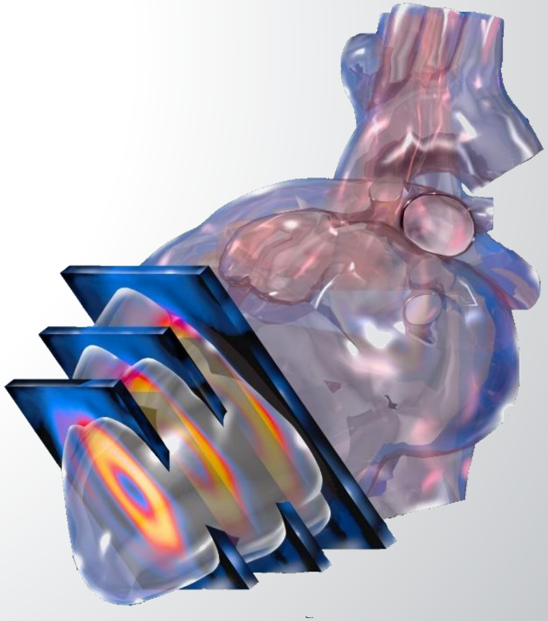
MASTER PhyMed

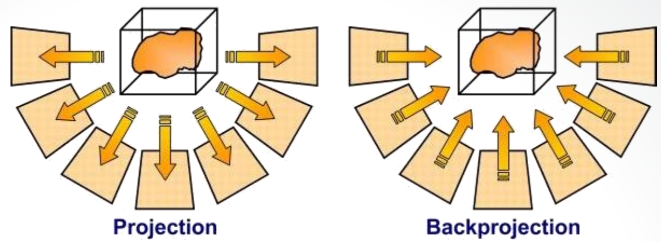
GMPH308 - Physique de l'imagerie médicale

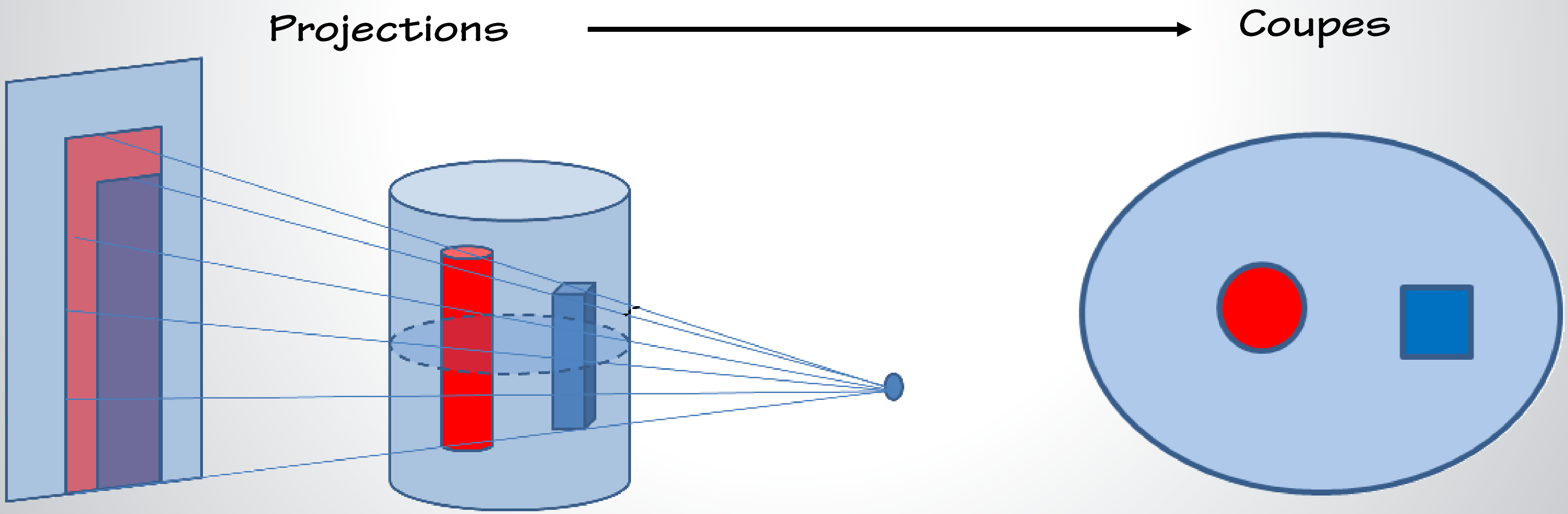
TOMOGRAPHIE D'EMISSION



Fayçal Ben Bouallègue
faybenb@hotmail.com
http://scinti.edu.umontpellier.fr

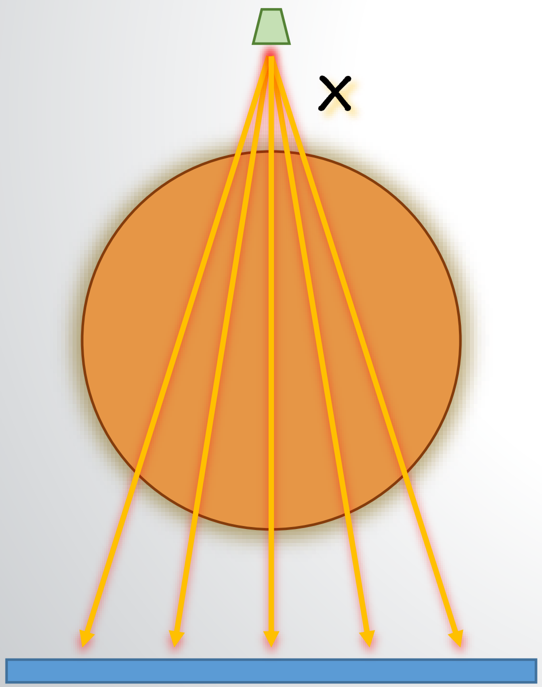
Tomographie



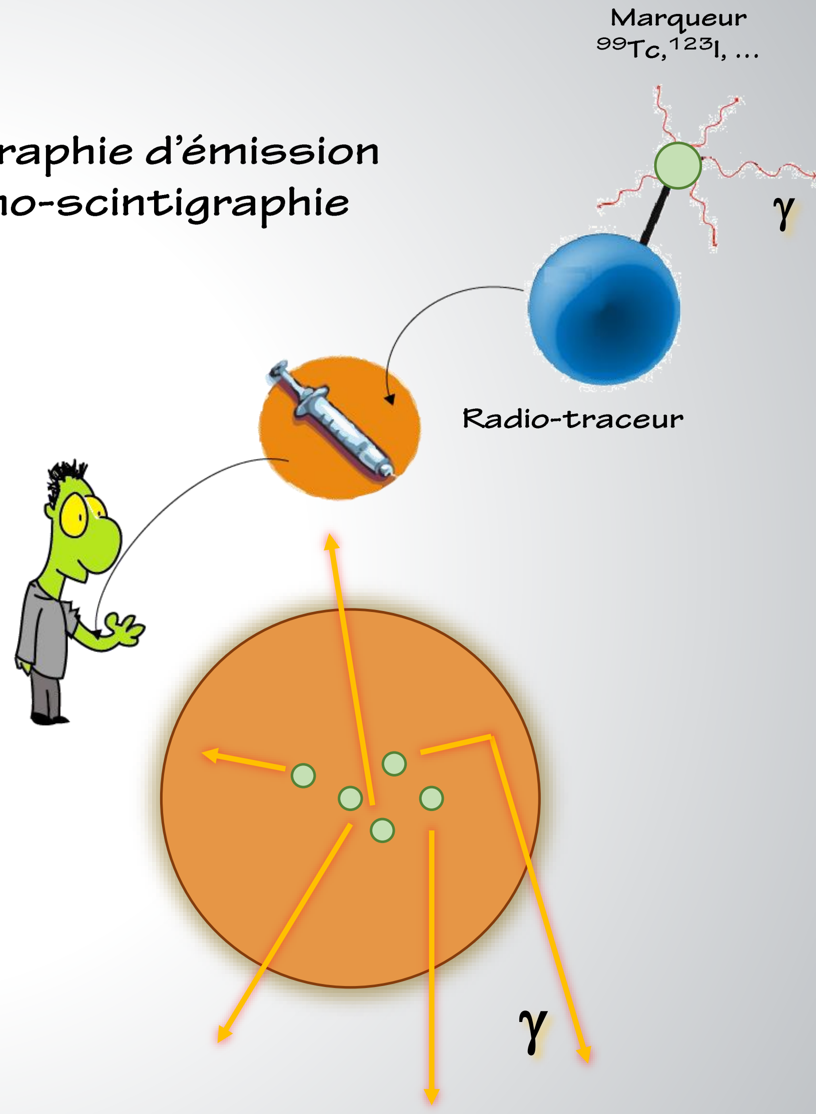


Tomographie

Tomographie de transmission



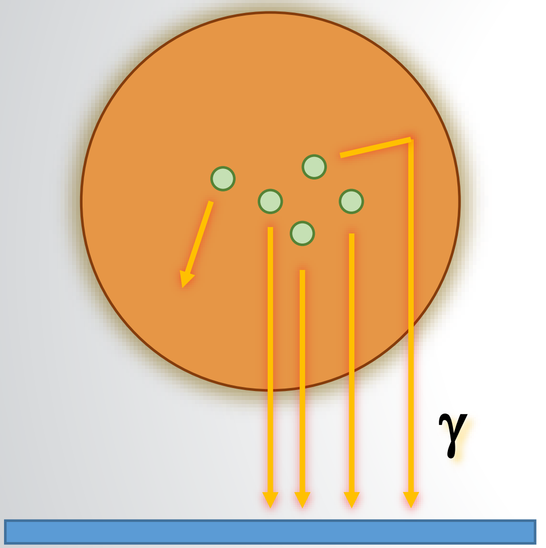
Tomographie d'émission
= tomo-scintigraphie



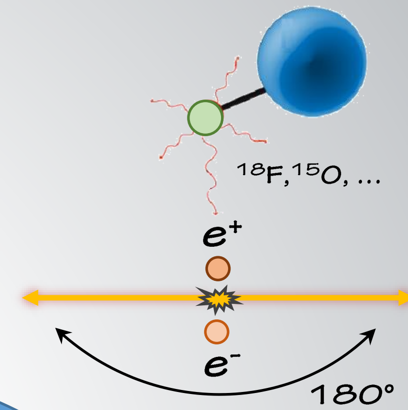
Tomographie

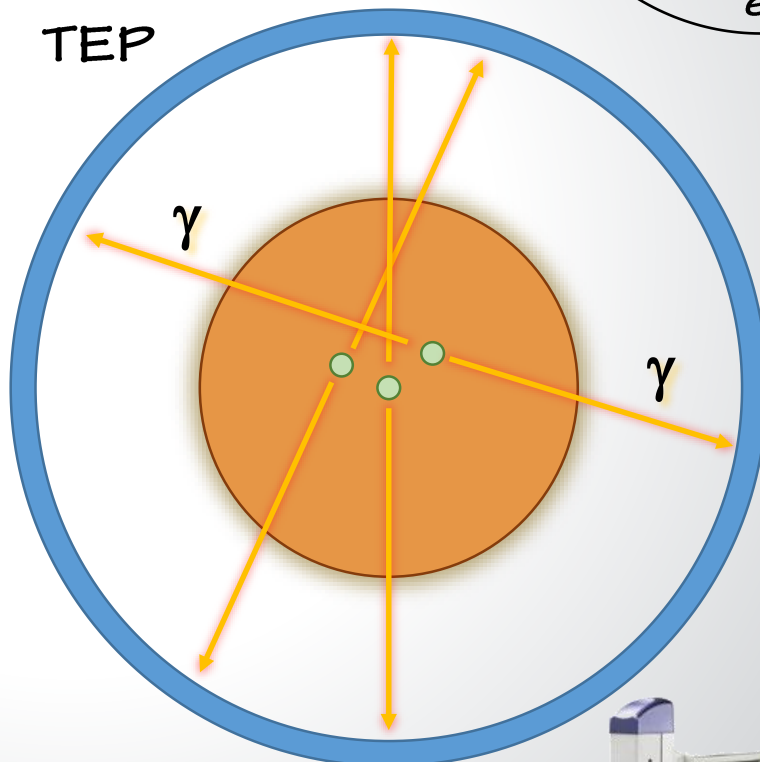
Tomographie d'émission

TEMP = SPECT



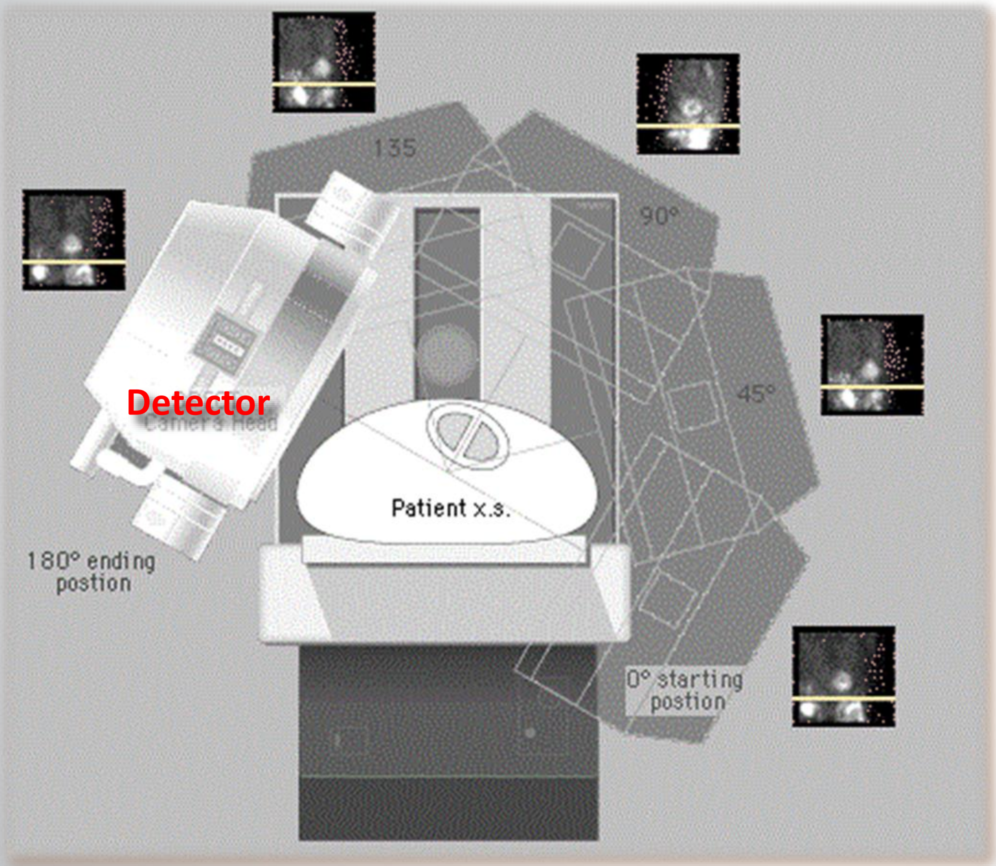
TEP

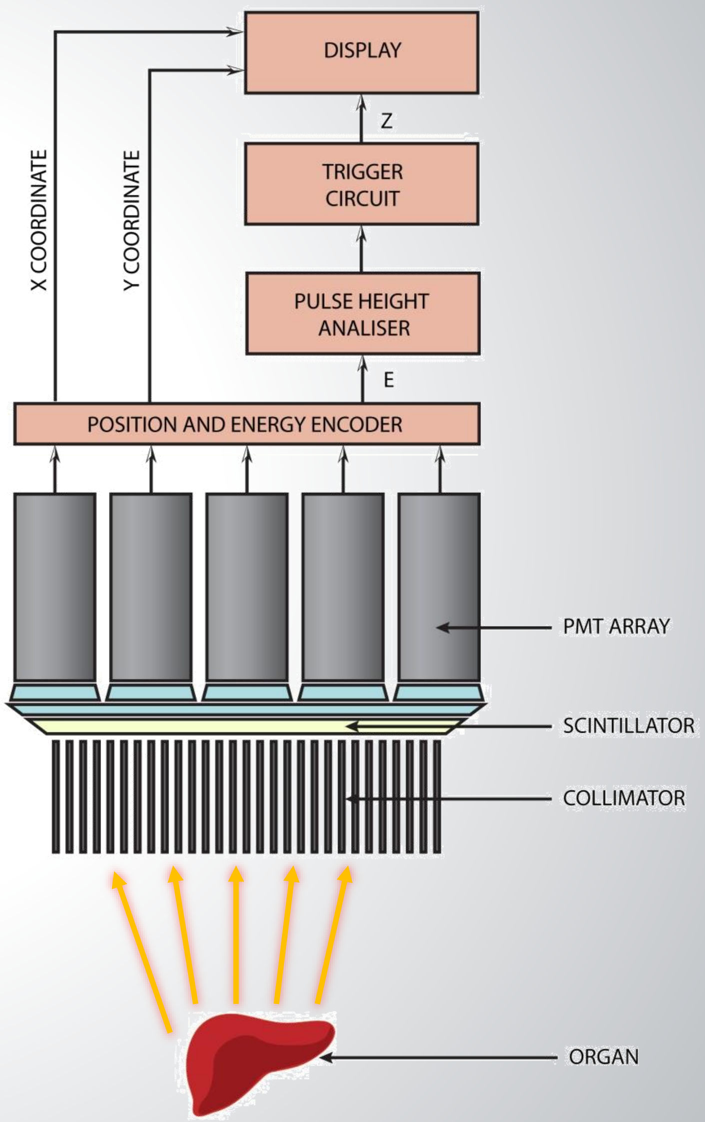




TEMP - acquisition

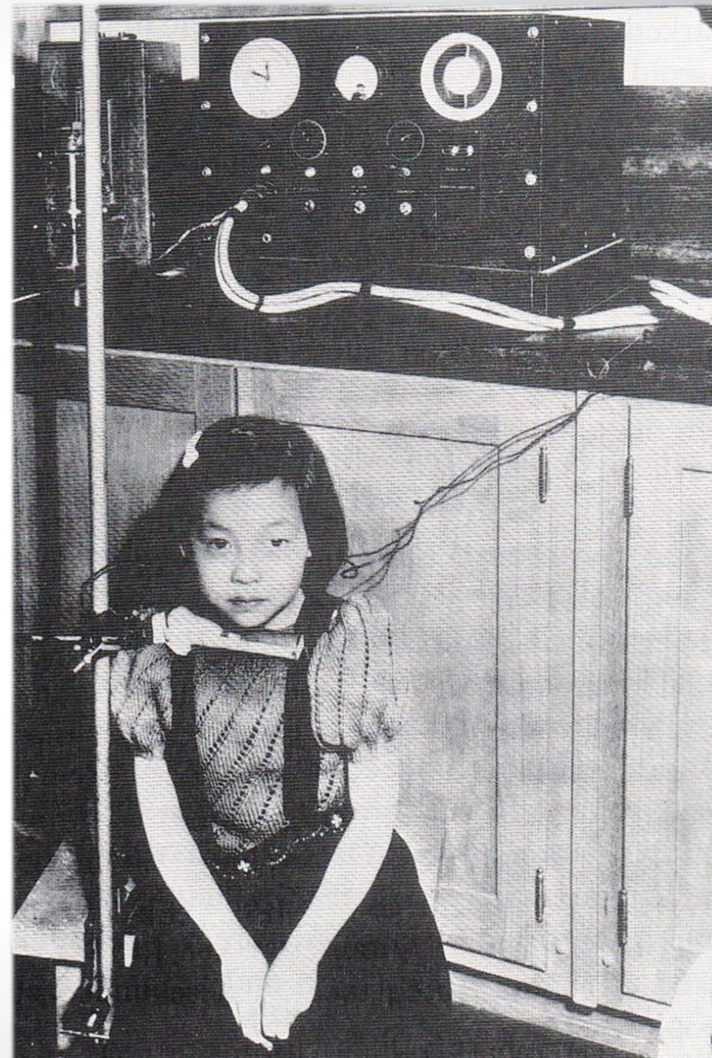
Gamma caméra d'Anger





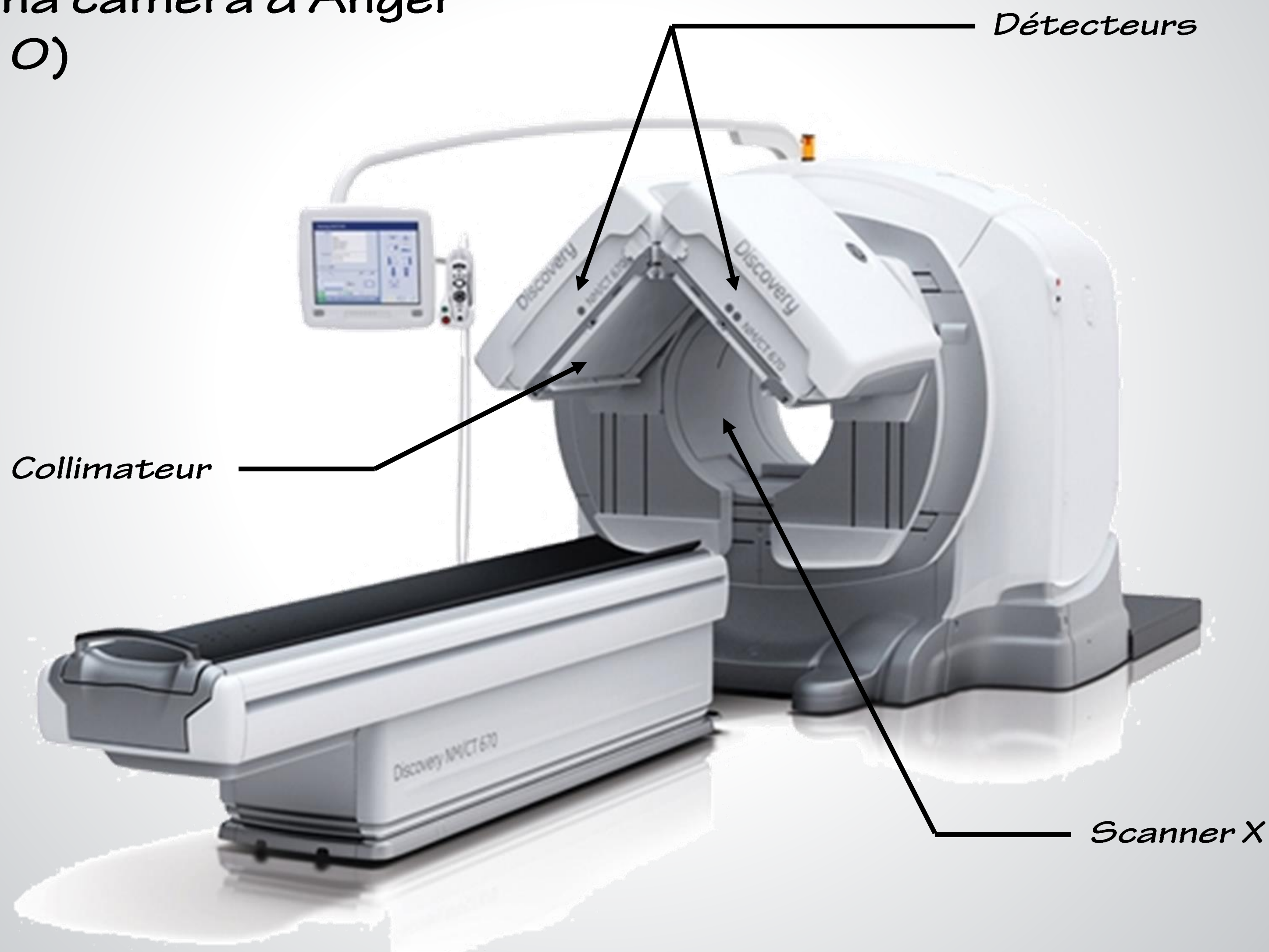
TEMP - acquisition

Gamma caméra d'Anger
(1958)



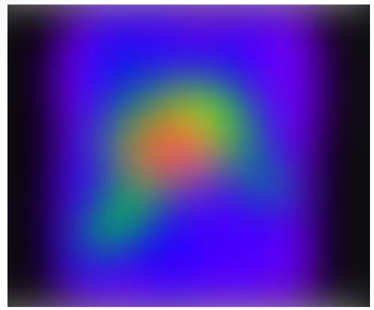
TEMP - acquisition

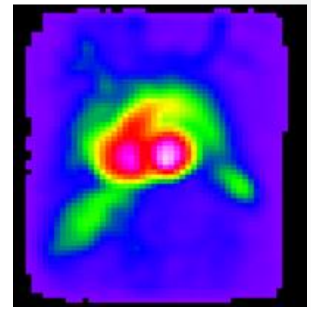
Gamma caméra d'Anger
(2010)



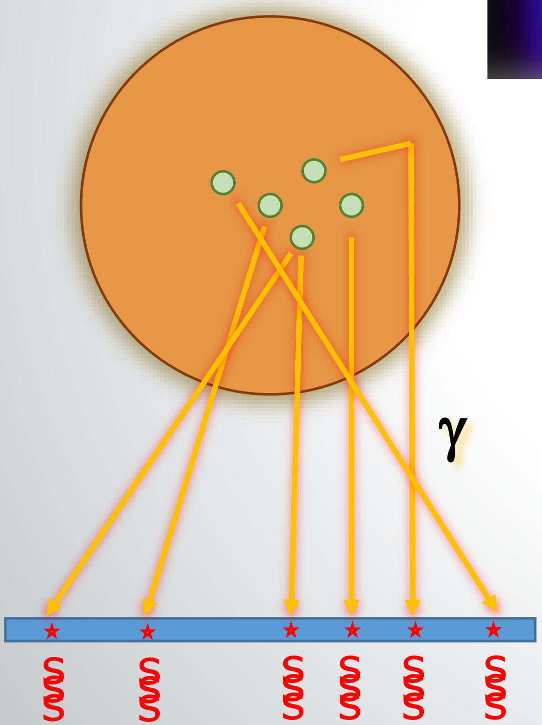
TEMP - acquisition

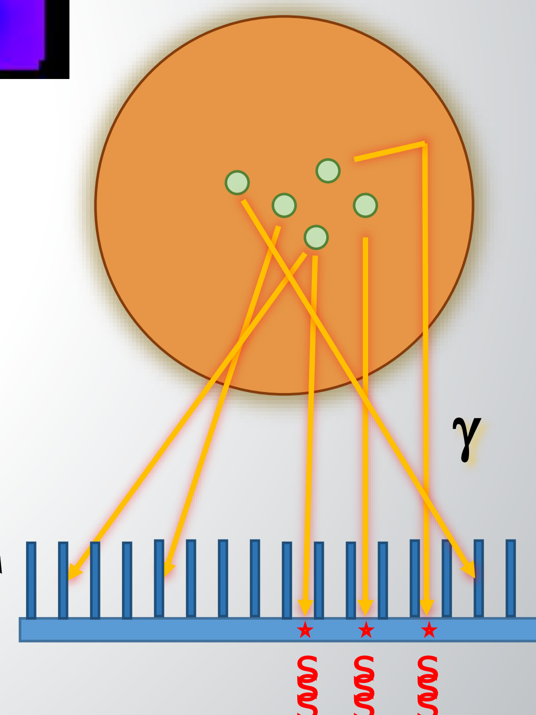
- Collimation





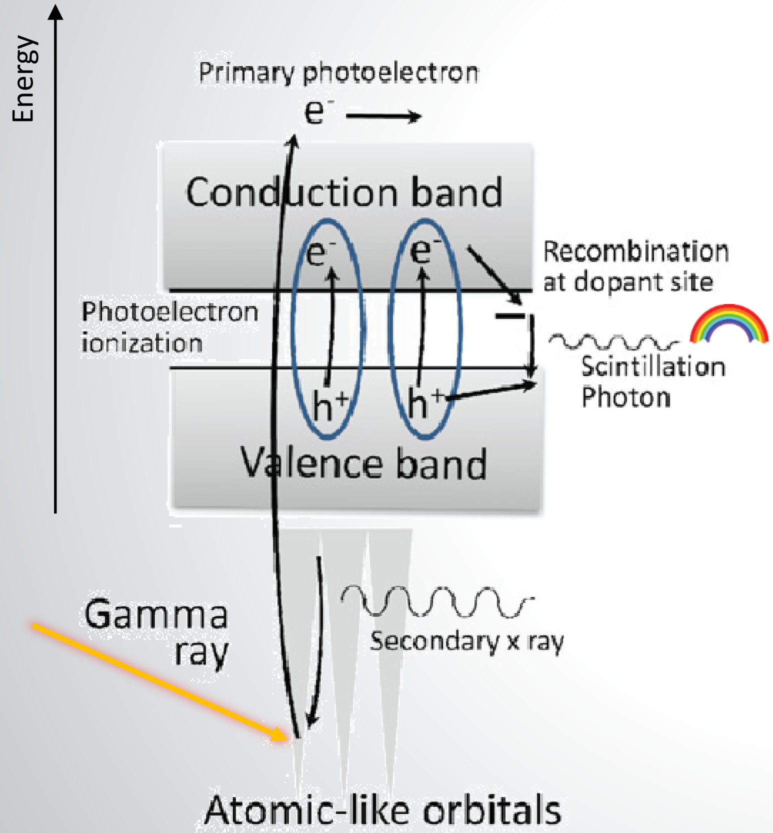
Compromis
résolution / sensibilité

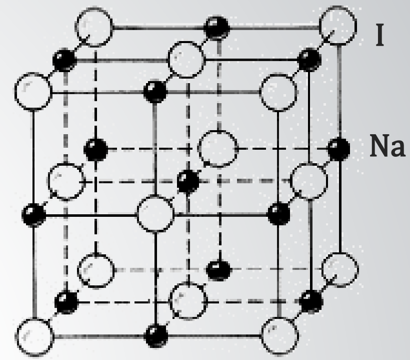




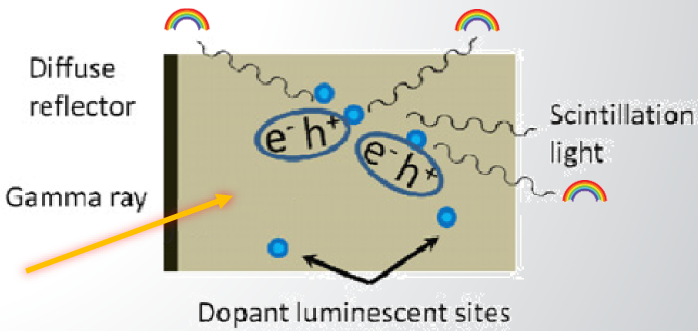
TEMP - acquisition

- Scintillation



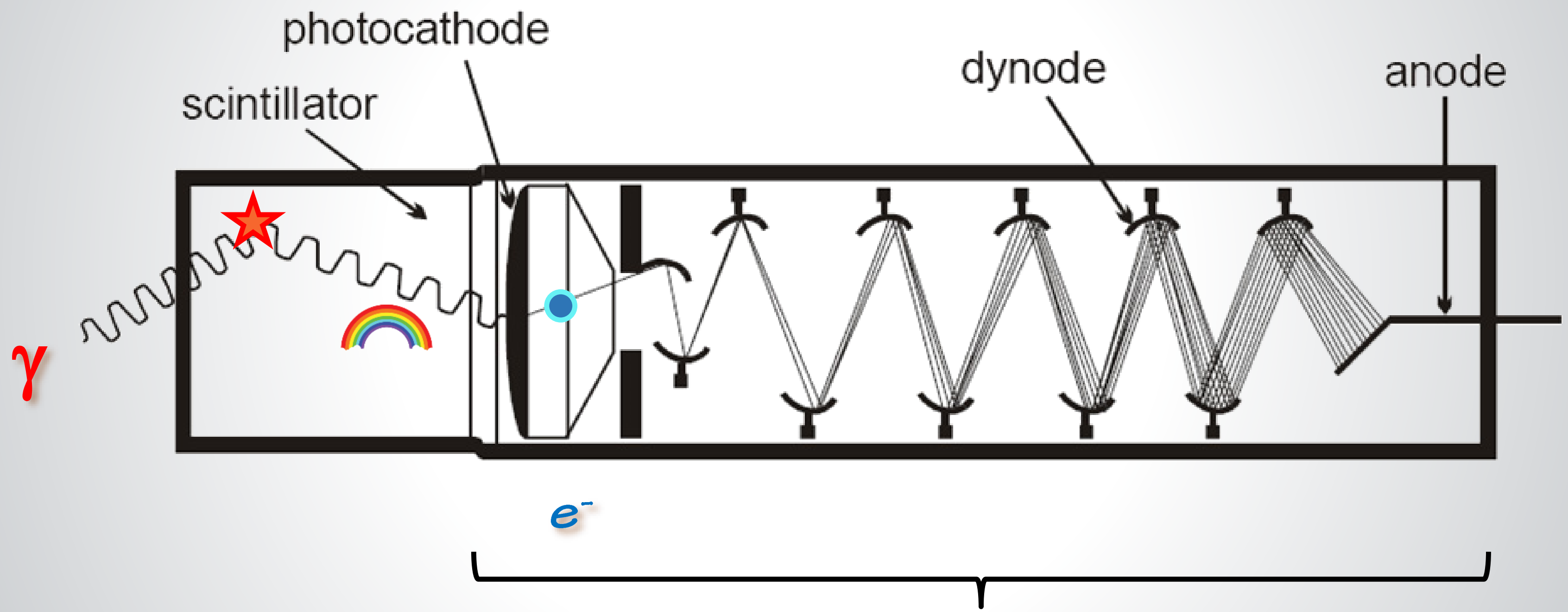


NaI(Tl)



