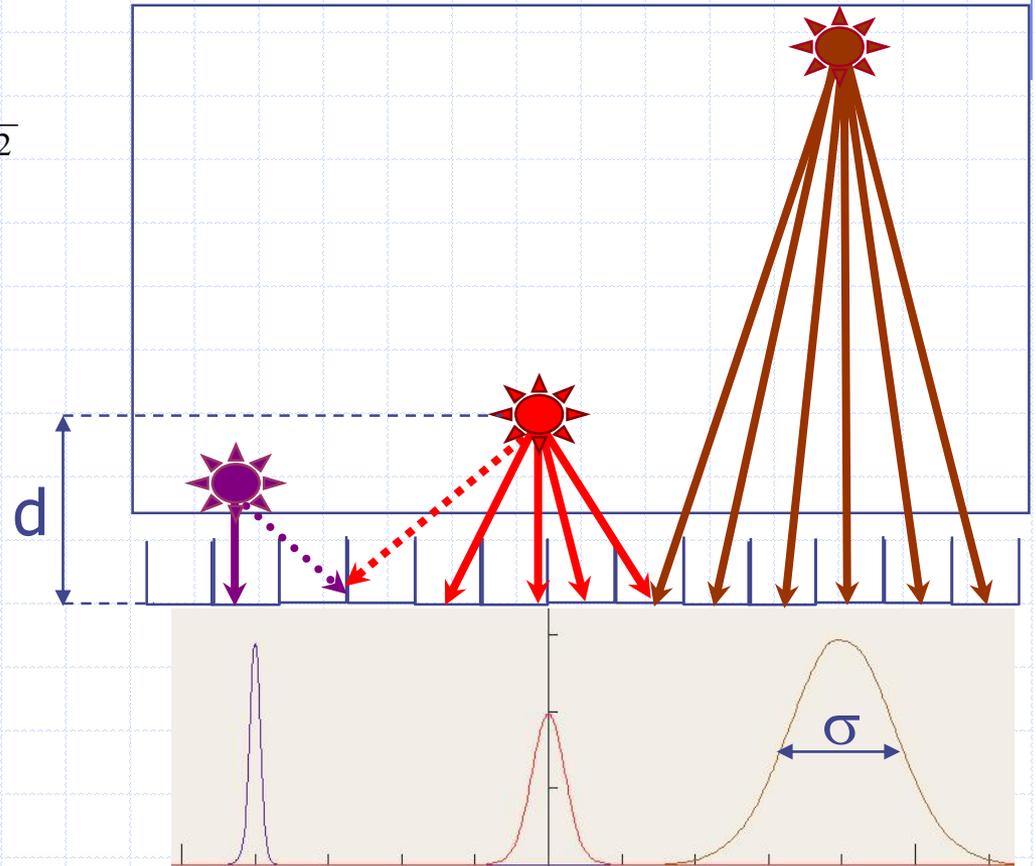
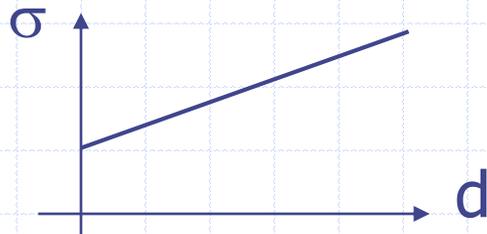
King et al. *JNM* 83;24

Metz Et al. *JNM* 73; 15

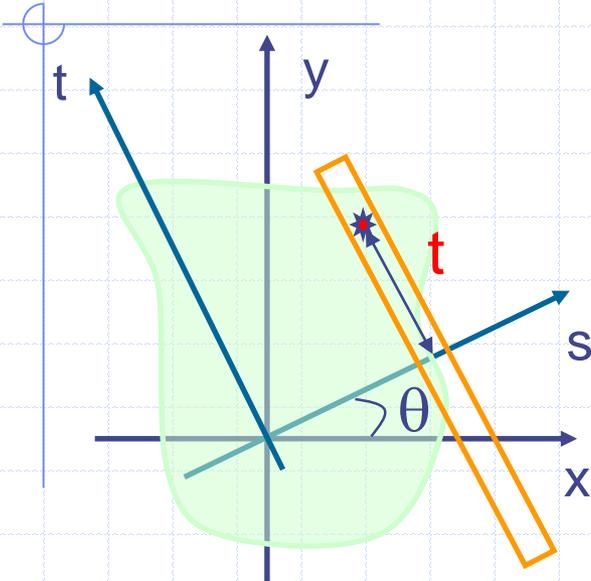
Réponse impulsionnelle en SPECT

$$\diamond h(i) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{i^2}{2\sigma^2}}$$

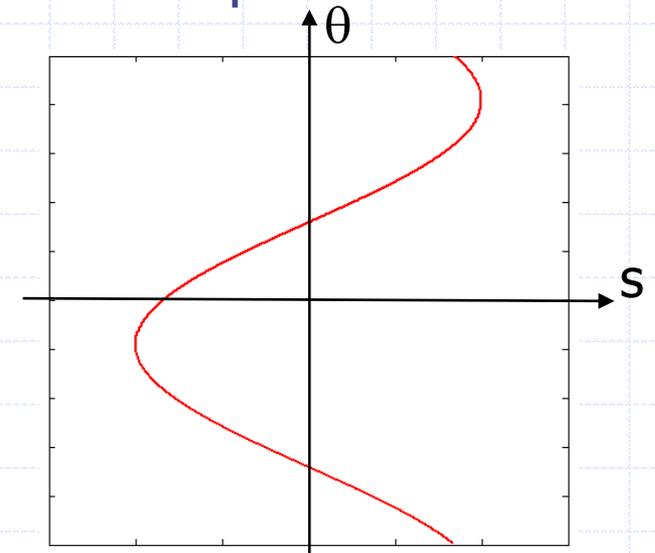
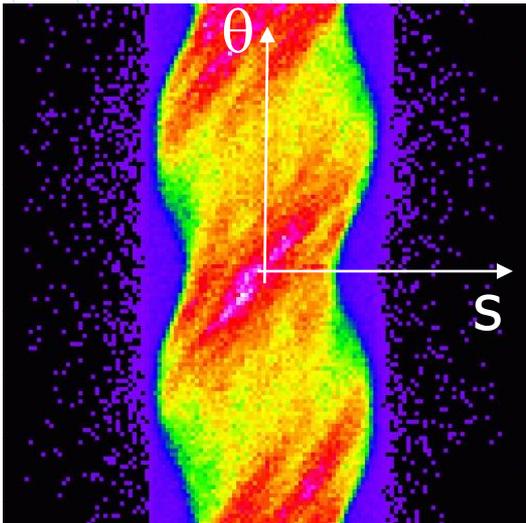
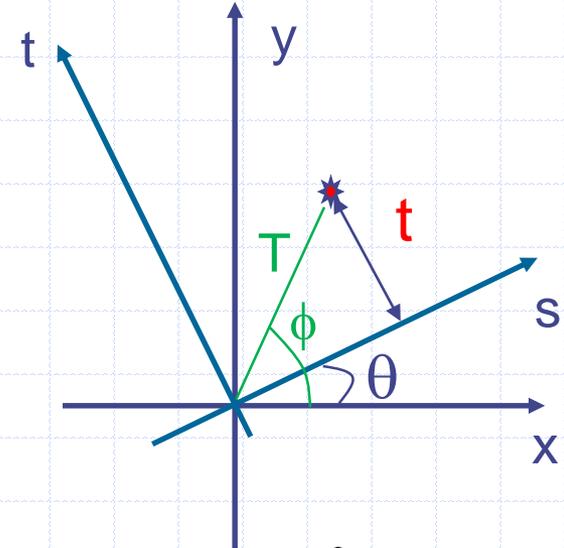
$$\diamond \sigma = \alpha \cdot d + \beta$$



