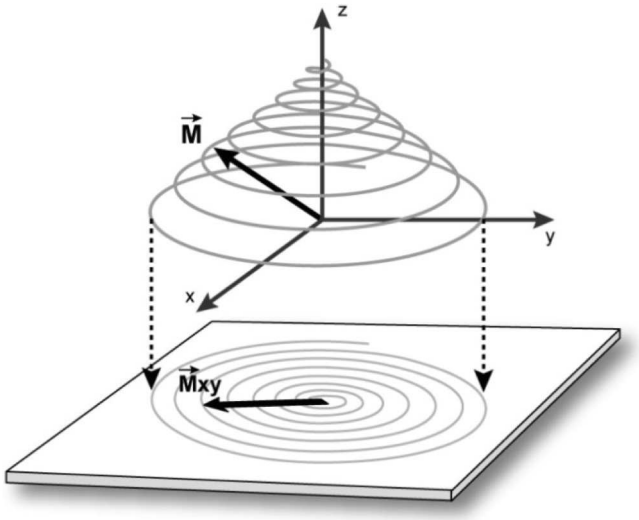
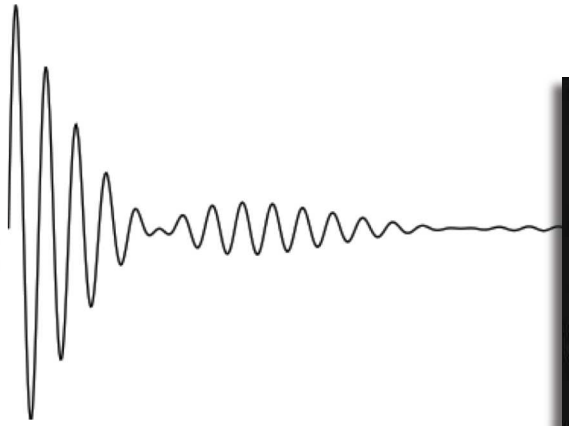
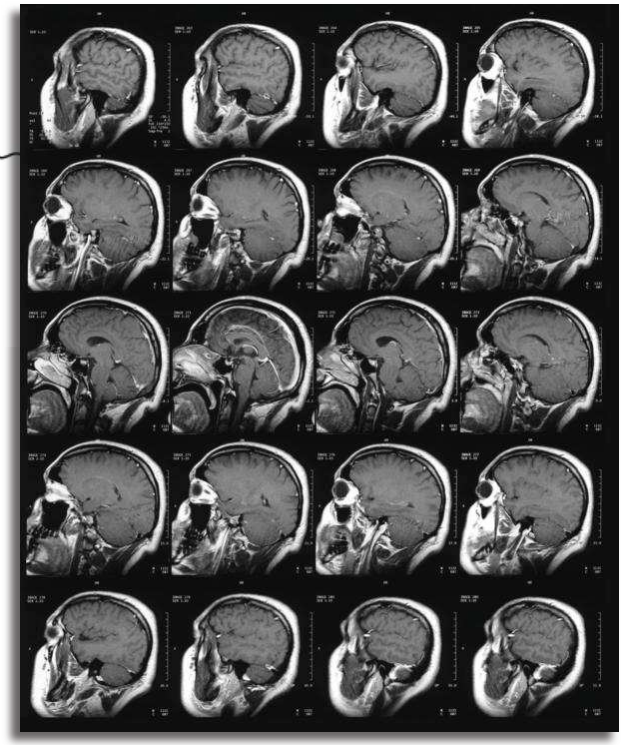
MAGNETISME & RMN

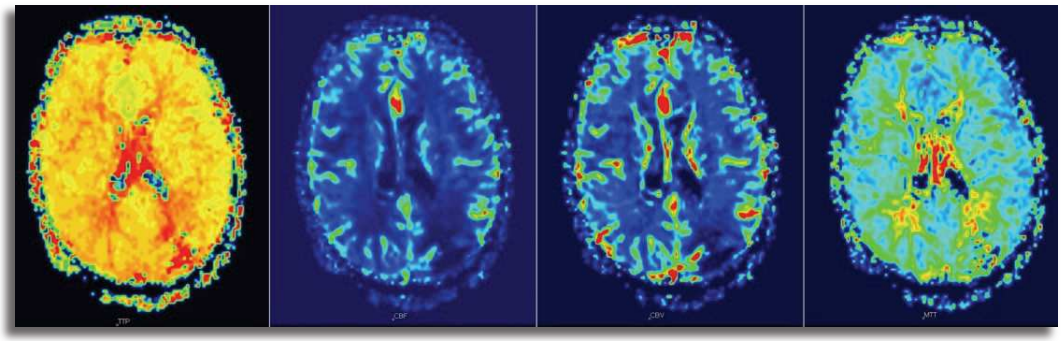
UE3 – Organisation des Appareils et des Systèmes