TEMP - acquisition

- Amplification



photomultiplier tube

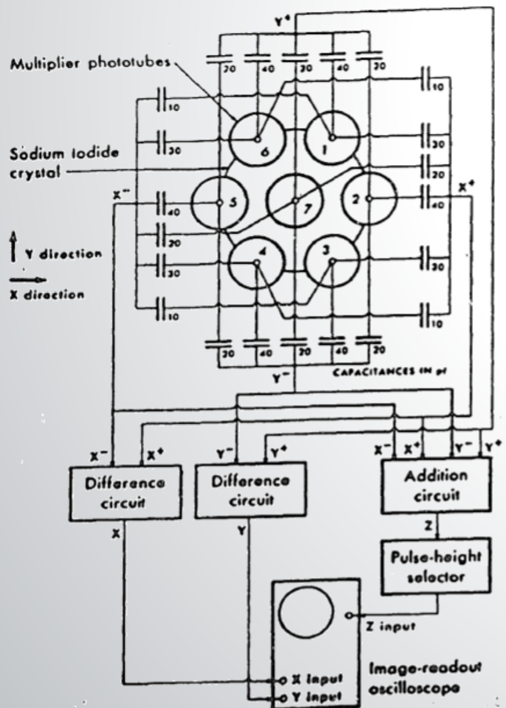
TEMP - acquisition

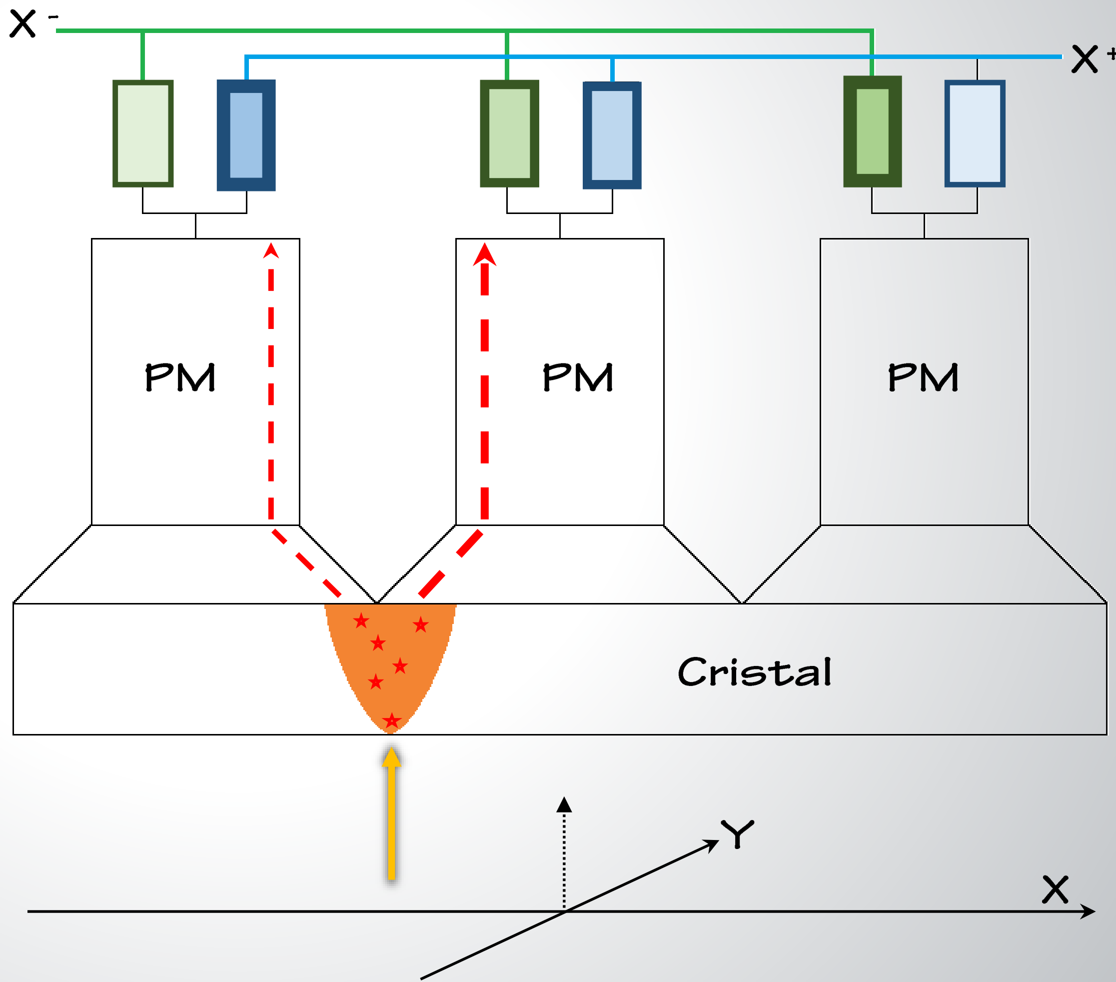
- Localisation

$$X = X^+ - X^-$$

$$Y = Y^+ - Y^-$$

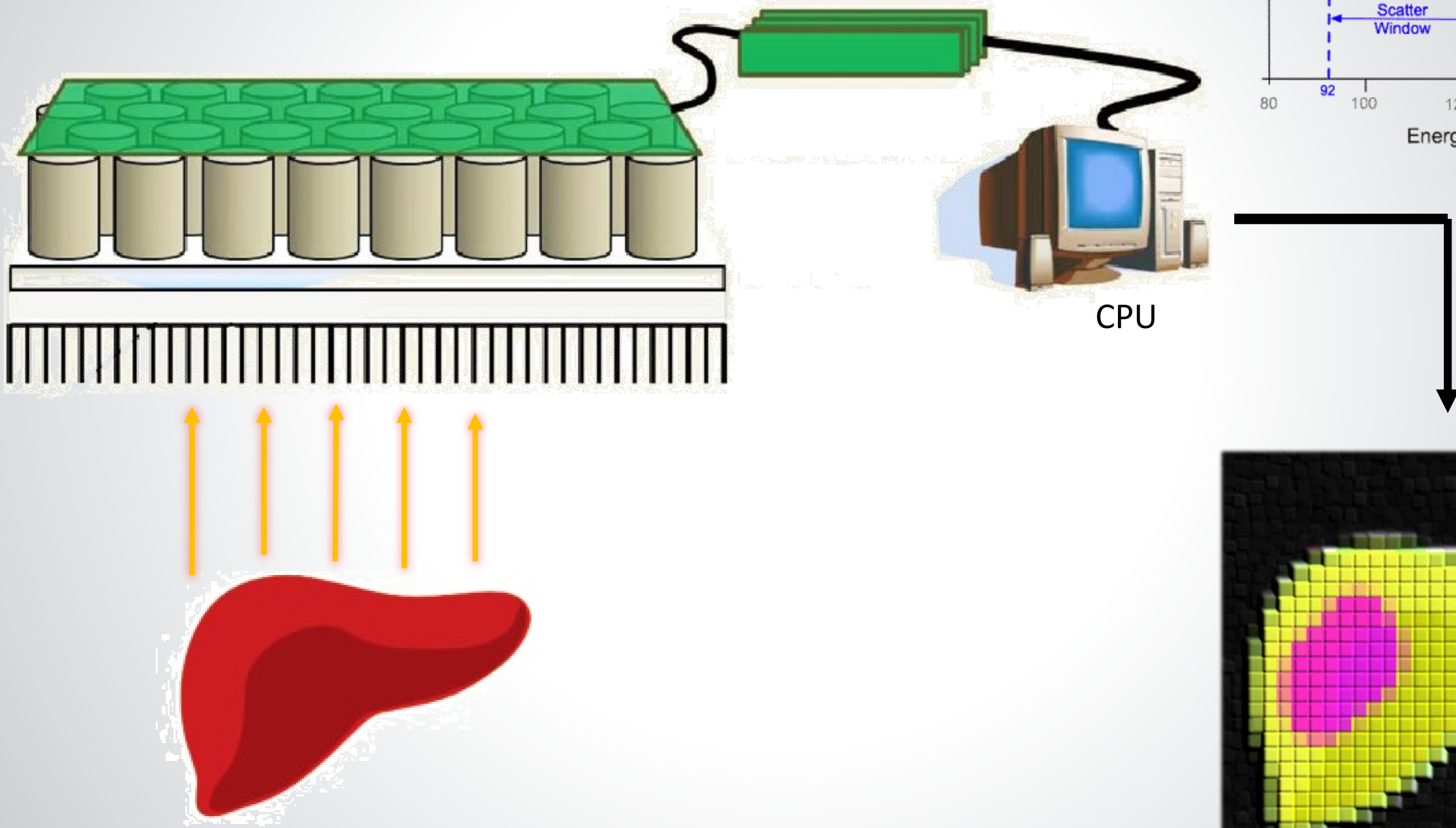
$$E = X^+ + X^- + Y^+ + Y^-$$

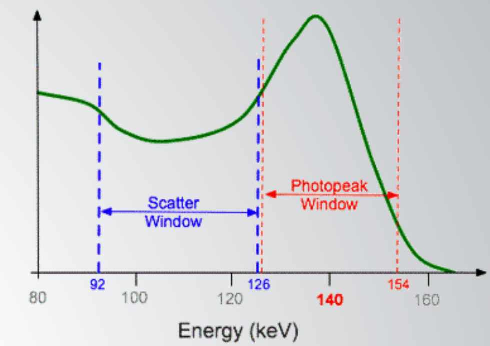


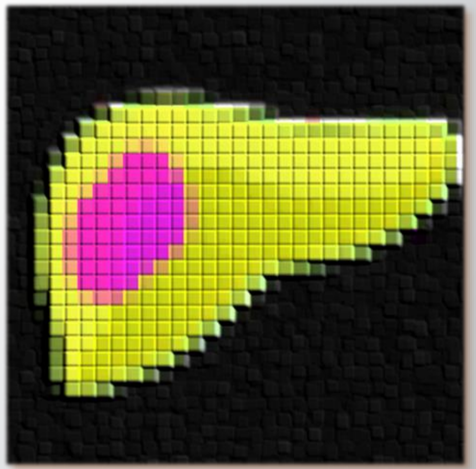


TEMP - acquisition

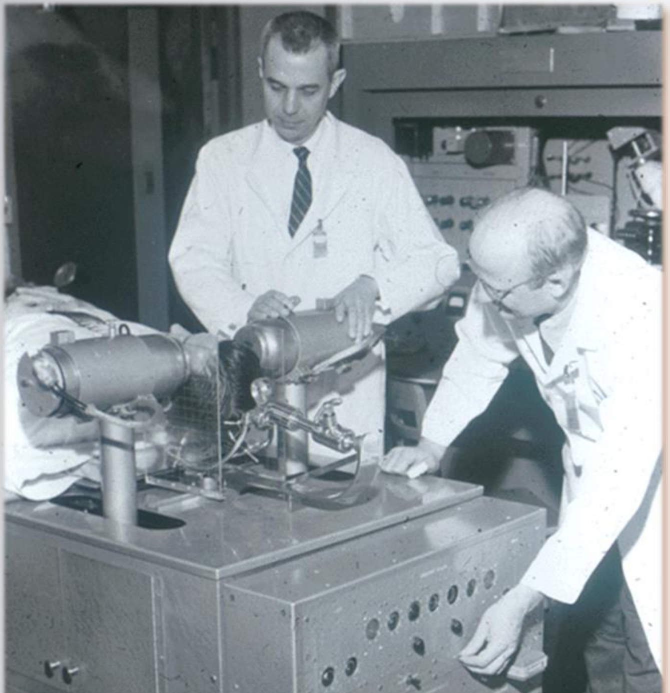
- Numérisation



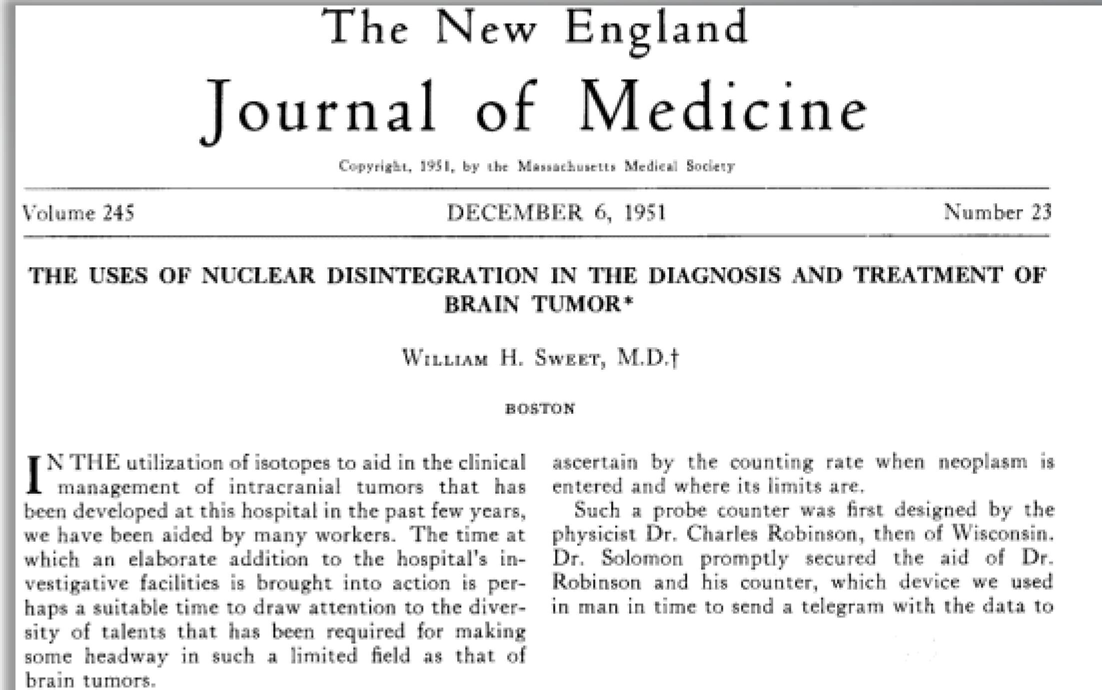




TEP - acquisition



G Brownell & C Burnham
Boston 1952



The New England
Journal of Medicine
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Volume 245 DECEMBER 6, 1951 Number 23

THE USES OF NUCLEAR DISINTEGRATION IN THE DIAGNOSIS AND TREATMENT OF BRAIN TUMOR*

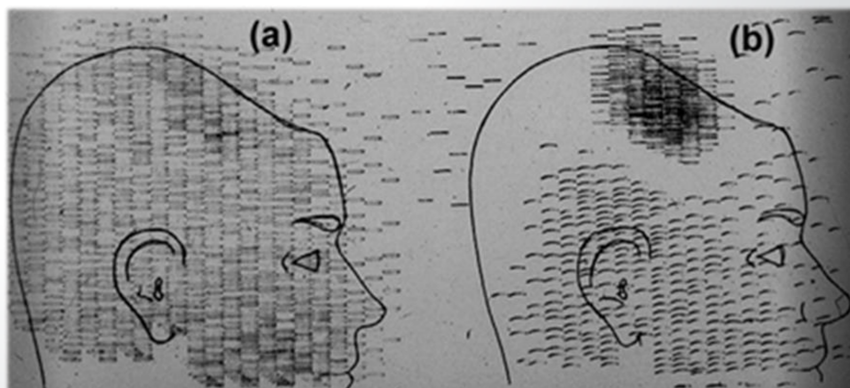
William H. Sweet, M.D.†

BOSTON

IN THE utilization of isotopes to aid in the clinical management of intracranial tumors that has been developed at this hospital in the past few years, we have been aided by many workers. The time at which an elaborate addition to the hospital's investigative facilities is brought into action is perhaps a suitable time to draw attention to the diversity of talents that has been required for making some headway in such a limited field as that of brain tumors.

ascertain by the counting rate when neoplasm is entered and where its limits are.

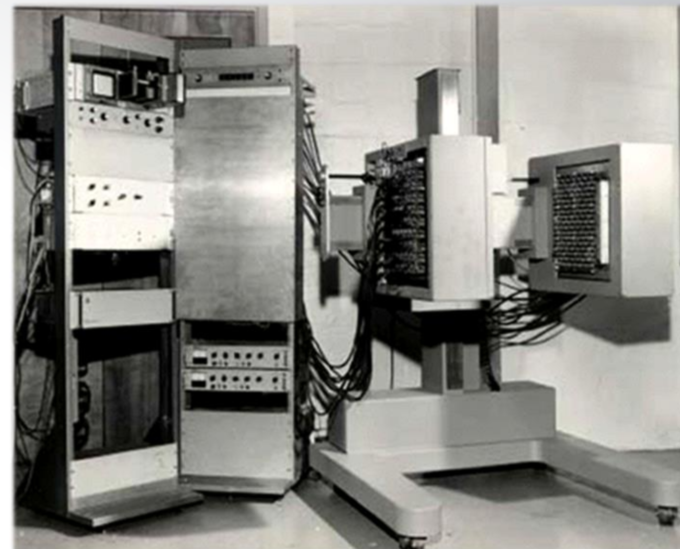
Such a probe counter was first designed by the physicist Dr. Charles Robinson, then of Wisconsin. Dr. Solomon promptly secured the aid of Dr. Robinson and his counter, which device we used in man in time to send a telegram with the data to