Relation fréquence-distance



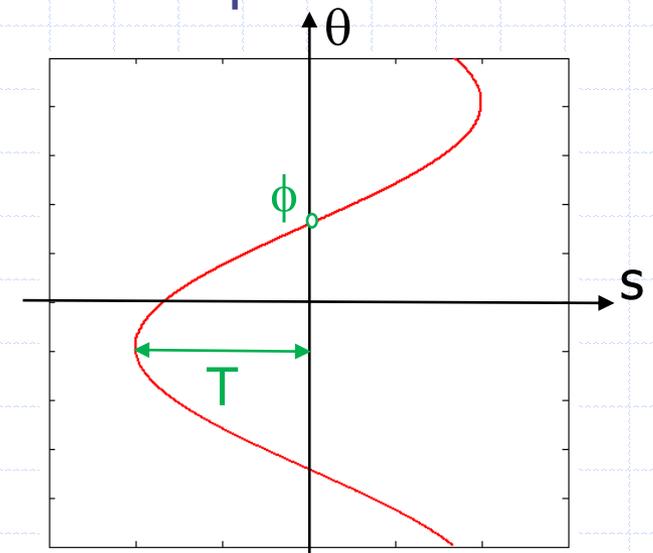
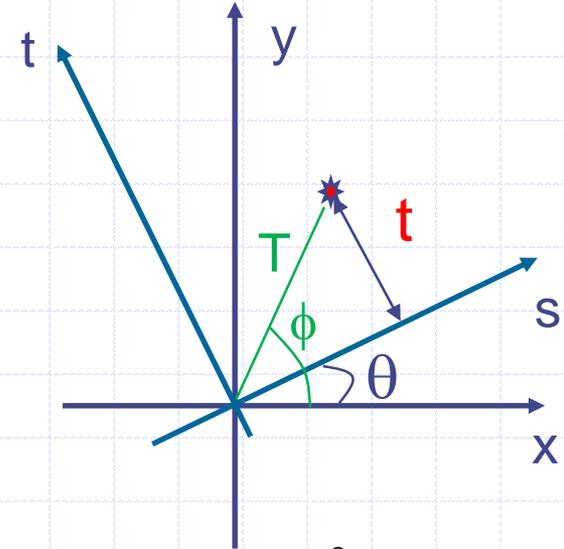
$$p(s, \theta) = \int f(i, j) dt$$



Relation fréquence-distance

$$s(\theta) = T \cdot \cos(\varphi - \theta)$$

$$s'(\theta) = T \cdot \sin(\theta - \varphi) = t$$

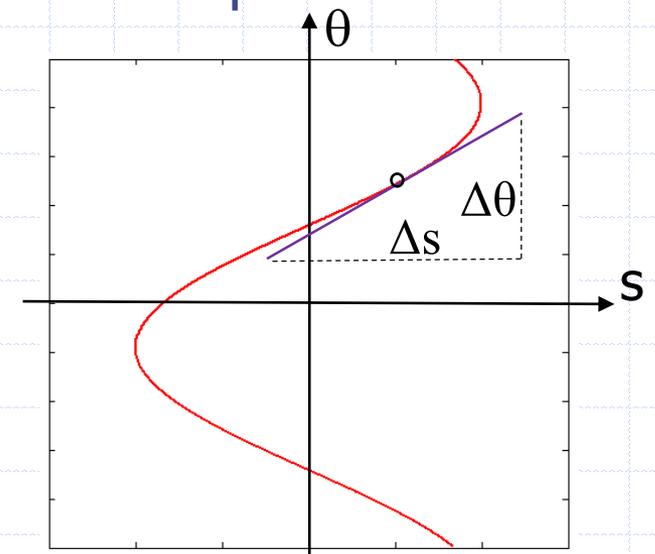
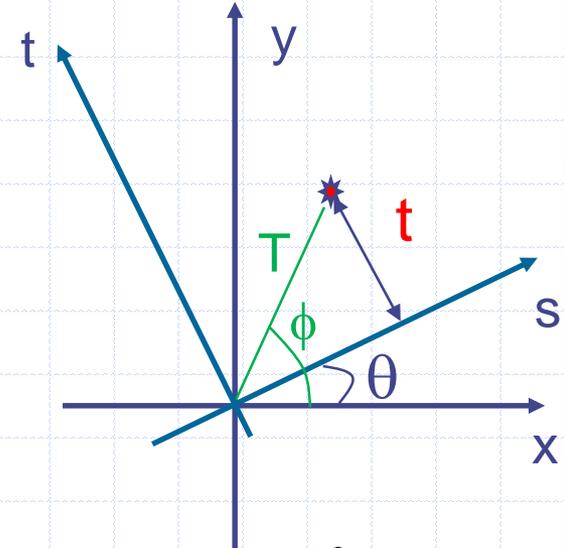