Bases physiques des méthodes d'exploration









Catherine Lozza- Fayçal Ben Bouallègue **2019–2020**

PACES

MAGNETISME & RMN

Plan du cours

Contexte

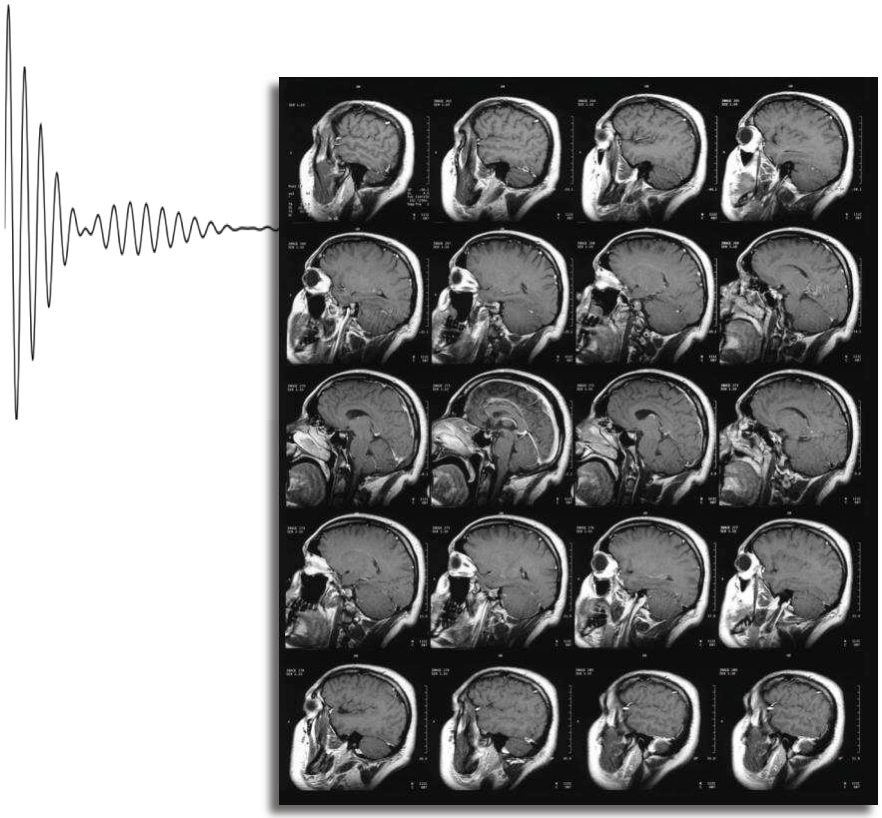
- **Champ et interaction électrique**
- **Champ et interaction magnétique**

Electron atomique
Mécanique quantique

- **Magnétisme dans la matière**
- **Magnétisme nucléaire**

RMN – principe
RMN – séquence
RMN – contraste

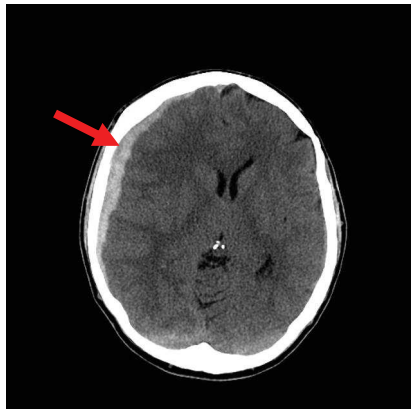
- **Applications : IRM & SRM**
- **Illustrations**

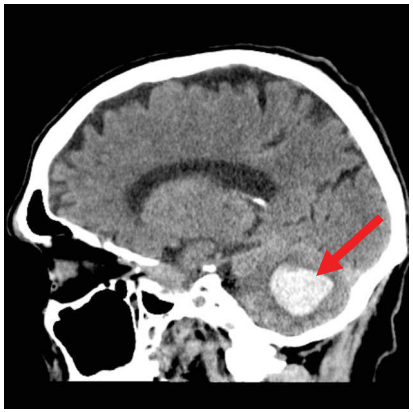


Contexte

Explorations morphologiques en coupe

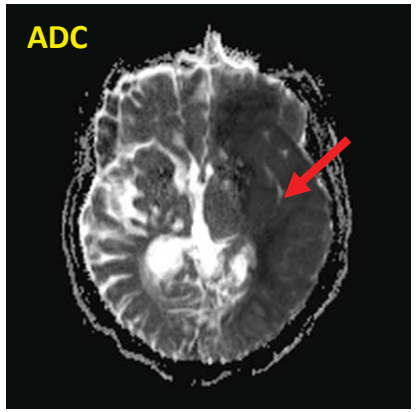
Scanner (TDM, tomodensitométrie)