TEP - acquisition

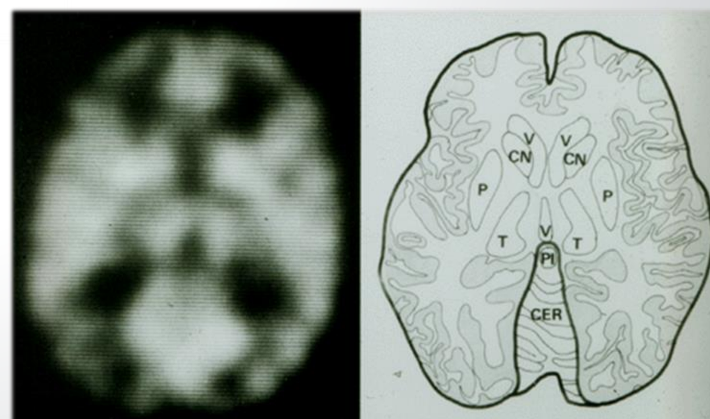
Multidétecteur
Mode tomographique

1975



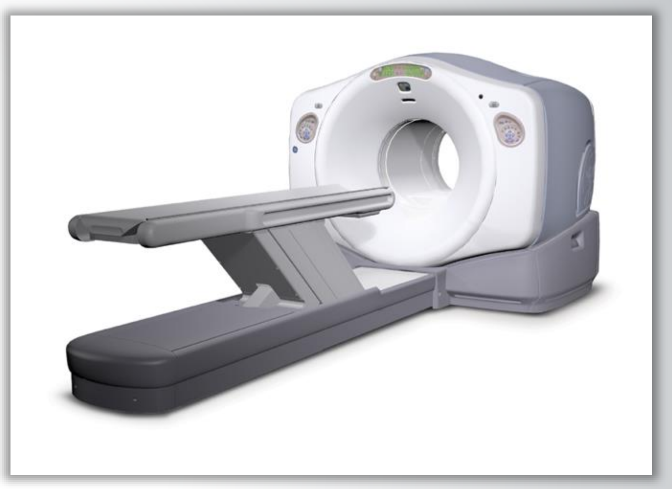
1970

^{18}FDG
1976

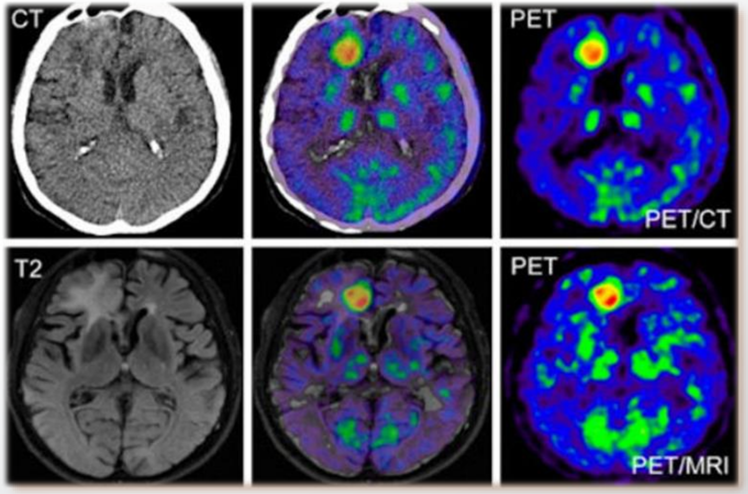


TEP - acquisition

Anneau de détecteurs
1980

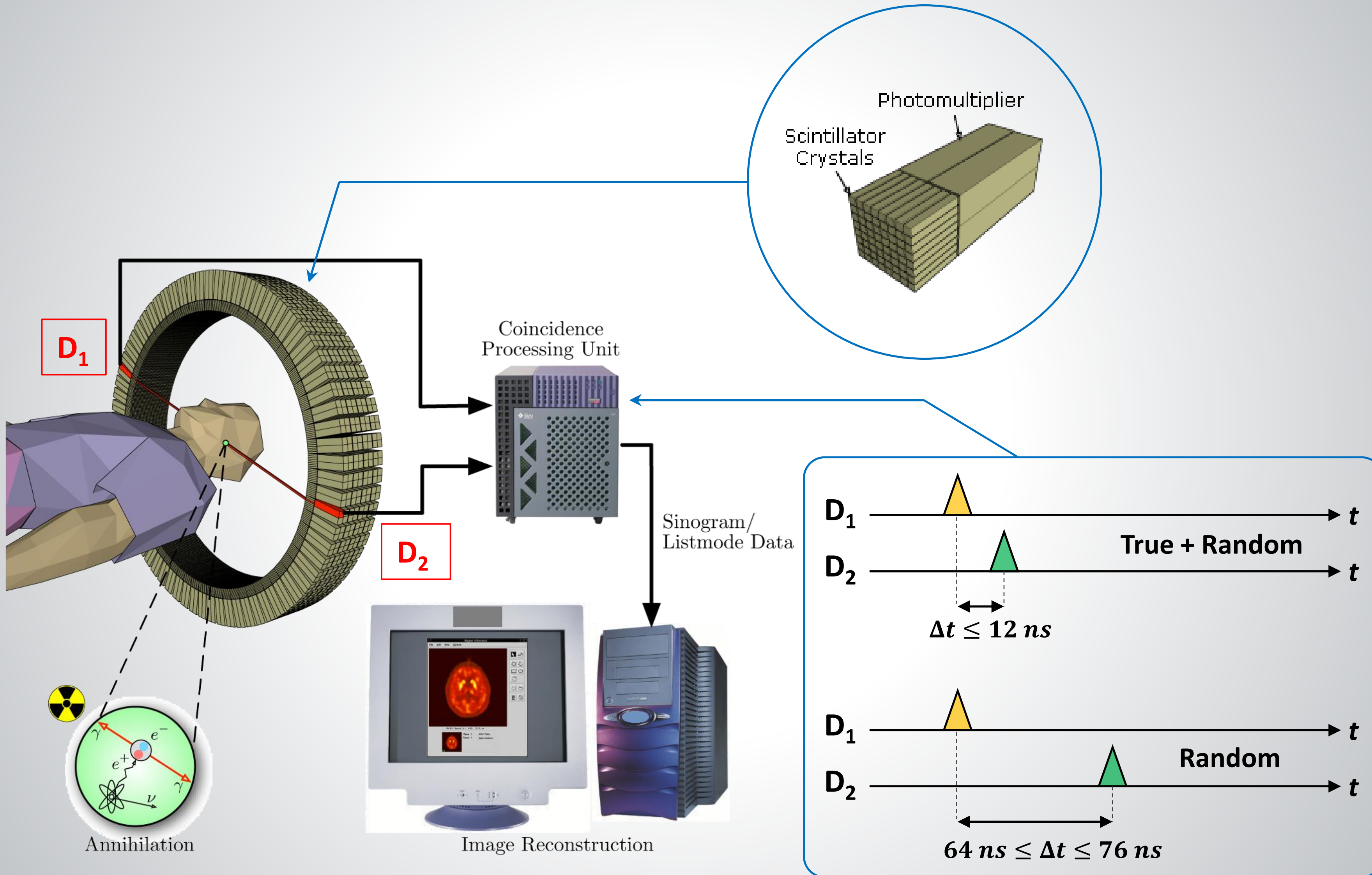


TEP-CT
1998

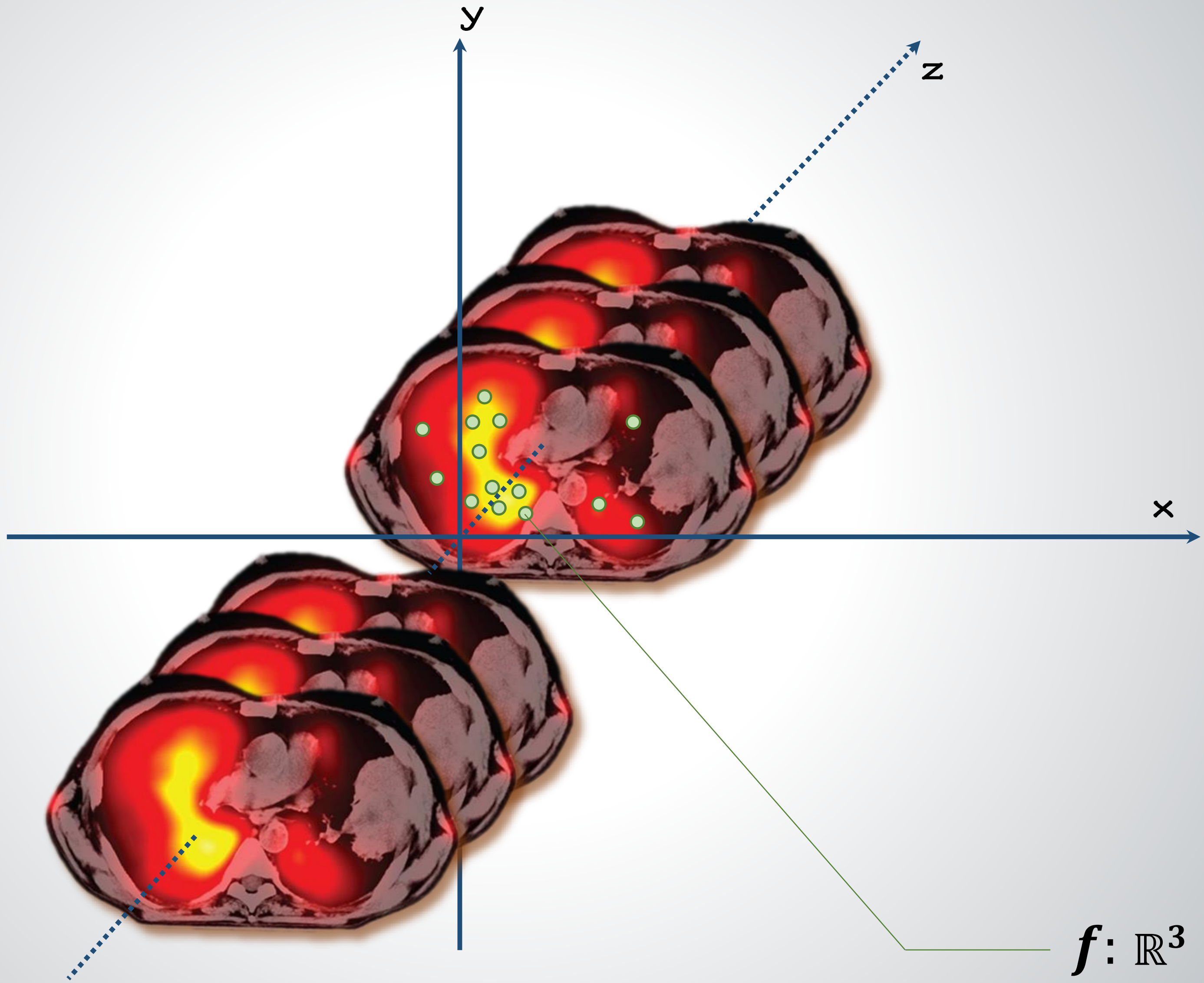


TEP-MR
2010

TEP - acquisition

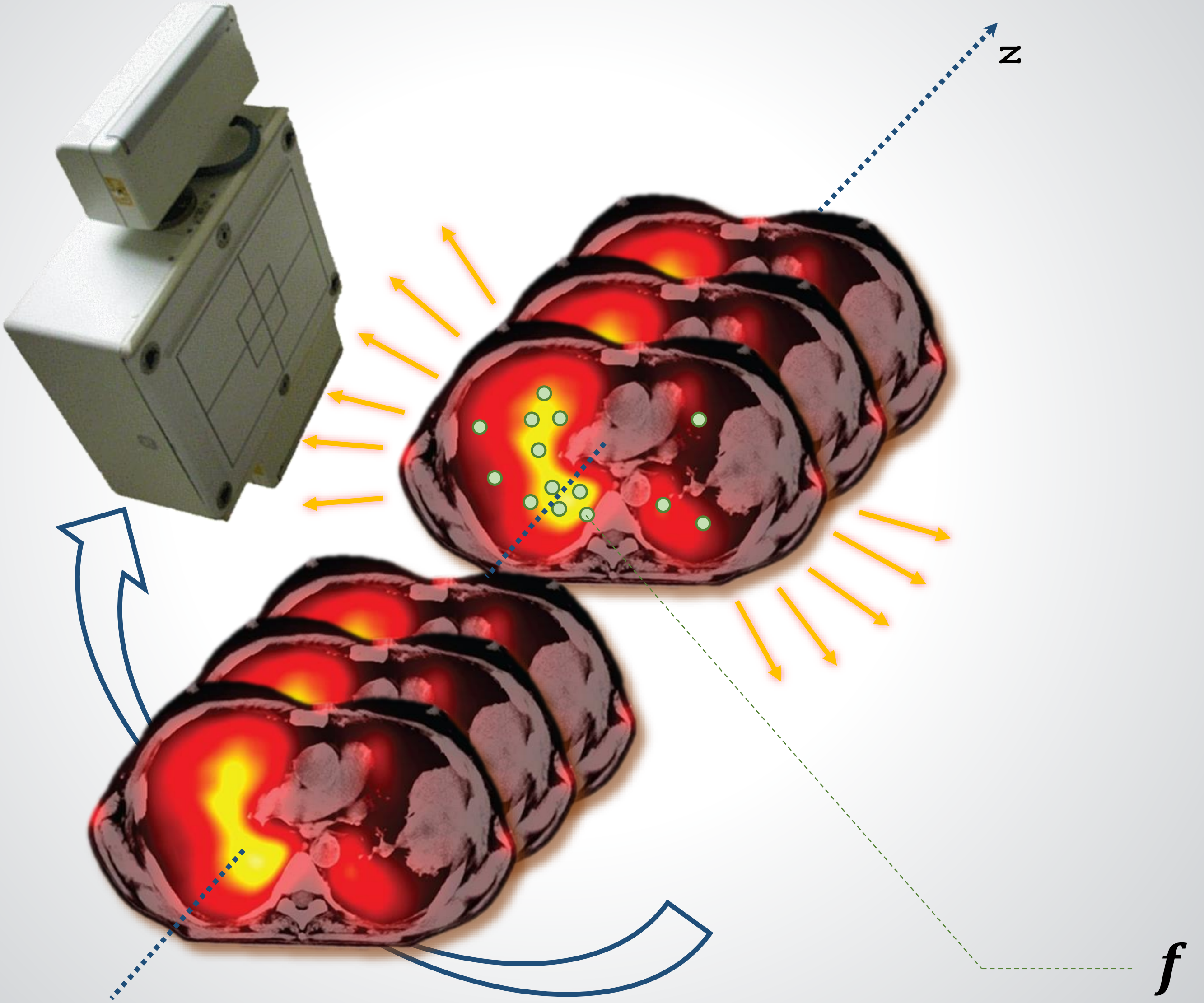


Problème tomographique

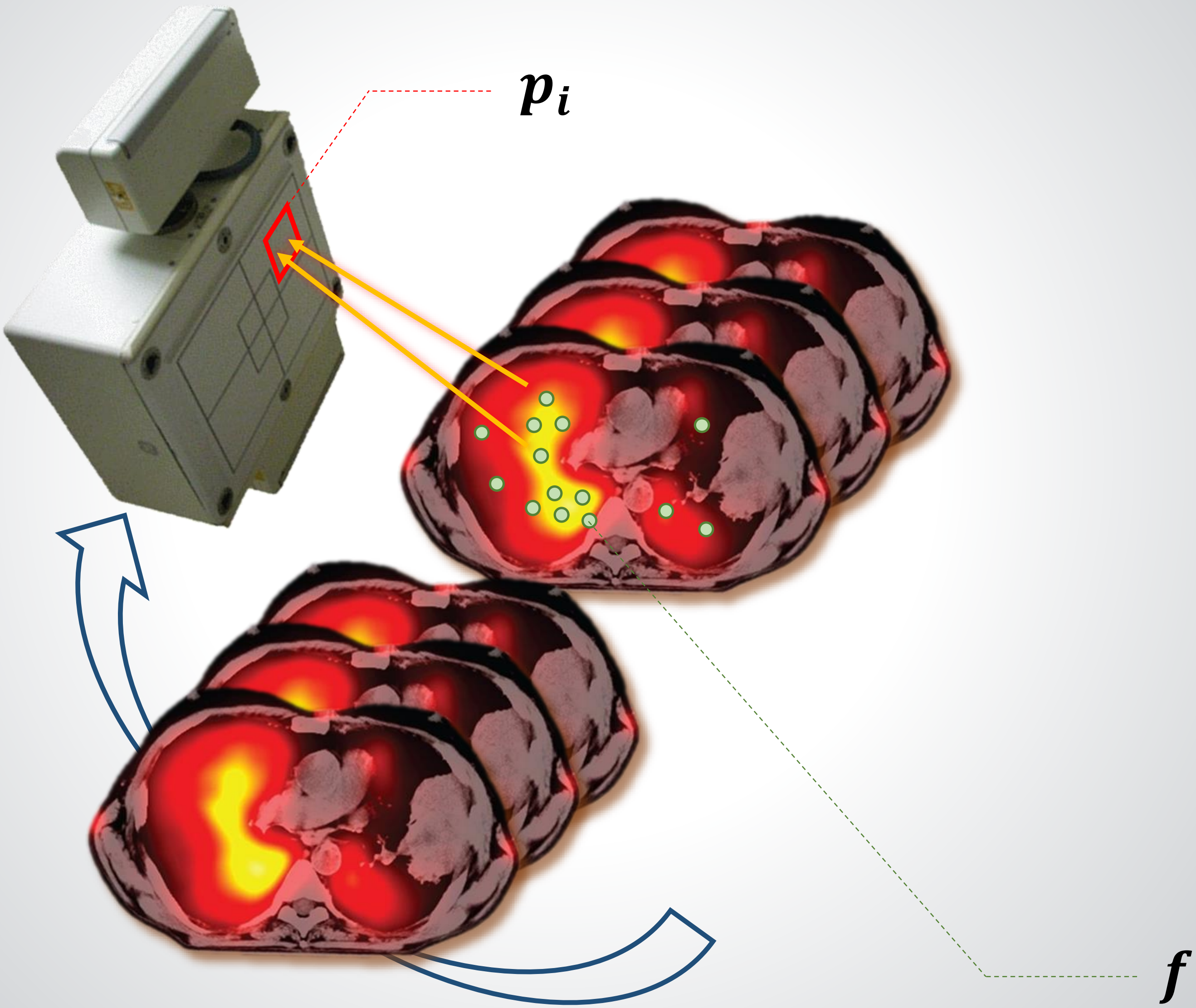


$f : \mathbb{R}^3 \to \mathbb{R}^+$

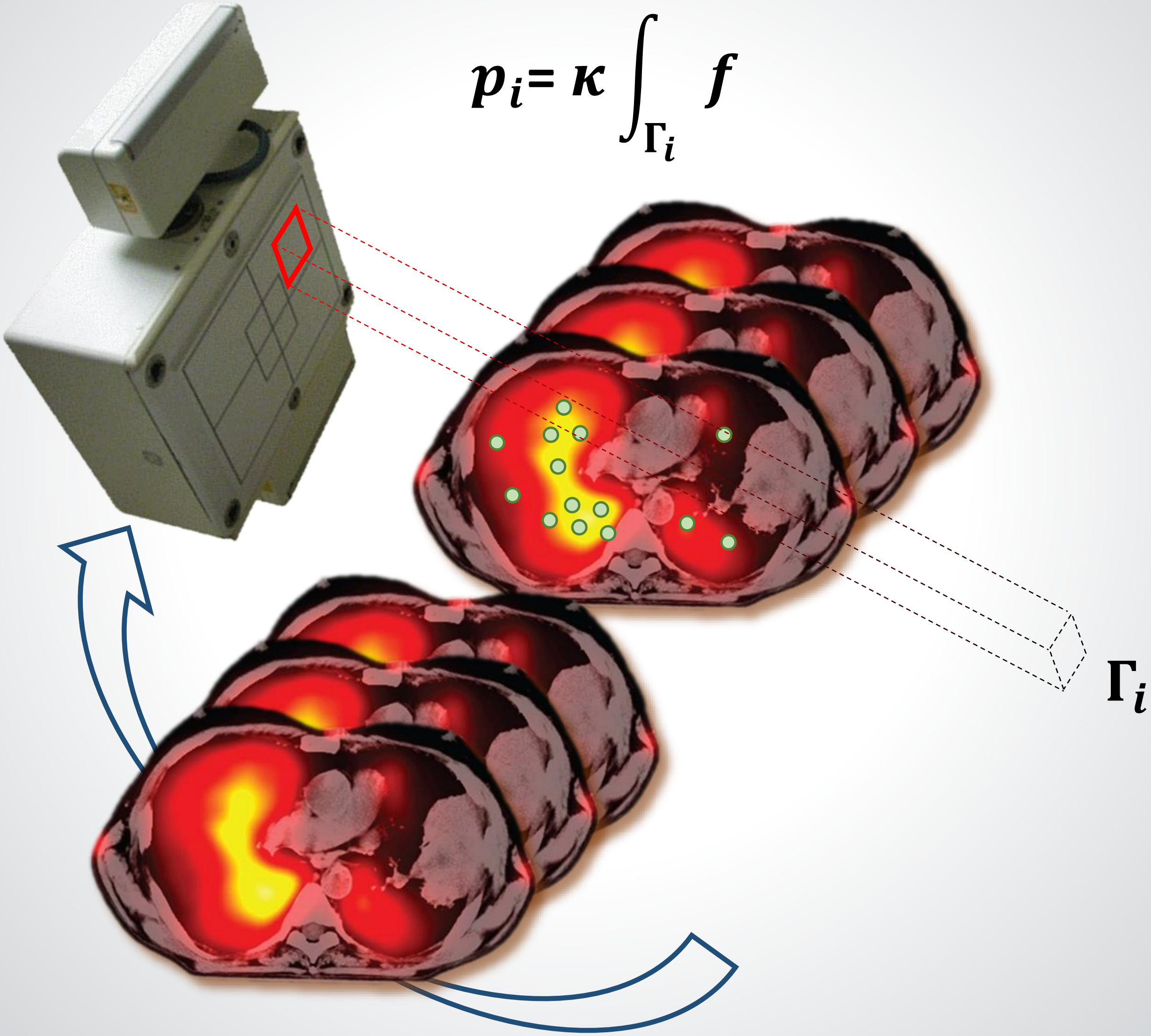
Problème tomographique



Problème tomographique

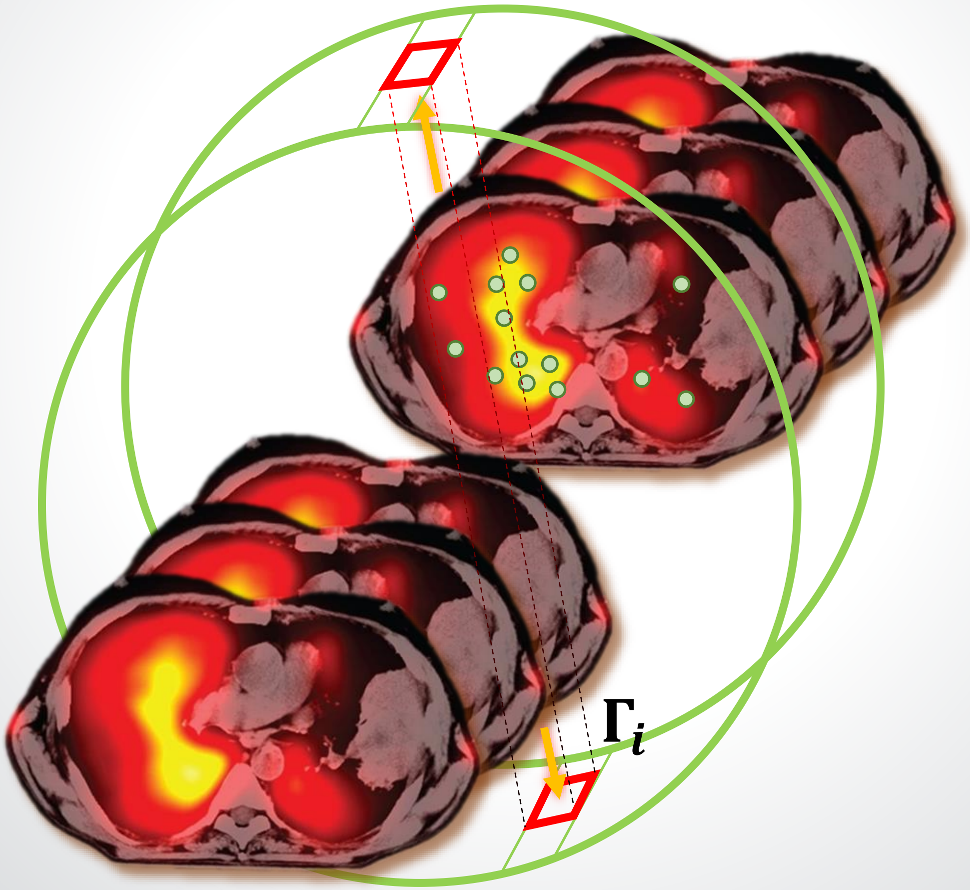


Problème tomographique



Problème tomographique

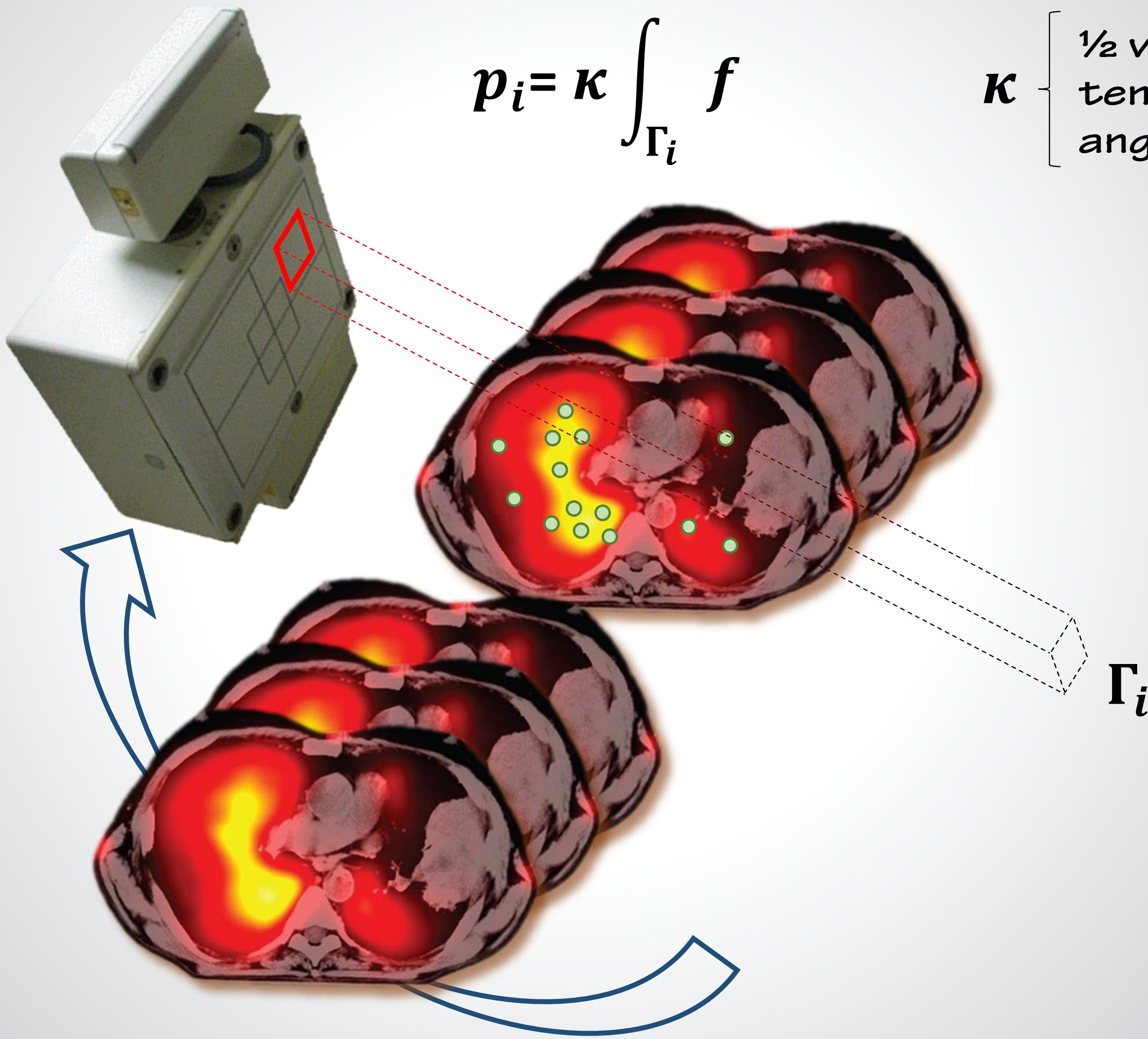
$$p_i = \kappa \int_{\Gamma_i} f$$



Problème tomographique

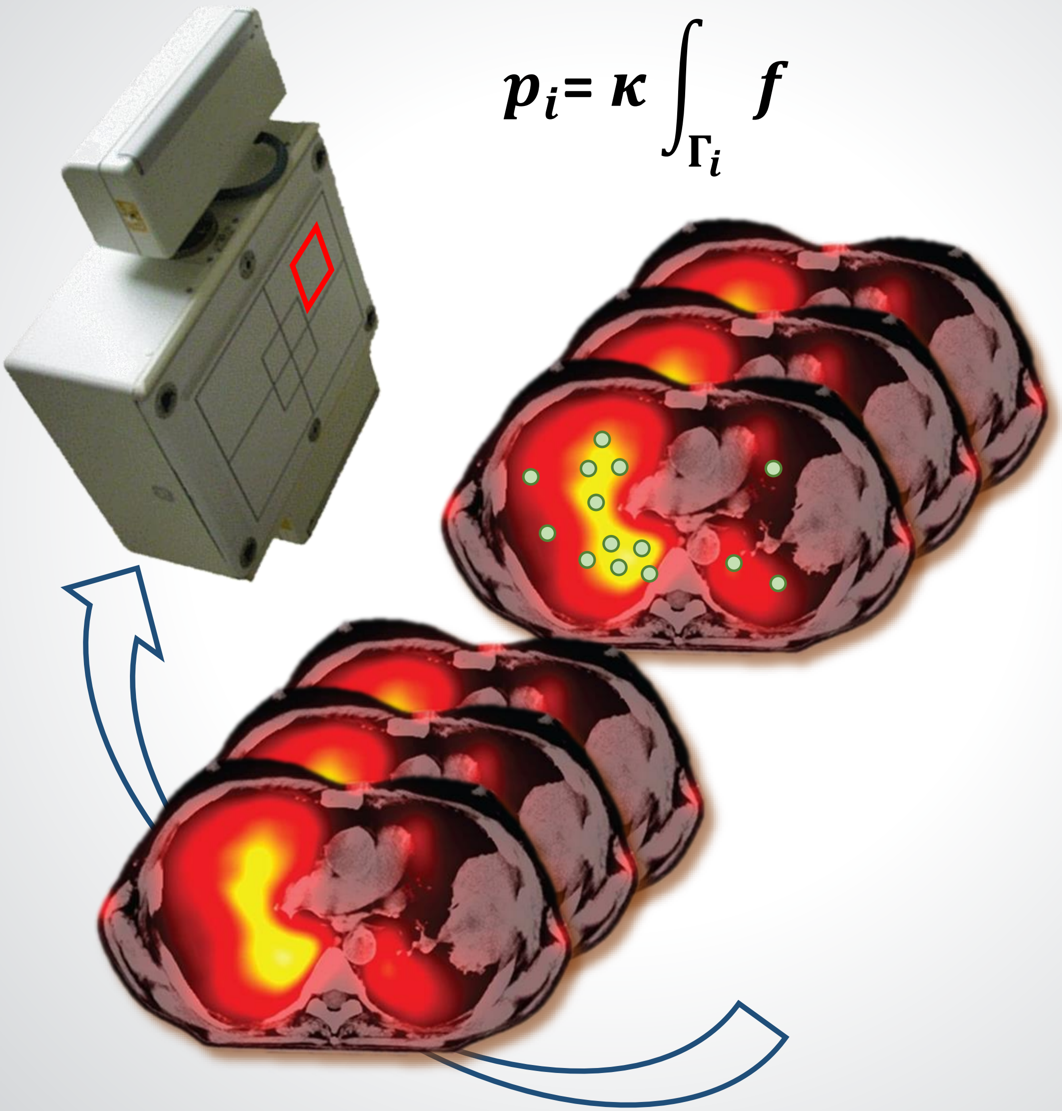
$$p_i = \kappa \int_{\Gamma_i} f$$

κ { ½ vie, temps de pose, angle solide }



Problème tomographique

$$p_i = \kappa \int_{\Gamma_i} f$$



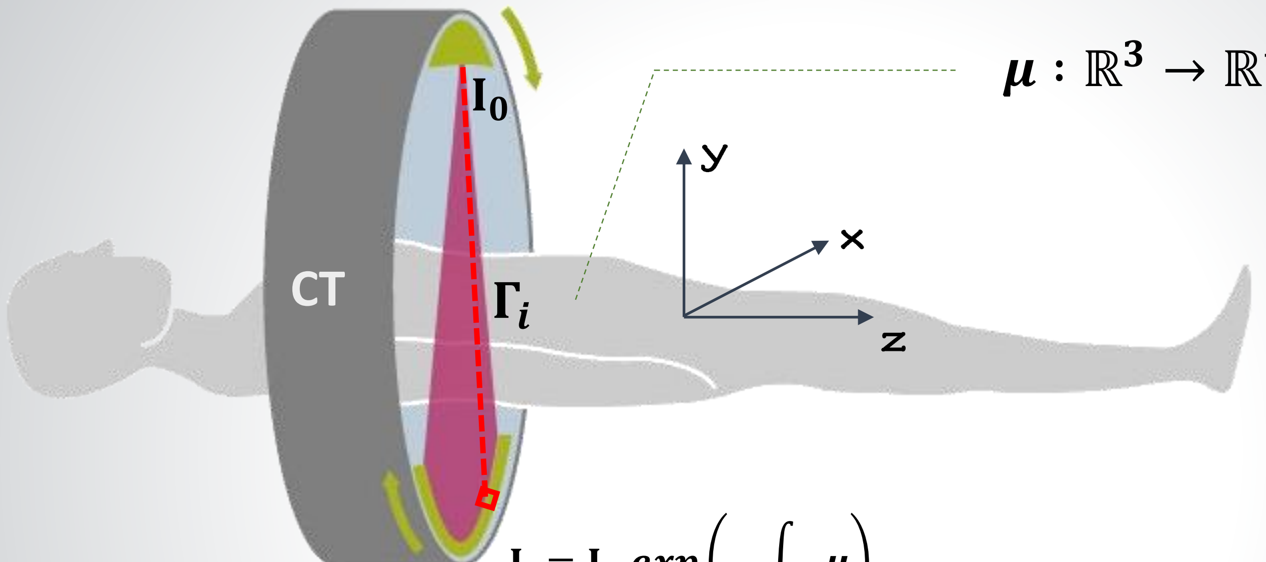
$p_i \quad (i = 1 \ldots N)$

?

f

Problème tomographique

$$\mu : \mathbb{R}^3 \rightarrow \mathbb{R}^+$$



$$\mathrm{I_i} = \mathrm{I_0}\, exp\left(-\int_{\Gamma_i} \mu\right)$$

$$p_i = -\,ln\left(\frac{\mathrm{I_i}}{\mathrm{I_0}}\right) = \int_{\Gamma_i} \mu$$

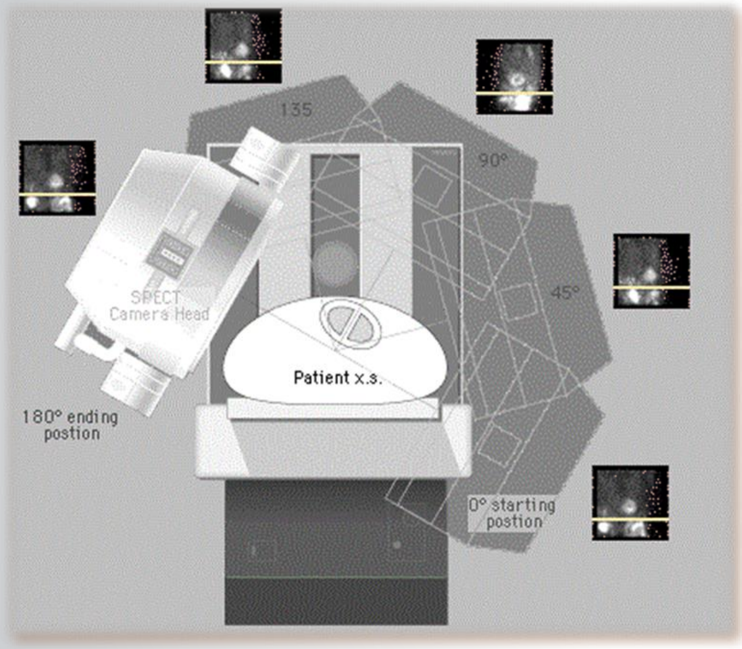
$p_i \quad (i = 1 \ldots \mathrm{N})$

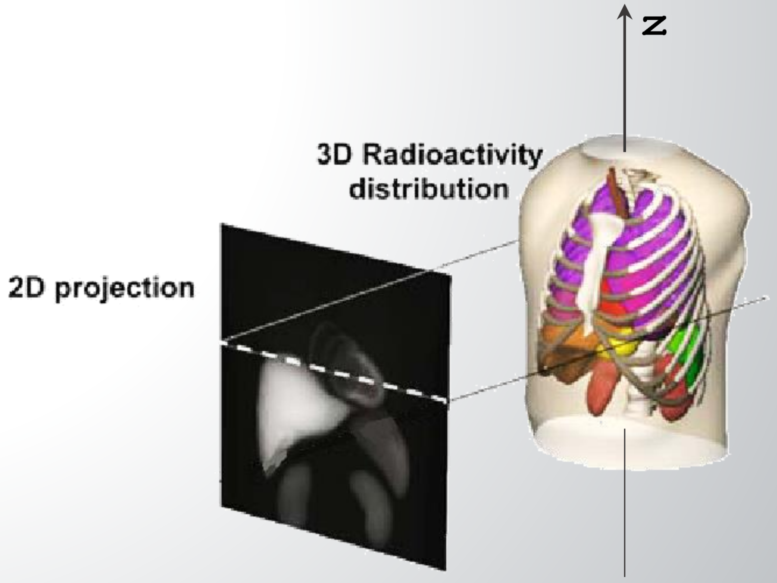
?

μ

Reconstruction

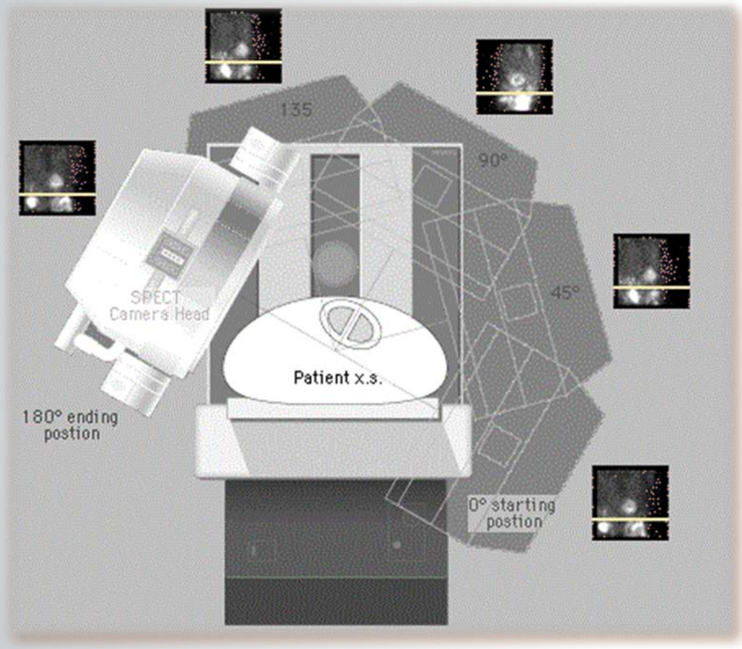
- Modèle analytique

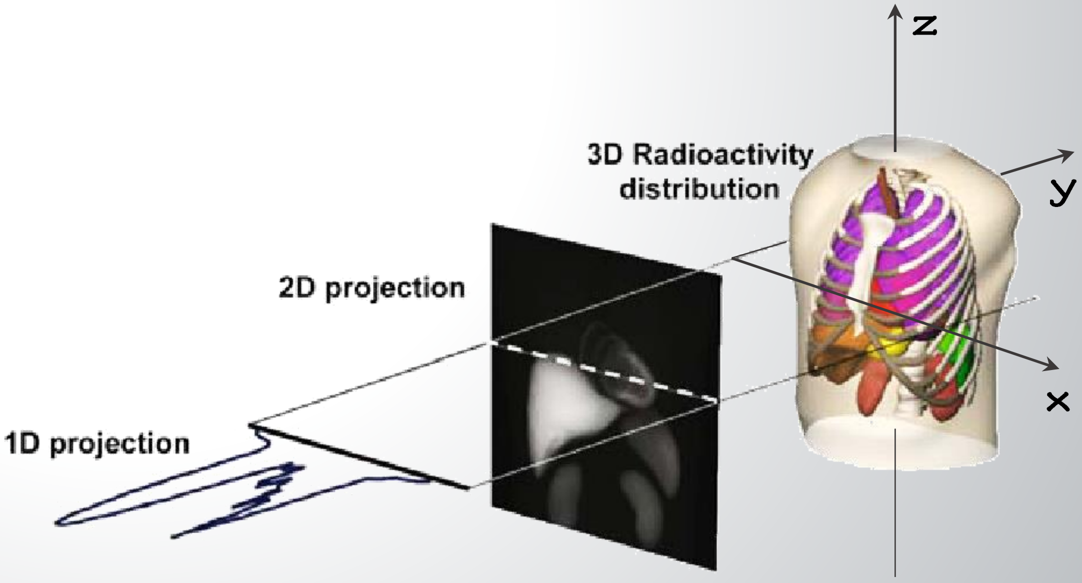




Reconstruction

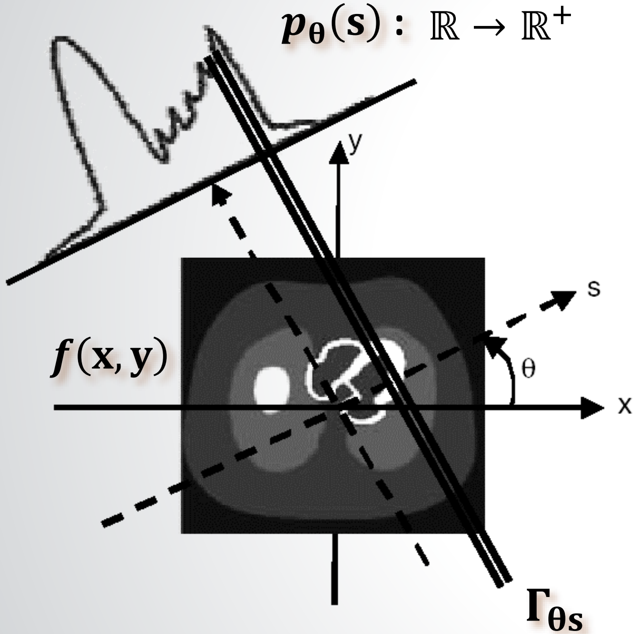
- Modèle analytique

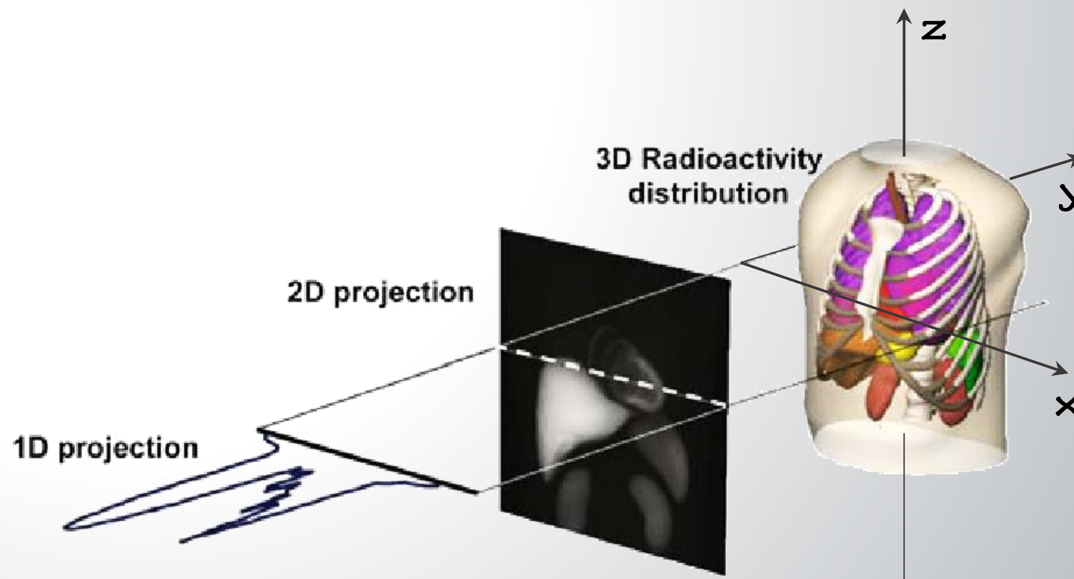




Reconstruction

- Modèle analytique

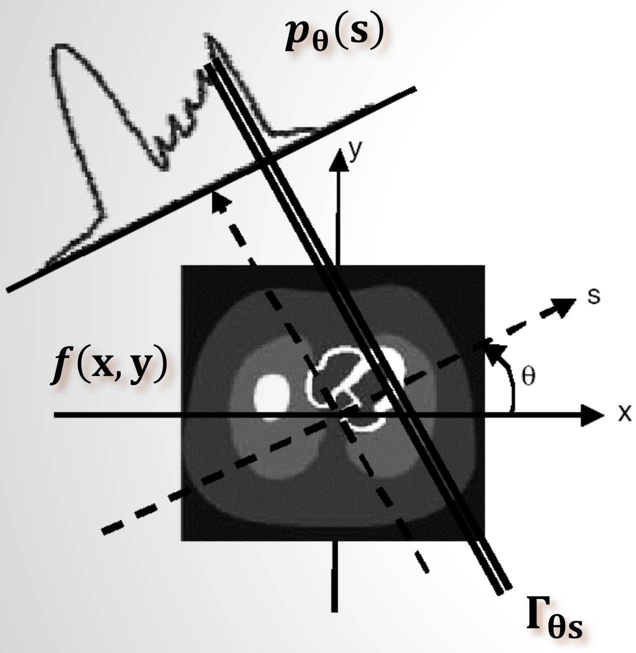


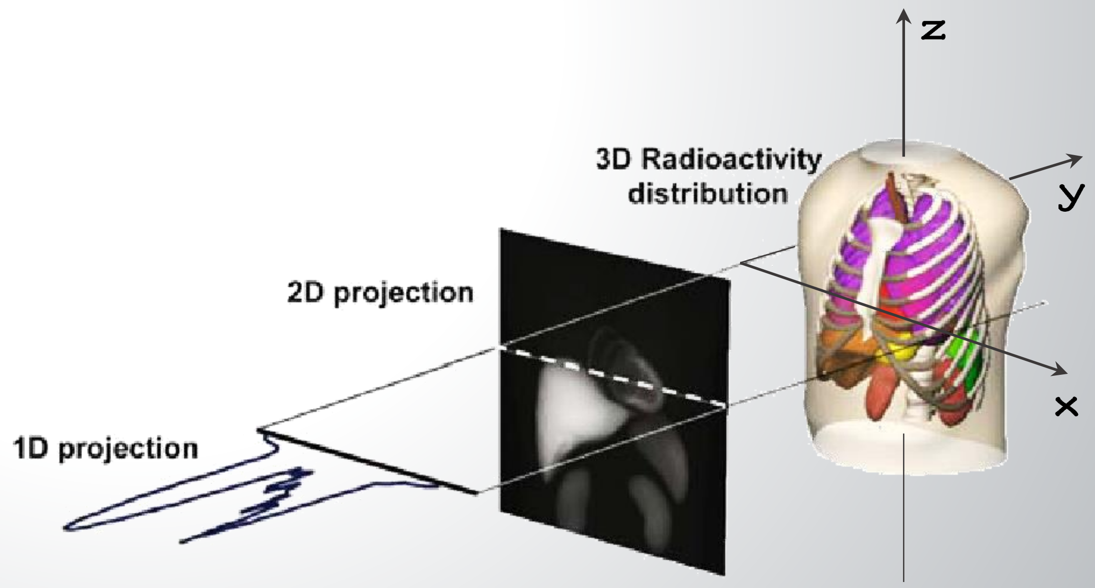


Reconstruction

- Modèle analytique

$$p_\theta(s) = \int_{\Gamma_{\theta s}} f(x, y)$$

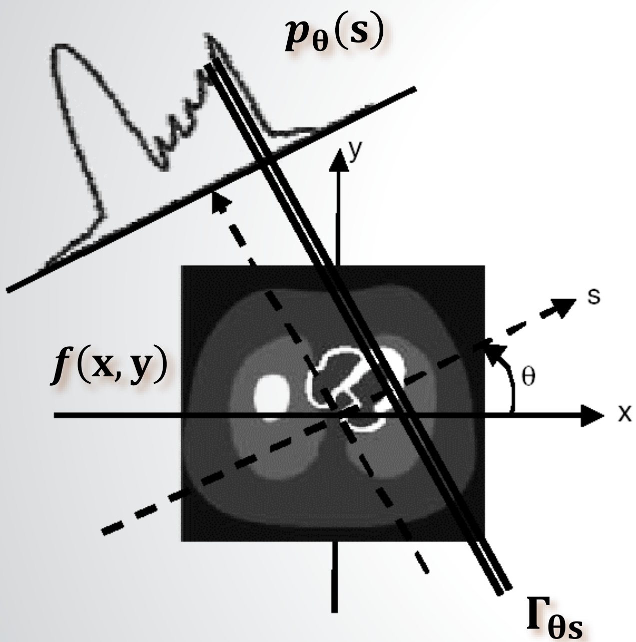


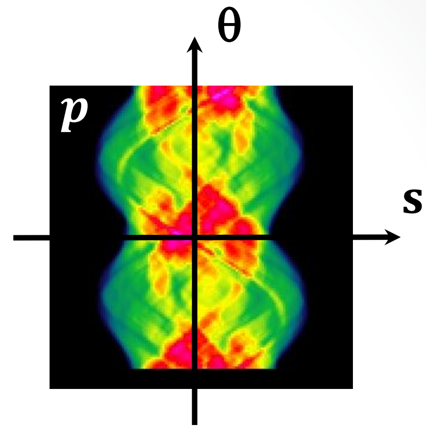


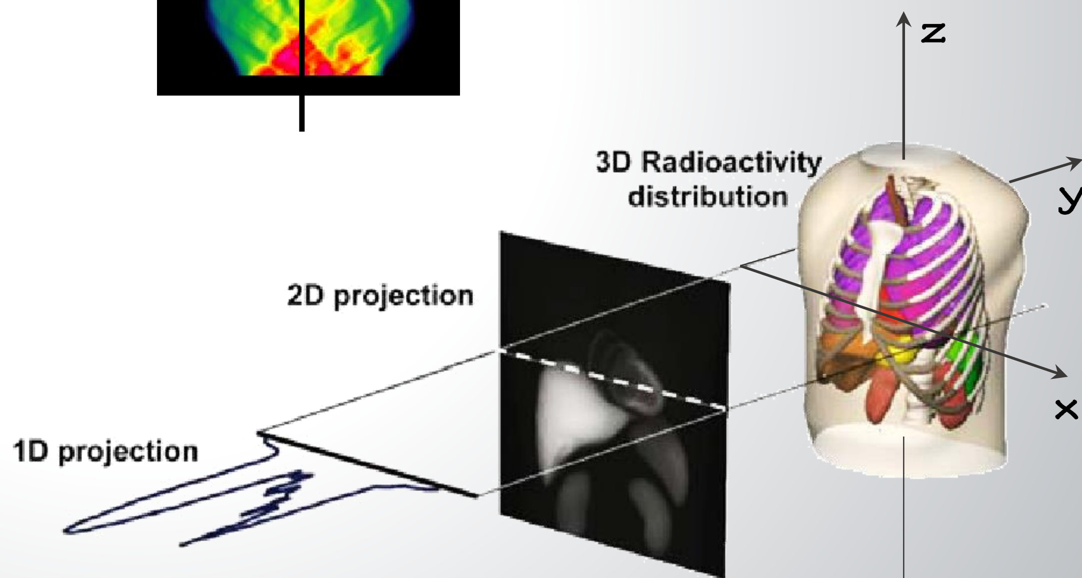
Reconstruction

- Modèle analytique

$$p_\theta(s) = \int_{\Gamma_{\theta s}} f(x, y)$$



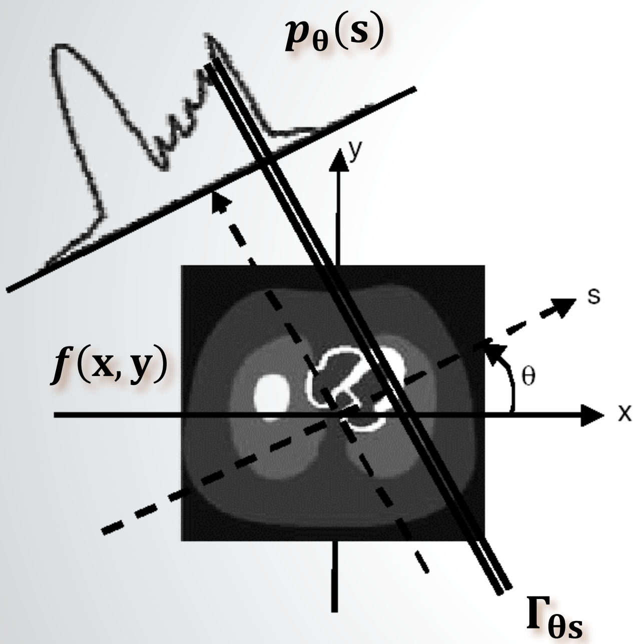




Reconstruction

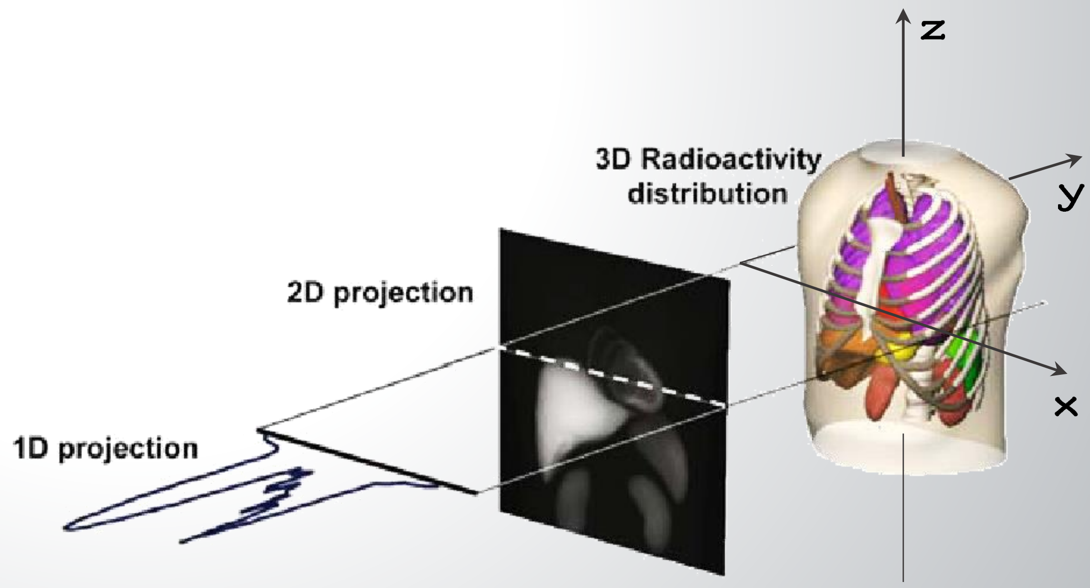
- Modèle analytique

$$p_\theta(s) = \int_{\Gamma_{\theta s}} f(x,y)$$



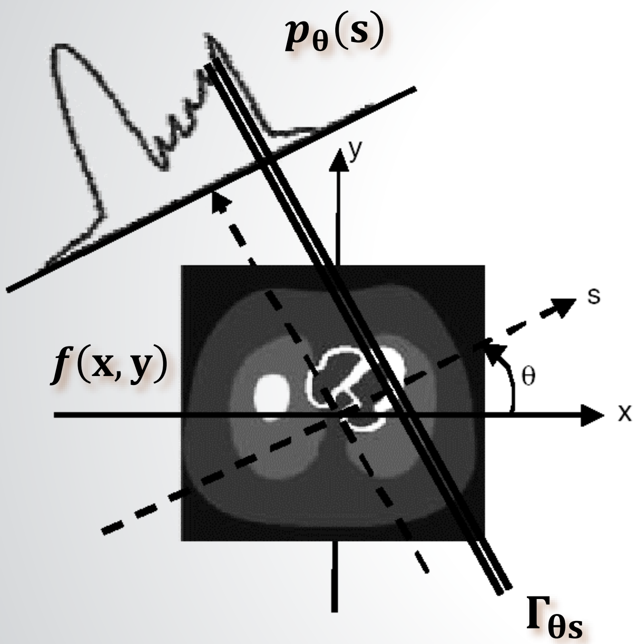
Projection (Radon) : $p = \mathrm{R}\, f$

Rétro-projection : R^*



Reconstruction

- Modèle analytique



Projection

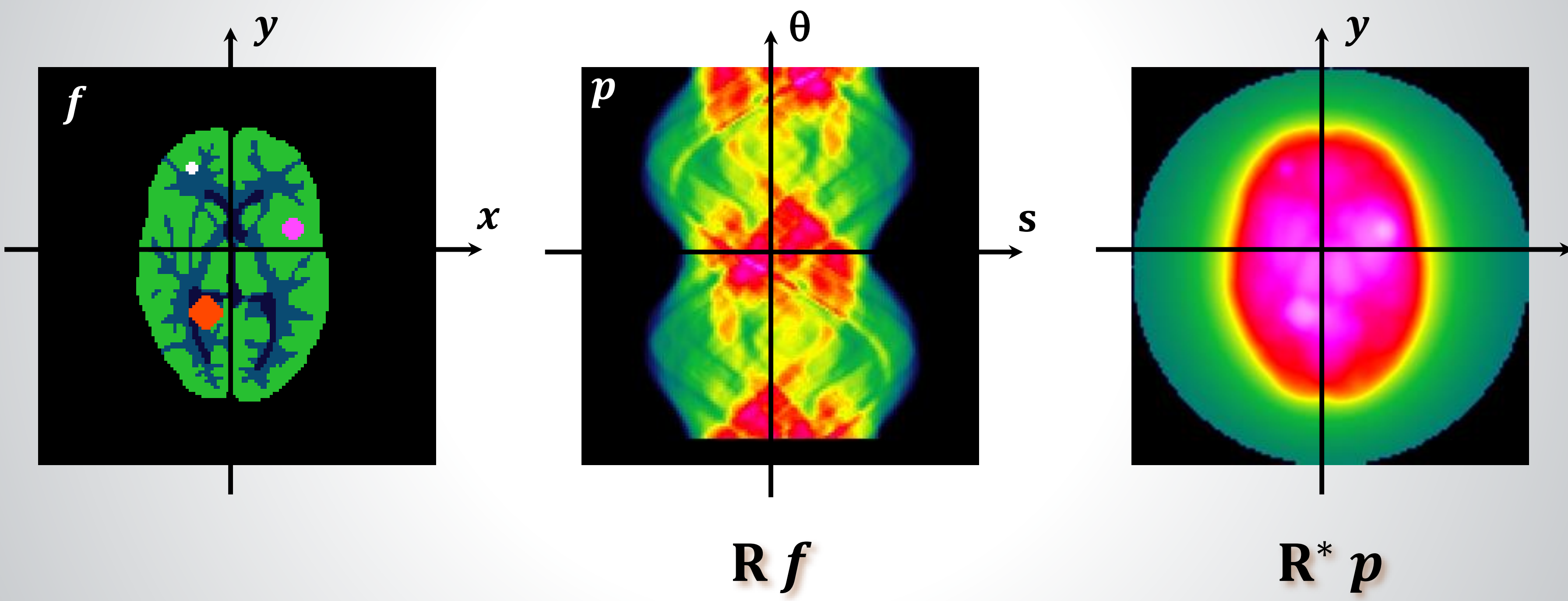
$$Rf_{\theta}(s) = \int_{\mathbb{R}} f(s\cos\theta - t\sin\theta, s\sin\theta + t\cos\theta)\, dt$$

Rétro-projection

$$R^{*}p(x, y) = \int_{0}^{\pi} p_{\theta}(x\cos\theta + y\sin\theta)\, d\theta$$

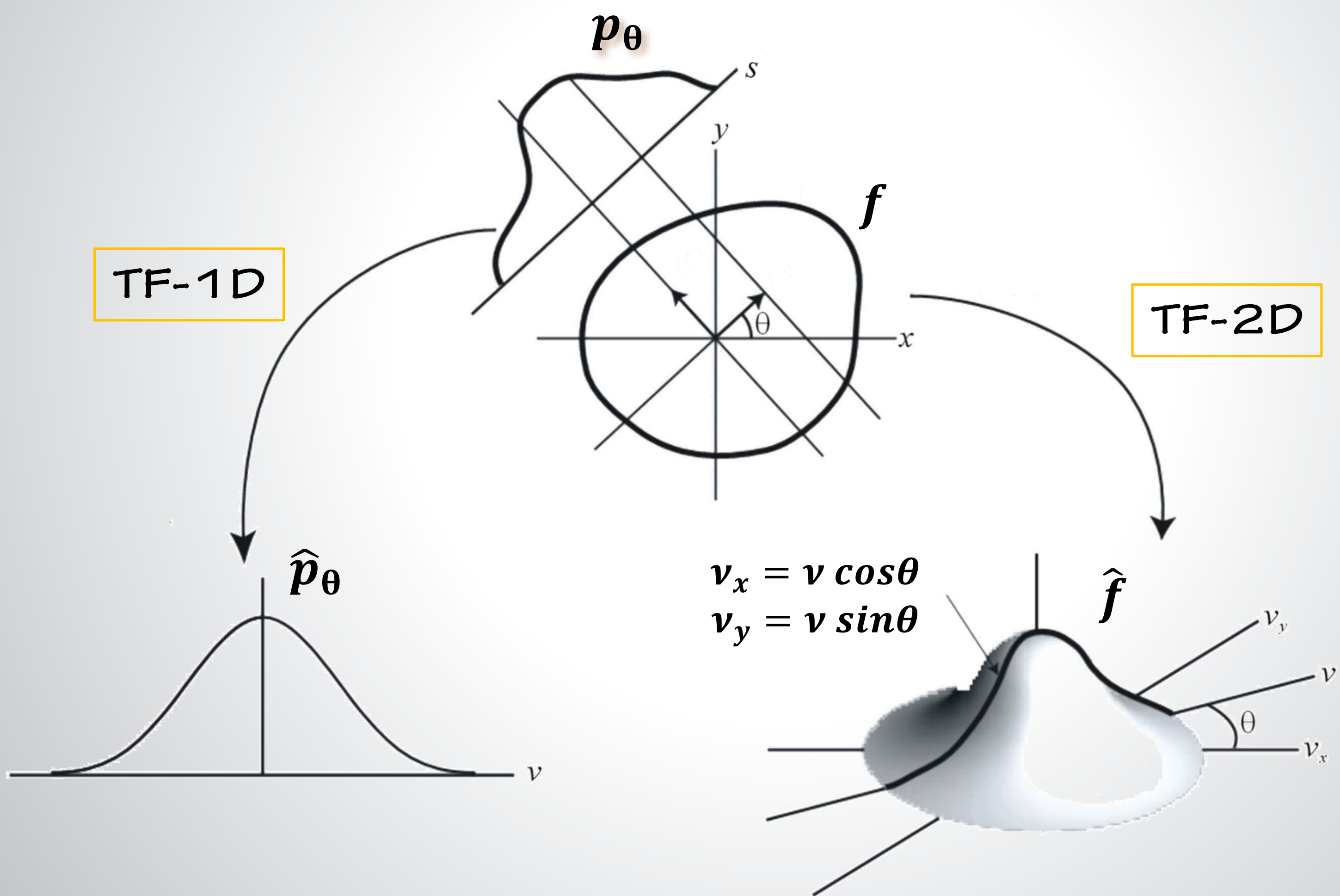
Reconstruction

- Modèle analytique



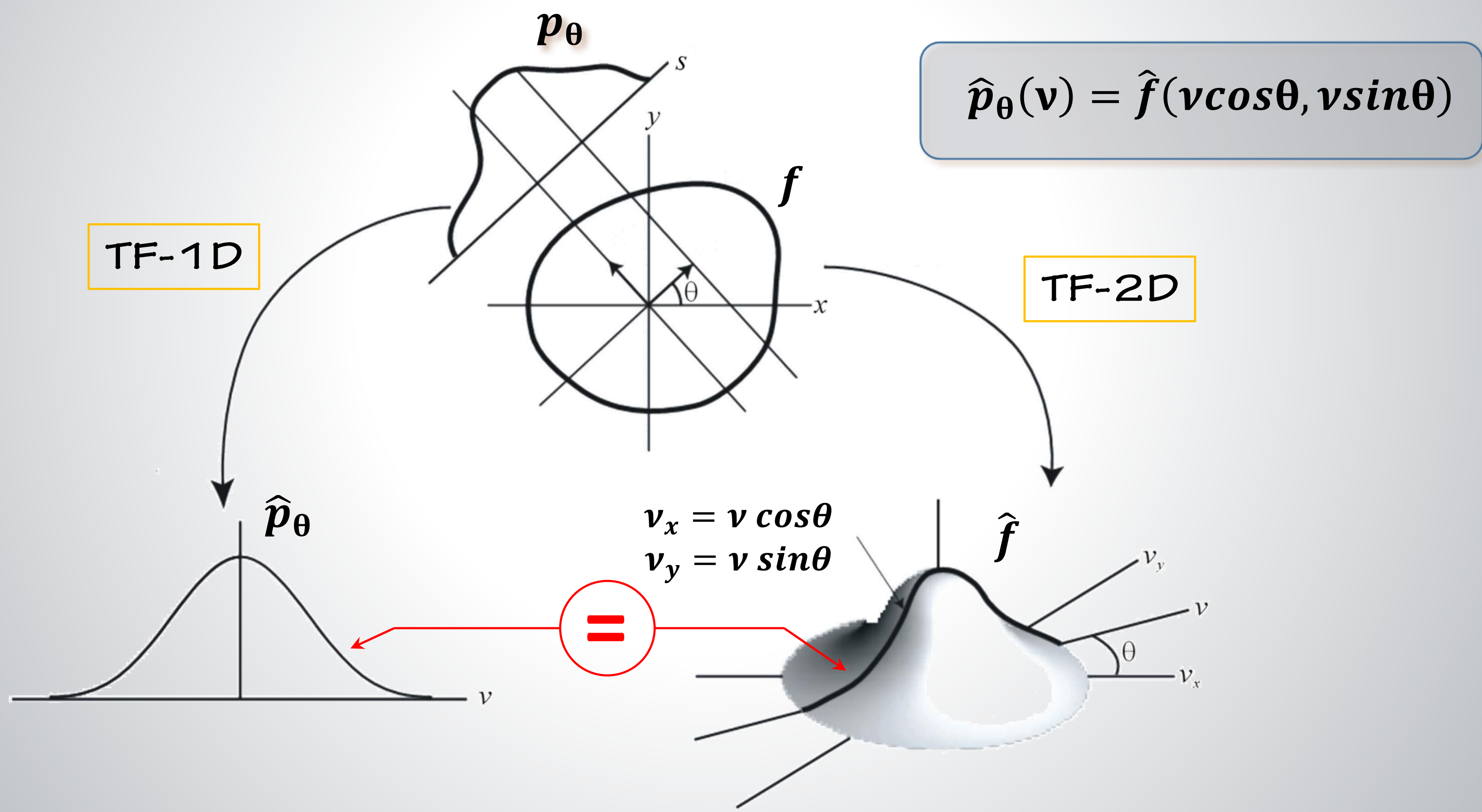
Reconstruction

- Modèle analytique — Synthèse de Fourier



Reconstruction

- Modèle analytique — Synthèse de Fourier



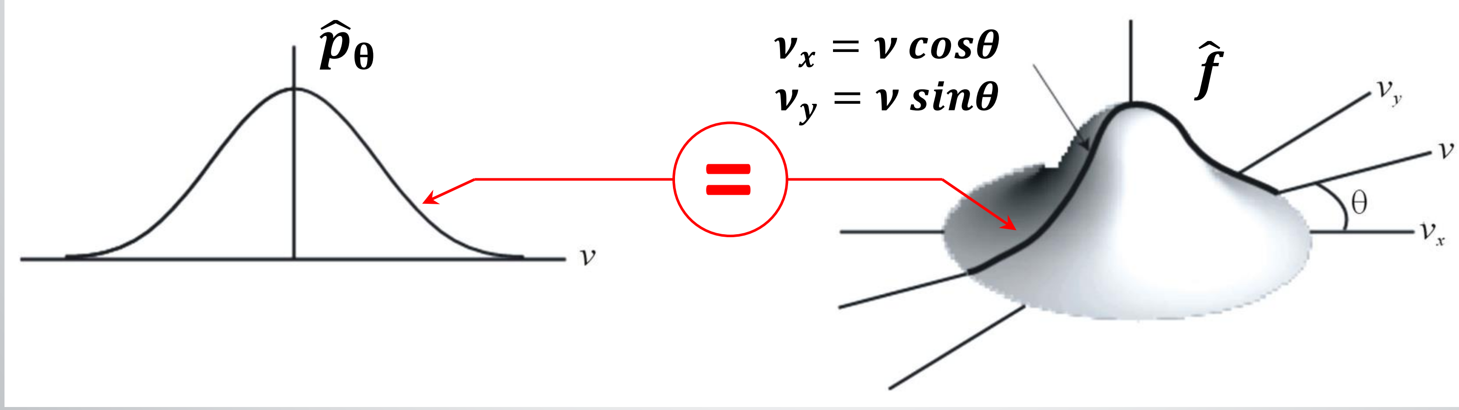
Reconstruction

- Modèle analytique — Synthèse de Fourier

$$\hat{p}_\theta(\mathrm{v}) = \int_{\mathbb{R}} p_\theta(s)\, e^{-ivs}\, ds = \iint_{\mathbb{R}^2} f(\mathrm{s}\cos\theta - \mathrm{t}\sin\theta\,, \mathrm{s}\sin\theta + \mathrm{t}\cos\theta)\, e^{-ivs}\, ds\, dt$$