Relation fréquence-distance

$$s(\theta) = T \cdot \cos(\varphi - \theta)$$

$$s'(\theta) = T \cdot \sin(\theta - \varphi) = t$$

$$\frac{\Delta s}{\Delta \theta} = t$$

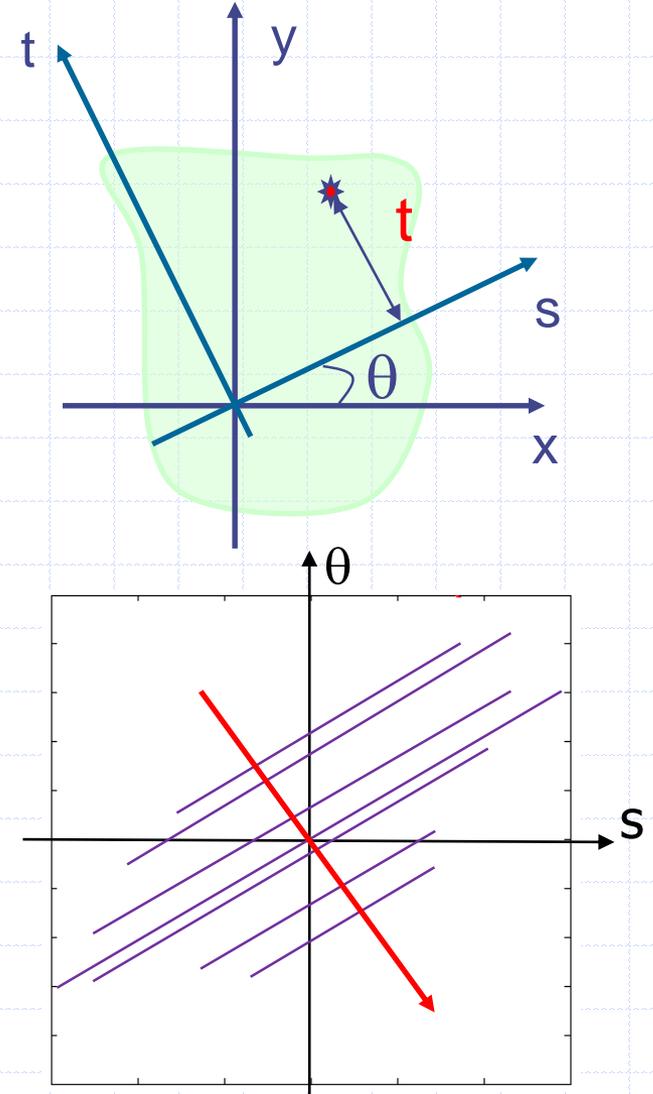


Relation fréquence-distance

$$s(\theta) = T \cdot \cos(\varphi - \theta)$$

$$s'(\theta) = T \cdot \sin(\theta - \varphi) = t$$

$$\frac{\Delta s}{\Delta \theta} = t$$

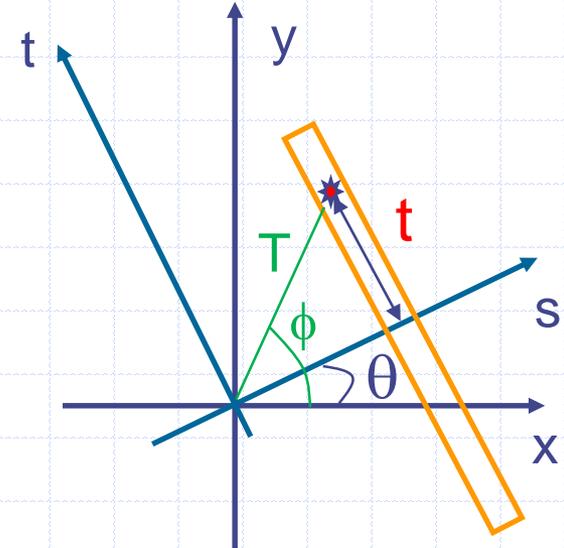


Relation fréquence-distance

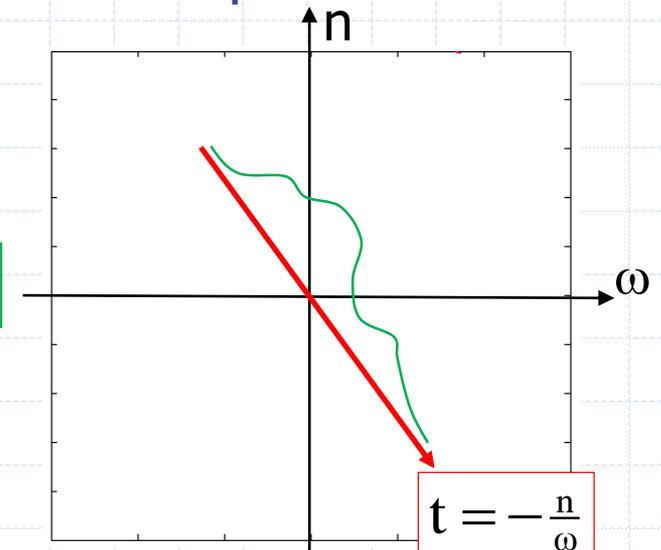
$$s(\theta) = T \cdot \cos(\varphi - \theta)$$

$$s'(\theta) = T \cdot \sin(\theta - \varphi) = t$$

$$\frac{\Delta s}{\Delta \theta} = t$$

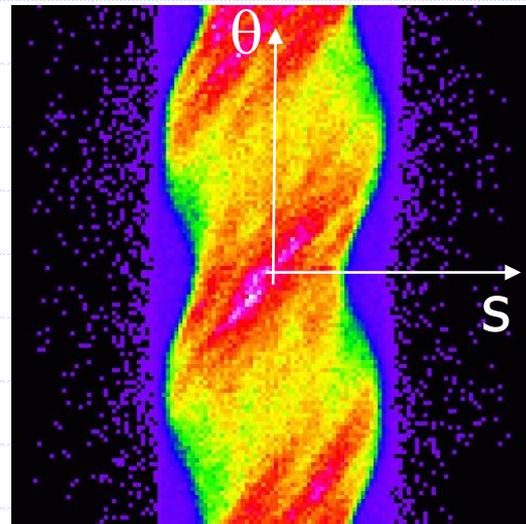
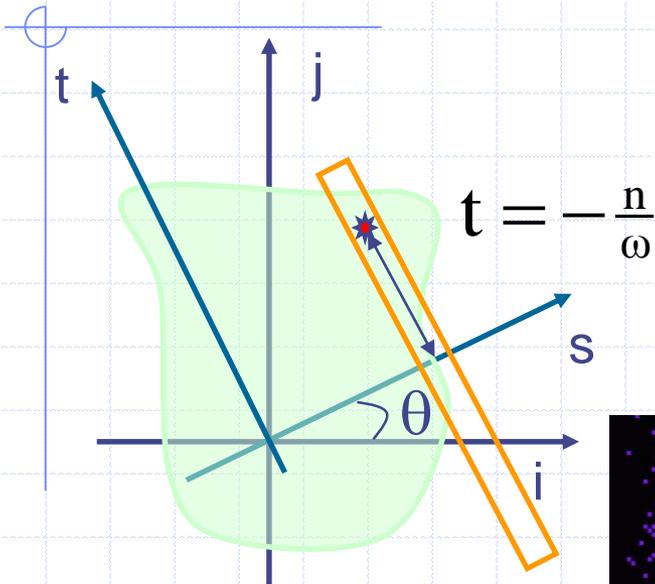


TF 2D



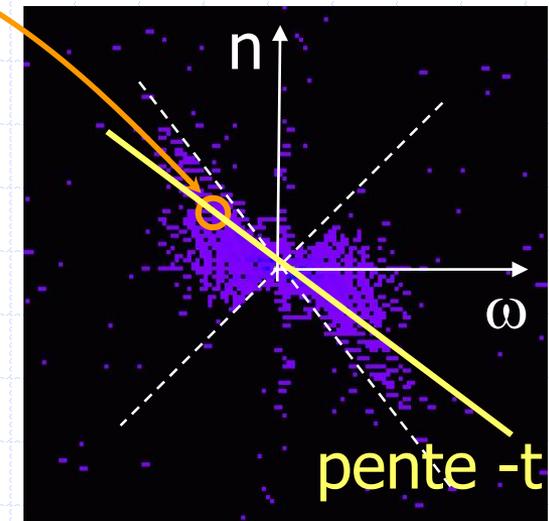
$$t = -\frac{n}{\omega}$$

Relation fréquence-distance



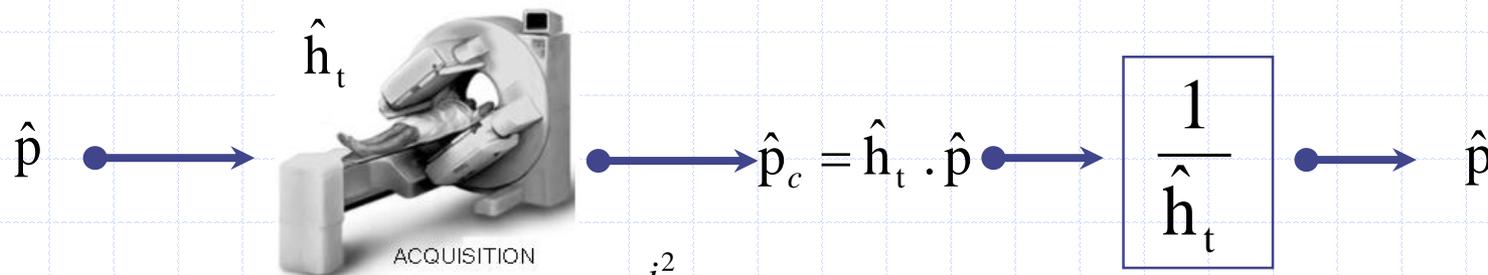
$$p(s, \theta) = \int \underbrace{f(i, j)}_{\text{orange box}} dt$$