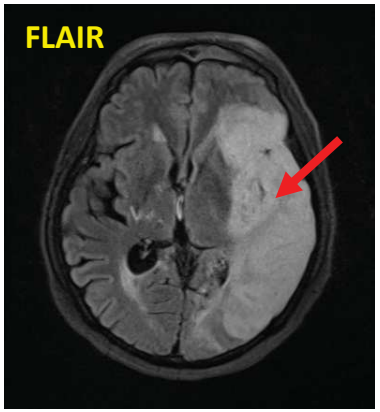


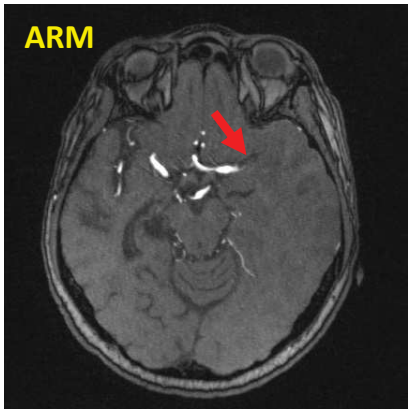


Carte de densité des tissus traversés par RX.
Comparaison signal émis / signal reçu

Résonance magnétique (IRM)







Contexte

Explorations morphologiques en coupe

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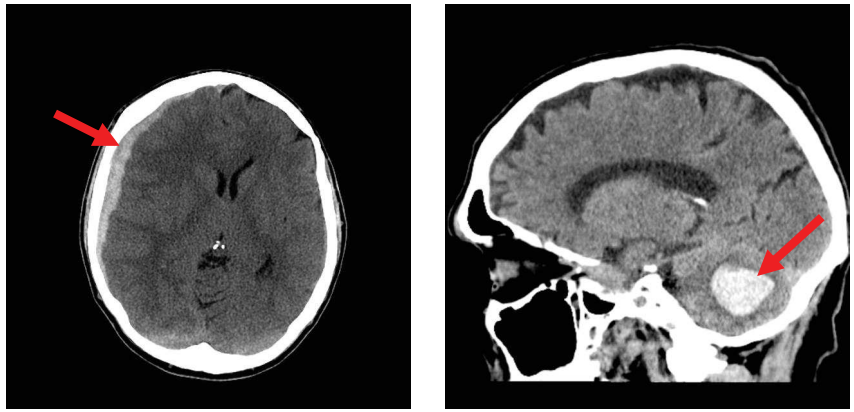
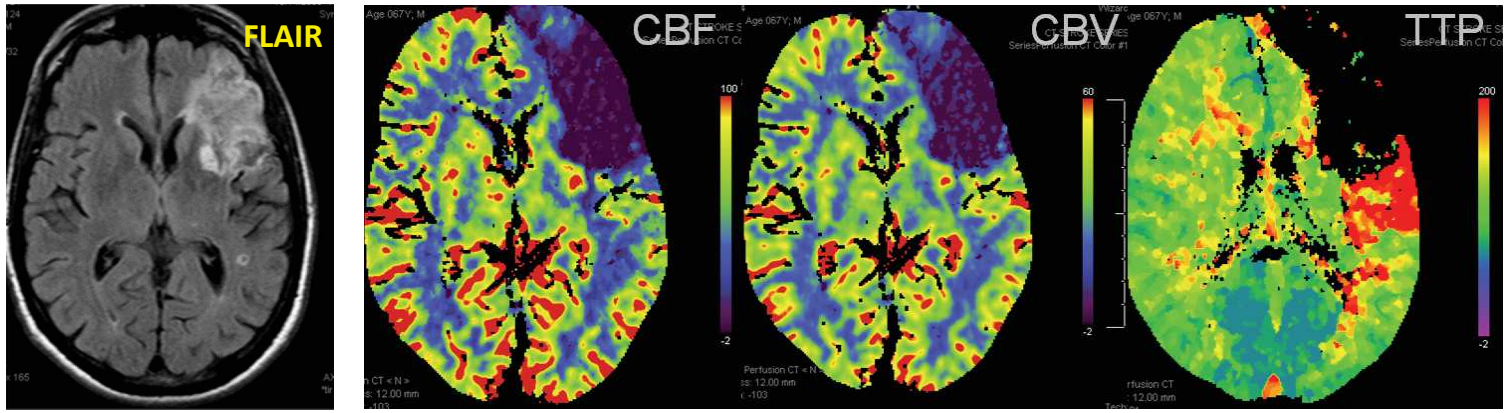