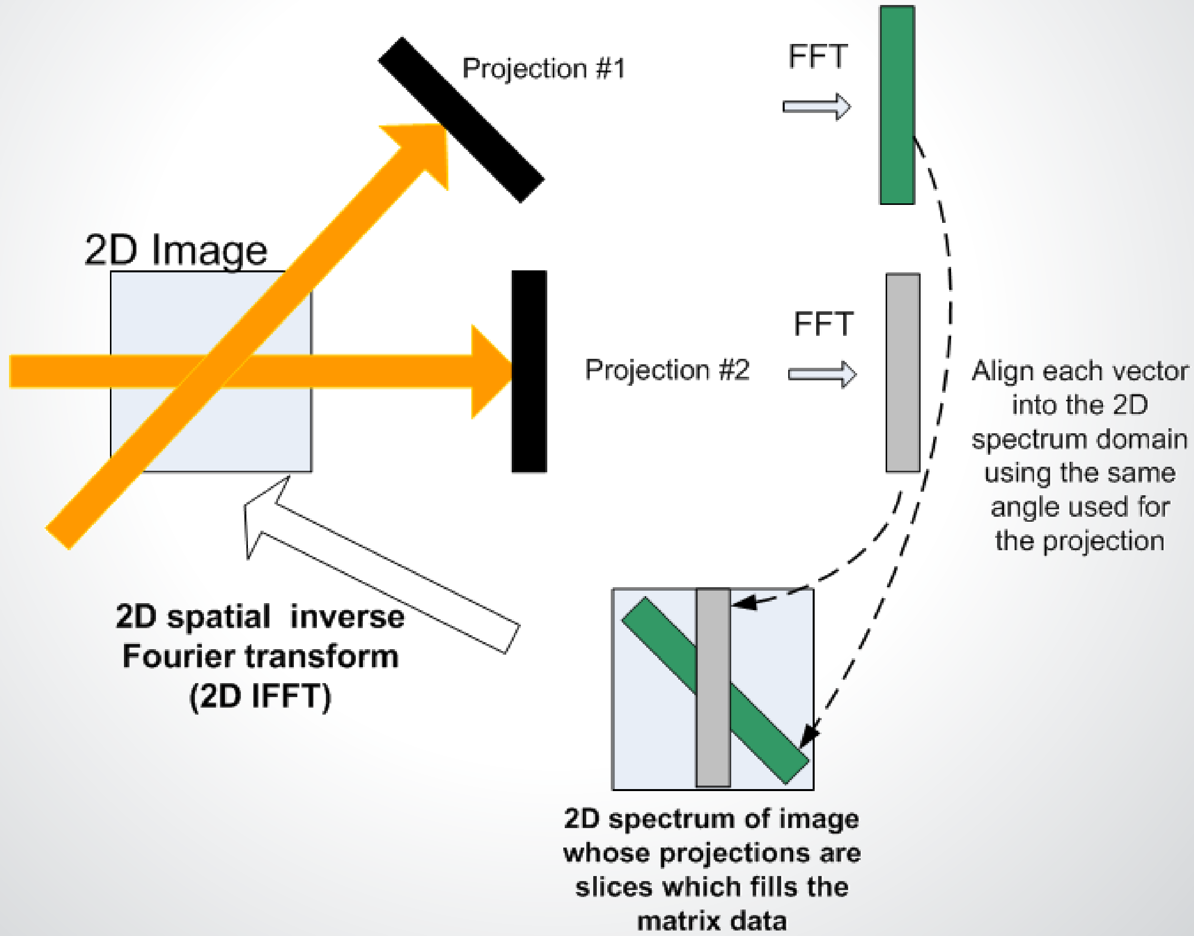
$$\begin{cases} x = \mathrm{s}\cos\theta - \mathrm{t}\sin\theta \\ y = \mathrm{s}\sin\theta + \mathrm{t}\cos\theta \end{cases} \qquad \begin{cases} \mathrm{s} = x\cos\theta + y\sin\theta \\ \mathrm{t} = x\sin\theta - y\cos\theta \end{cases}$$

$$= \iint_{\mathbb{R}^2} f(x,y)\, e^{-iv(x\cos\theta + y\sin\theta)}\, dx\, dy = \hat{f}(v\cos\theta, v\sin\theta)$$



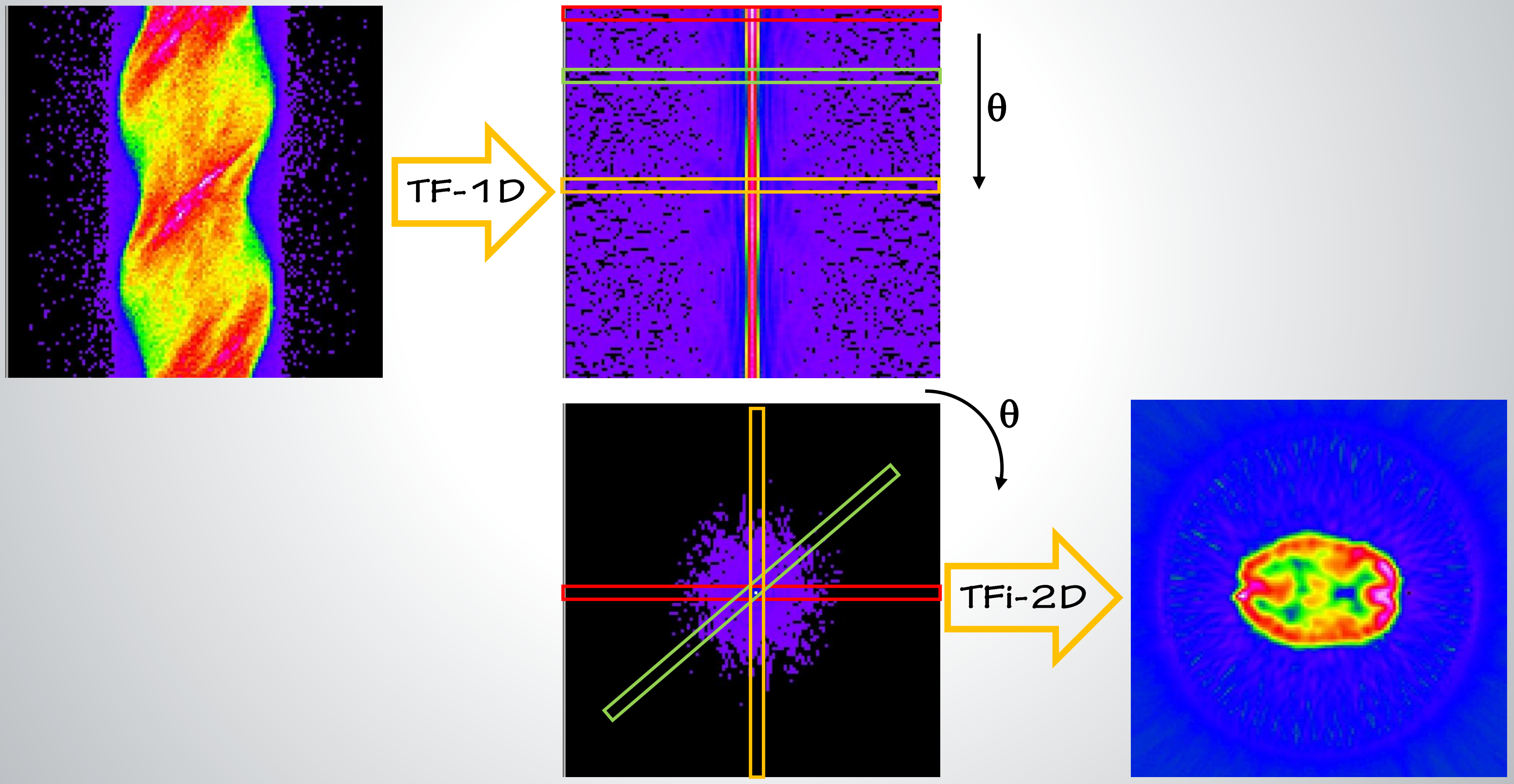
Reconstruction

- Modèle analytique — Synthèse de Fourier



Reconstruction

- Modèle analytique — Synthèse de Fourier



Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu \cos\theta$$
$$\nu_y = \nu \sin\theta$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$\boldsymbol{f} = \iint_{\mathbb{R}^2} \hat{\boldsymbol{f}}\, \boldsymbol{e}^{i(x\nu_x + y\nu_y)}\, \boldsymbol{d\nu_x d\nu_y}$$

$$\nu_x = \nu\, cos\theta$$
$$\nu_y = \nu\, sin\theta$$

$$\boldsymbol{f} = \int \boldsymbol{d\theta} \int \hat{\boldsymbol{f}}\, \boldsymbol{e}^{i\nu(xcos\theta + ysin\theta)}\, |\boldsymbol{\nu}|\, \boldsymbol{d\nu}$$

Reconstruction

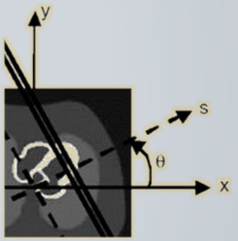
- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(xv_x + yv_y)}\, dv_x dv_y$$

$$v_x = v\, cos\theta$$
$$v_y = v\, sin\theta$$

$$f = \int d\theta \int \hat{f}\, e^{iv(xcos\theta + ysin\theta)}\, |v|\, dv$$

$= s$



Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu\, cos\theta$$
$$\nu_y = \nu\, sin\theta$$

$$f = \int d\theta \int \hat{f}\, e^{i\nu(xcos\theta + ysin\theta)}\, |\nu|\, d\nu$$

$$\hat{f}(\nu cos\theta, \nu sin\theta) = \hat{p}_\theta(\nu)$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu\, cos\theta$$
$$\nu_y = \nu\, sin\theta$$

$$f = \int d\theta \int \hat{f}\, e^{i\nu(xcos\theta + ysin\theta)}\, |\nu|\, d\nu$$

$$\hat{f}(\nu cos\theta, \nu sin\theta) = \hat{p}_\theta(\nu)$$

$$f = \int d\theta \int d\nu\, e^{is\nu}\, |\nu|\, \hat{p}$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu \cos\theta$$
$$\nu_y = \nu \sin\theta$$

$$f = \int d\theta \int \hat{f}\, e^{i\nu(x\cos\theta + y\sin\theta)}\, |\nu|\, d\nu$$

$$\hat{f}(\nu\cos\theta, \nu\sin\theta) = \hat{p}_\theta(\nu)$$

$$f = \int d\theta \int d\nu\, e^{is\nu}\, |\nu|\, \hat{p}$$

$$\widehat{\mathrm{H}p} = |\nu|\, \hat{p}$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

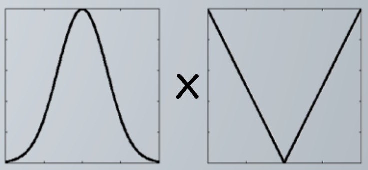
$$\nu_x = \nu\, cos\theta$$
$$\nu_y = \nu\, sin\theta$$

$$f = \int d\theta \int \hat{f}\, e^{i\nu(x cos\theta + y sin\theta)}\, |\nu|\, d\nu$$

$$\hat{f}(\nu cos\theta, \nu sin\theta) = \hat{p}_\theta(\nu)$$

$$f = \int d\theta \int d\nu\, e^{is\nu} |\nu|\, \hat{p}$$

$$\widehat{\mathrm{H}p} = |\nu|\, \hat{p}$$



Filtre rampe

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(xv_x + yv_y)}\, dv_x dv_y$$

$$v_x = v\, cos\theta$$
$$v_y = v\, sin\theta$$

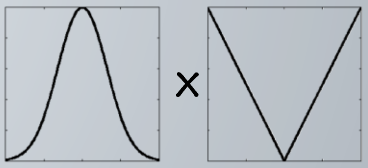
$$f = \int d\theta \int \hat{f}\, e^{iv(xcos\theta + ysin\theta)}\, |v|\, dv$$

$$\hat{f}(vcos\theta, vsin\theta) = \hat{p}_\theta(v)$$

$$f = \int d\theta \int dv\, e^{isv}\, |v|\, \hat{p}$$

$$\widehat{Hp} = |v|\, \hat{p}$$

TF inv - 1D



Filtre rampe

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu \cos\theta$$
$$\nu_y = \nu \sin\theta$$

$$f = \int d\theta \int \hat{f}\, e^{i\nu(x\cos\theta + y\sin\theta)}\, |\nu|\, d\nu$$

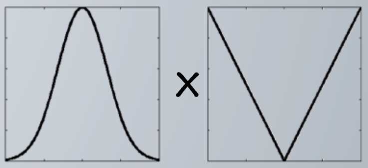
$$\hat{f}(\nu\cos\theta, \nu\sin\theta) = \hat{p}_\theta(\nu)$$

$$f = \int d\theta \int d\nu\, e^{is\nu}\, |\nu|\, \hat{p}$$

$$\widehat{\mathrm{H}p} = |\nu|\, \hat{p}$$

TF inv - 1D

Rétro-projection



Filtre rampe

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$f = \iint_{\mathbb{R}^2} \hat{f}\, e^{i(x\nu_x + y\nu_y)}\, d\nu_x d\nu_y$$

$$\nu_x = \nu \cos\theta$$
$$\nu_y = \nu \sin\theta$$

$$f = \int d\theta \int \hat{f}\, e^{i\nu(x\cos\theta + y\sin\theta)}\, |\nu|\, d\nu$$

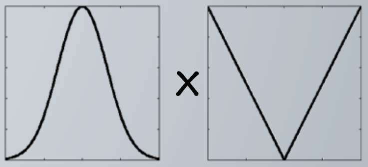
$\hat{f}(\nu\cos\theta, \nu\sin\theta) = \hat{p}_\theta(\nu)$

$$f = \int d\theta \int d\nu\, e^{is\nu}\, |\nu|\, \hat{p}$$

$\widehat{\mathrm{H}p} = |\nu|\, \hat{p}$

TF inv - 1D

Rétro-projection



Filtre rampe

$$f = \mathrm{R}^* \mathrm{H}p$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$\mathbf{H}\boldsymbol{p} = \mathbf{TF}^{-1}\left(\widehat{\boldsymbol{h}}\,\widehat{\boldsymbol{p}}\right)$$

$$\widehat{\boldsymbol{h}}(\boldsymbol{\nu}) = |\boldsymbol{\nu}|$$

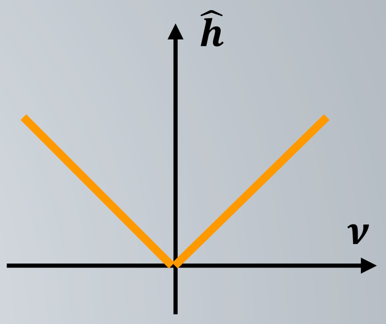
$$\mathbf{H}\boldsymbol{p} = \boldsymbol{h} * \boldsymbol{p}$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$\mathbf{H}p = \mathbf{TF}^{-1}\left(\hat{h}\,\hat{p}\right)$$

$$\hat{h}(\nu) = |\nu|$$



$$\mathbf{H}p = h * p$$

$$h(s) = \int_{\mathbb{R}} \hat{h}(\nu)\; e^{i\nu s}\, d\nu$$

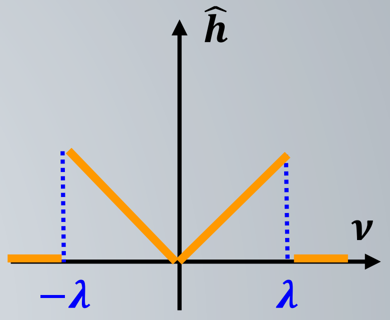
Reconstruction

- Modèle analytique — Rétro-projection filtrée

$$\mathbf{H}p = \mathbf{TF}^{-1}\left(\hat{h}\,\hat{p}\right)$$

$$\mathbf{H}p = h * p$$

$$h(s) = \int_{-\lambda}^{\lambda} \hat{h}(\nu)\; e^{i\nu s}\, d\nu$$



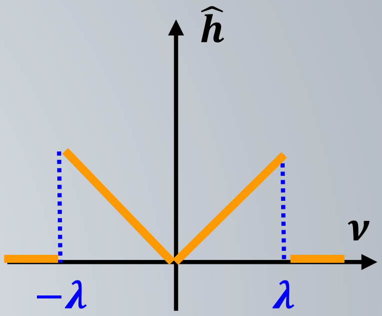
Reconstruction

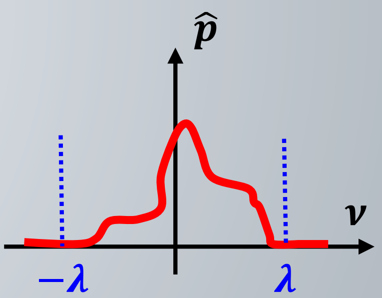
- Modèle analytique — Rétro-projection filtrée

$$\mathbf{H}p = \mathrm{TF}^{-1}(\hat{h}\,\hat{p})$$

$$\mathbf{H}p = h * p$$

$$h(s) = \int_{-\lambda}^{\lambda} \hat{h}(\nu)\ e^{i\nu s}\, d\nu$$





$$\lambda \geq \nu_{max} = 2\pi\, F_{max} = \pi$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

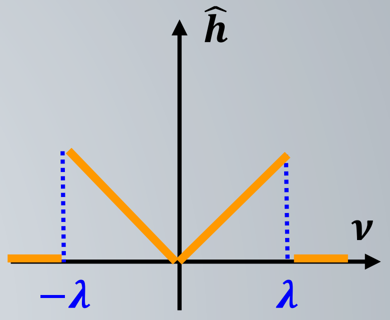
$$\mathbf{H}p = \mathrm{TF}^{-1}\left(\hat{h}\,\hat{p}\right)$$

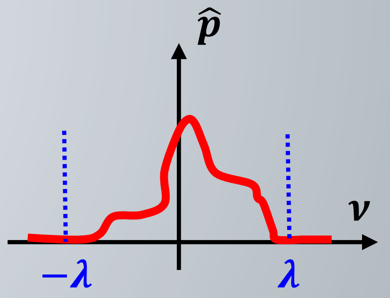
$$\mathbf{H}p = h * p$$

$$h(s) = \int_{-\lambda}^{\lambda} \hat{h}(\nu)\; e^{i\nu s}\, d\nu$$

$$= -\int_{-\pi}^{0} \nu\, e^{i\nu s}\, d\nu + \int_{0}^{\pi} \nu\, e^{i\nu s}\, d\nu$$

$$= \frac{2\pi}{s} sin(\pi s) + \frac{2}{s^2}\left(cos(\pi s) - 1\right)$$



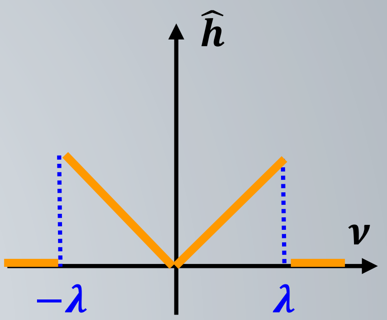


$$\lambda \geq \nu_{max}$$
$$= 2\pi\, F_{max} = \pi$$

Reconstruction

- Modèle analytique — Rétro-projection filtrée

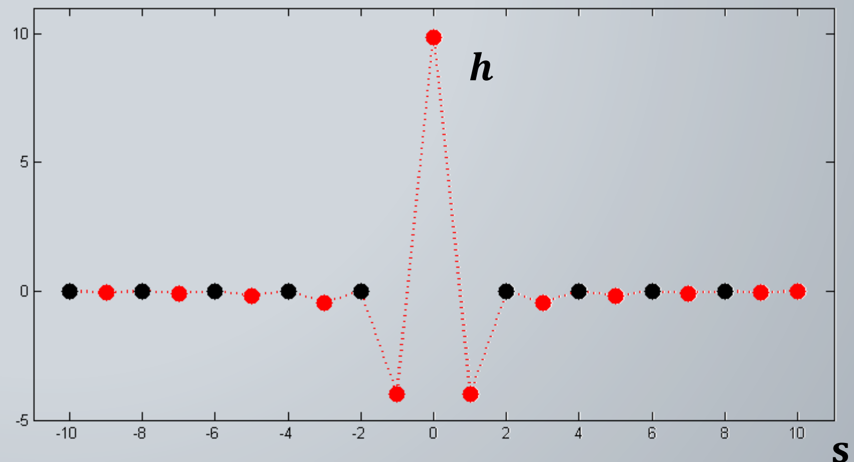
$$\mathrm{H}p = \mathrm{TF}^{-1}(\hat{h}\,\hat{p})$$



$$\mathrm{H}p = h * p$$

$$h(s) = \int_{-\lambda}^{\lambda} \hat{h}(\nu)\; e^{i\nu s}\, d\nu$$

$$= \begin{cases} \pi^2 \quad en\ \ 0 \\ 0 \quad pour\ s\ pair \\ -\dfrac{4}{s^2} \quad pour\ s\ impair \end{cases}$$



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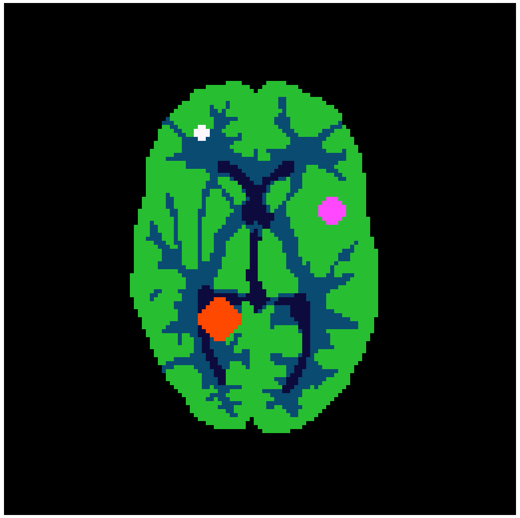
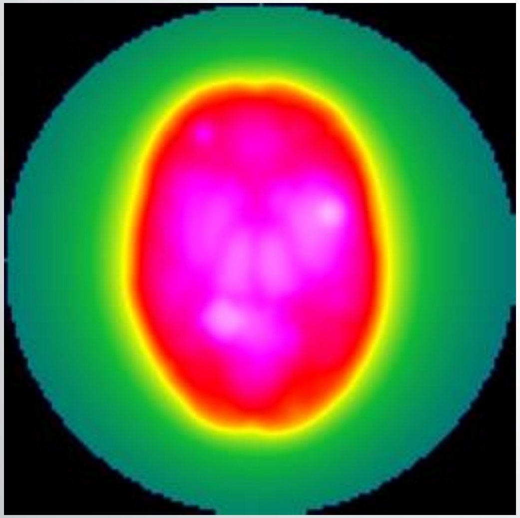
Reconstruction

- Modèle analytique — Rétro-projection filtrée

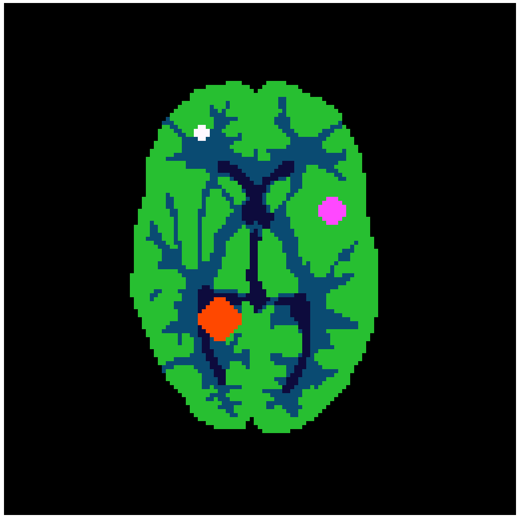


$$f = \mathbf{R}^{*} p$$

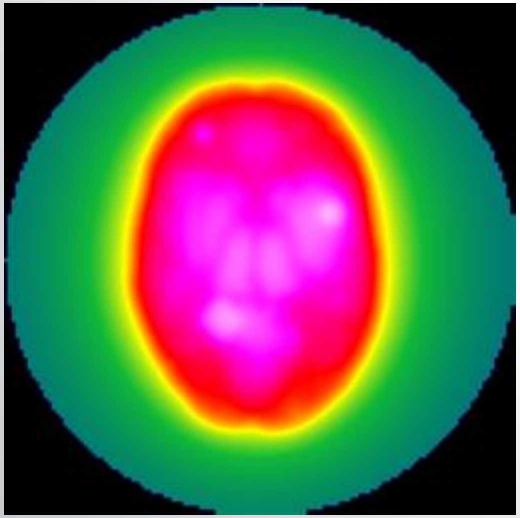


Reconstruction

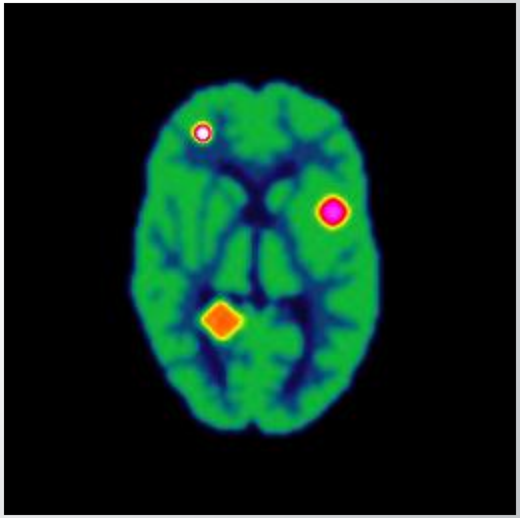
- Modèle analytique — Rétro-projection filtrée