$\xrightarrow{\text{TF}_2}$



$$\hat{p}(\omega, n)$$

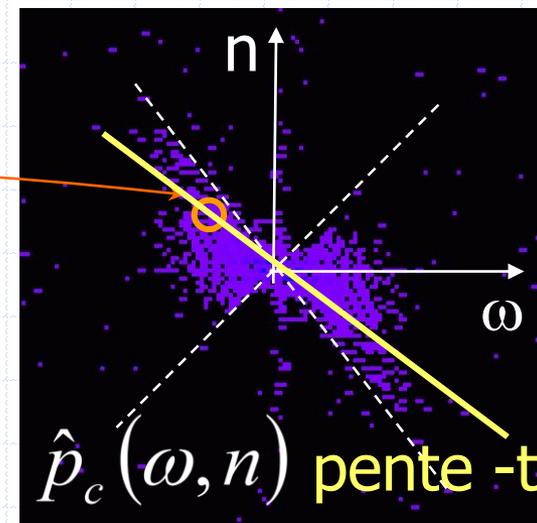
Déconvolution en TEMP



$$h_t(i) = \frac{1}{\sqrt{2\pi}\sigma_t} e^{-\frac{i^2}{2\sigma_t^2}}$$

$$\hat{p}_c(\omega, n) = \hat{h}_{\frac{n}{\omega}}(\omega) \cdot \hat{p}(\omega, n)$$

$$\hat{p}(\omega, n) = \frac{1}{\hat{h}_{\frac{n}{\omega}}(\omega)} \cdot \hat{p}_c(\omega, n)$$

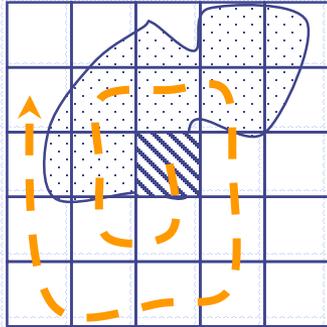


Filtrage non linéaire

Il faut abandonner le fait de réaliser des moyennes pondérées, d'utiliser un masque identique sur toute l'image, ou les deux.

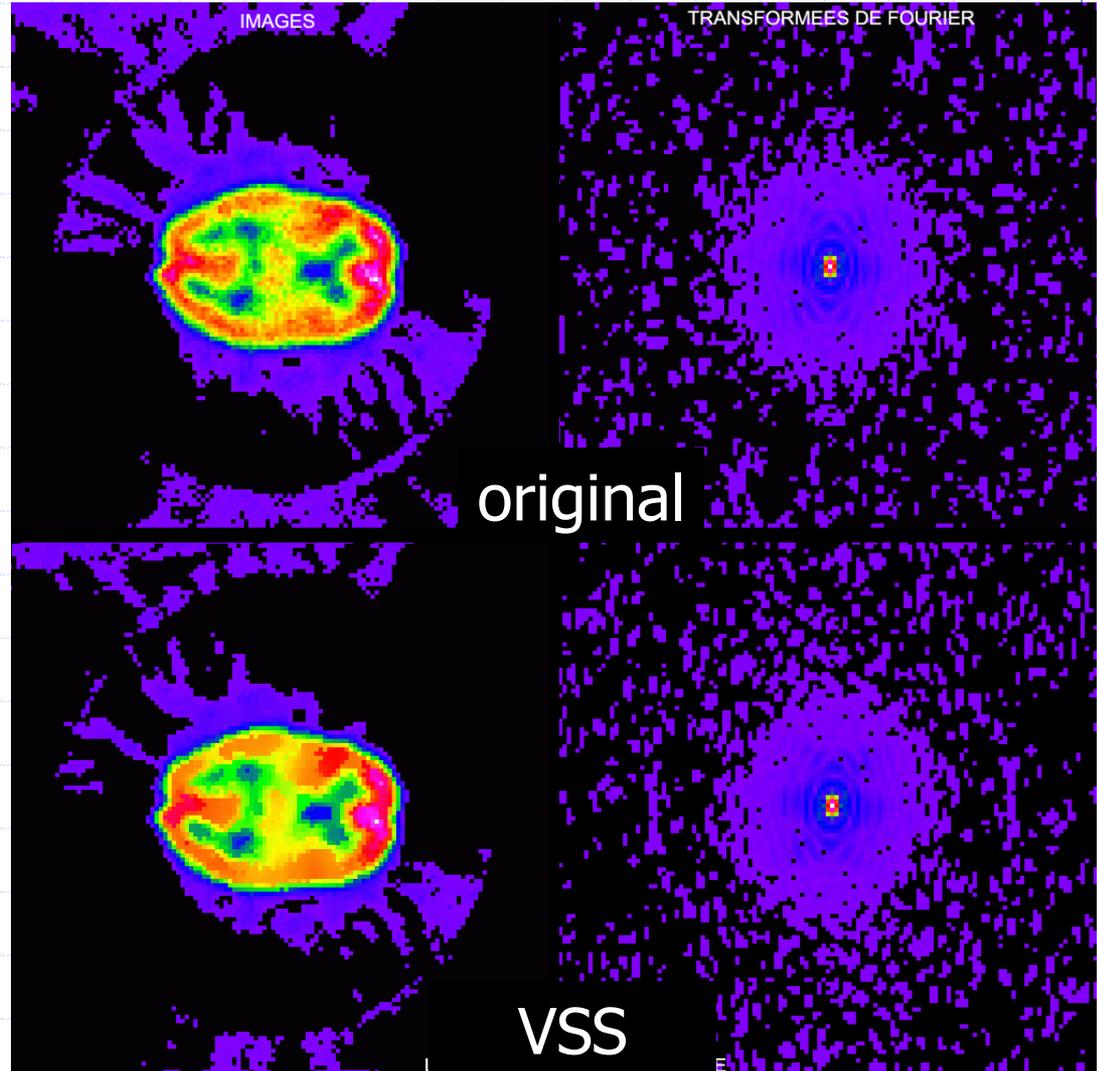
- ◆ Masque variable : VSS
- ◆ Pas de moyenne :
 - ◆ Médian, ouverture et fermeture morphologiques
- ◆ Pas de moyenne et masque variable :
 - ◆ Centre, filtres géodésiques, filtres adaptés...