Résonance magnétique (IRM)

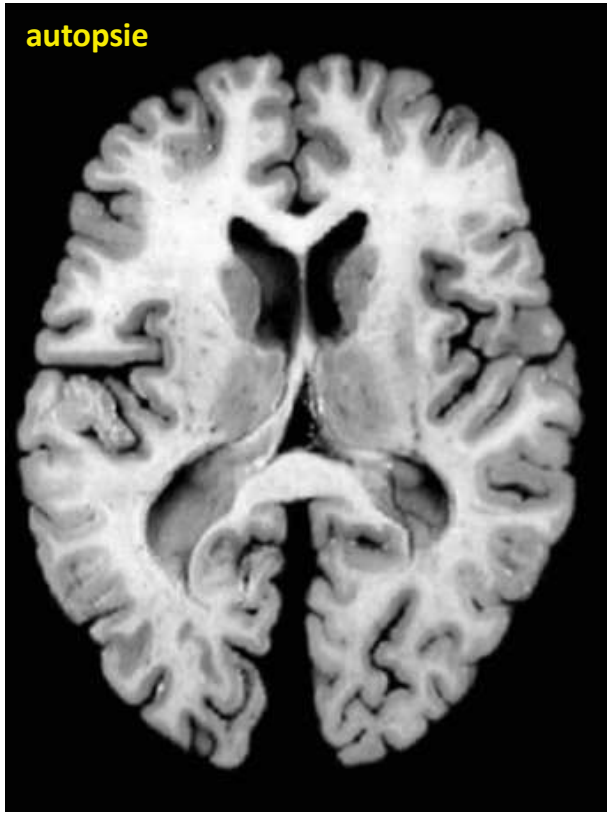


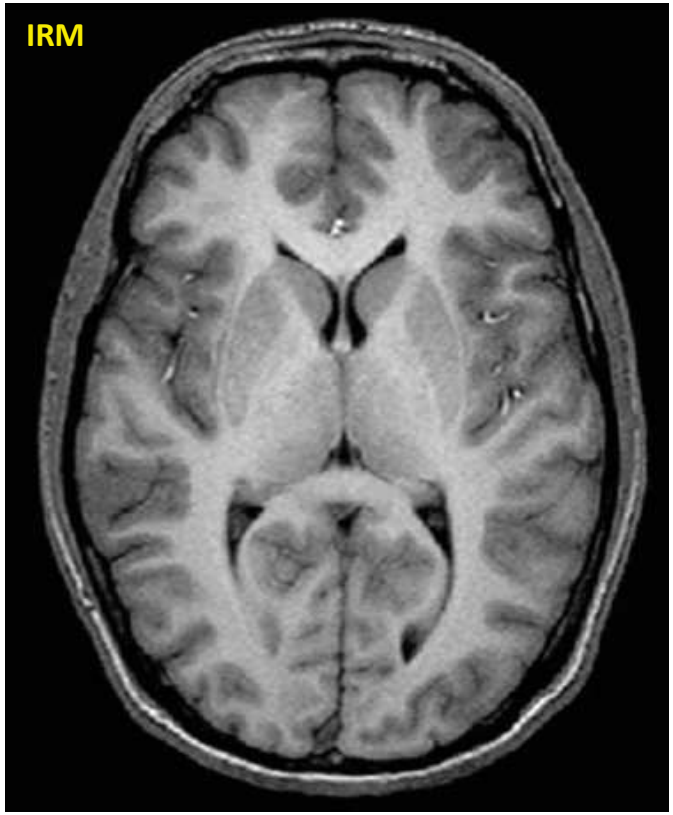
neuroangio.org

Contexte

Explorations morphologiques en coupe

IRM : excellent contraste des tissus mous





Contexte

Explorations morphologiques en coupe

IRM = examen de référence

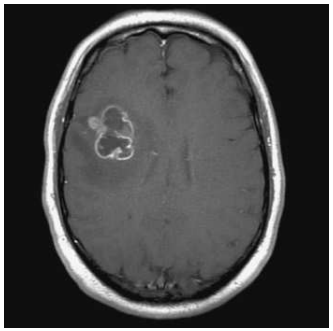
- Neurologie : neuro-vasculaire
  tumeurs
  pathologies SB
  infections
  MAV