$$f = R^* p$$



$$f = R^* Hp$$

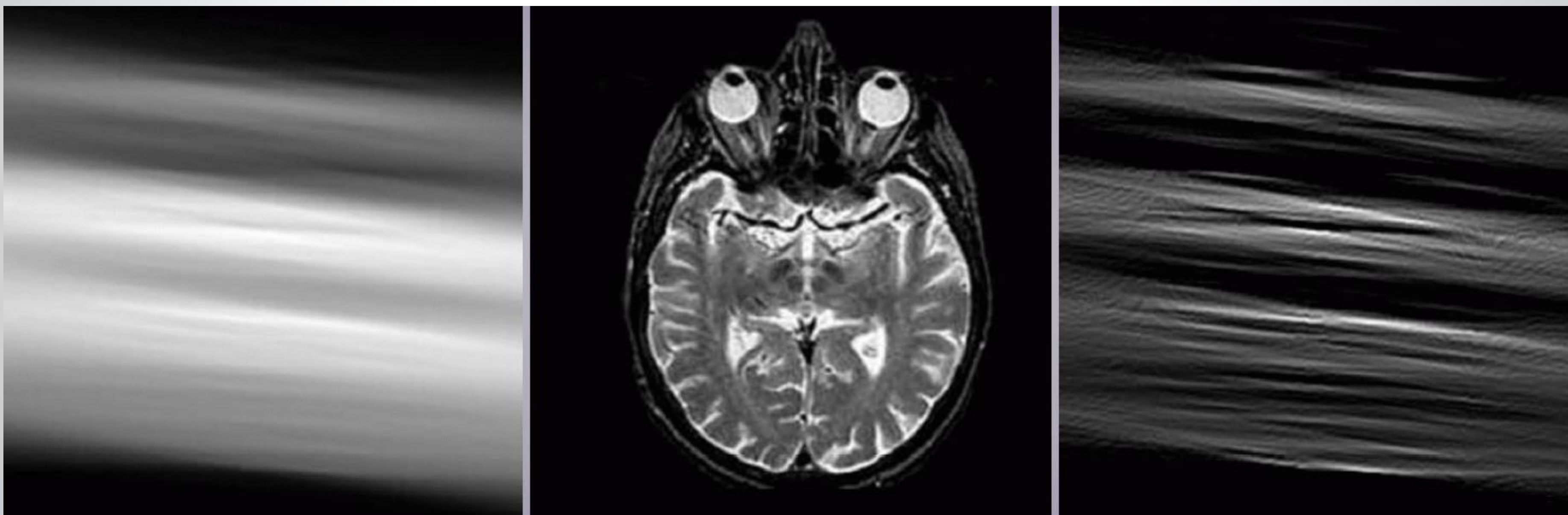


Reconstruction

- Modèle analytique — Rétro-projection filtrée

$f = \mathrm{R}^* p$

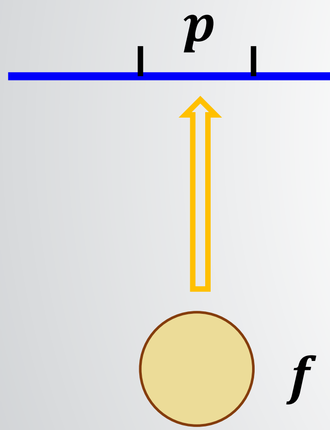
$f = \mathrm{R}^* \mathrm{H} p$



Reconstruction

- Modèle analytique — Limites

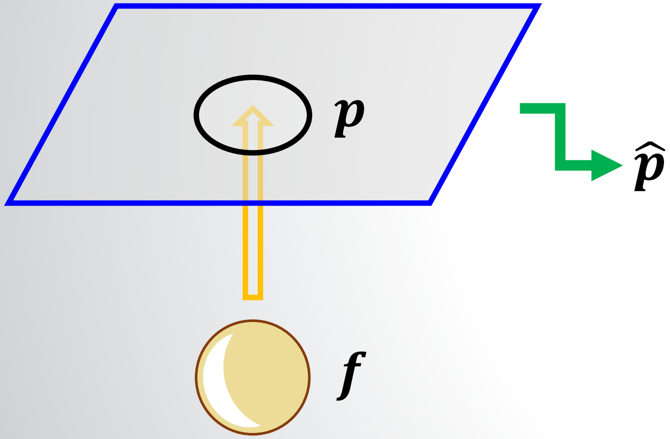
Troncature des données



Reconstruction

- Modèle analytique — Limites

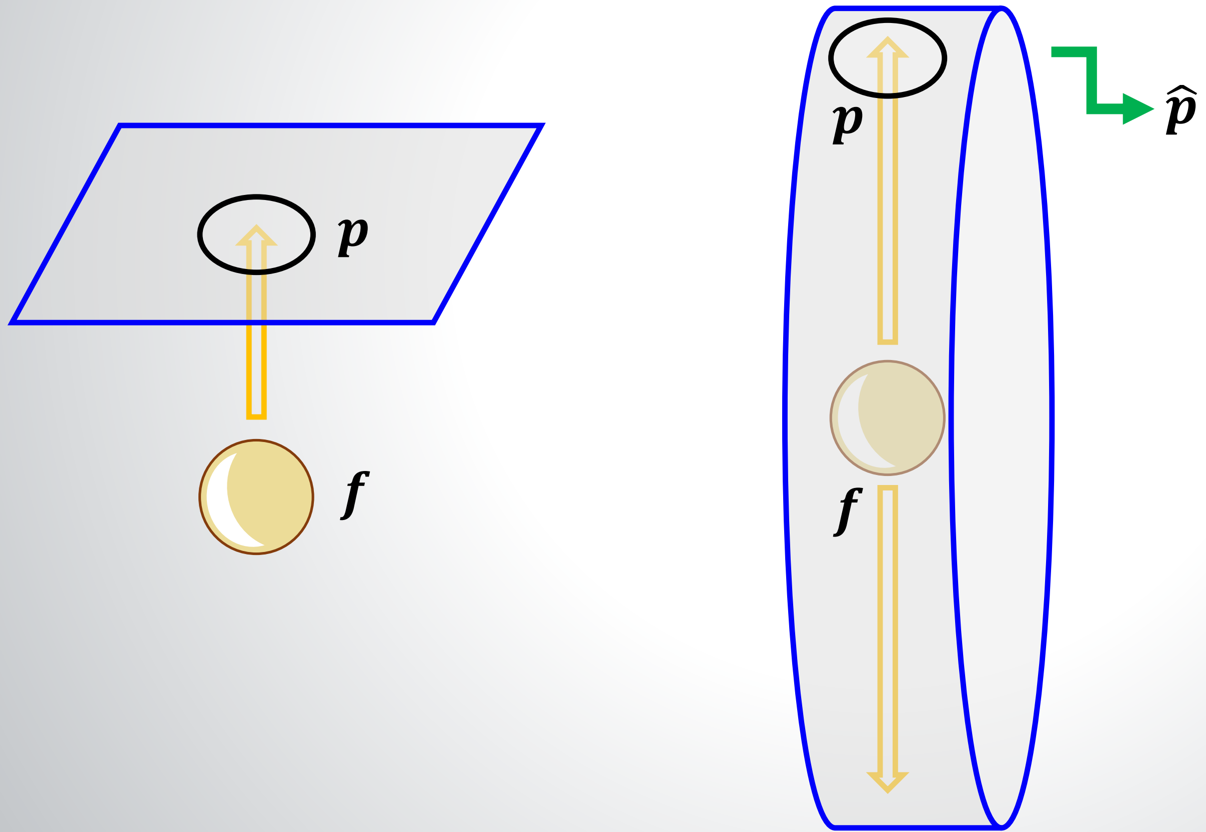
Troncature des données



Reconstruction

- Modèle analytique — Limites

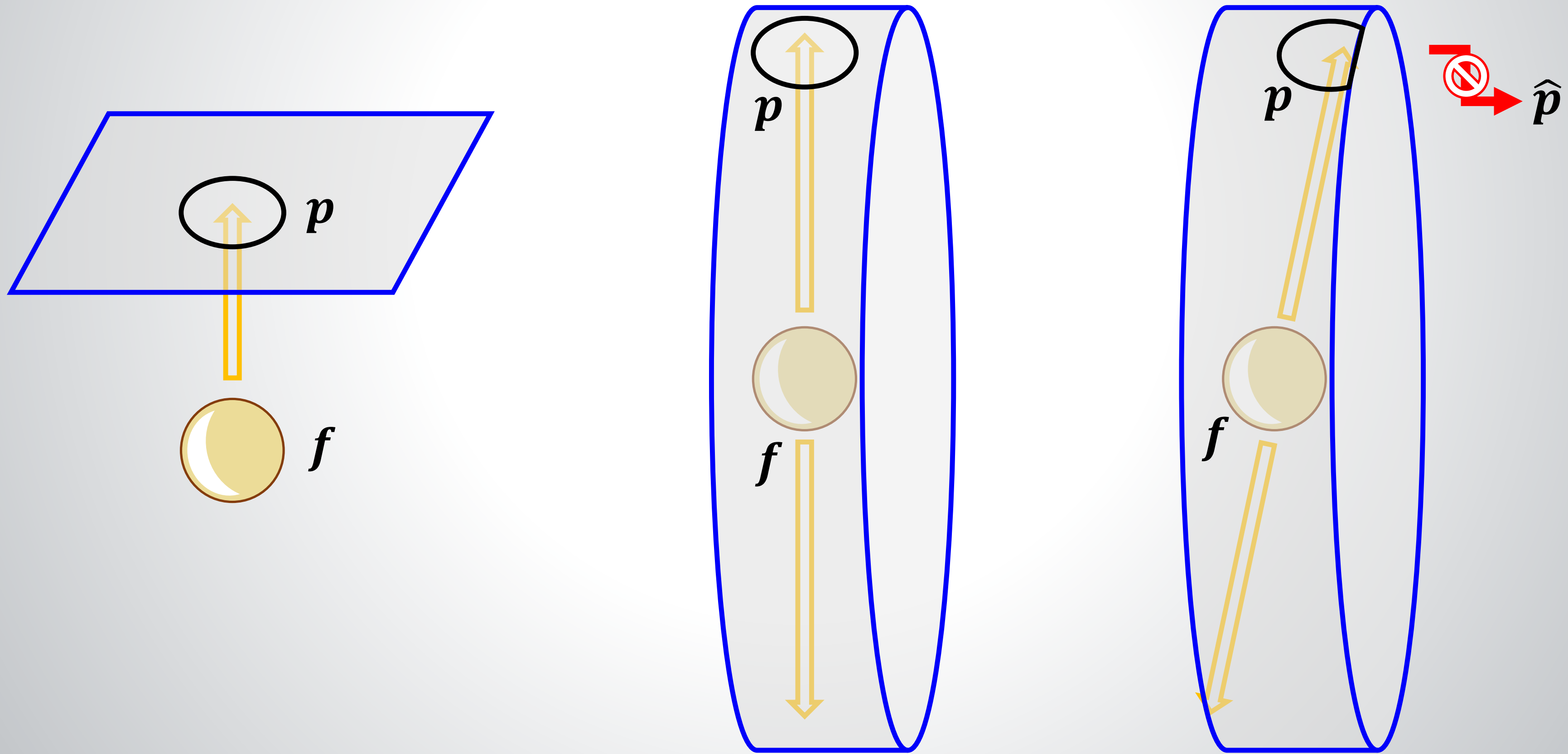
Troncature des données



Reconstruction

- Modèle analytique — Limites

Troncature des données



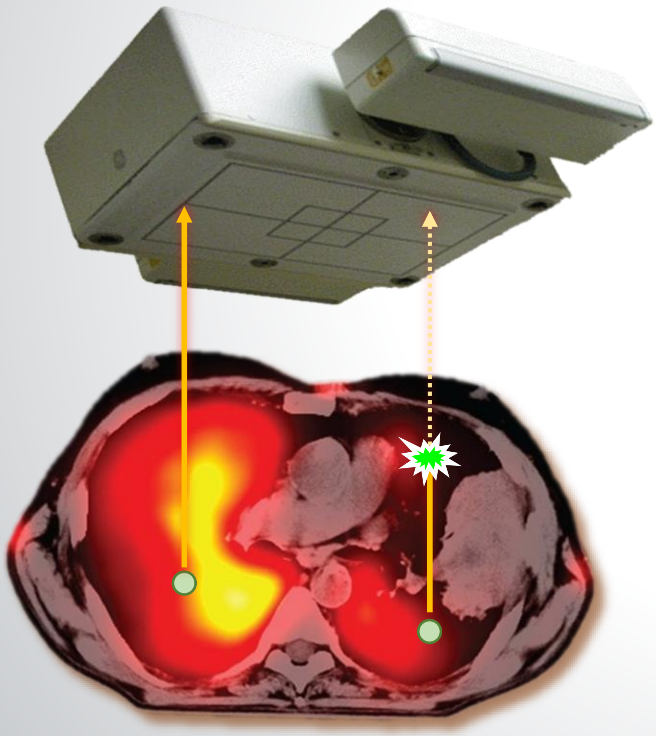
Reconstruction

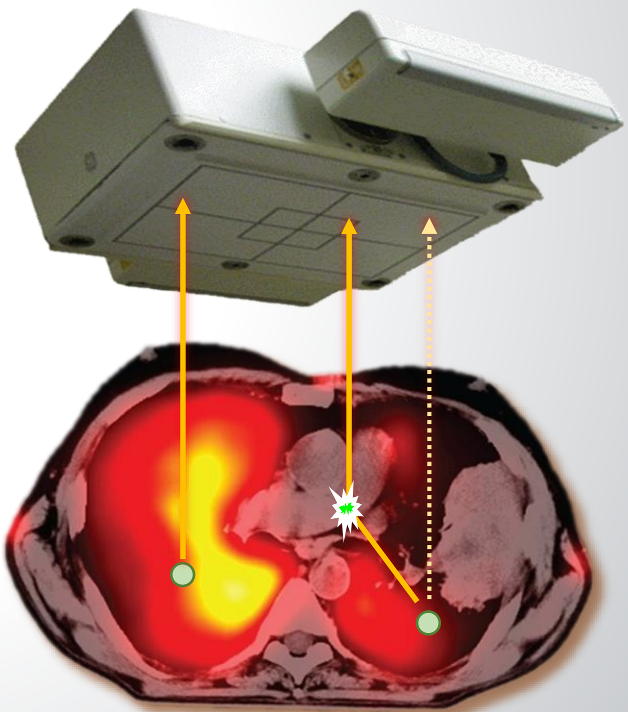
■ Modèle analytique — Limites

Interactions γ - matière

Absorption

Diffusion

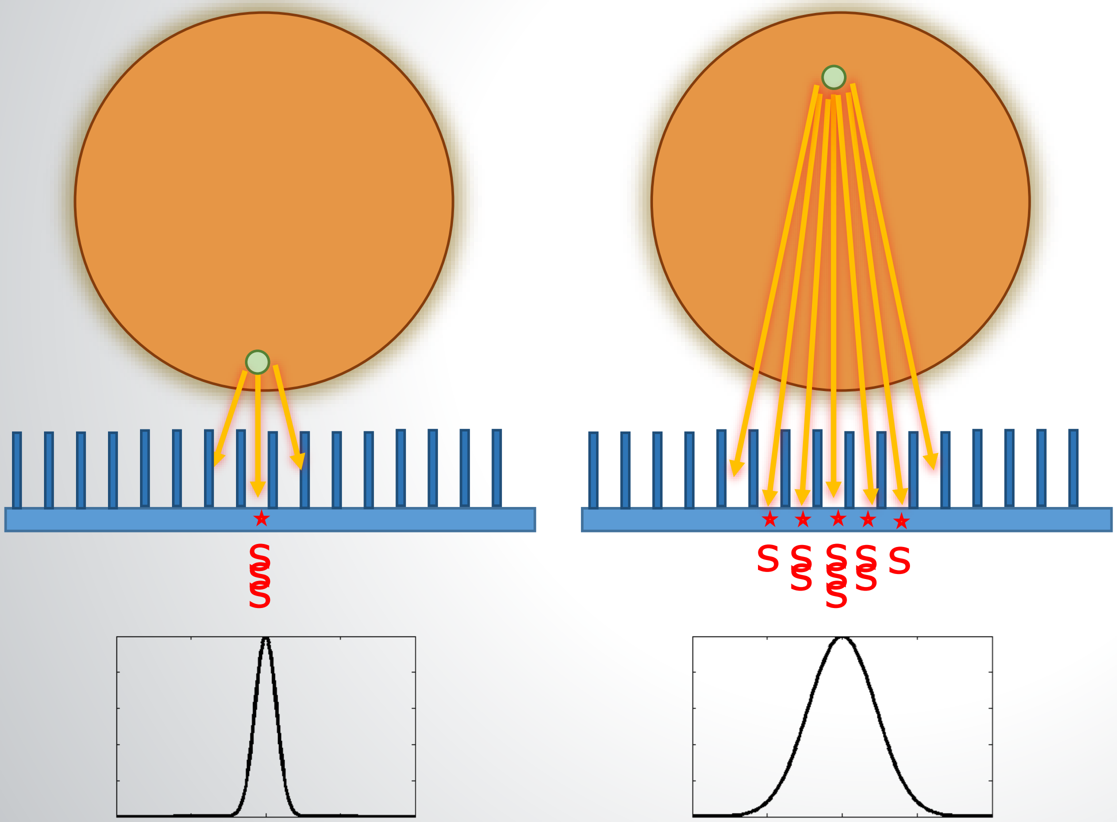


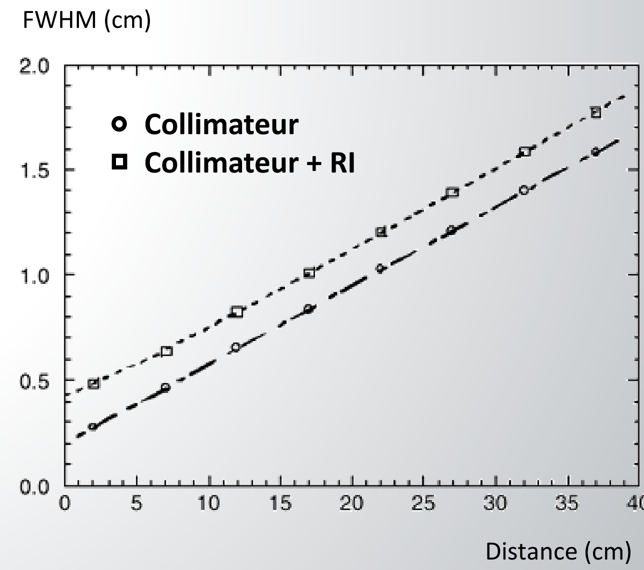


Reconstruction

- Modèle analytique — Limites

Réponse du détecteur





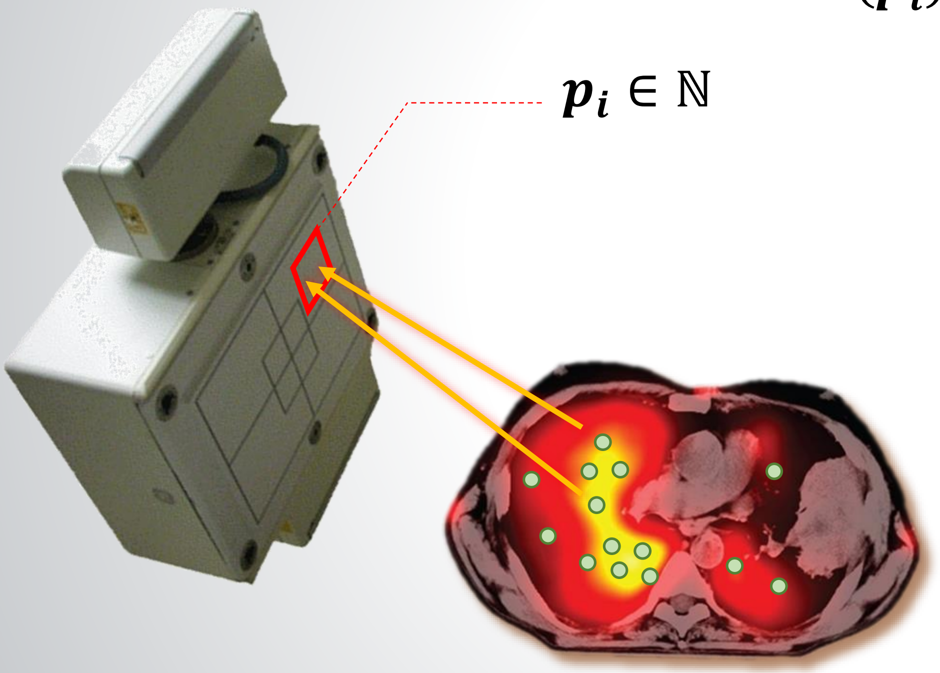
Reconstruction

- Modèle analytique — Limites

Bruit statistique

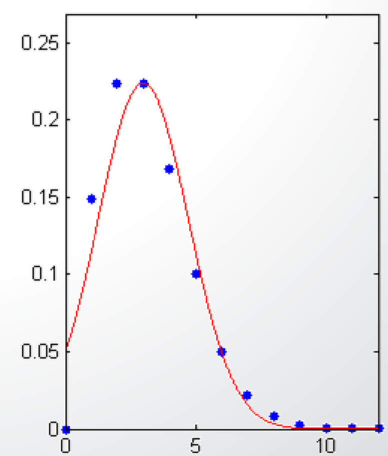
$$E(\boldsymbol{p_i}) = \boldsymbol{\kappa} \int_{\boldsymbol{\Gamma_i}} \boldsymbol{f}$$

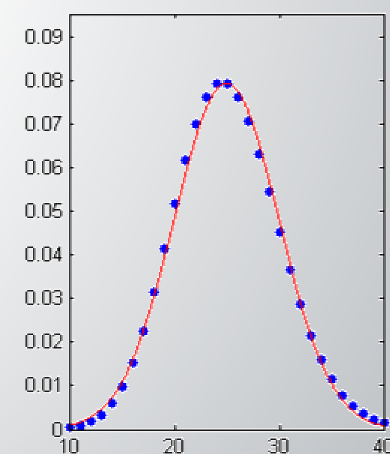
$\boldsymbol{p_i} \in \mathbb{N}$



$$Var(\boldsymbol{p_i}) = E(\boldsymbol{p_i})$$

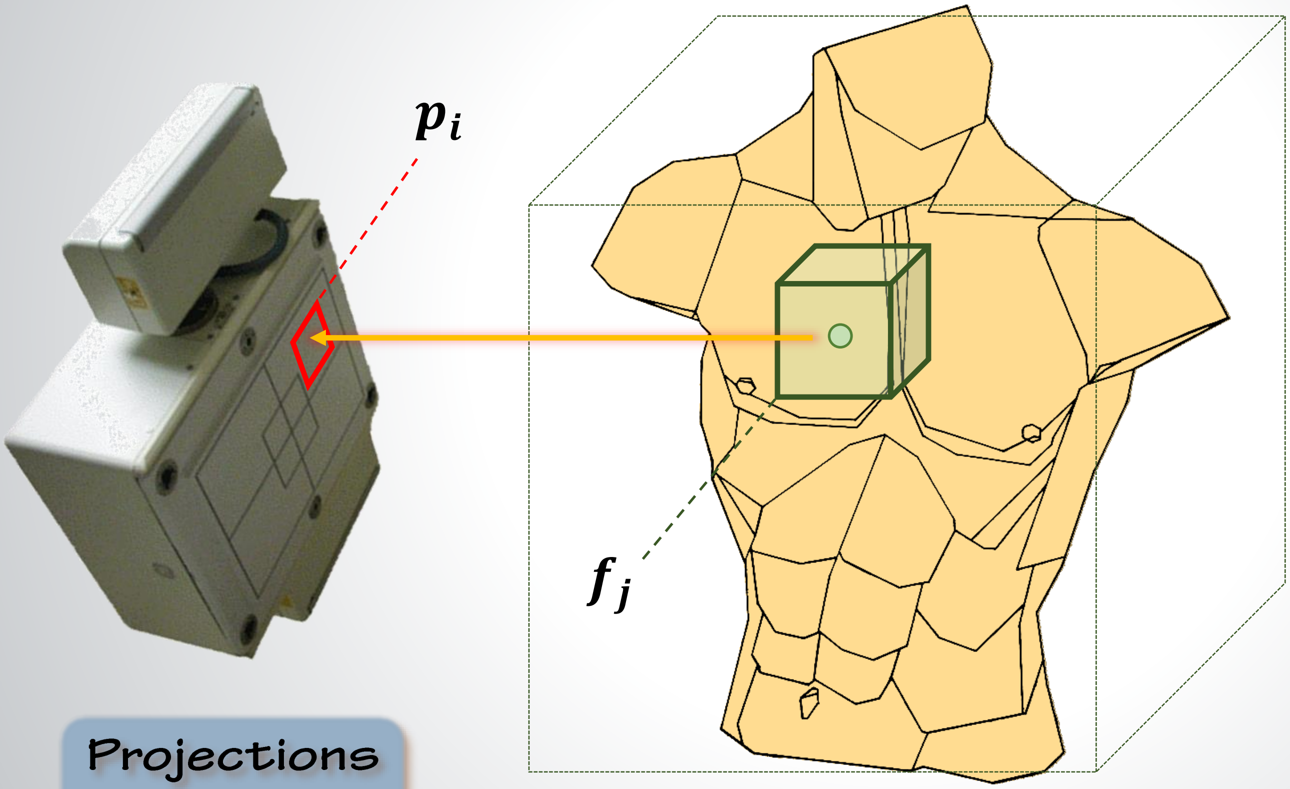
$$SNR = \frac{E(\boldsymbol{p_i})}{\sigma(\boldsymbol{p_i})} \approx \sqrt{\boldsymbol{p_i}}$$





Reconstruction

- Modèle algébrique

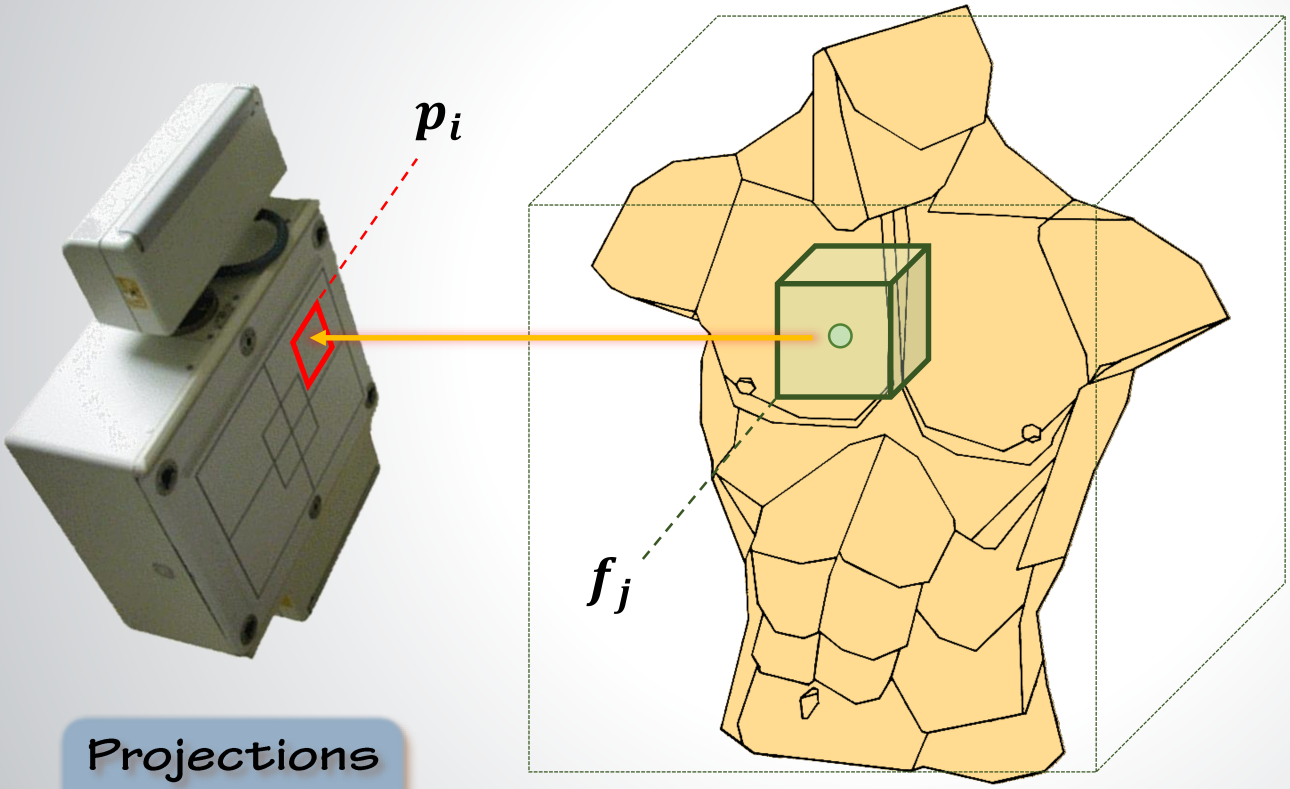


Projections
$(i = 1 \ldots N)$

Voxels / pixels $(j = 1 \ldots M)$

Reconstruction

- Modèle algébrique



Projections
$(i = 1 \dots \mathrm{N})$

Voxels / pixels $(j = 1 \dots \mathrm{M})$

Matrice système

$$\mathbf{R} \in \mathbb{R}^{\mathbf{N} \times \mathbf{M}}$$

$$\mathbf{R}_{ij} = \wp(j \rightarrow i)$$

-> Géométrie

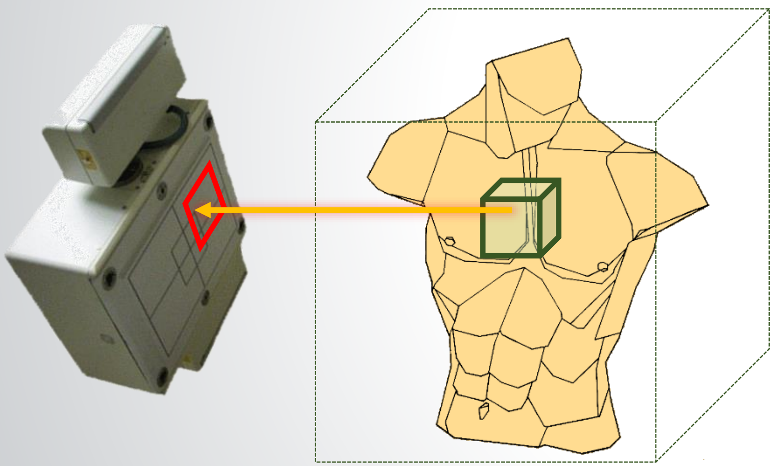
-> Atténuation

-> Réponse du détecteur

Reconstruction

- Modèle algébrique

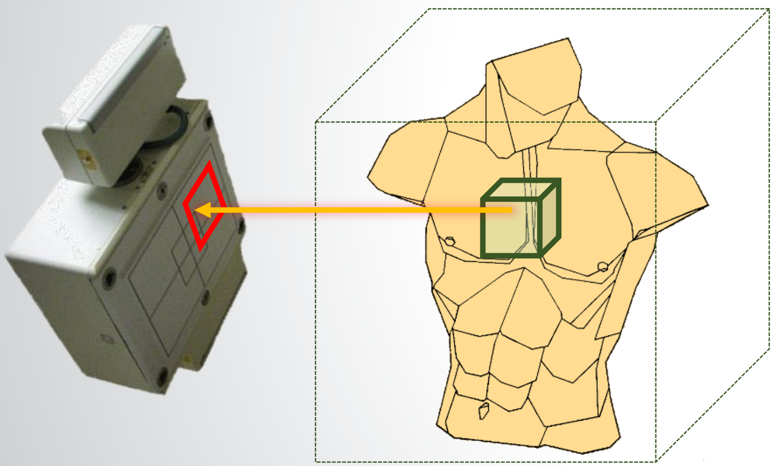
$$p = \mathbf{R}f + s + n$$

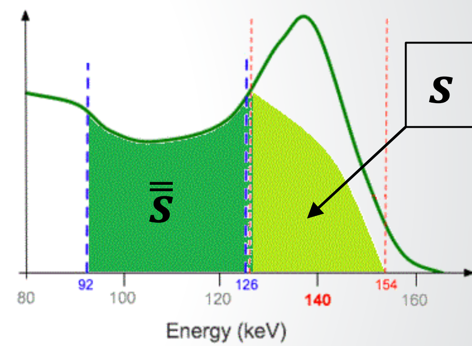


Reconstruction

- Modèle algébrique

$$p = \mathrm{R}f + s + n$$





$$s = \kappa\, \bar{\bar{s}}$$

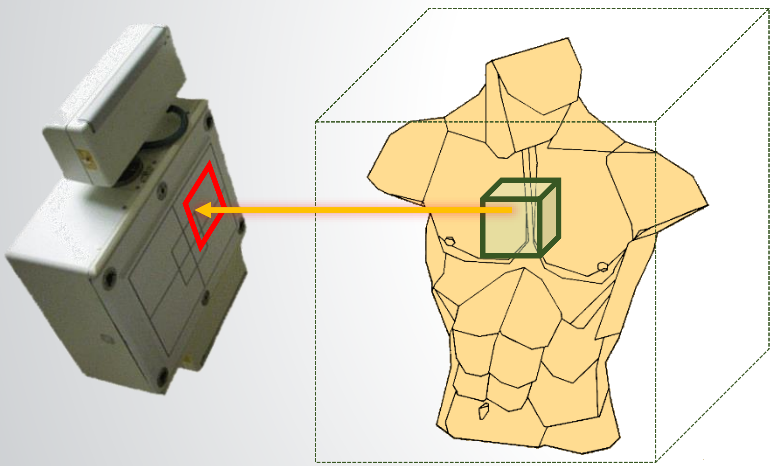
s

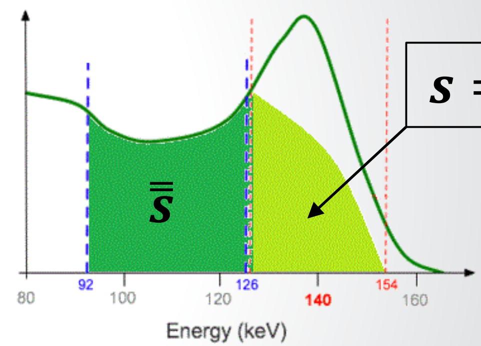
$$s \approx p * h$$

Reconstruction

- Modèle algébrique

$$p = \mathrm{R}f + s + n$$





$$s = \kappa\,\bar{\bar{s}}$$

s :

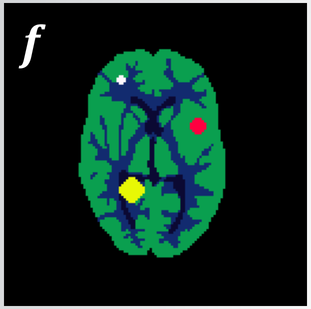
$$s = \kappa\,\bar{\bar{s}}$$

$$s \approx p * h$$

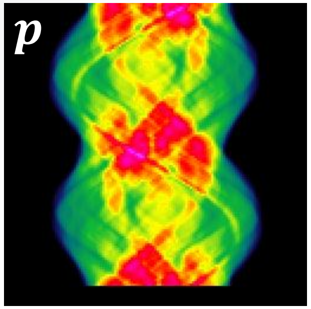
$$E(n) = 0 \;;\; V(n) \approx p$$

Reconstruction

- Modèle algébrique



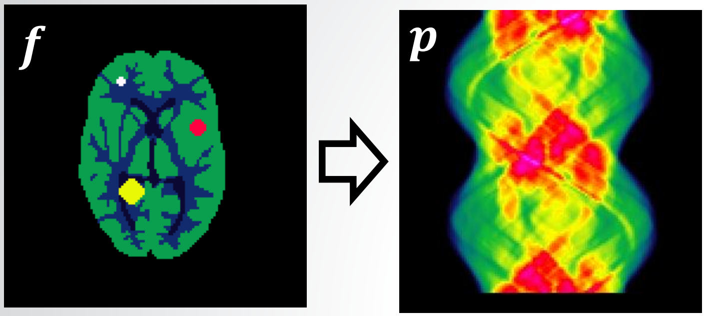
objet



$$\boldsymbol{p} = \mathbf{R}\boldsymbol{f} + \boldsymbol{n}$$

Reconstruction

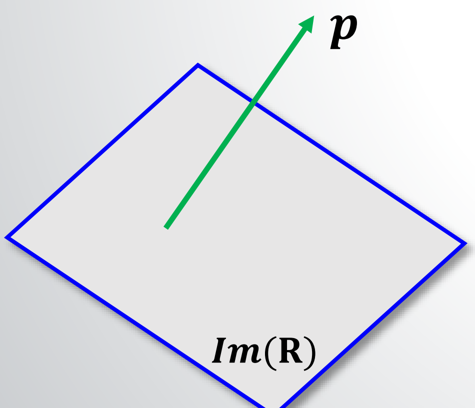
- Modèle algébrique



objet

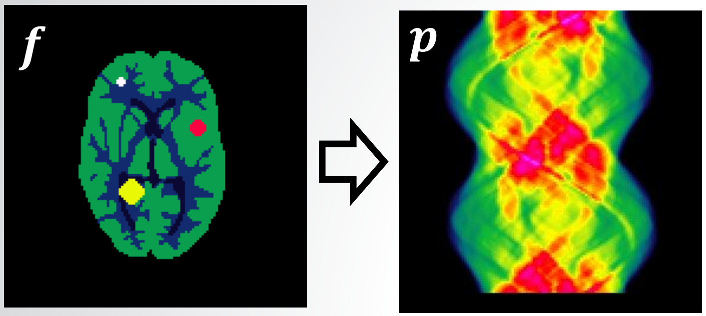
$$\boldsymbol{p} = \mathrm{R}\boldsymbol{f} + \boldsymbol{n}$$

$$\boldsymbol{p} \notin \boldsymbol{Im}(\mathrm{R}) \qquad \nexists \bar{\boldsymbol{f}} \;:\; \mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{p}$$



Reconstruction

- Modèle algébrique

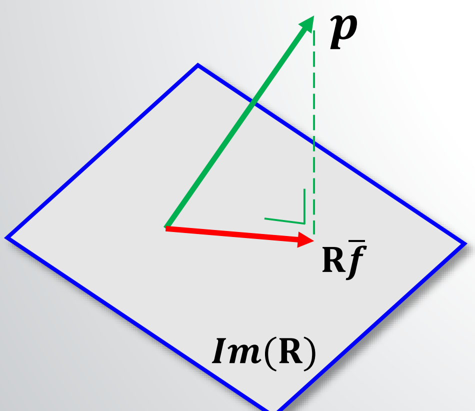


objet

$$\boldsymbol{p} = \mathbf{R}\boldsymbol{f} + \boldsymbol{n}$$

$$\boldsymbol{p} \notin \boldsymbol{Im}(\mathbf{R}) \qquad \nexists \bar{\boldsymbol{f}} \; : \; \mathbf{R}\bar{\boldsymbol{f}} = \boldsymbol{p}$$

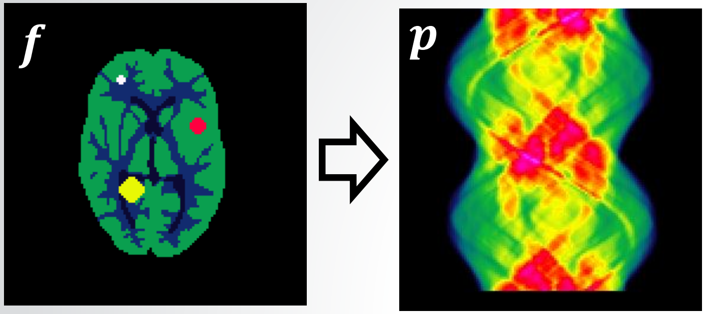
$$\bar{\boldsymbol{f}} \; : \mathbf{R}\bar{\boldsymbol{f}} = \mathbf{P}_{\boldsymbol{Im}(\mathbf{R})}(\boldsymbol{p})$$



$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in \Omega}{argmin} \, \{\|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|\}$$

Reconstruction

- Modèle algébrique



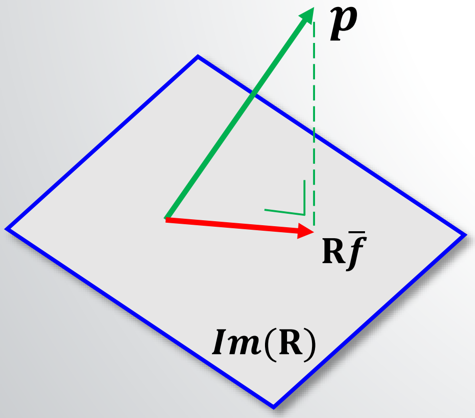
objet

$$\boldsymbol{p} = \mathrm{R}\boldsymbol{f} + \boldsymbol{n}$$

$$\boldsymbol{p} \notin \boldsymbol{Im}(\mathrm{R}) \qquad \nexists \bar{\boldsymbol{f}} \ : \ \mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{p}$$

$$\bar{\boldsymbol{f}} \ : \mathrm{R}\bar{\boldsymbol{f}} = \mathrm{P}_{\boldsymbol{Im}(\mathrm{R})}(\boldsymbol{p})$$

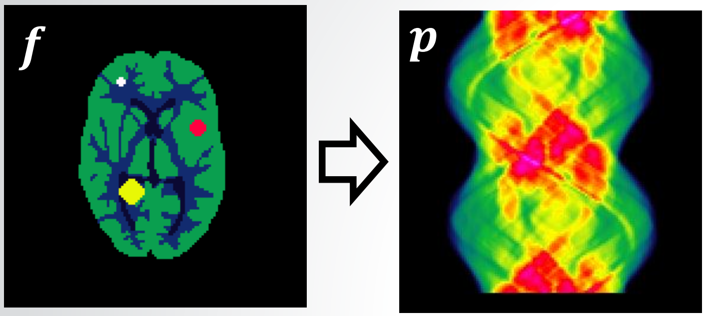
$$\forall \boldsymbol{w} \in \boldsymbol{Im}(\boldsymbol{R}) : \ \boldsymbol{w}^{\mathrm{T}}\mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{p}$$



$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\Omega}{argmin} \ \{\|\mathrm{R}\boldsymbol{f} - \boldsymbol{p}\|\}$$

Reconstruction

- Modèle algébrique



objet

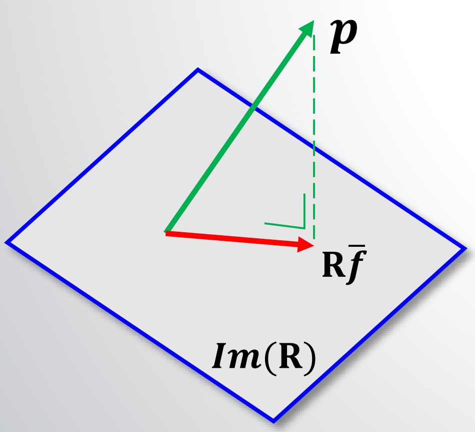
$$p = \mathrm{R}f + n$$

$$p \notin Im(\mathrm{R}) \qquad \nexists \bar{f} \;:\; \mathrm{R}\bar{f} = p$$

$$\bar{f} \;:\mathrm{R}\bar{f} = \mathrm{P}_{Im(\mathrm{R})}(p)$$

$$\forall w \in Im(R) : \; w^{\mathrm{T}}\mathrm{R}\bar{f} = w^{\mathrm{T}}p$$

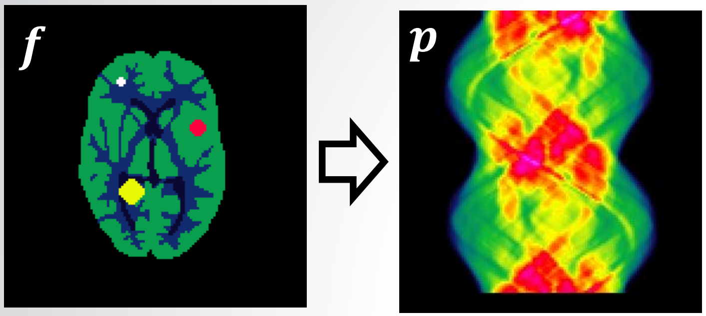
$$\forall g \in \Omega : \; (\mathrm{R}g)^{\mathrm{T}}\mathrm{R}\bar{f} = (\mathrm{R}g)^{\mathrm{T}}p$$



$$\bar{f} = \underset{f \in \Omega}{argmin} \{\|\mathrm{R}f - p\|\}$$

Reconstruction

- Modèle algébrique



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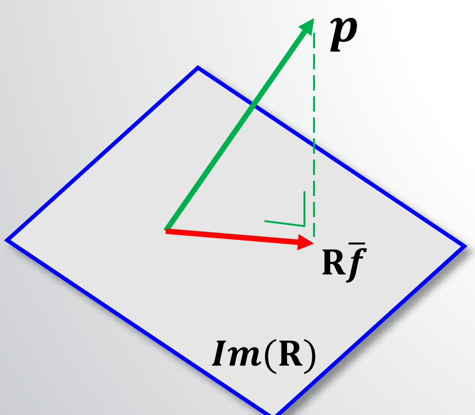

$$\bar{\boldsymbol{f}} = \underset{f \in \Omega}{argmin} \ \{\|\mathrm{R}\boldsymbol{f} - \boldsymbol{p}\|\}$$

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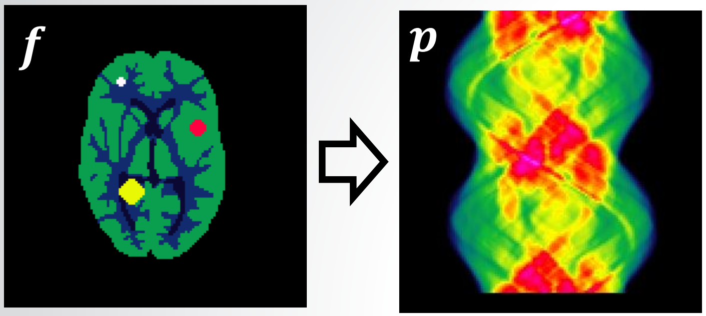
$$\forall \boldsymbol{g} \in \boldsymbol{\Omega} : \ (\mathrm{R}\boldsymbol{g})^{\mathrm{T}}\mathrm{R}\bar{\boldsymbol{f}} = (\mathrm{R}\boldsymbol{g})^{\mathrm{T}}\boldsymbol{p}$$

$$\forall \boldsymbol{g} \in \boldsymbol{\Omega} : \ \boldsymbol{g}^{\mathrm{T}}\mathrm{R}^{*}\mathrm{R}\bar{\boldsymbol{f}} = \boldsymbol{g}^{\mathrm{T}}\mathrm{R}^{*}\boldsymbol{p}$$

Reconstruction

■ Modèle algébrique



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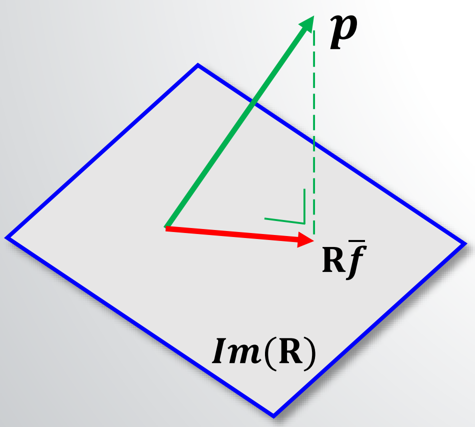

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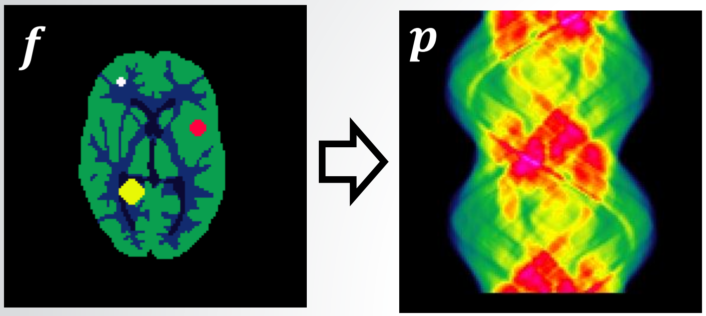
$$\mathrm{R}^{*}\mathrm{R}\bar{\boldsymbol{f}} = \mathrm{R}^{*}\boldsymbol{p}$$

Reconstruction

- Modèle algébrique



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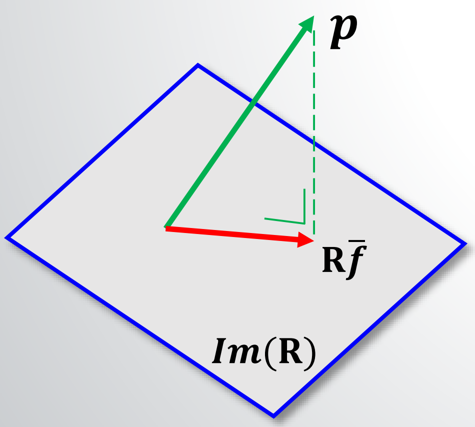

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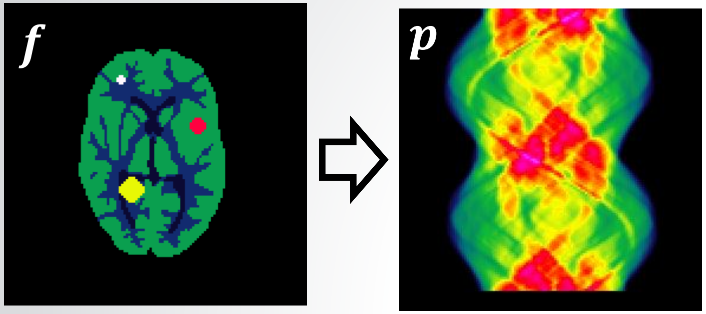
$$\mathrm{R}^{*}\mathrm{R}\bar{\boldsymbol{f}} = \mathrm{R}^{*}\boldsymbol{p}$$

$$\bar{\boldsymbol{f}} = (\mathrm{R}^{*}\mathrm{R})^{-1}\mathrm{R}^{*}\boldsymbol{p}$$

Reconstruction

- Modèle algébrique



objet

$$\boldsymbol{p} = \mathbf{R}\boldsymbol{f} + \boldsymbol{s} + \boldsymbol{n}$$

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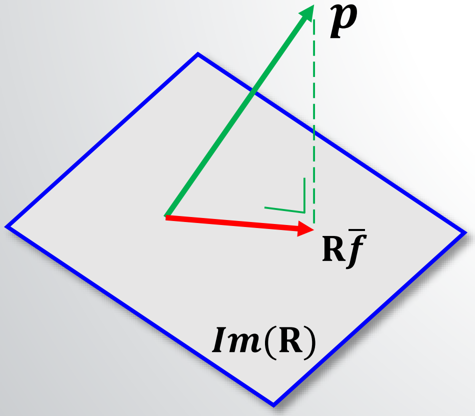