Lissage sur masque adapté (VSS)

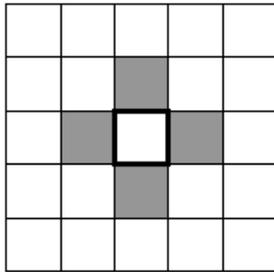


Accumulation pour
moyenne des

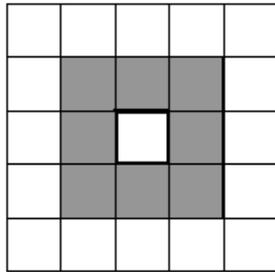
$$s(i',j') \in s(i,j) \pm 2\sqrt{s(i,j)}$$



Filtre médian

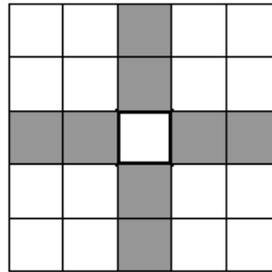


Connexité 4

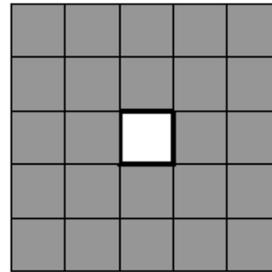


Connexité 8

Taille 1



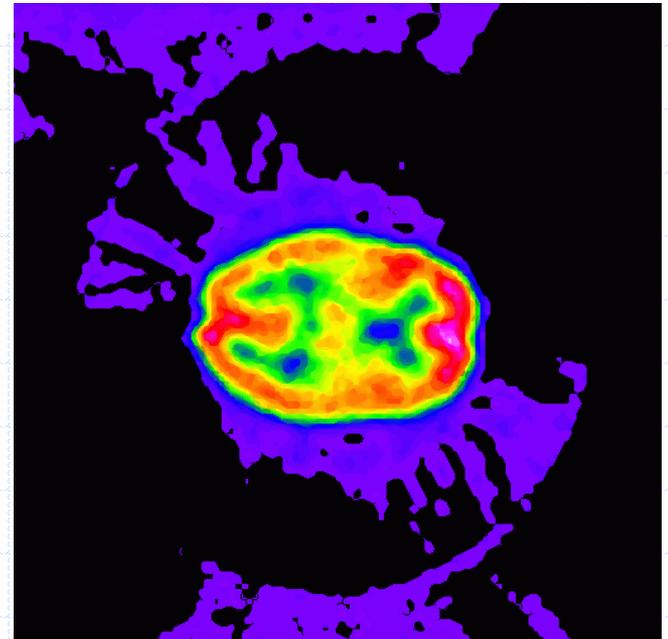
Connexité 4



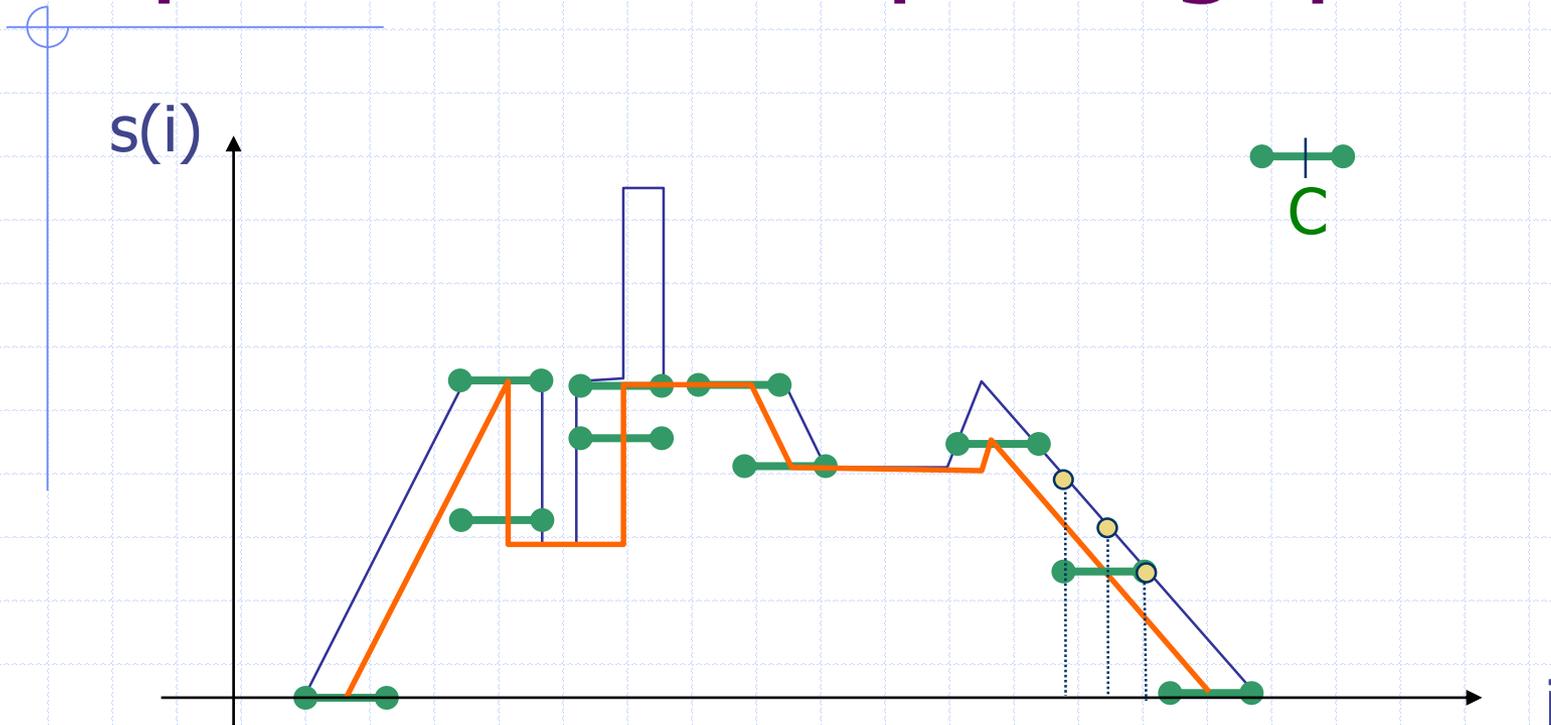
Connexité 8

Taille 2

On remplace $s(i,j)$ par la valeur médiane dans un voisinage fixe de (i,j)



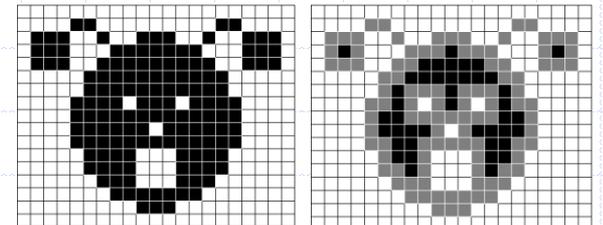
Opérateurs morphologiques



érosion

$$\varepsilon_B(s)(i, j) = \inf_{(i', j') \in B_{i, j}} s(i', j')$$

$$\varepsilon_B(G) = \{x / C_x \subset G\}$$



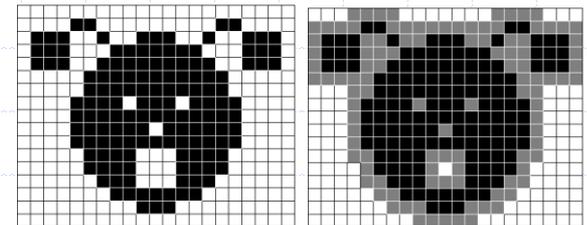
Opérateurs morphologiques



dilatation

$$\delta_B(s)(i, j) = \sup_{(i', j') \in B_{i, j}} s(i', j')$$

$$\delta_B(G) = \{ \mathbf{x} / C_x \cap G \neq \emptyset \}$$



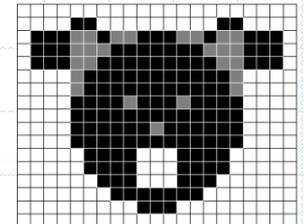
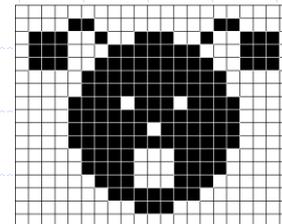
Filtres morphologiques