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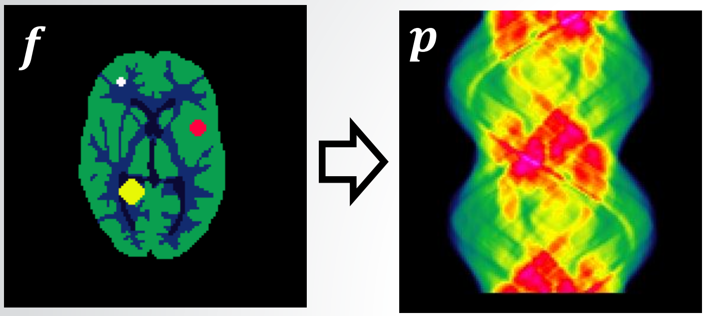
Reconstruction

- Modèle algébrique



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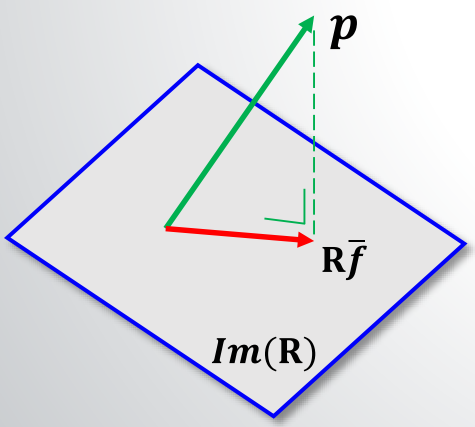


$$\bar{\boldsymbol{f}} = \underset{f \in \Omega}{argmin} \ \{\|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|\}$$

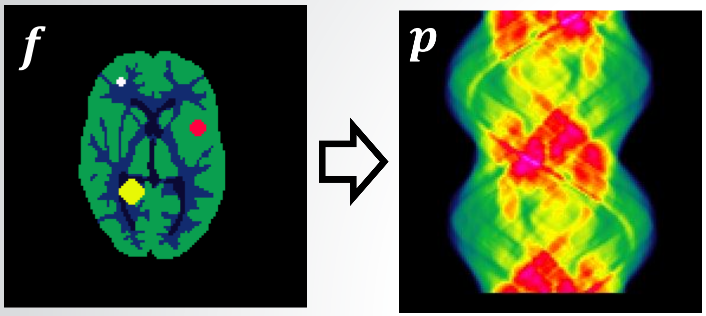
$$\bar{\boldsymbol{f}} = (\mathbf{R}^*\mathbf{R})^{-1}\mathbf{R}^*\boldsymbol{p}$$

$dim(\mathbf{R}^*\mathbf{R}) = \mathcal{O}(10^5)$

$\kappa(\mathbf{R}^*\mathbf{R}) \gg$

Reconstruction

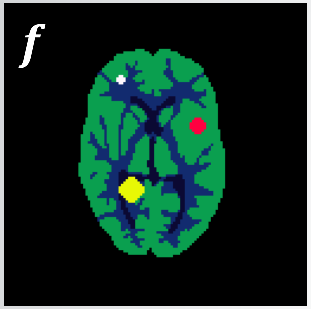
- Méthodes itératives



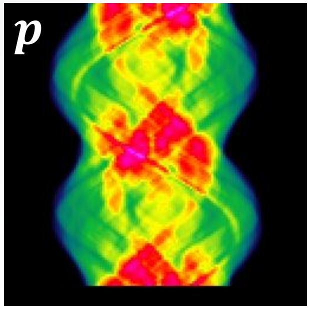
objet

Reconstruction

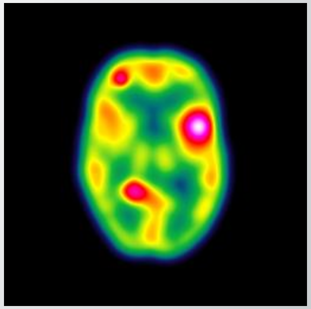
- Méthodes itératives



objet



estimation



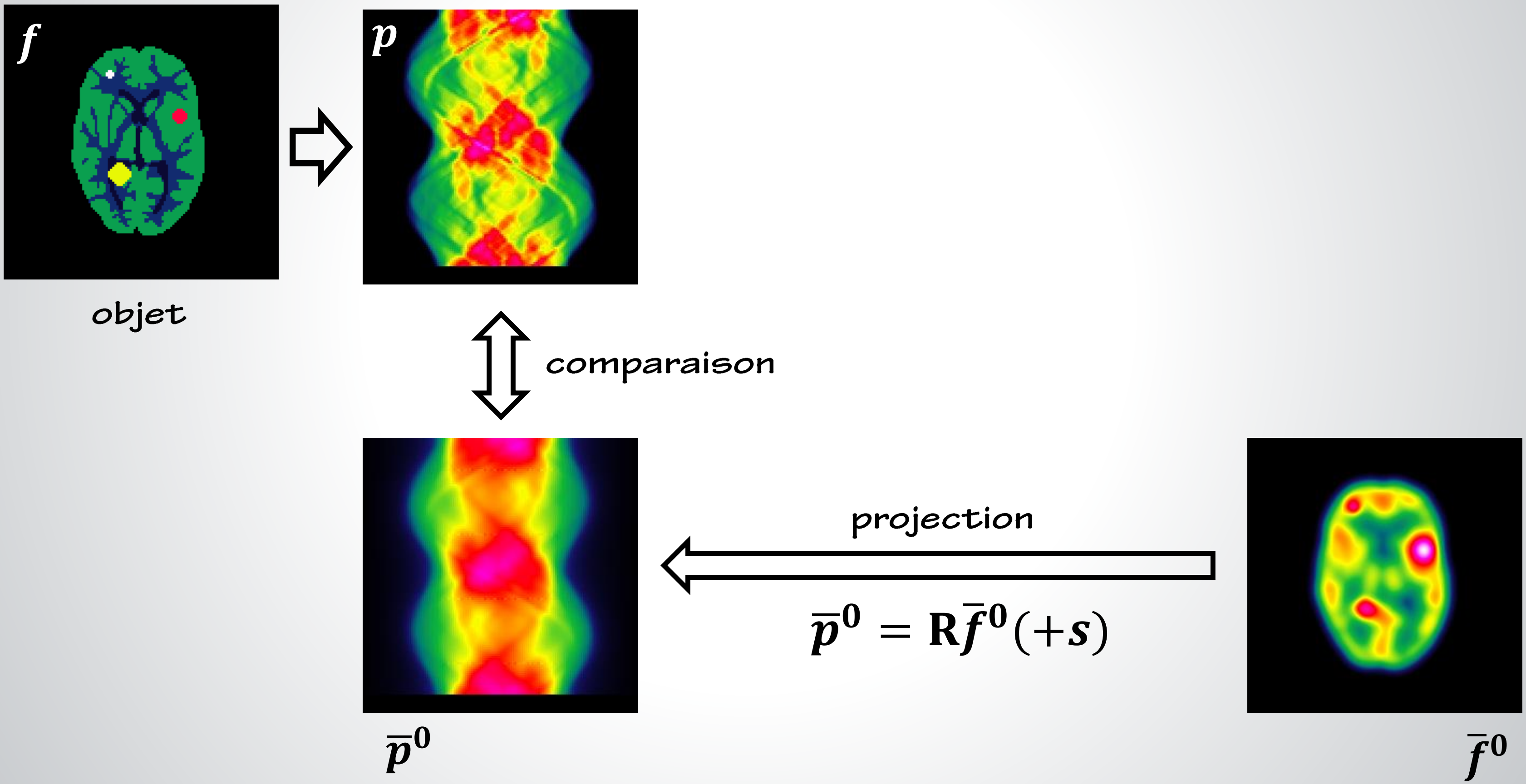
$\bar{f}^0$

projection

$$\bar{p}^0 = \mathrm{R}\bar{f}^0(+s)$$

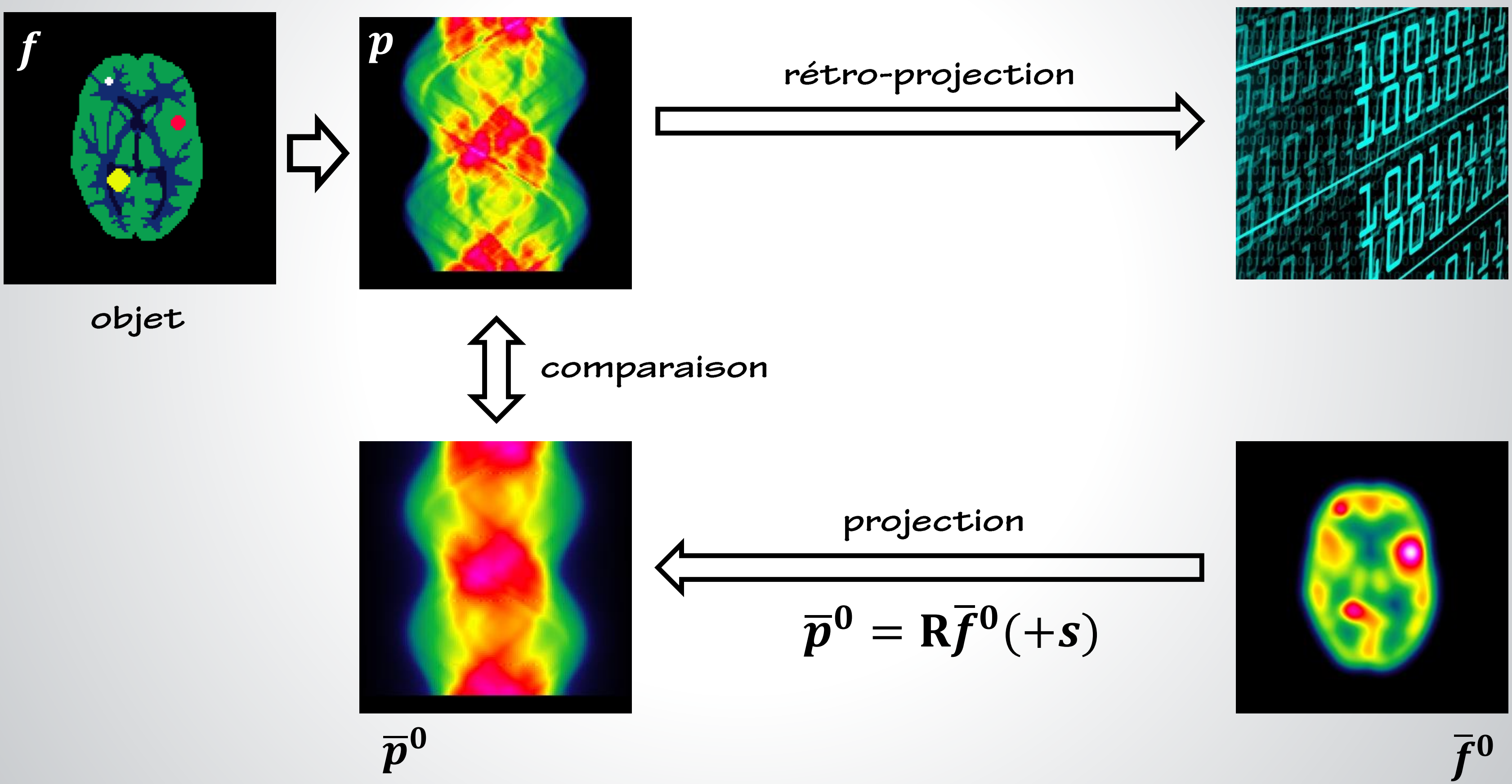
Reconstruction

- Méthodes itératives



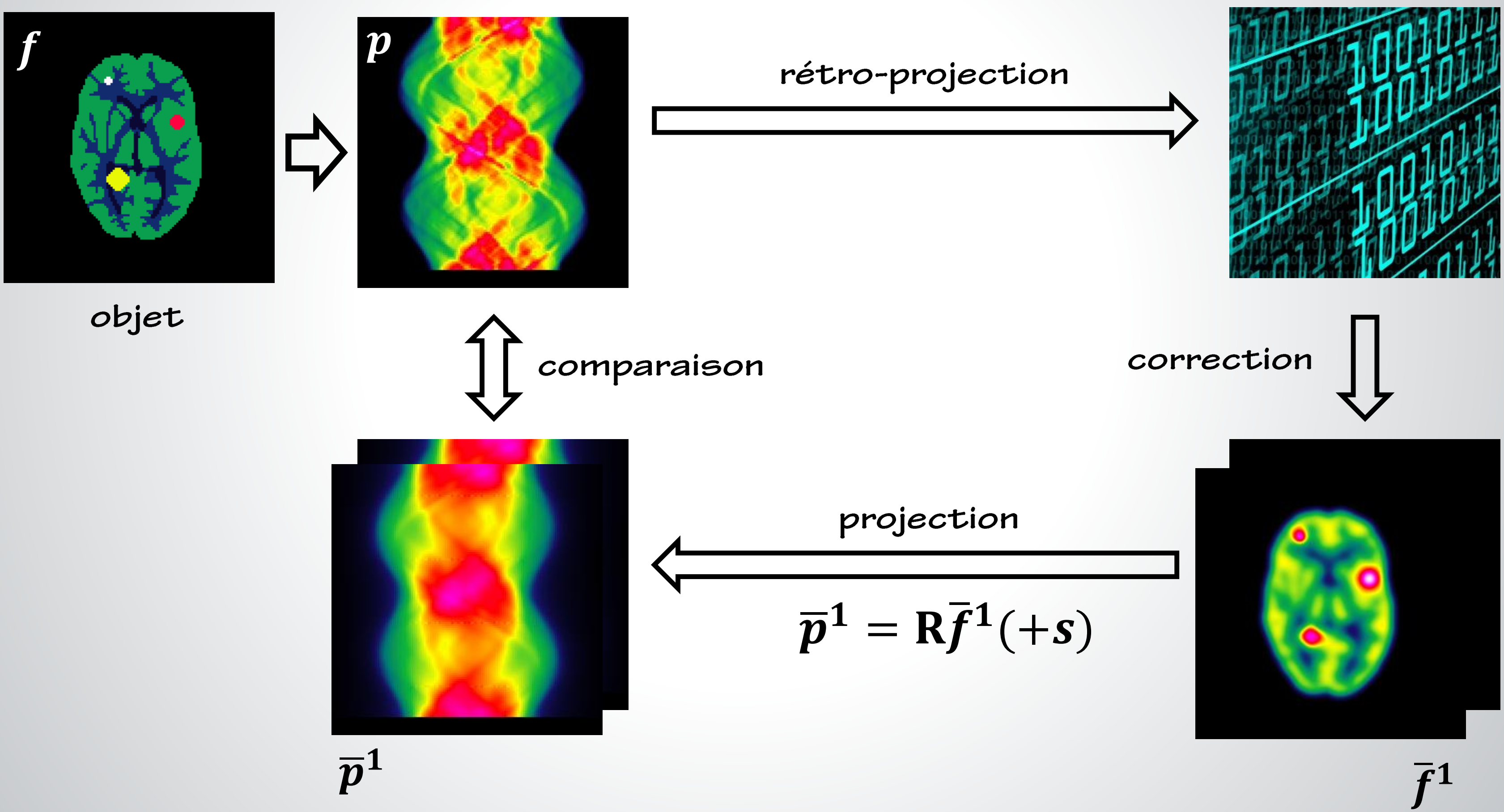
Reconstruction

- Méthodes itératives



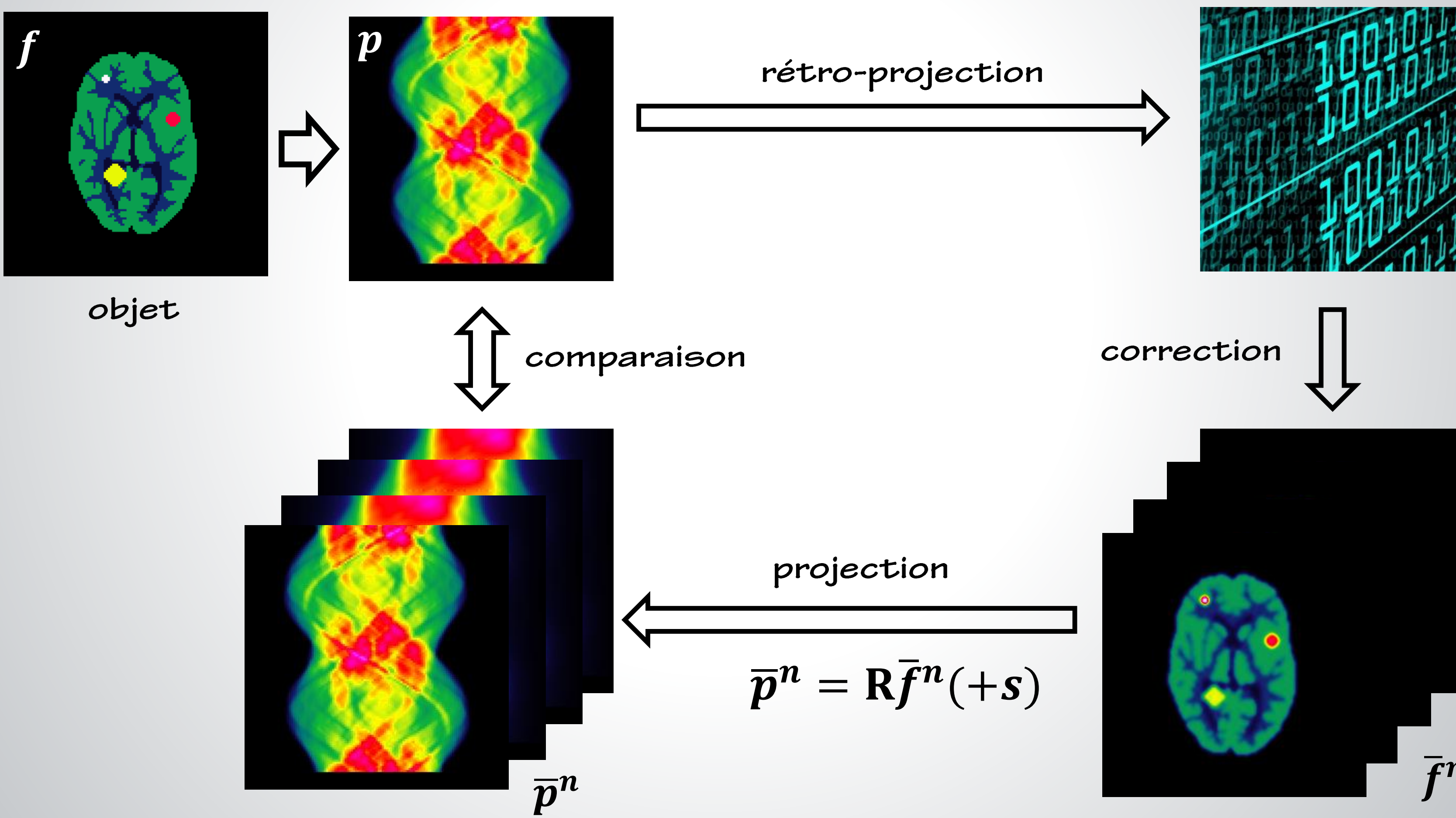
Reconstruction

- Méthodes itératives



Reconstruction

- Méthodes itératives



Reconstruction

- Méthodes itératives

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in \boldsymbol{\Omega}}{argmin} \{J(\boldsymbol{f})\}$$

Méthode **LS**

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2$$

Reconstruction

- Méthodes itératives

Méthode LS

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in \boldsymbol{\Omega}}{argmin} \, \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2$$

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Reconstruction

- Méthodes itératives

Méthode LS

$$\bar{f} = \underset{f \in \Omega}{argmin} \{J(f)\}$$

$$J(f) = \|\mathrm{R}f - p\|^2$$

$$\bar{f} = (\mathrm{R}^*\mathrm{R})^{-1}\,\mathrm{R}^*\,p$$

$$\bar{f}^{n+1} = \bar{f}^{n} + \eta\,\mathrm{R}^*\left(p - \mathrm{R}\,\bar{f}^{n}\right)$$

Reconstruction

- Méthodes itératives

Méthode LS

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\boldsymbol{\Omega}}{argmin} \{J(\boldsymbol{f})\}$$

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$$\bar{\boldsymbol{f}}^{n+1} = \bar{\boldsymbol{f}}^{n} + \boldsymbol{\eta}\,\mathbf{R}^*\left(\boldsymbol{p} - \mathbf{R}\,\bar{\boldsymbol{f}}^{n}\right)$$

correction additive

Reconstruction

- Méthodes itératives

Méthode LS

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f}\in\boldsymbol{\Omega}}{argmin}\,\{J(\boldsymbol{f})\}$$

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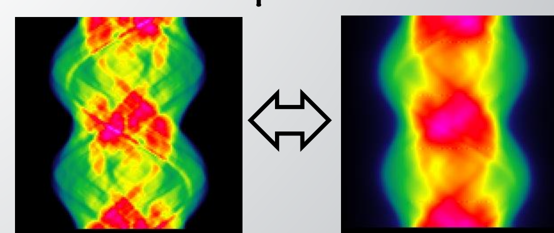
comparaison



Reconstruction

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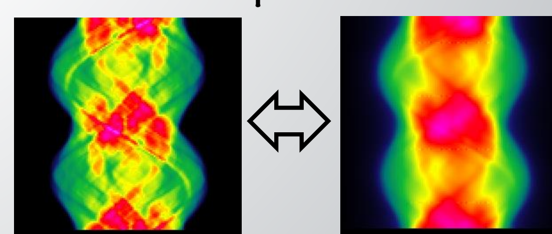
rétro-projection

comparaison



Reconstruction

- Méthodes itératives

Méthode ML

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in \boldsymbol{\Omega}}{argmin} \{ J(\boldsymbol{f}) \}$$

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{p}|\boldsymbol{f}) \}$$

Reconstruction

- Méthodes itératives

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$$\breve{\boldsymbol{p}}_i = (\mathbf{R}\boldsymbol{f})_i$$

Méthode ML

Reconstruction

- Méthodes itératives

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$$J(\boldsymbol{f}) = \sum_i \{ \mathbf{R}\boldsymbol{f}_i - \boldsymbol{p}_i \, log(\mathbf{R}\boldsymbol{f}_i) \}$$

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Reconstruction

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$$\bar{\boldsymbol{f}}^{n+1} = \bar{\boldsymbol{f}}^{n} \times \left(\mathbf{R}^* \frac{\boldsymbol{p}}{\mathbf{R}\, \bar{\boldsymbol{f}}^{n}} \right)$$

Reconstruction

- Méthodes itératives

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correction multiplicative

Reconstruction

- Méthodes itératives

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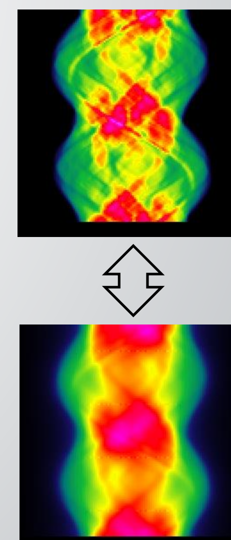

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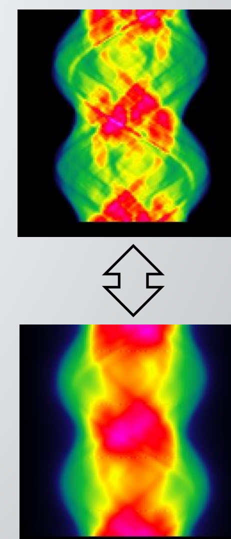
rétro-projection

comparaison



Régularisation

$$\boldsymbol{p} = \mathbf{R}\boldsymbol{f}$$

$\mathbf{R}$ est mal conditionné : $\kappa(\mathbf{R}) \gg$

Régularisation

$$\boldsymbol{p} = \mathbf{R}\boldsymbol{f}$$

$\mathbf{R}$ est mal conditionné : $\kappa(\mathbf{R}) \gg$

$$\frac{\|\Delta \boldsymbol{f}\|}{\|\boldsymbol{f}\|} \leq \kappa(\mathbf{R}) \boxed{\frac{\|\Delta \boldsymbol{p}\|}{\|\boldsymbol{p}\|}} \approx \sigma\left(\frac{1}{\sqrt{\boldsymbol{p}}}\right)$$

Régularisation

$$\boldsymbol{p} = \mathbf{R}\boldsymbol{f}$$

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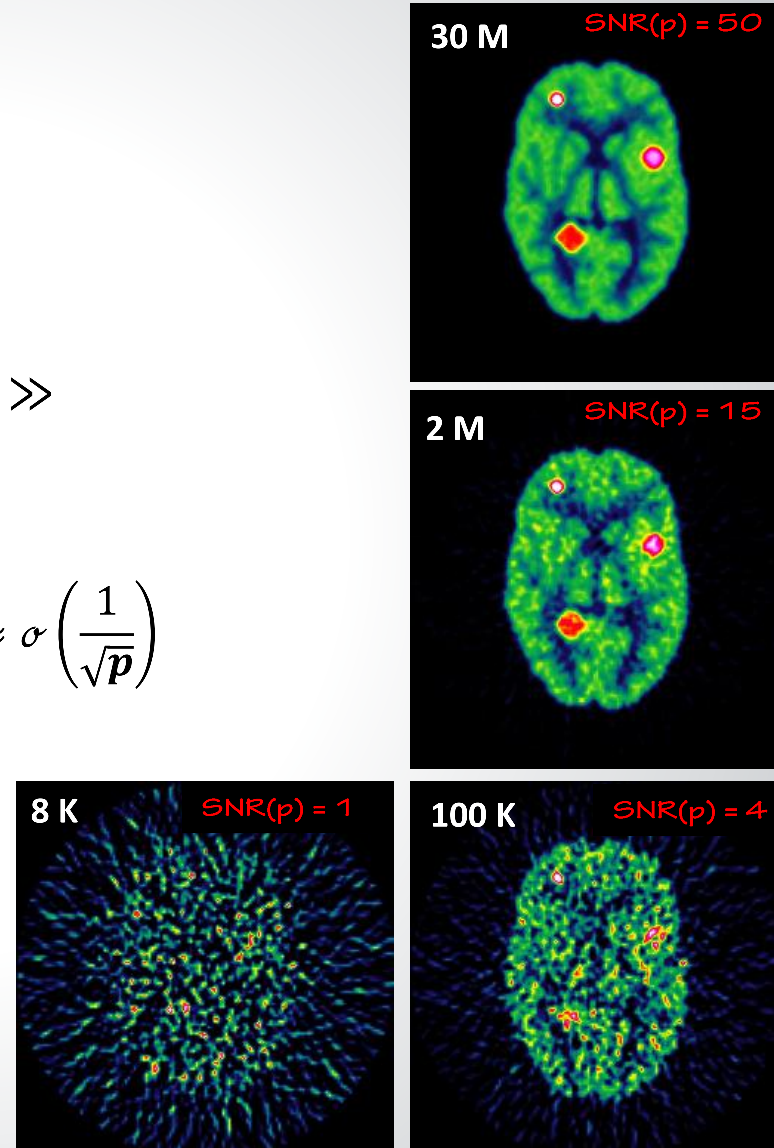
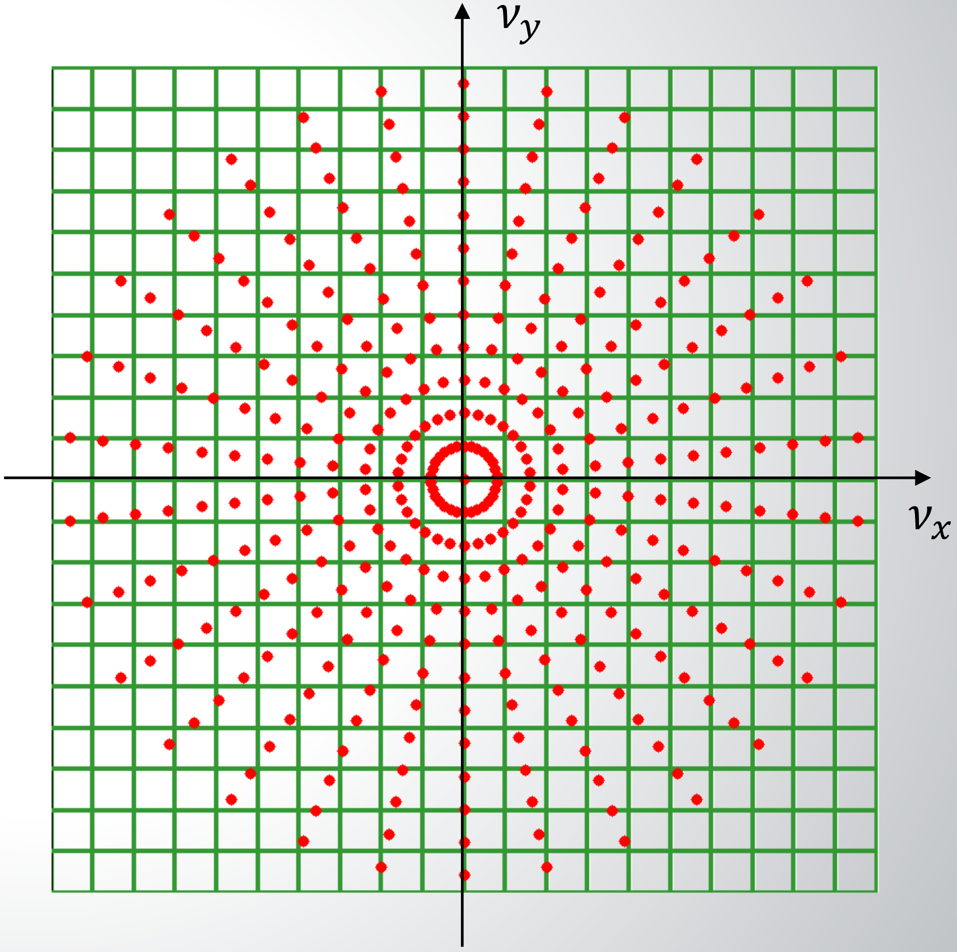
⇨ Sensibilité au bruit



Régularisation

- Reconstruction analytique

$$\hat{f}(\nu cos\theta, \nu sin\theta) = \hat{p}_\theta(\nu)$$



Régularisation

- Reconstruction analytique

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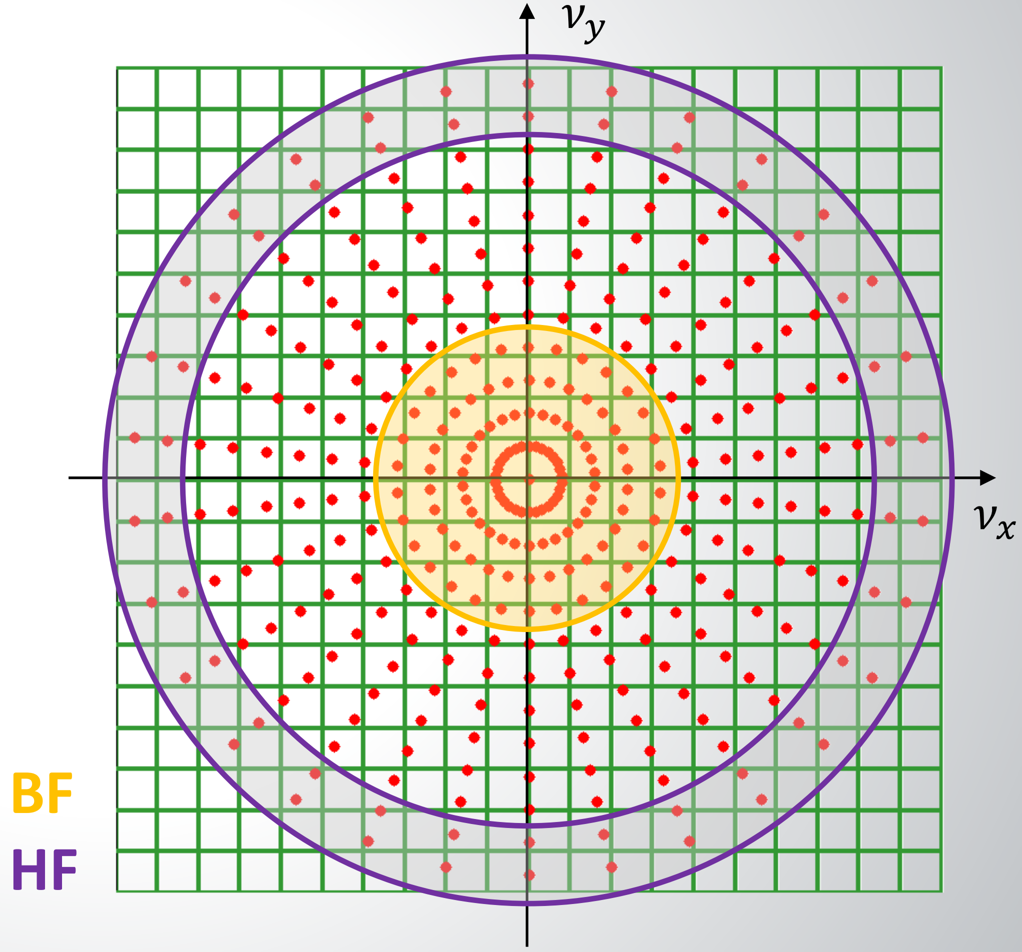
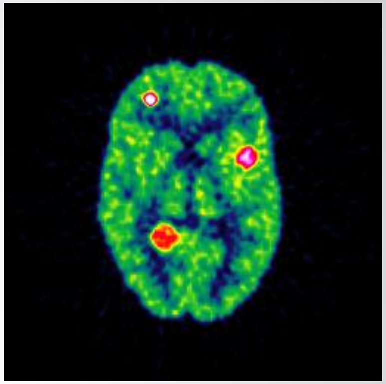


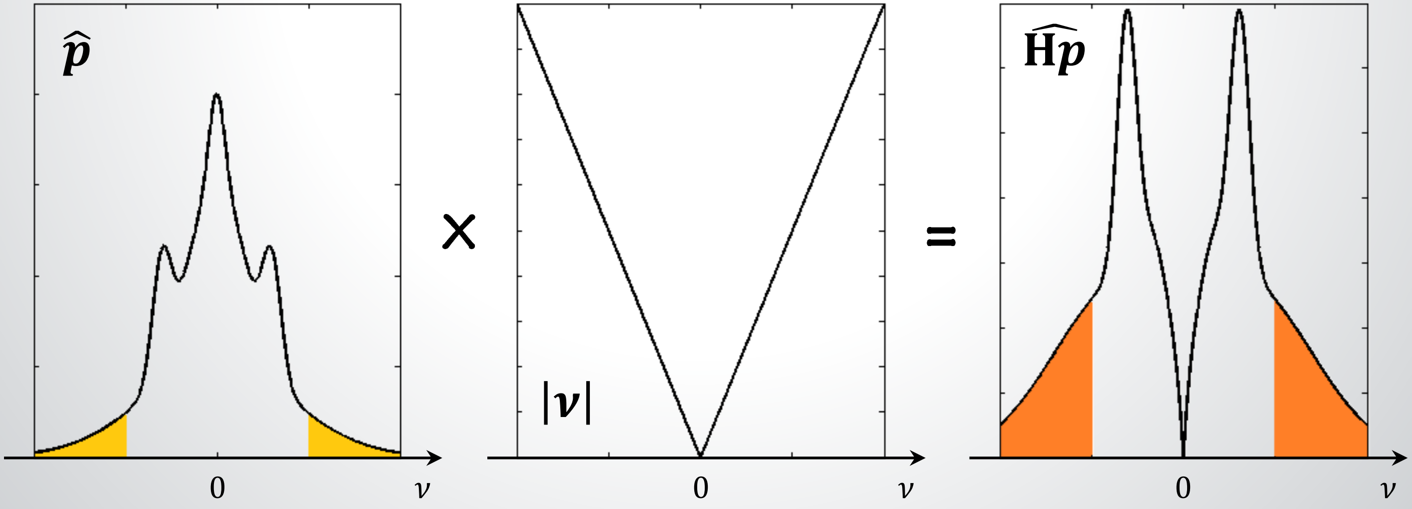
Régularisation

- Reconstruction analytique

$$f = \mathrm{R}^* \, \mathrm{H}p$$

$$\widehat{\mathrm{H}p} = |\nu| \, \hat{p}$$





Régularisation

- Reconstruction analytique

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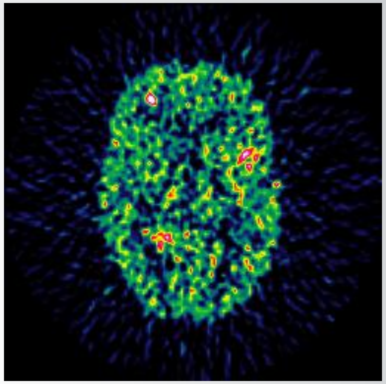
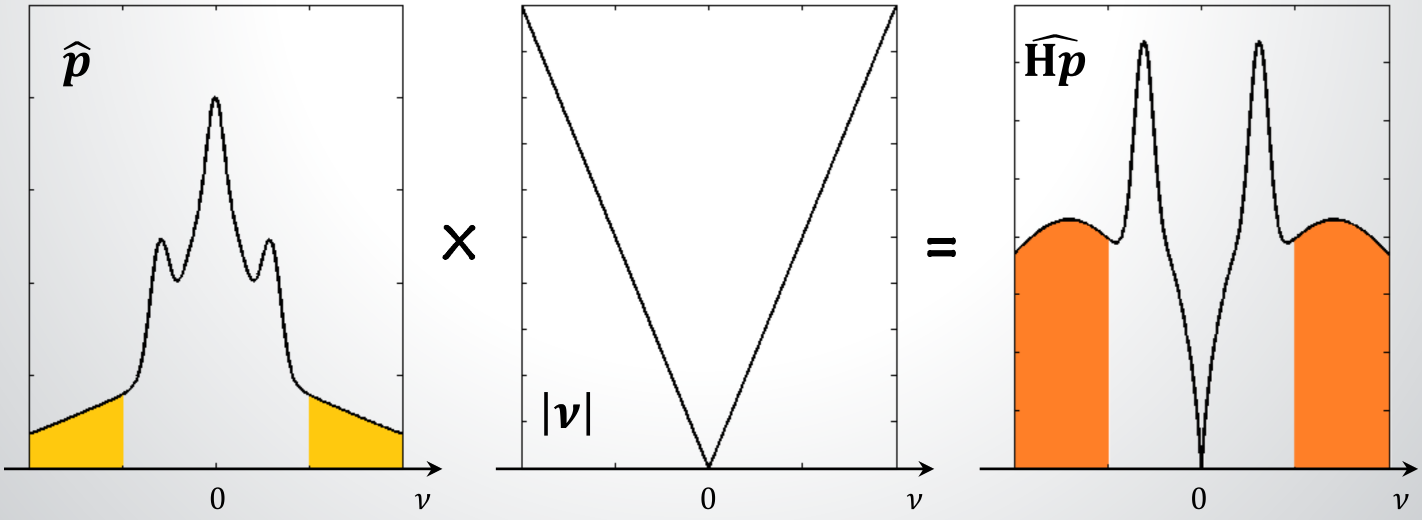
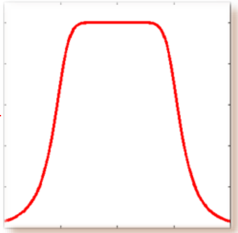