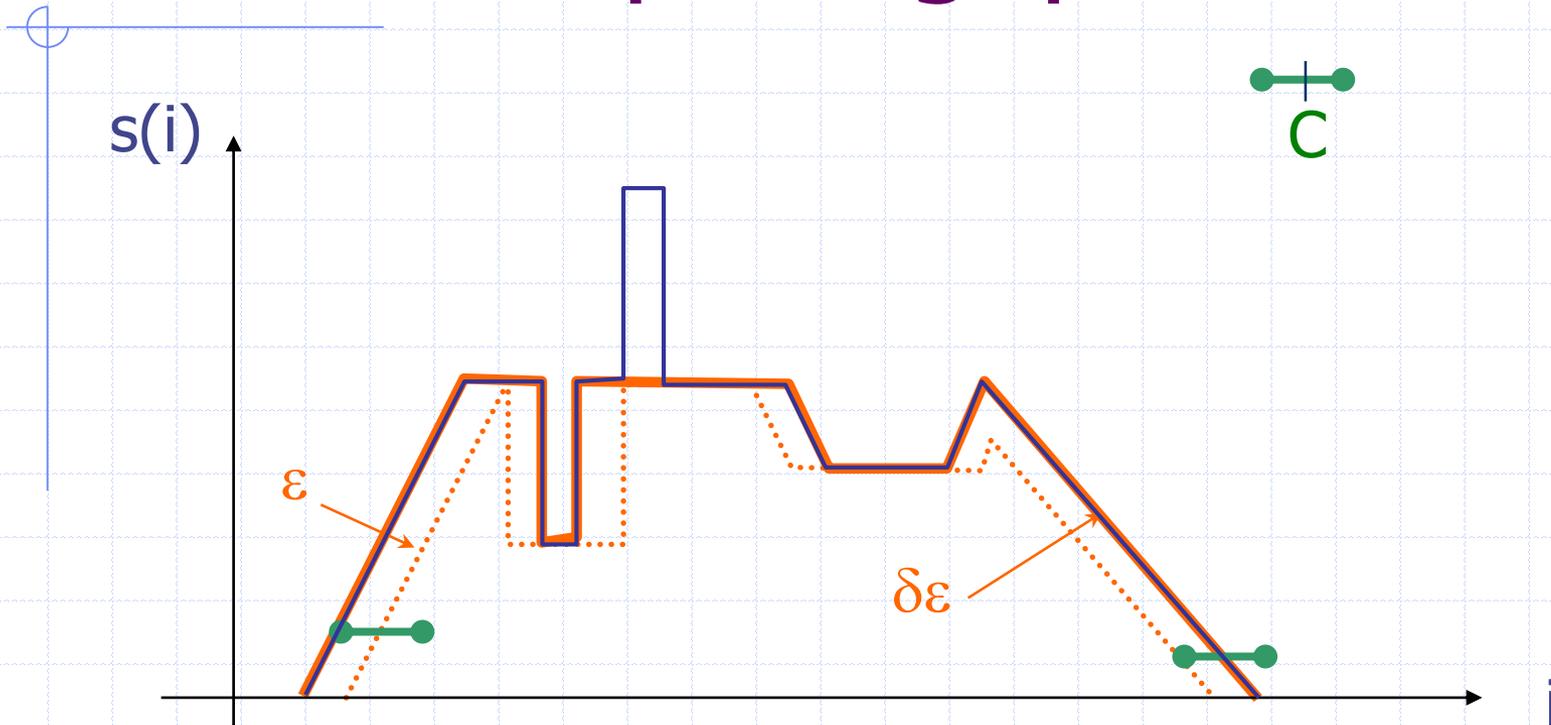
fermeture

$$\varphi_B(s)(i, j) = (\varepsilon_B \circ \delta_B)(s)(i, j)$$

$$\varphi_B(G) = \cup \{ C_x / C_x \cap G = \emptyset \}^c$$



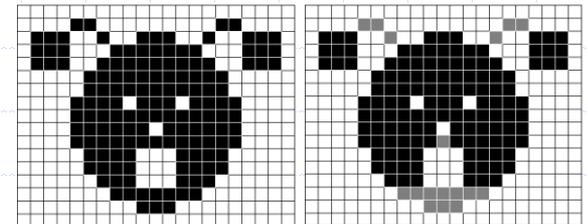
Filtres morphologiques



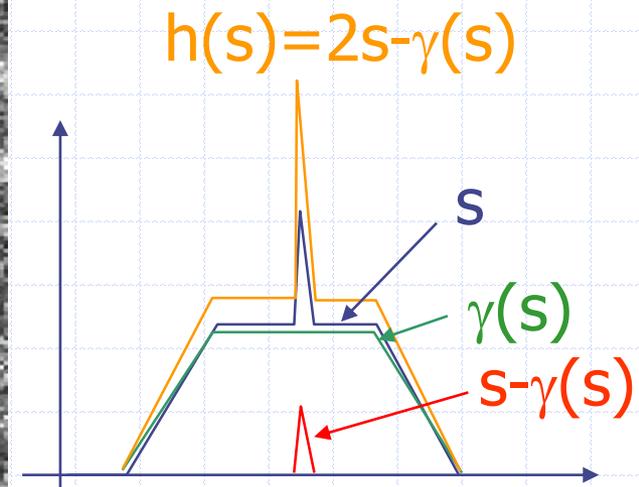
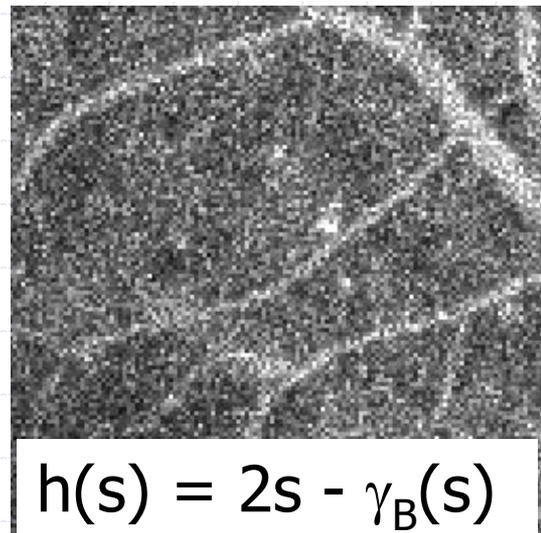
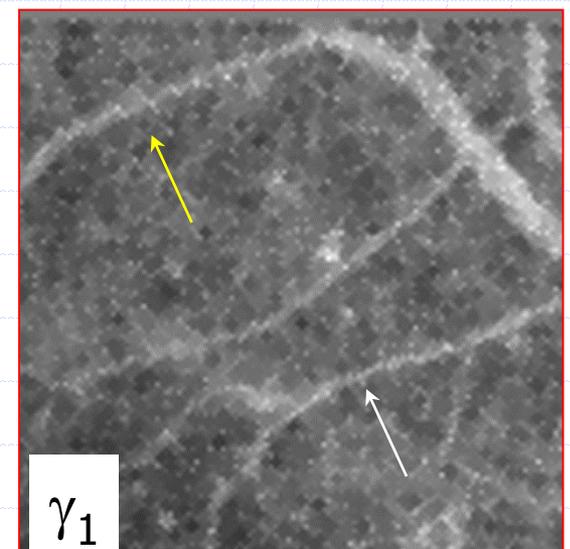
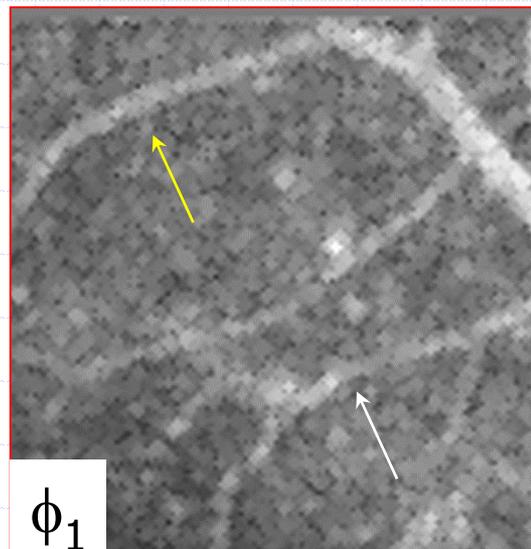
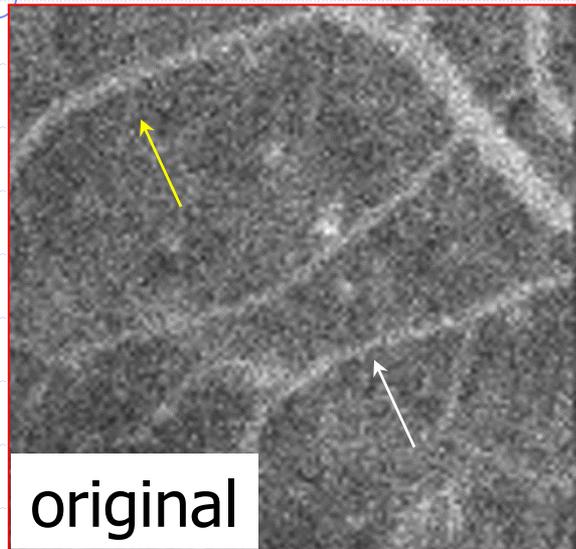
ouverture