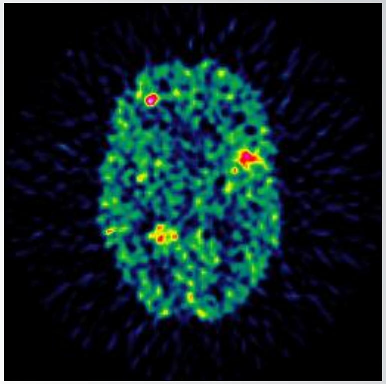


Régularisation

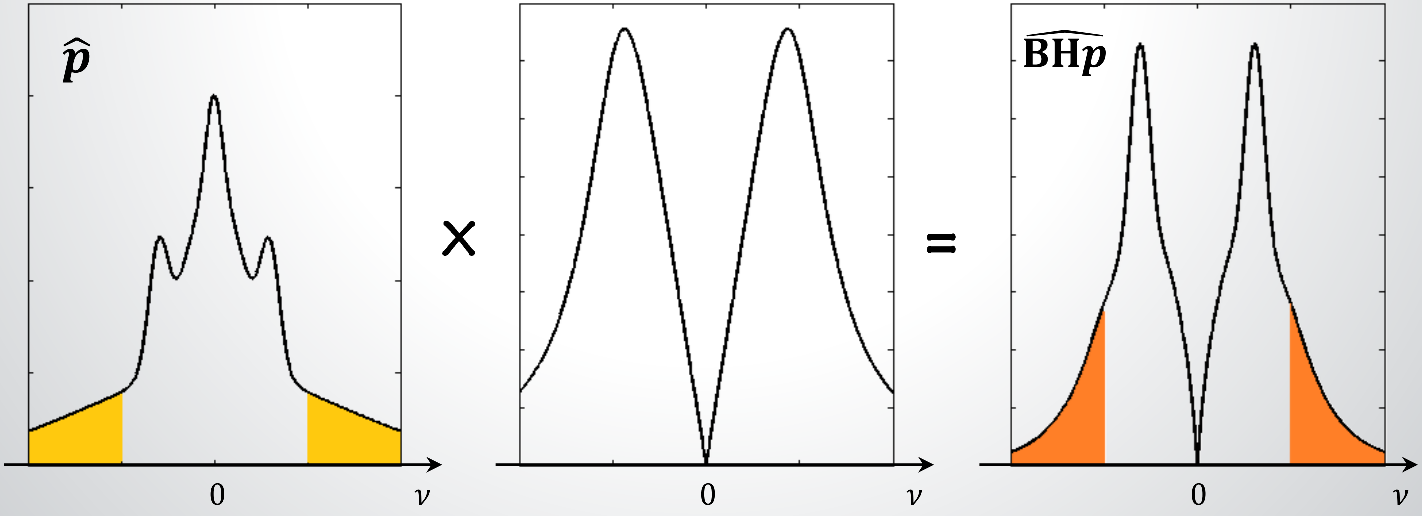
- Reconstruction analytique

$$f = \mathrm{R}^* \, \mathrm{BH}p \qquad \widehat{\mathrm{BH}p} = \frac{|\nu| \, \hat{p}}{\sqrt{1 + \left(\frac{\nu}{\nu_c}\right)^{\beta}}}$$





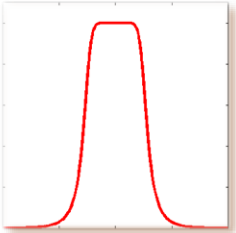
$\nu_c = 0{,}5$

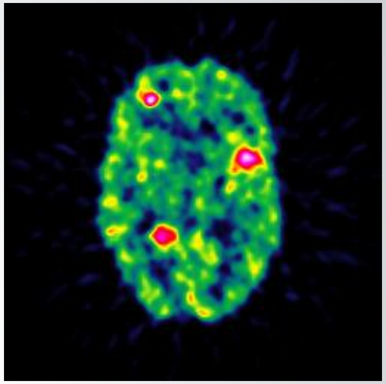


Régularisation

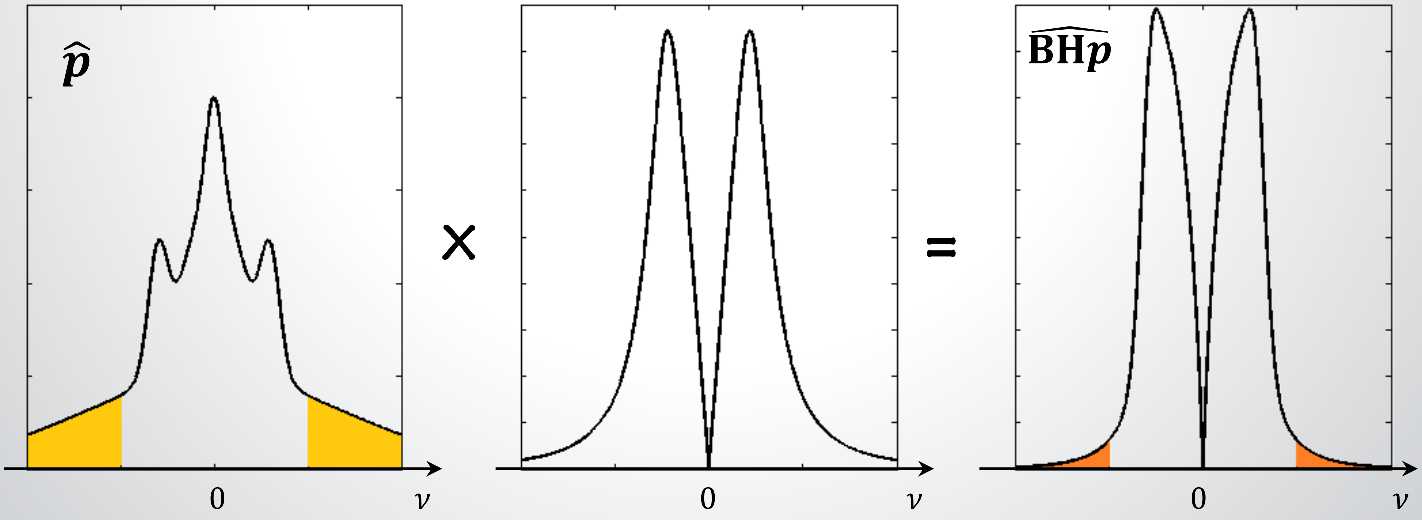
- Reconstruction analytique

$$f = \mathrm{R}^* \, \mathrm{BH}p \qquad \widehat{\mathrm{BH}p} = \frac{|\nu| \, \hat{p}}{\sqrt{1 + \left(\frac{\nu}{\nu_c}\right)^{\beta}}}$$



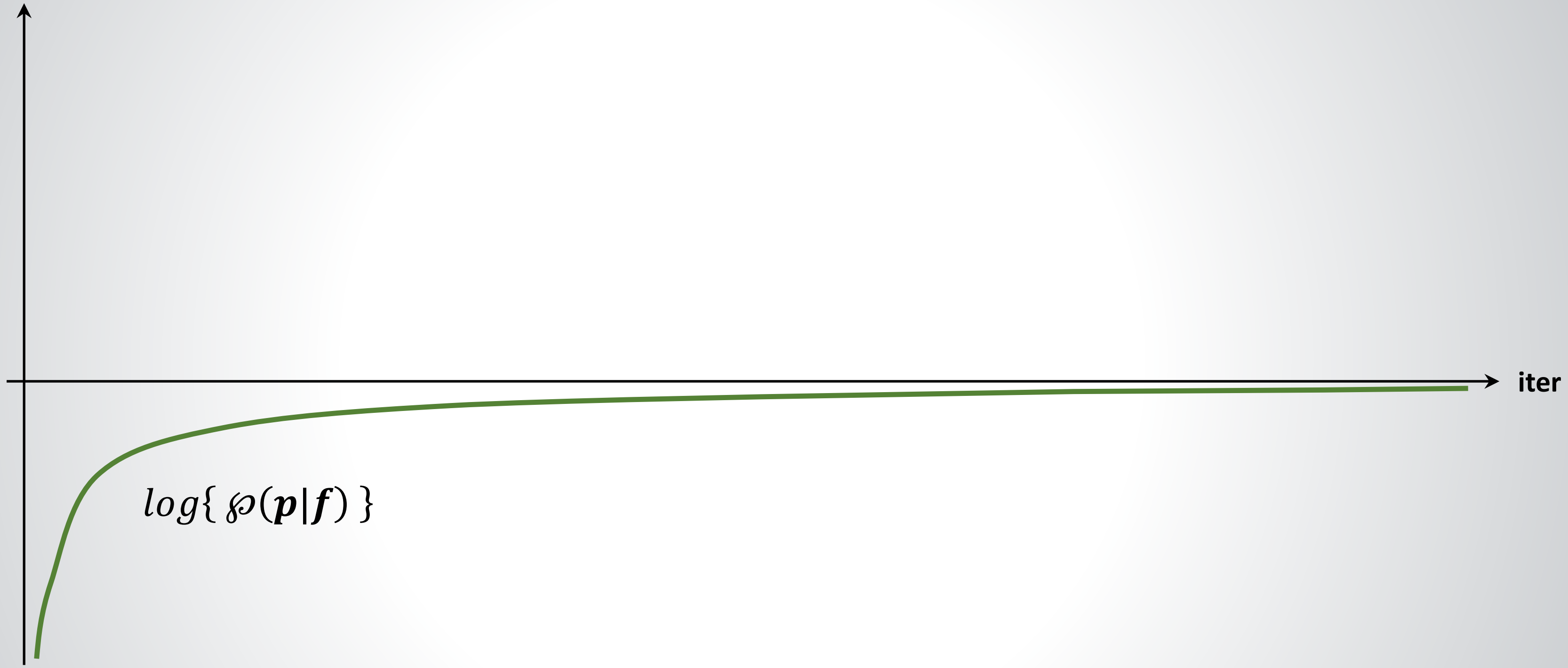


$\nu_c = 0{,}25$



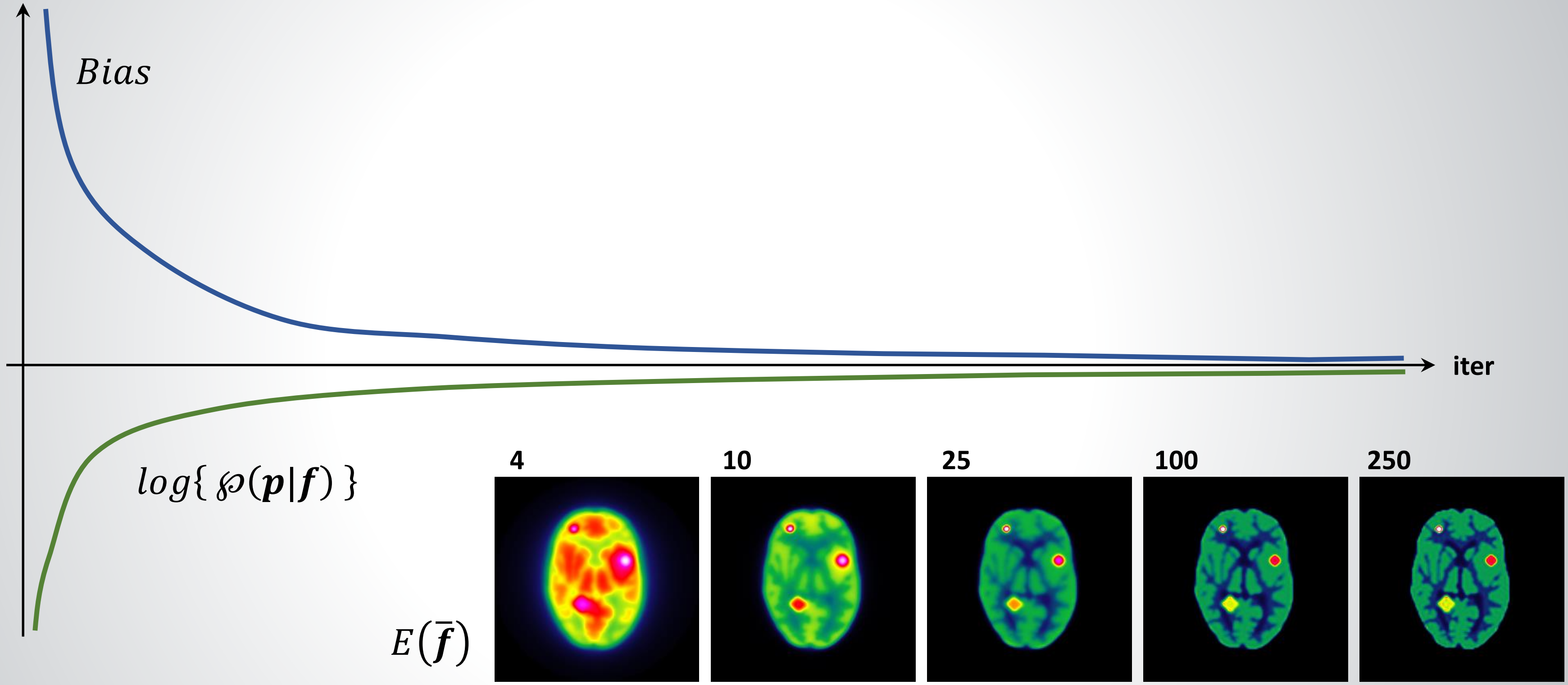
Régularisation

- Reconstruction itérative



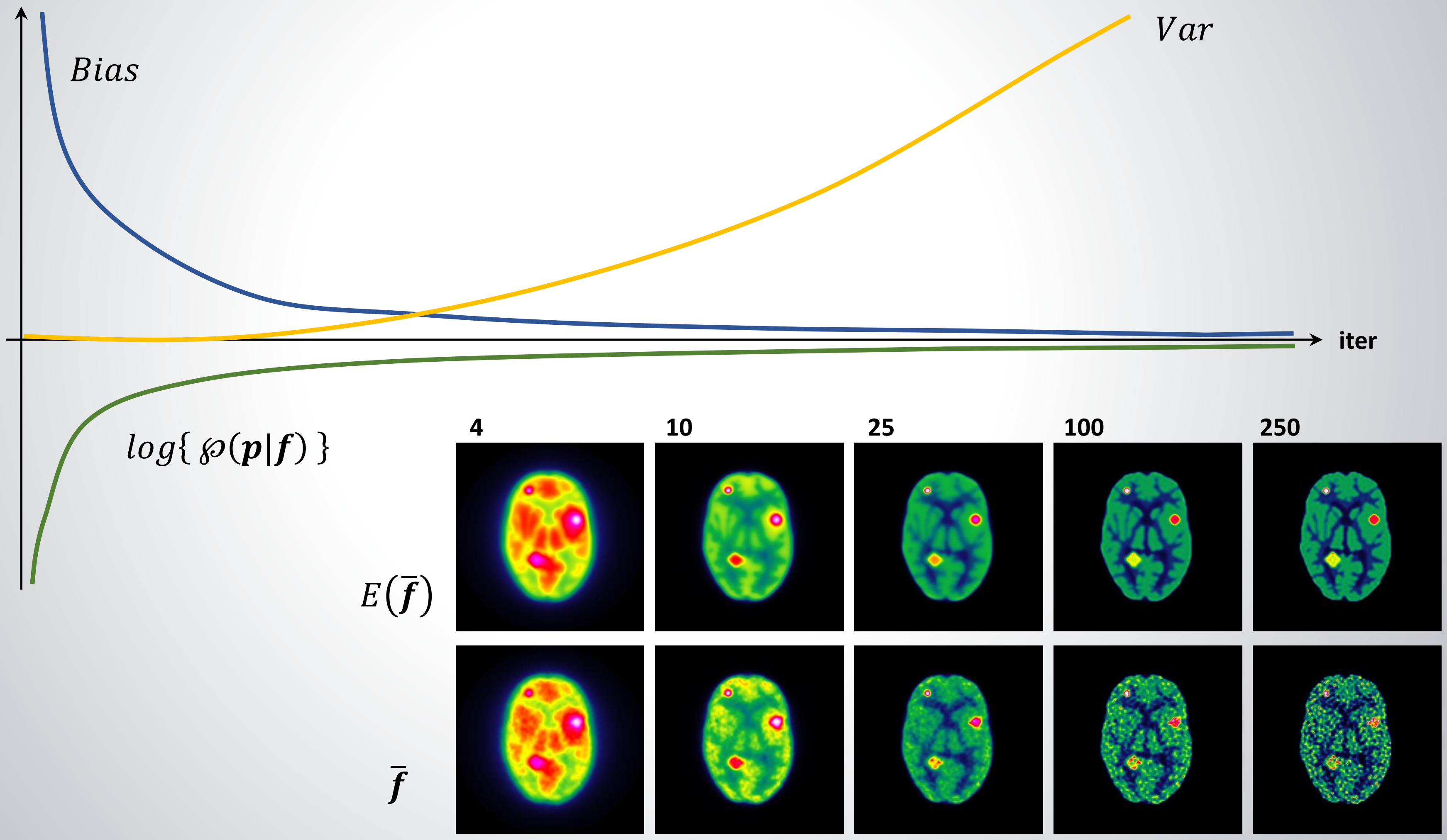
Régularisation

- Reconstruction itérative



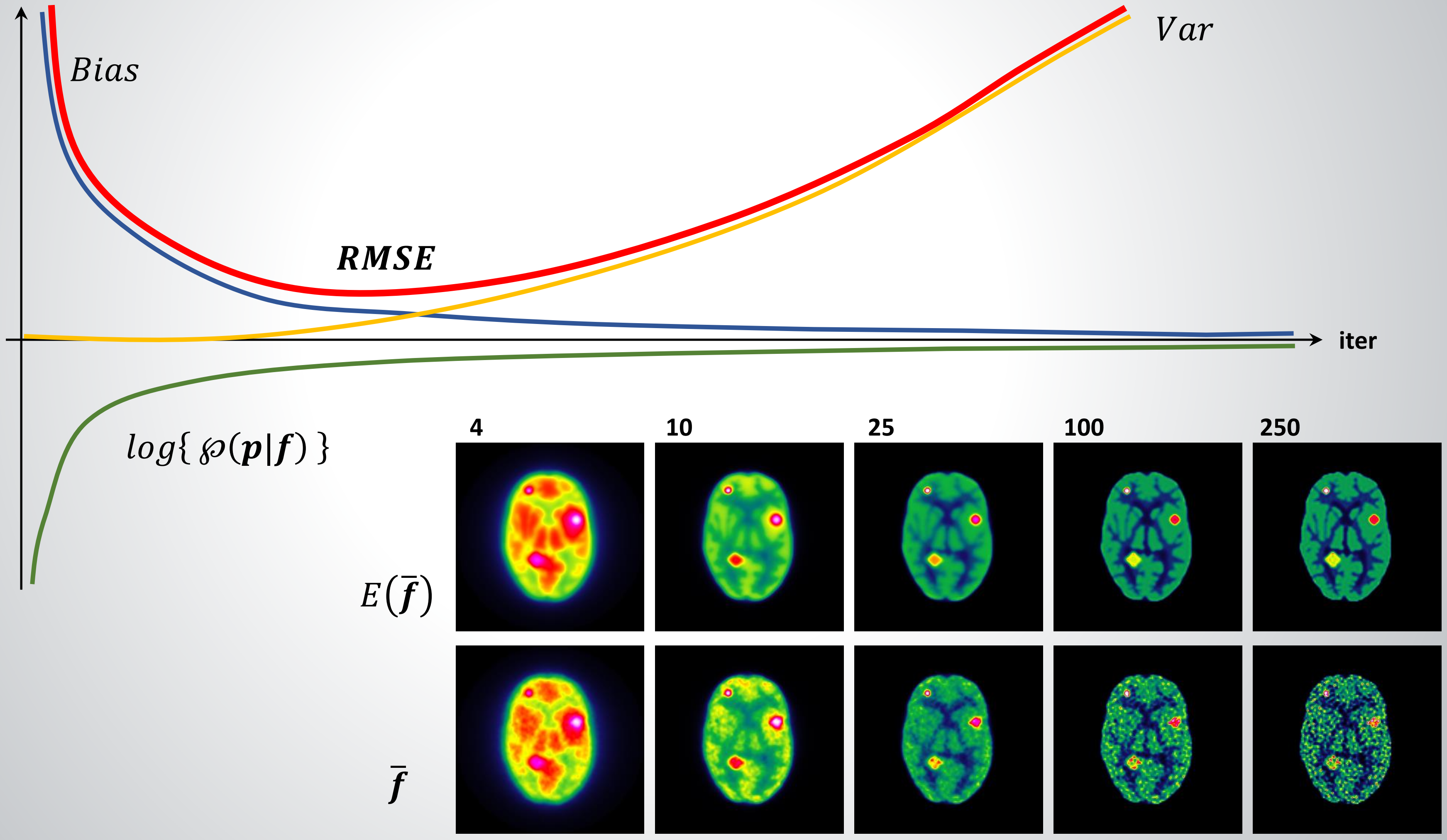
Régularisation

- Reconstruction itérative



Régularisation

- Reconstruction itérative

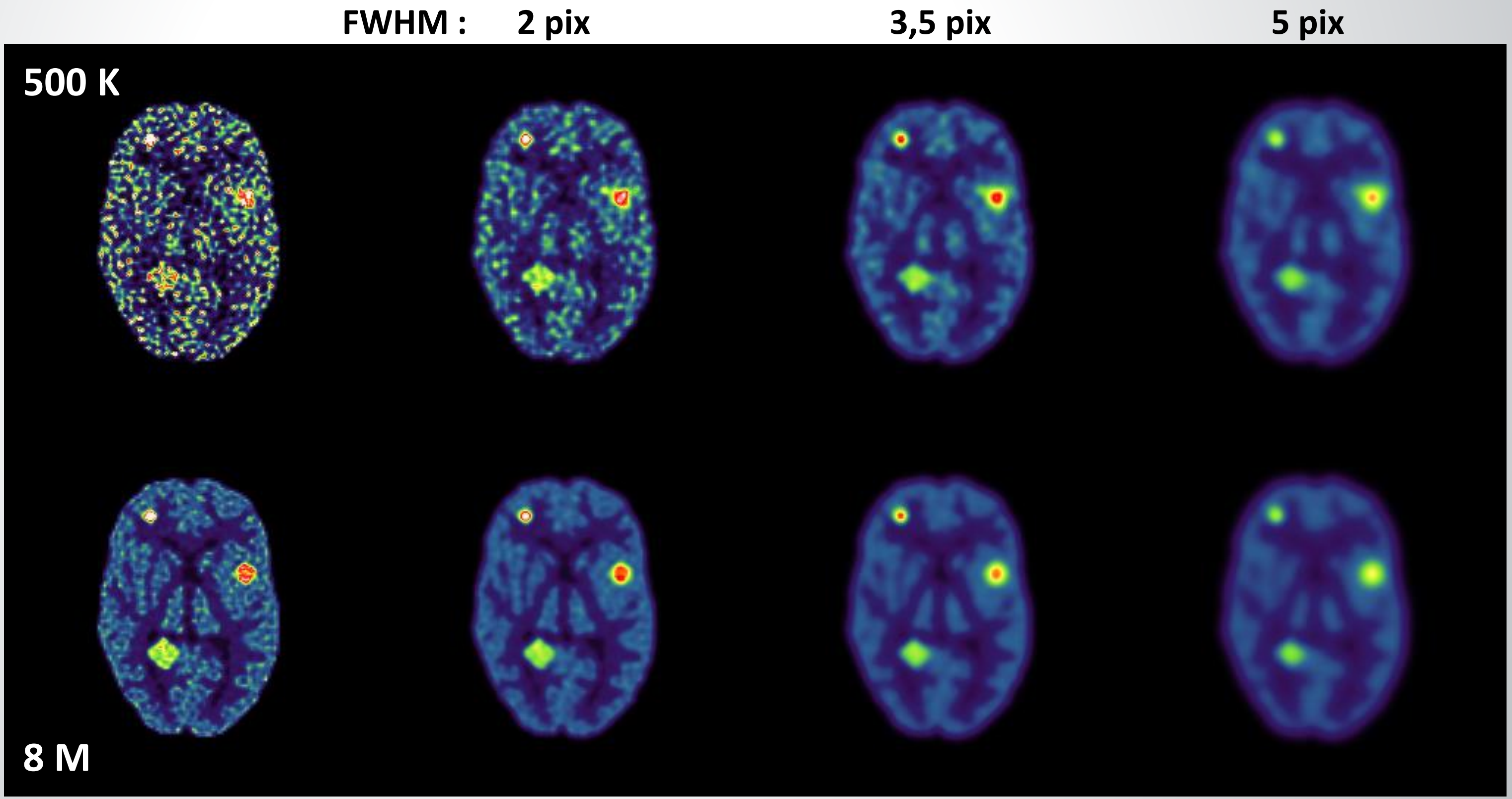


Régularisation

- Reconstruction itérative

Post-filtrage

$h * \bar{f}$



Régularisation

- Reconstruction itérative

Tikhonov

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in C}{argmin} \, \{ J(\boldsymbol{f}) \}$$

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2 + \rho(\boldsymbol{f})$$

Régularisation

- Reconstruction itérative

Tikhonov

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin}\ \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2 + \rho(\boldsymbol{f})$$

Adéquation
Surjectivité
↓ biais

Régularisation

- Reconstruction itérative

Tikhonov

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in C}{argmin} \, \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2 + \rho(\boldsymbol{f})$$

Adéquation
Surjectivité
↓ biais

Régularisation
Injectivité
↓ variance

Régularisation

- Reconstruction itérative

Tikhonov

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \{ J(\boldsymbol{f}) \}$$

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2 + \beta \|\boldsymbol{f}\|^{\mathbf{2}}$$

Régularisation

- Reconstruction itérative

Tikhonov

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \, \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\mathbf{R}\boldsymbol{f} - \boldsymbol{p}\|^2 + \beta\|\boldsymbol{f}\|^2$$

$$\bar{\boldsymbol{f}} = (\mathbf{R}^*\mathbf{R} + \beta\mathbf{I})^{-1}\,\mathbf{R}^*\,\boldsymbol{p}$$

$$\bar{\boldsymbol{f}}^{n+1} = (1-\beta)\bar{\boldsymbol{f}}^{n} + \boldsymbol{\eta}\,\mathbf{R}^*\left(\boldsymbol{p} - \mathbf{R}\,\bar{\boldsymbol{f}}^{n}\right)$$

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \; \{ J(\boldsymbol{f}) \}$$

Posterior

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{f}|\boldsymbol{p}) \}$$

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin}\ \{J(\boldsymbol{f})\}$$

Posterior

$$J(\boldsymbol{f}) = -log\{\ \wp(\boldsymbol{f}|\boldsymbol{p})\ \}$$

$$\wp(\boldsymbol{f}|\boldsymbol{p}) = \frac{\wp(\boldsymbol{p}|\boldsymbol{f})\ \wp(\boldsymbol{f})}{\wp(\boldsymbol{p})}$$

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \, \{ J(\boldsymbol{f}) \}$$

Posterior

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{f}|\boldsymbol{p}) \}$$

$$\wp(\boldsymbol{f}|\boldsymbol{p}) = \frac{\wp(\boldsymbol{p}|\boldsymbol{f}) \, \wp(\boldsymbol{f})}{\wp(\boldsymbol{p})}$$

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{p}|\boldsymbol{f}) \} - log\{ \wp(\boldsymbol{f}) \}$$

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{\boldsymbol{f} \in C}{argmin} \{ J(\boldsymbol{f}) \}$$

Posterior

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{f}|\boldsymbol{p}) \}$$

Likelihood

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{p}|\boldsymbol{f}) \} - log\{ \wp(\boldsymbol{f}) \}$$

Adéquation
Surjectivité
↓ biais

Régularisation

- Reconstruction itérative

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Likelihood

Prior

$$J(\boldsymbol{f}) = -log\{\ \wp(\boldsymbol{p}|\boldsymbol{f})\ \} - log\{\ \wp(\boldsymbol{f})\ \}$$

Adéquation
Surjectivité
↓ biais

Régularisation
Injectivité
↓ variance

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin}\ \{J(\boldsymbol{f})\}$$

Posterior

$$J(\boldsymbol{f}) = -log\{\ \wp(\boldsymbol{f}|\boldsymbol{p})\ \}$$

Likelihood | Prior

$$J(\boldsymbol{f}) = -log\{\ \wp(\boldsymbol{p}|\boldsymbol{f})\ \} - log\{\ \wp(\boldsymbol{f})\ \}$$

$$\wp(\boldsymbol{f}) = \kappa\ e^{-\beta \mathrm{U}}$$

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \, \{J(\boldsymbol{f})\}$$

Posterior

$$J(\boldsymbol{f}) = -log\{\, \wp(\boldsymbol{f}|\boldsymbol{p})\, \}$$

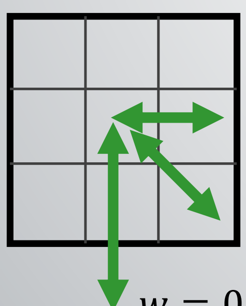
Likelihood

Prior

$$J(\boldsymbol{f}) = -log\{\, \wp(\boldsymbol{p}|\boldsymbol{f})\, \} - log\{\, \wp(\boldsymbol{f})\, \}$$

$$\mathrm{U} = \sum_{i,j} w_{ij} \left\| \boldsymbol{f}_i - \boldsymbol{f}_j \right\|^2$$

$$\wp(\boldsymbol{f}) = \kappa \, e^{-\beta \mathrm{U}}$$



$w_{1st} = 1$

$w_{2nd} = 1/\sqrt{2}$

$w = 0$

Régularisation

- Reconstruction itérative

MAP

$$\bar{\boldsymbol{f}} = \underset{f \in C}{argmin} \, \{ J(\boldsymbol{f}) \}$$

Posterior

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{f}|\boldsymbol{p}) \}$$

Likelihood — Prior

$$J(\boldsymbol{f}) = -log\{ \wp(\boldsymbol{p}|\boldsymbol{f}) \} - log\{ \wp(\boldsymbol{f}) \}$$

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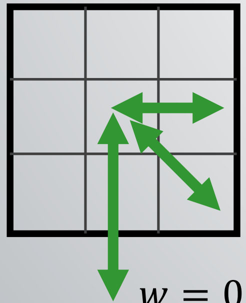


$w_{1st} = 1$

$w_{2nd} = 1/\sqrt{2}$

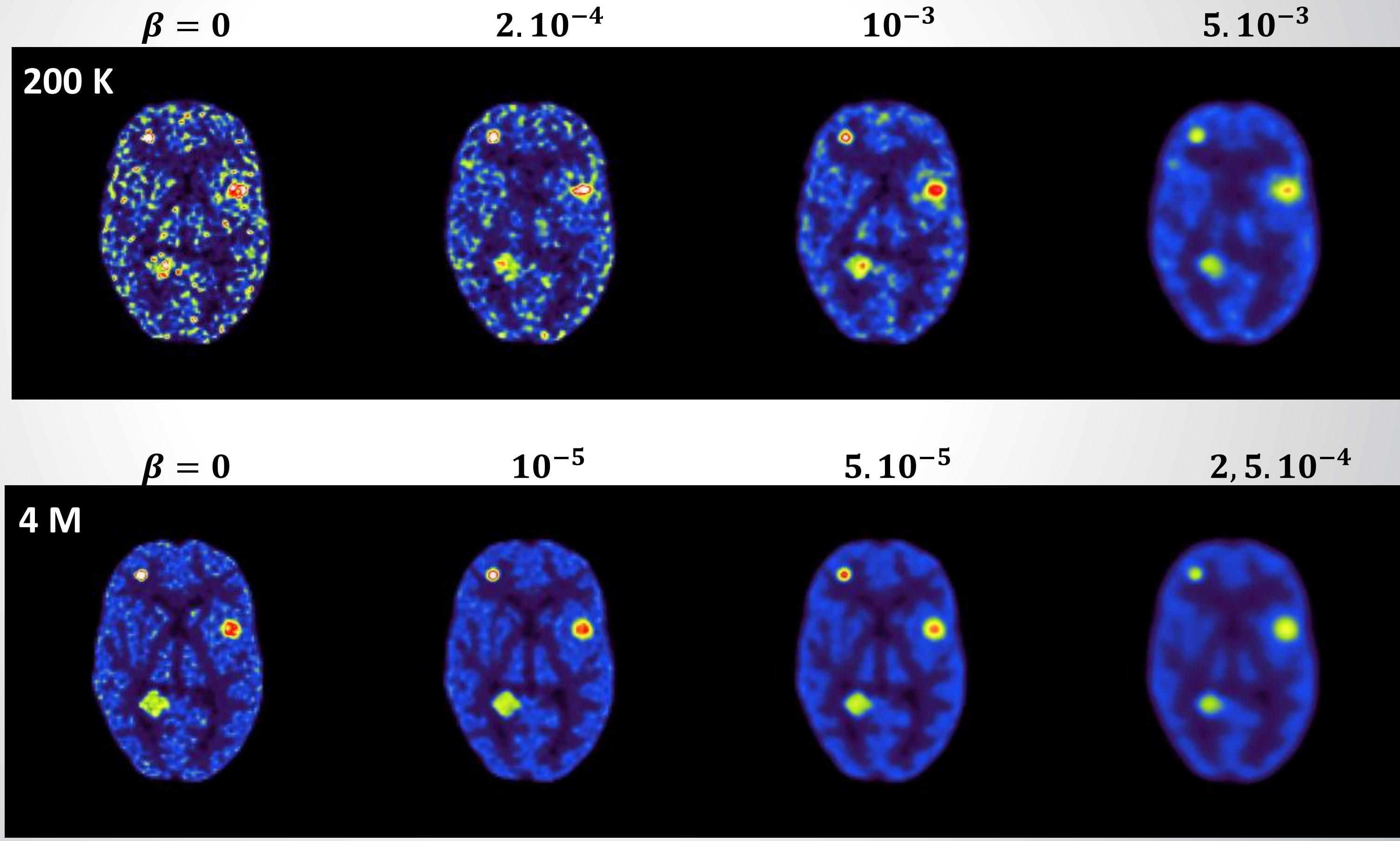
$w = 0$

$$\bar{\boldsymbol{f}}^{n+1} = \frac{\bar{\boldsymbol{f}}^{n}}{1 + \beta \, \boldsymbol{\nabla}\mathrm{U}} \times \left(\mathbf{R}^* \frac{\boldsymbol{p}}{\mathbf{R} \, \bar{\boldsymbol{f}}^{n}} \right)$$

Régularisation

- Reconstruction itérative

MAP



Régularisation

- Reconstruction itérative

MAP

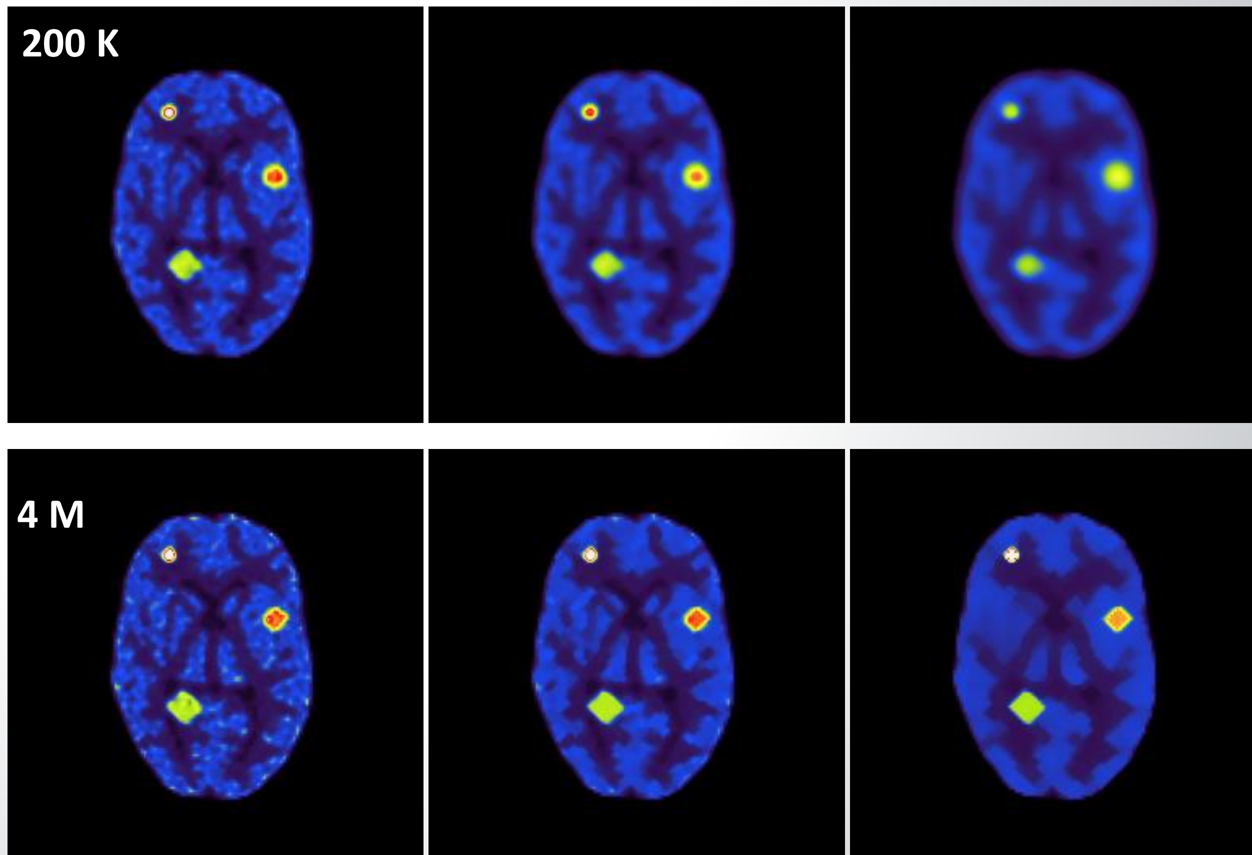
Quad. prior

$$U = \sum_{i,j} w_{ij} \left\| \boldsymbol{f}_i - \boldsymbol{f}_j \right\|^2$$

Median prior

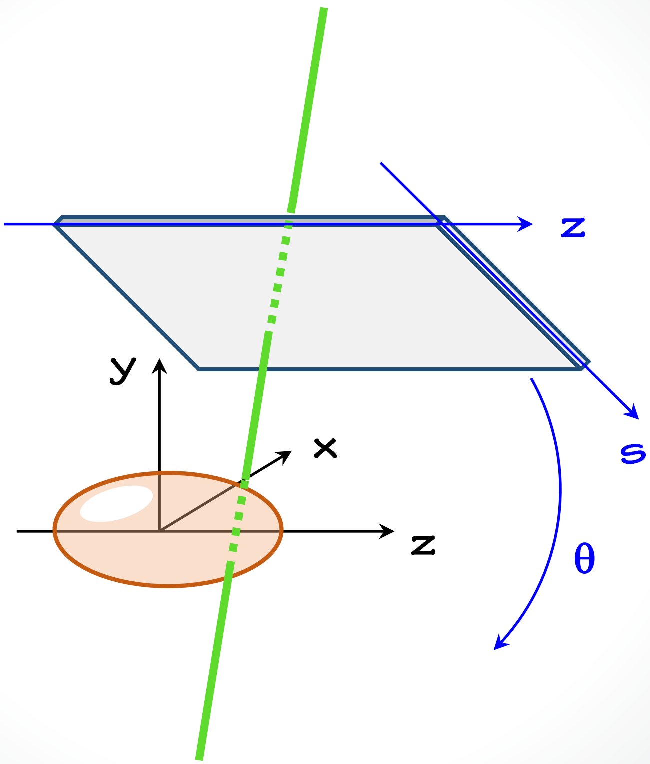
$$U = \sum_{i,j} w_{ij} \left| \boldsymbol{f}_i - \boldsymbol{f}_j \right|$$

« edge-preserving »



TEP 3D

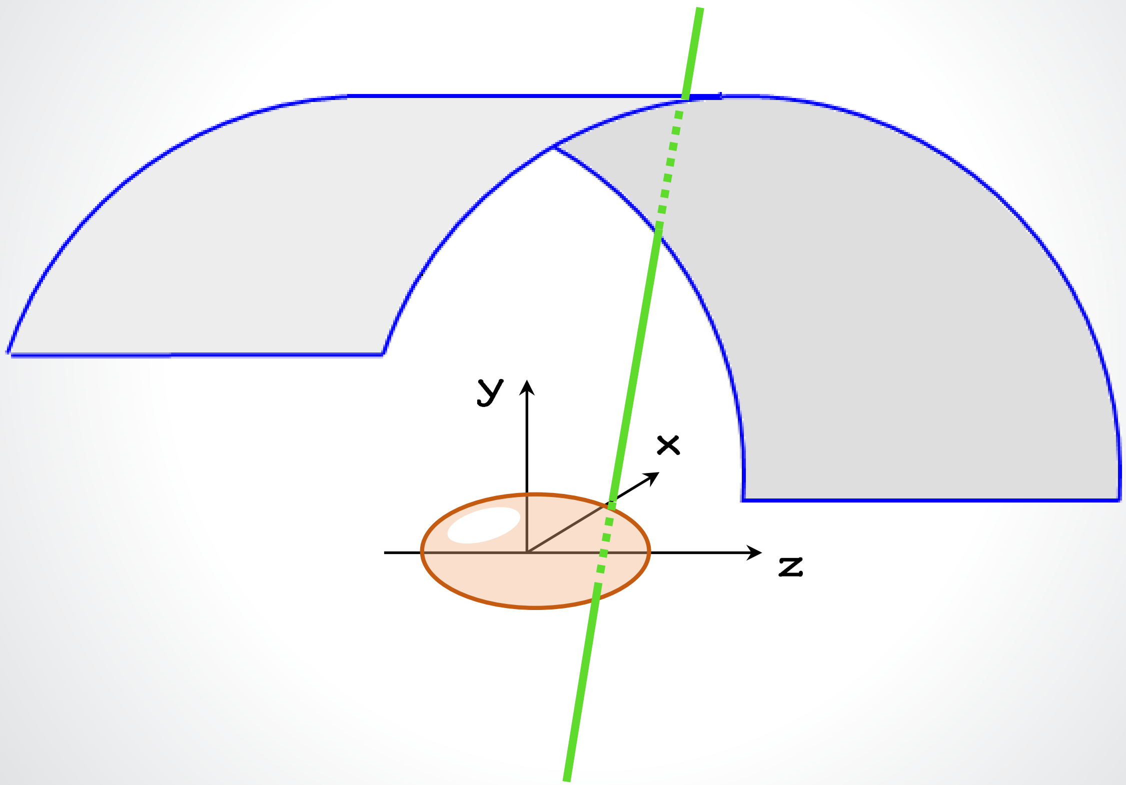
- Encodage TEMP



$p(z, \theta, s)$

TEP 3D

- Encodage TEP

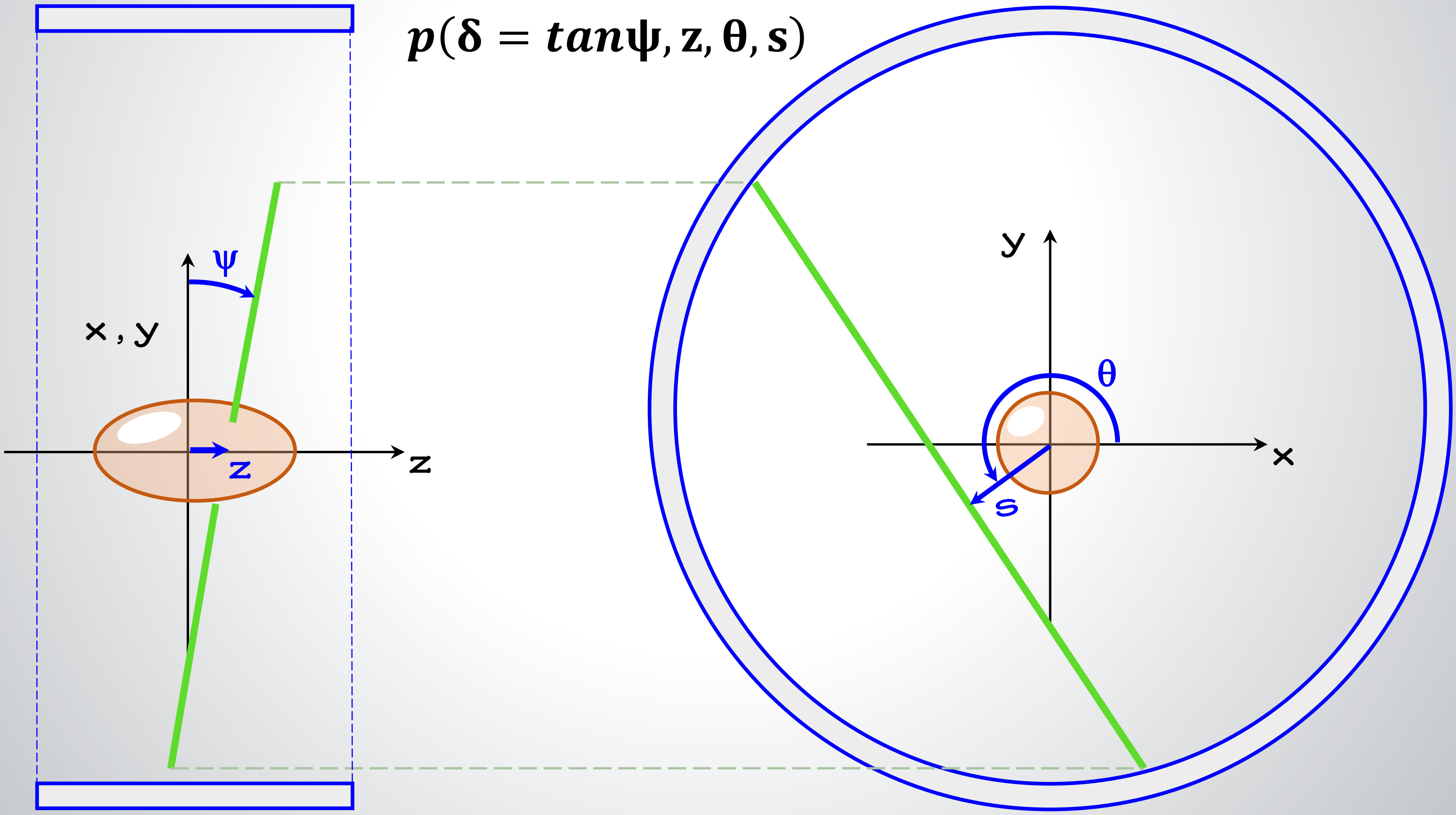


$p(\delta, z, \theta, s)$

TEP 3D

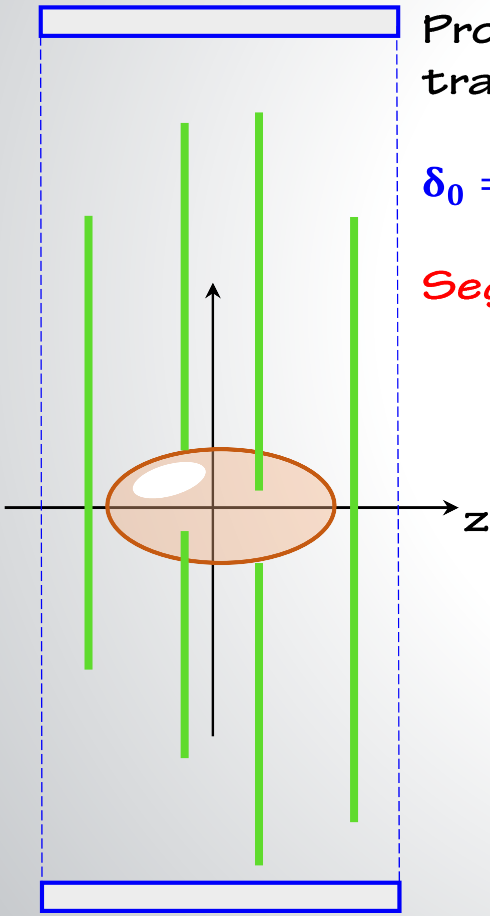
- Encodage TEP

$$p(\delta = tan\psi, z, \theta, s)$$



TEP 3D

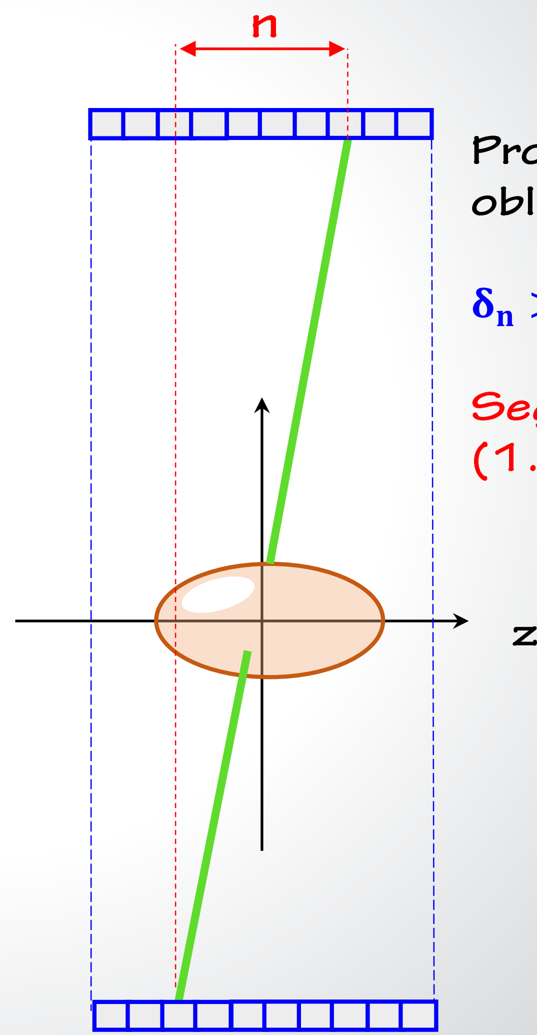
- Encodage TEP



Projections transverses :

$\delta_0 = 0$

Segment 0



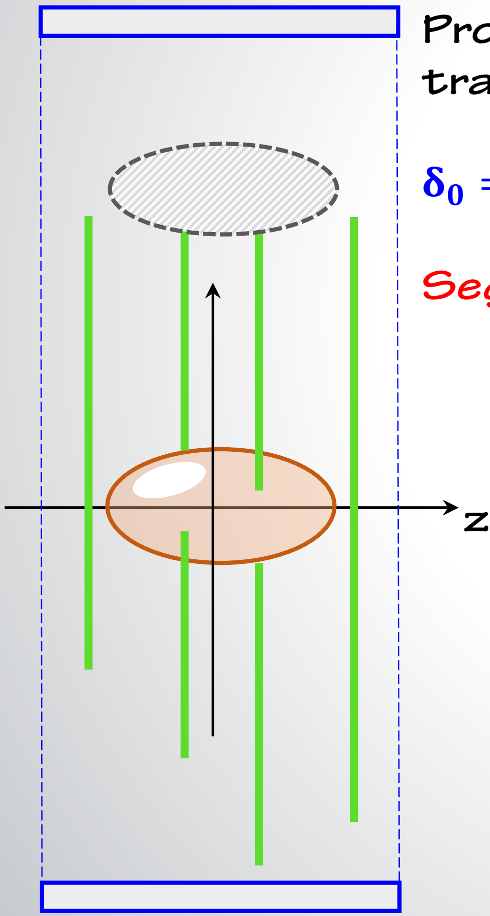
Projections obliques :

$\delta_n > 0$

Segment n (1…N)

TEP 3D

- Encodage TEP



Projections transverses :

$\delta_0 = 0$

Segment 0

COMPLETES

&

SUFFISANTES

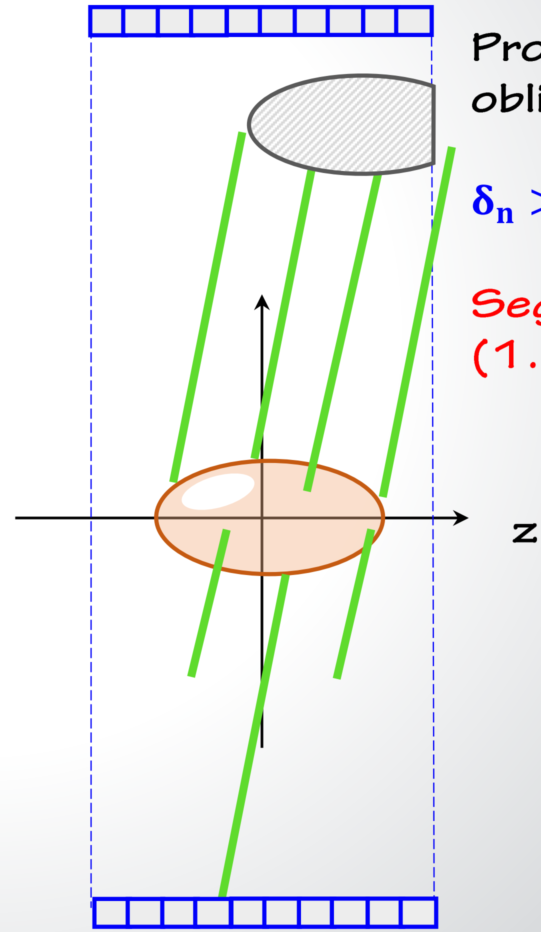
$$p(0, z, \theta, s) \rightarrow f(x, y, z)$$

TEP 3D

- Encodage TEP

TRONQUEES

$$p(\delta_{n>0}, z, \theta, s) \nrightarrow f(x, y, z)$$



Projections obliques :

$\delta_n > 0$

Segment n (1…N)

TEP 3D

- Encodage TEP

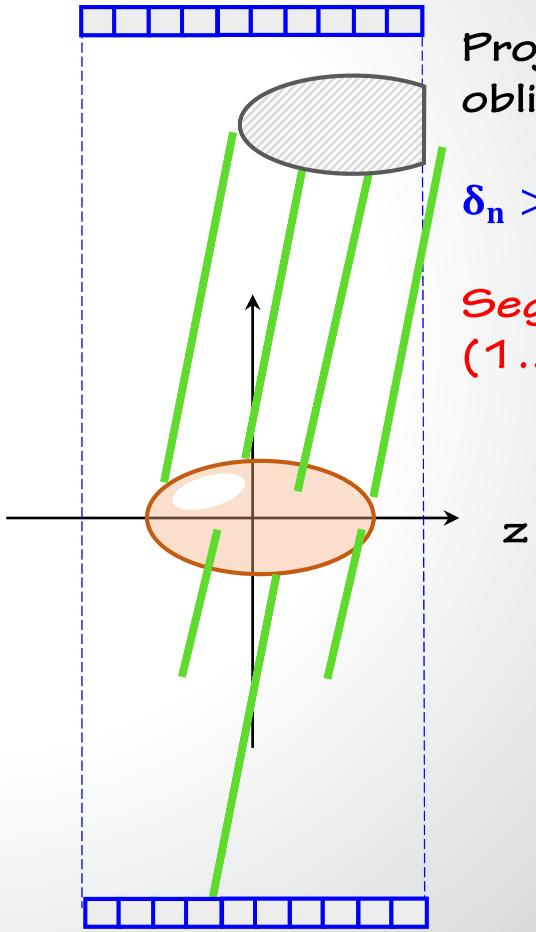
TRONQUEES

&

REDONDANTES

$$p(\delta_{n>0}, z, \theta, s) \nrightarrow f(x, y, z)$$

$$\left.\begin{array}{l} p(0, z, \theta, s) \\ p(\delta_1, z, \theta, s) \\ p(\delta_2, z, \theta, s) \\ p(\delta_3, z, \theta, s) \\ \vdots \end{array}\right\} \rightarrow f(x, y, z)$$



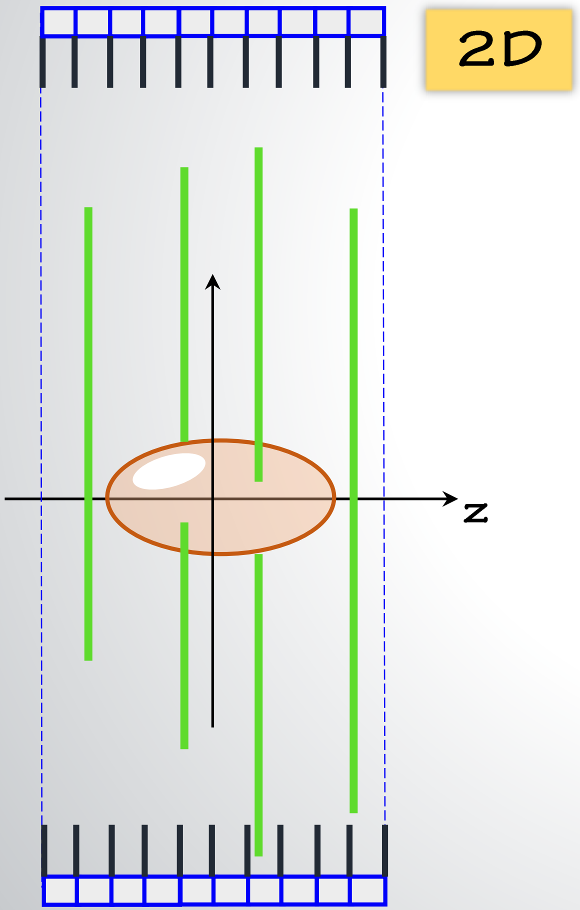
Projections obliques :

$\delta_n > 0$

Segment n (1…N)

TEP 3D

- Reconstruction



$$p(0, z, \theta, s) \rightarrow f(x, y, z)$$

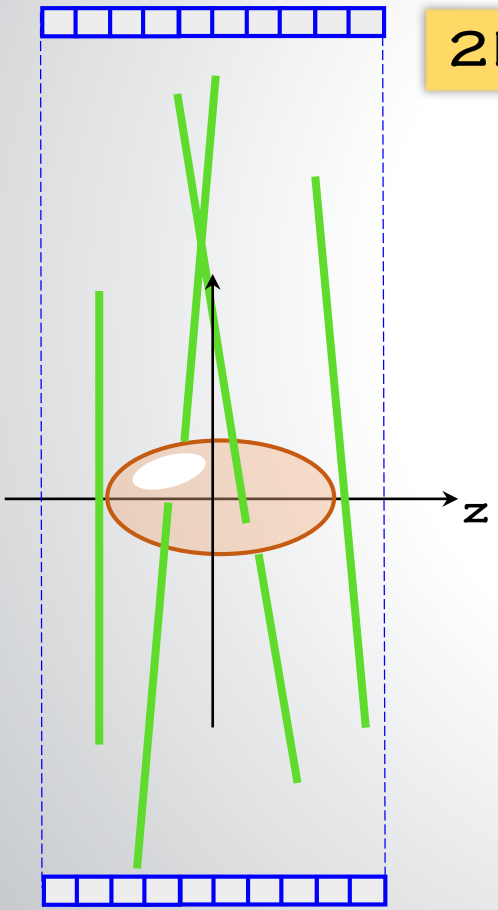
Collimation

Très rapide

SNR ↘

TEP 3D

- Reconstruction



2D ½

$$\underbrace{p(0, z, \theta, s)\ , p(\delta_1, z, \theta, s)\ , p(\delta_2, z, \theta, s)\ , \ldots}$$

$$\downarrow$$

$$p(0, z, \theta, s)$$

$$\downarrow$$

$$f(x, y, z)$$

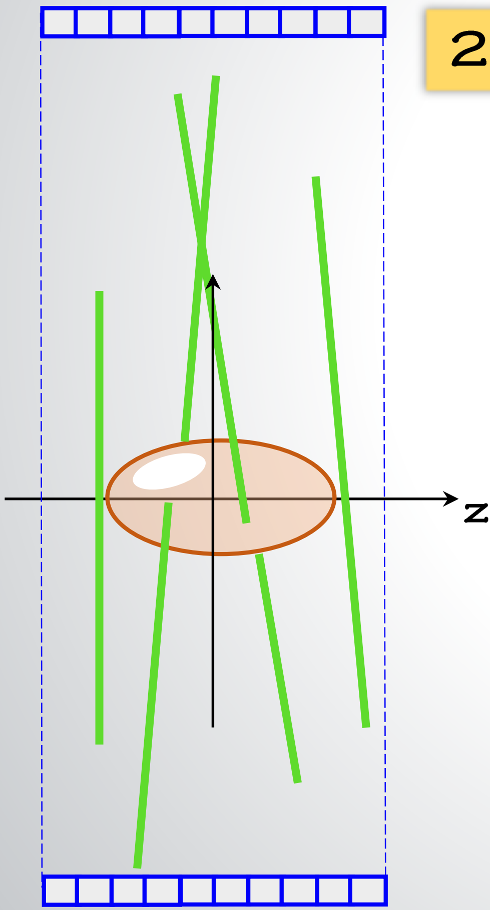
« Rebinning »

Rapide

Approximatif

TEP 3D

- Reconstruction



2D ½

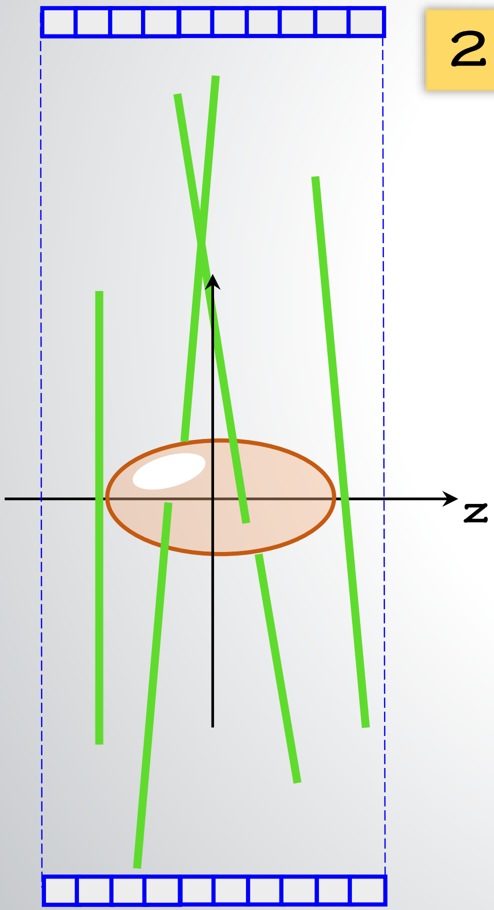
REBINNING EXACT

$$\hat{p}(0, \underbrace{\zeta, k, \omega}) = e^{-i\,k\,atan\left(\frac{\delta\zeta}{\omega'}\right)}\ \hat{p}(\delta, \underbrace{\zeta, k, \omega'})$$

$$\omega' = \sqrt{\omega^2 - \delta^2\zeta^2}$$

TEP 3D

- Reconstruction



2D ½

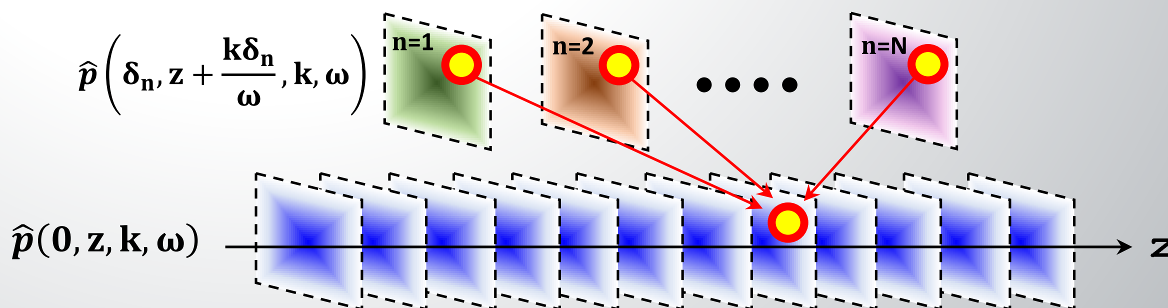
REBINNING EXACT

$$\hat{p}(0, \underbrace{\zeta, k, \omega}) = e^{-i\,k\,atan\left(\frac{\delta\zeta}{\omega'}\right)}\ \hat{p}(\delta, \underbrace{\zeta, k, \omega'})$$

$$\omega' = \sqrt{\omega^2 - \delta^2\zeta^2}$$

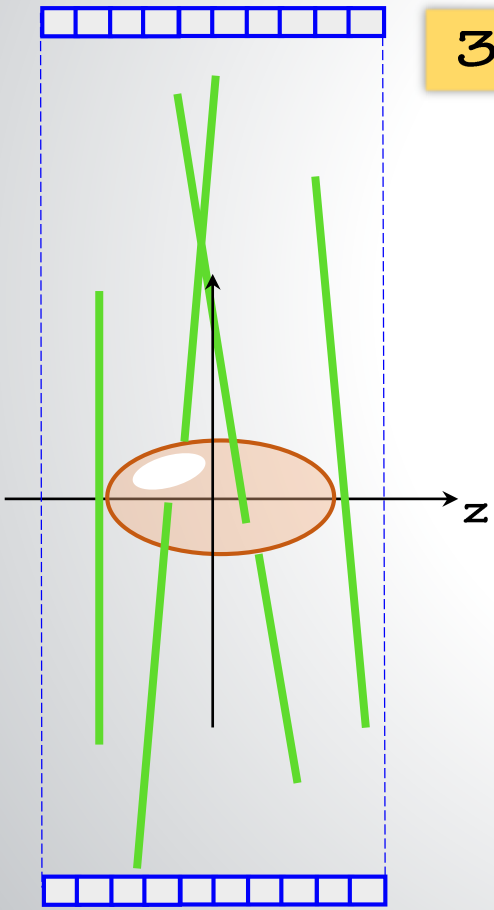
REBINNING APPROCHE

$$\hat{p}(0, z, \underbrace{k, \omega}) = \hat{p}\left(\delta, z + \frac{k\delta}{\omega}, \underbrace{k, \omega}\right)$$



TEP 3D

- Reconstruction



3D

$$\left.\begin{array}{c} p(0, z, \theta, s) \\ p(\delta_1, z, \theta, s) \\ p(\delta_2, z, \theta, s) \\ p(\delta_3, z, \theta, s) \\ \vdots \end{array}\right\} \rightarrow f(x, y, z)$$

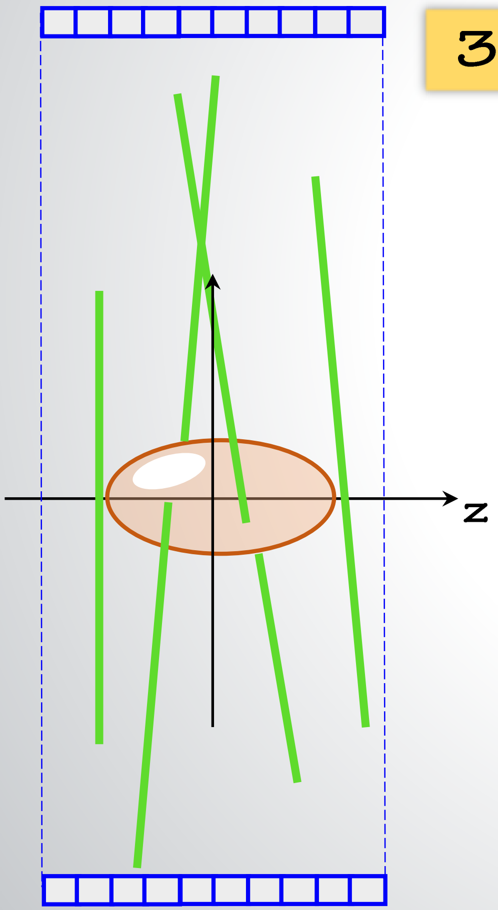
3D vrai

Reconstruction
analytique ou itérative

SNR ↗

TEP 3D

- Reconstruction



3D

ALGEBRIQUE - ITERATIF

$$\boldsymbol{p}(\delta, z, \theta, s) = \mathbf{R}_{3D}\, \boldsymbol{f}\,(x, y, z)$$

$$\mathbf{R}_{3D} : \mathbb{R}^3 \rightarrow \mathbb{R}^2 \times C_1 \times \{0 \ldots \mathrm{N}\}$$

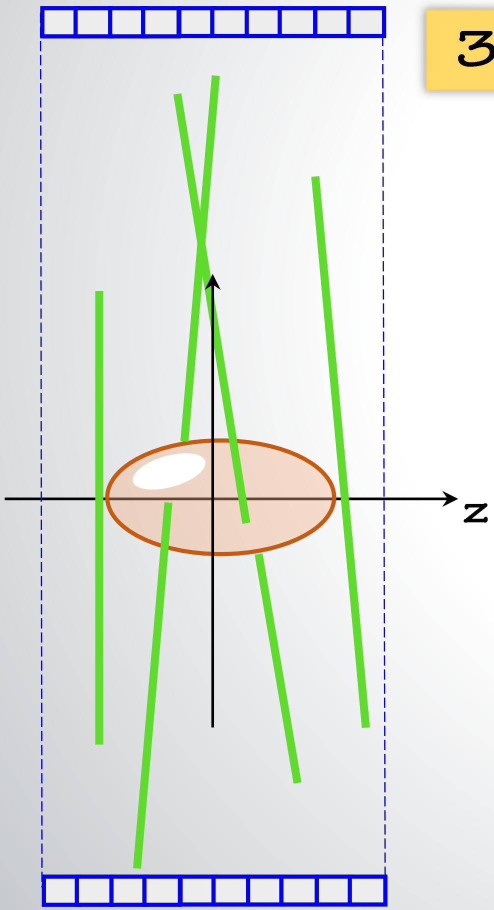
$$\boldsymbol{Dim}(\mathbf{R}_{3D}) \;\; >>$$

Stockage ?

Temps de calcul ...

TEP 3D

- Reconstruction



3D

ANALYTIQUE : Fourier ou RPF

!! Complétion des données obliques !!

$p(0, z, \theta, s) \rightarrow \tilde{f}(x, y, z)$

$\downarrow \mathbf{R}\,\tilde{f}$

$\tilde{p}(\delta_{n>0}, z, \theta, s)$ $\quad p(\delta_{n>0}, z, \theta, s)$

Fusion

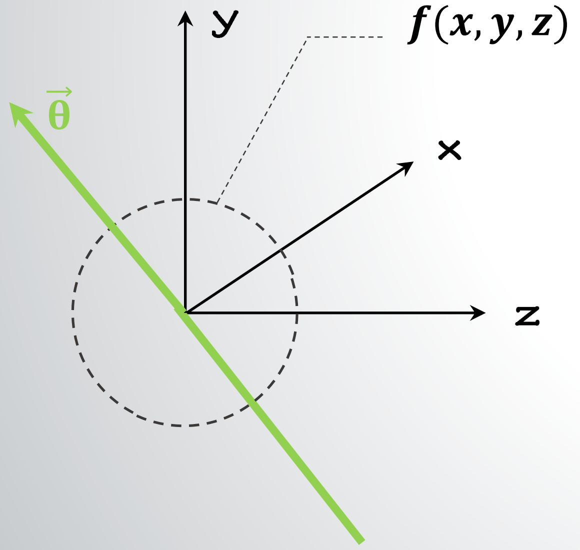
$$\left.\begin{array}{c} p(\delta_{n>0}, z, \theta, s) \\ p(0, z, \theta, s) \end{array}\right\} \rightarrow f(x, y, z)$$

TEP 3D

- Reconstruction

3D

Synthèse de Fourier 3D

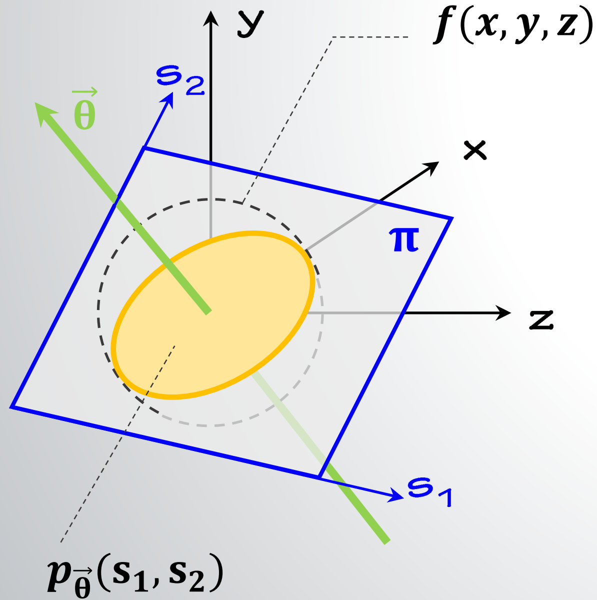


TEP 3D

- Reconstruction

3D

Synthèse de Fourier 3D

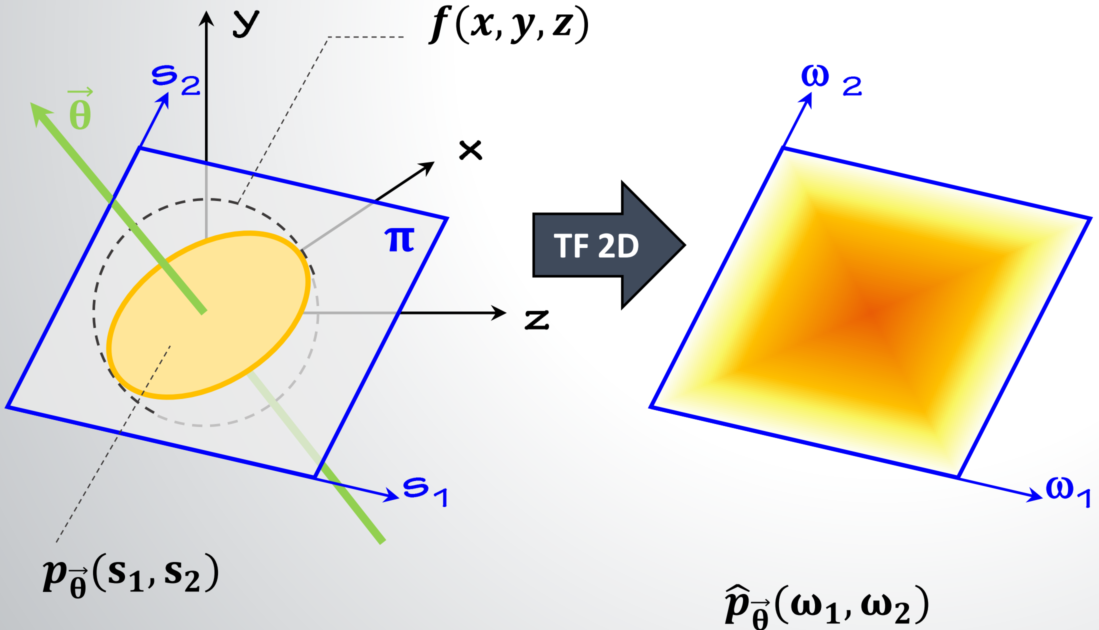


TEP 3D

- Reconstruction

3D

Synthèse de Fourier 3D



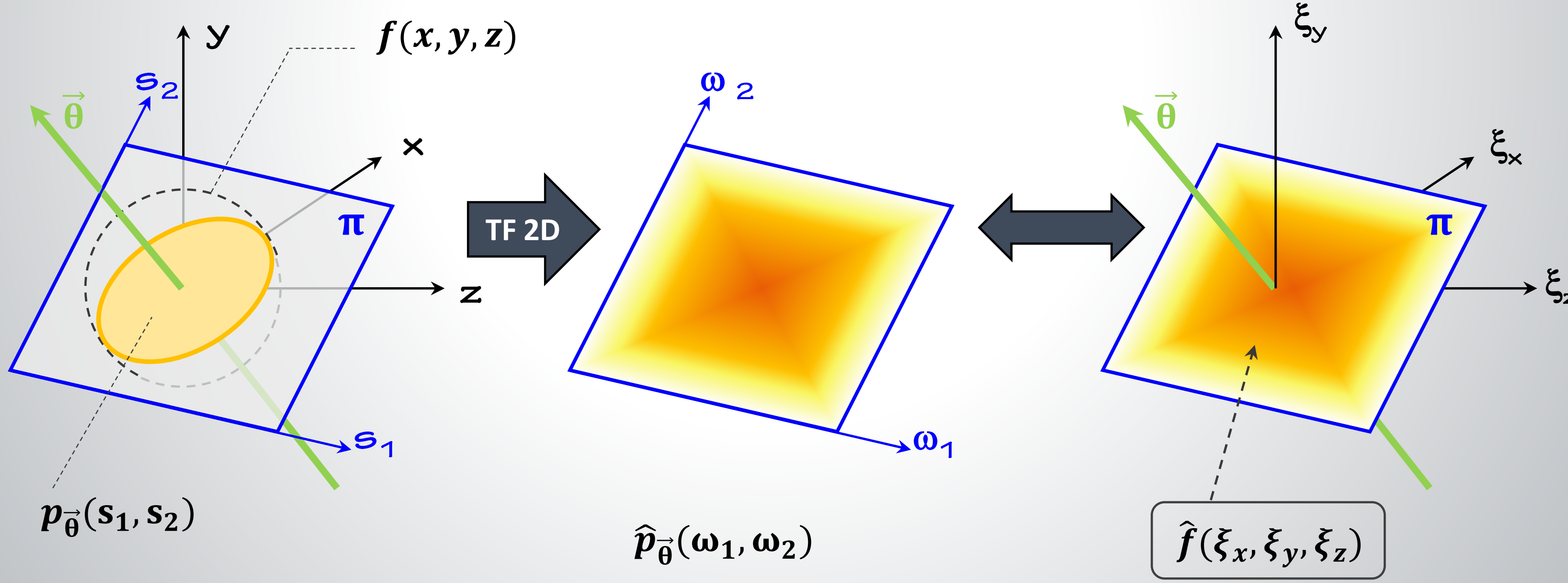
TEP 3D

- Reconstruction

3D

Synthèse de Fourier 3D

$$\hat{f}\left(\left[\xi_x, \xi_y, \xi_z\right] = R_{\vec{\theta}}[\omega_1, \omega_2]\right) = \hat{p}_{\vec{\theta}}(\omega_1, \omega_2)$$



TEP 3D

- Reconstruction

3D

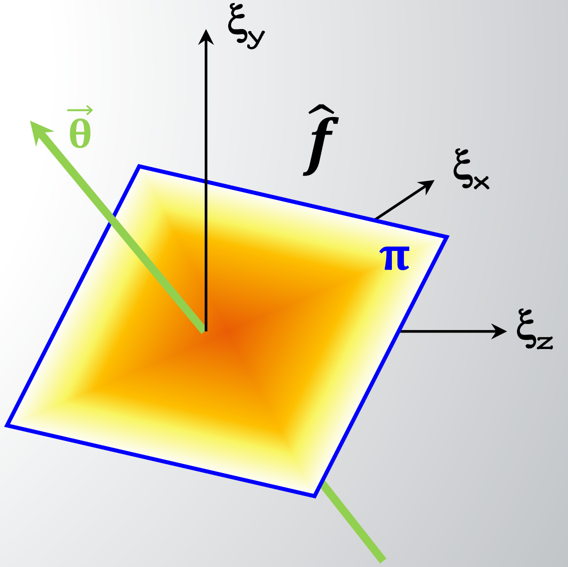
Synthèse de Fourier 3D

$$\hat{f}([\xi_x, \xi_y, \xi_z] = R_{\vec{\theta}}[\omega_1, \omega_2]) = \hat{p}_{\vec{\theta}}(\omega_1, \omega_2)$$

Condition d'ORLOV

Nécessaire et suffisante pour que π décrive $\mathbb{R}^3$

$$\vec{\theta} \in \Omega \subset S^2$$
$$\exists\, C \in S^2 : C \subset \Omega$$
$$\forall\, C \in S^2 : \Omega \cap C \neq \emptyset$$



TEP 3D

- Reconstruction

3D

Synthèse de Fourier 3D

$$\hat{f}\left(\left[\xi_x, \xi_y, \xi_z\right] = R_{\vec{\theta}}[\omega_1, \omega_2]\right) = \hat{p}_{\vec{\theta}}(\omega_1, \omega_2)$$

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