$$\gamma_B(s)(i, j) = (\delta_B \circ \varepsilon_B)(s)(i, j)$$

$$\gamma_B(G) = \cup \{ C_x / C_x \subset G \}$$



Exemples de filtres morphologiques

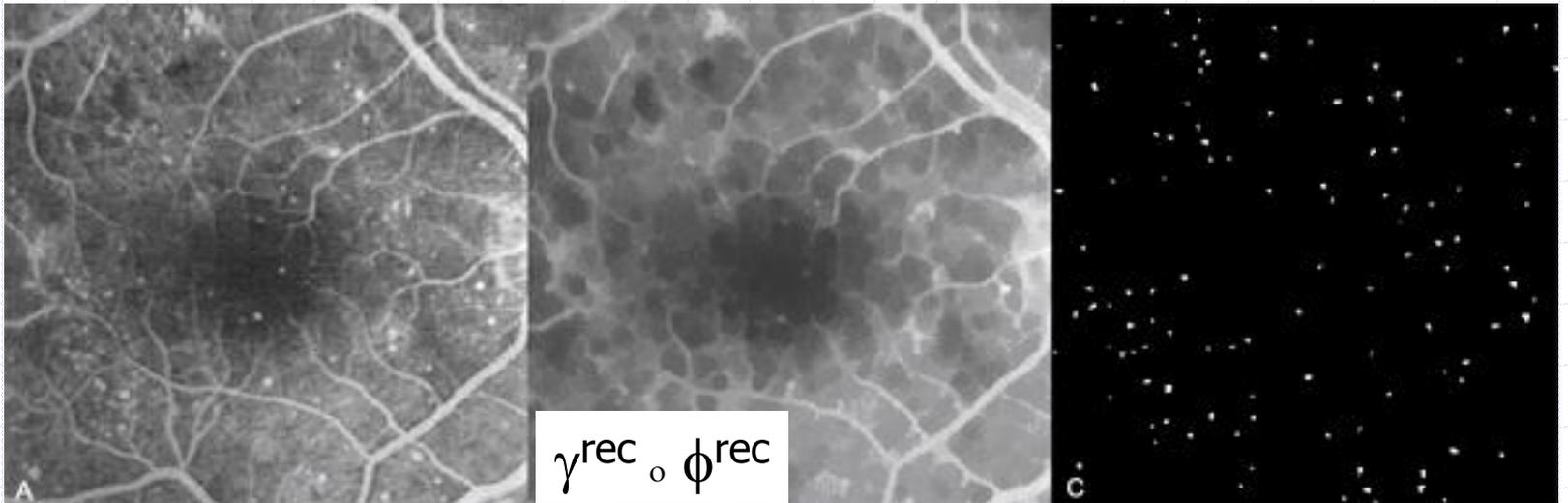


Filtres géodésiques

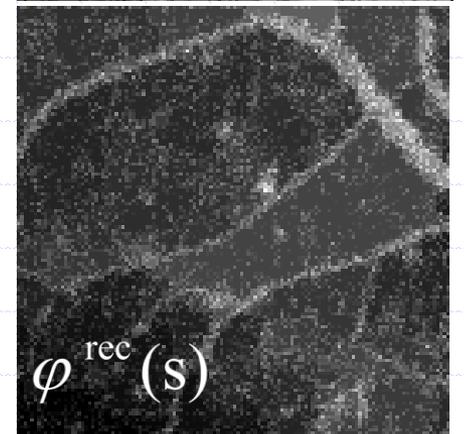
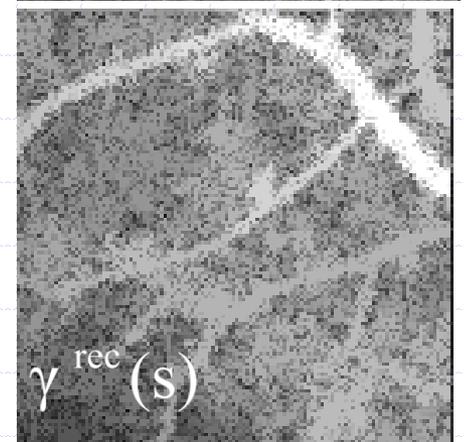
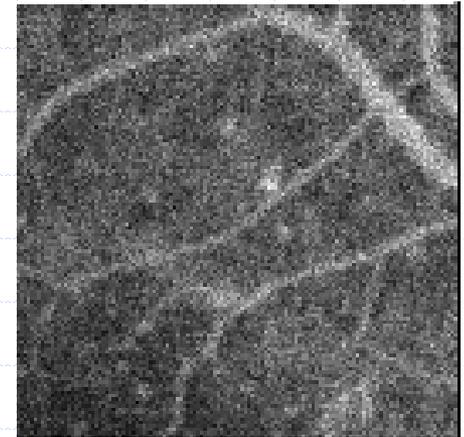
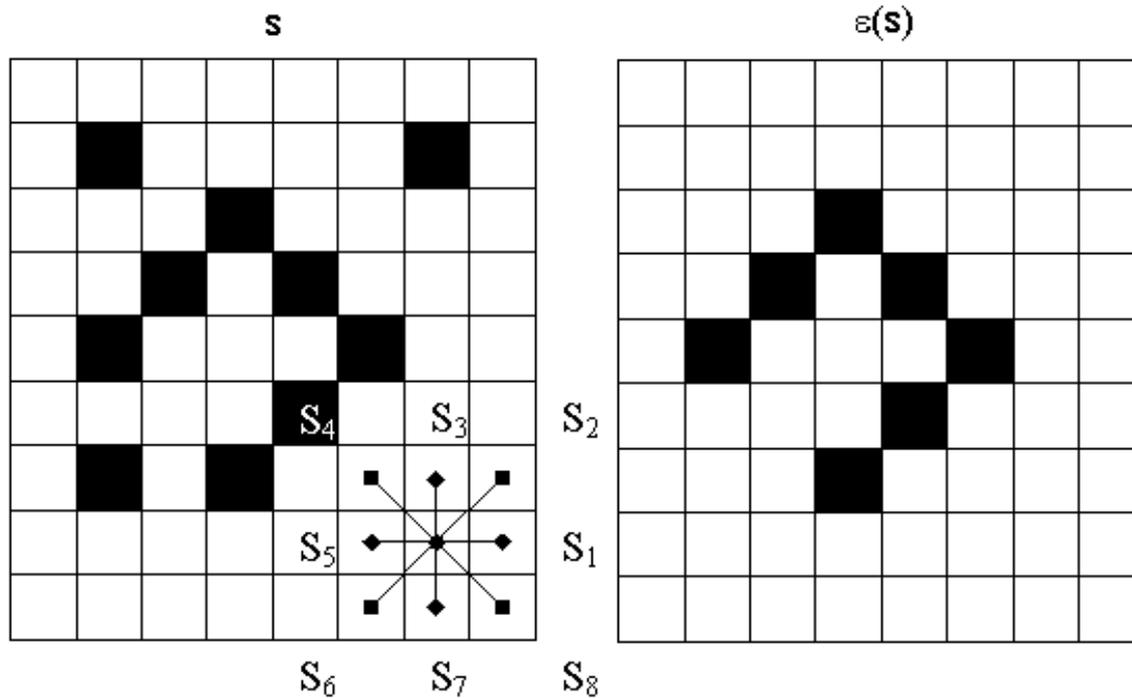
$$\Delta_s(m) = \wedge[\delta_B(m), s], \quad \text{où } m = \varepsilon_B(s)$$

$$E_s(m) = \vee[\varepsilon_B(m), s], \quad \text{où } m = \delta_B(s)$$

$$\gamma^{\text{rec}}(s) = \Delta_s^\infty(m) \quad \text{et} \quad \phi^{\text{rec}}(s) = E_s^\infty(m)$$



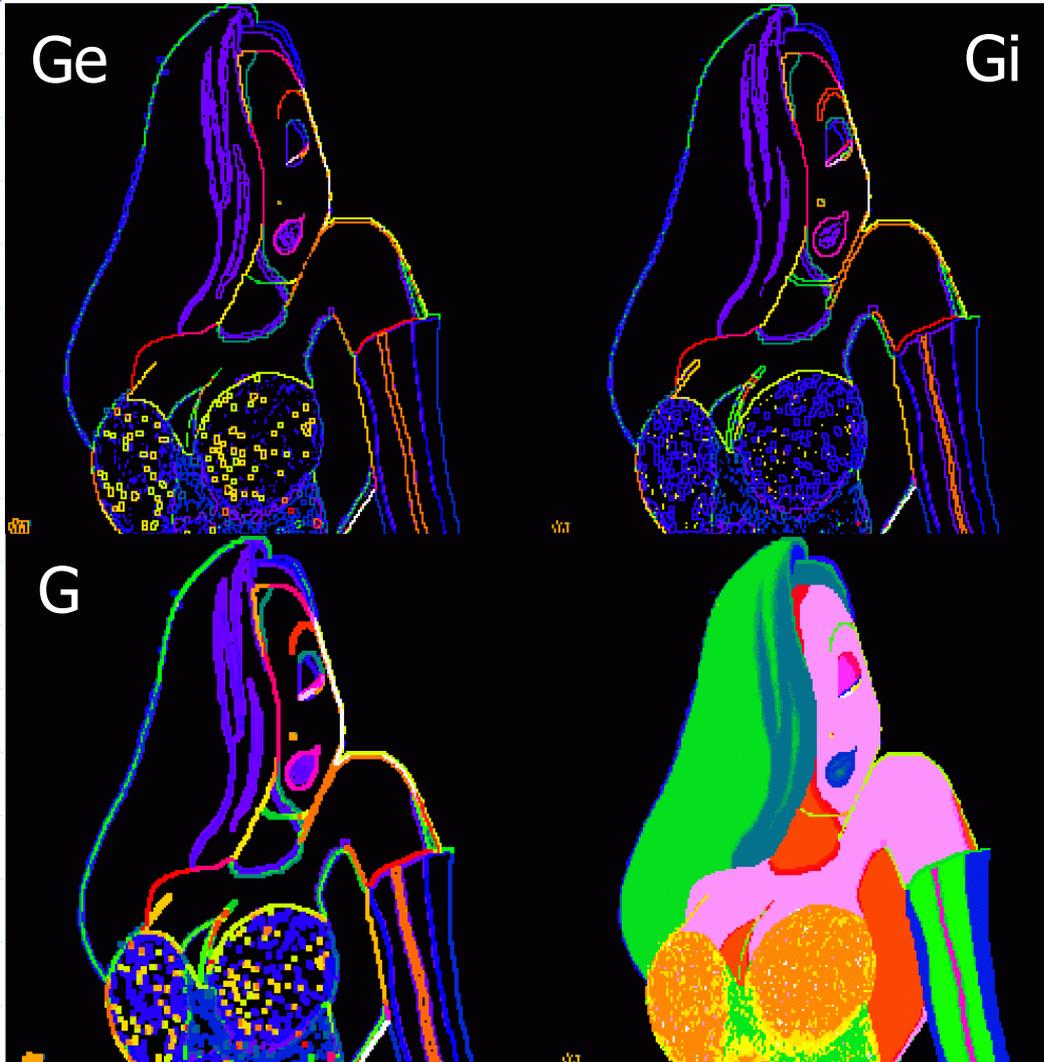
Sup et Inf d'opérateurs



$$\varepsilon(s)(i,j) = \vee_k (\varepsilon_{S_k})(s)(i,j)$$

$$\delta(s)(i,j) = \wedge_k (\delta_{S_k})(s)(i,j)$$

Gradients morphologiques



$$G_B^e(f) = \delta_B(f) - f$$

$$G_B^i(f) = f - \varepsilon_B(f)$$

$$G_B(f) = \delta_B(f) - \varepsilon_B(f)$$