- Cardiologie : fonction VG/VD
  cardiopathies…

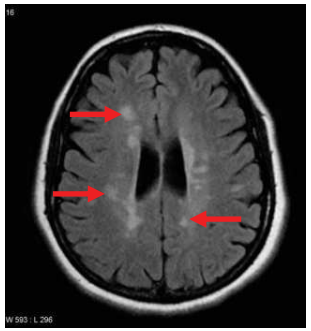
- Ostéo-articulaire
- Rachis

- Abdomen : foie, pancréas
- Pelvis : rectum, prostate

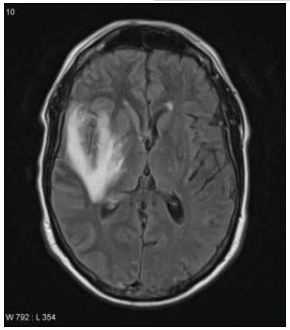
- Sein



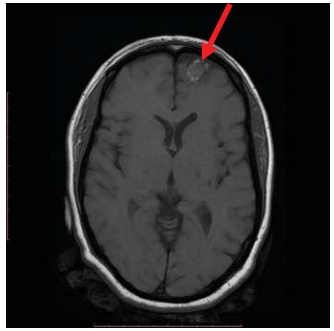
glioblastome



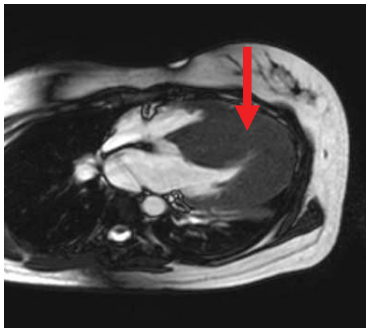
SEP



Encéphalite HSV



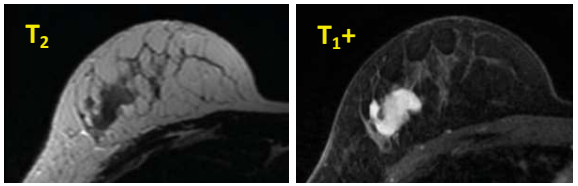
cavernome



CMH

Compression médullaire

HNF



CLI sein droit

Objectifs

- **Expliciter les propriétés magnétiques de l'électron / du proton : spin, moment magnétique, énergie magnétique.**
- **Décrire le comportement d'un dipôle magnétique dans un champ magnétique externe : précession, fréquence de Larmor, alignement.**
- **Exposer le principe d'une expérience de RMN : préparation (aimantation), perturbation (résonance), recueil du signal.**
- **Détailler les étapes d'une séquence d'acquisition RMN ainsi que les différents paramètres ajustables (temps de répétition, temps d'écho, angle de bascule).**
- **Décrire les bases physiques des phénomènes de relaxation (T_1 et T_2).**
- **Analyser la manière dont le choix des paramètres ajustables influe sur le contraste dans l'image IRM.**

Rappels & conventions

Grandeurs scalaires :

x, y, z, t

m, q, e

$V(x, y, z)$

$\hbar = h/2\pi$

Grandeurs vectorielles :

$\mathbf{x} = (x, y, z)$

$\mathbf{F}, \mathbf{\Gamma}$

$\mathbf{B}(\mathbf{x})$

$\mathbf{L} = (L_x, L_y, L_z)$

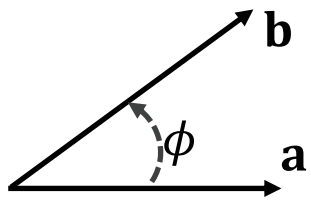
$\omega = |\boldsymbol{\omega}|$

$$\boldsymbol{\nabla} f = \left(\frac{\partial f}{\partial x}\,; \frac{\partial f}{\partial y}\,; \frac{\partial f}{\partial z}\right)$$

Produit scalaire :

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$$

$$= a\, b \cos(\phi)$$



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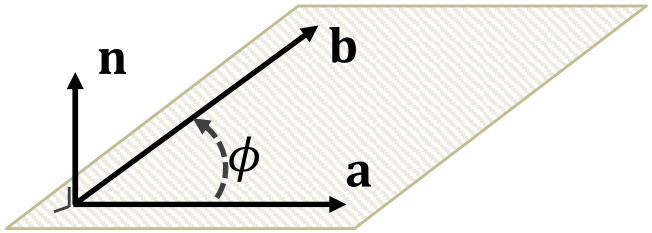
Produit scalaire :

$$\mathbf{a} \cdot \mathbf{b} = a\, b \cos(\phi)$$

Produit vectoriel :

$$\mathbf{a} \wedge \mathbf{b} = a\, b \sin(\phi)\, \mathbf{n}$$



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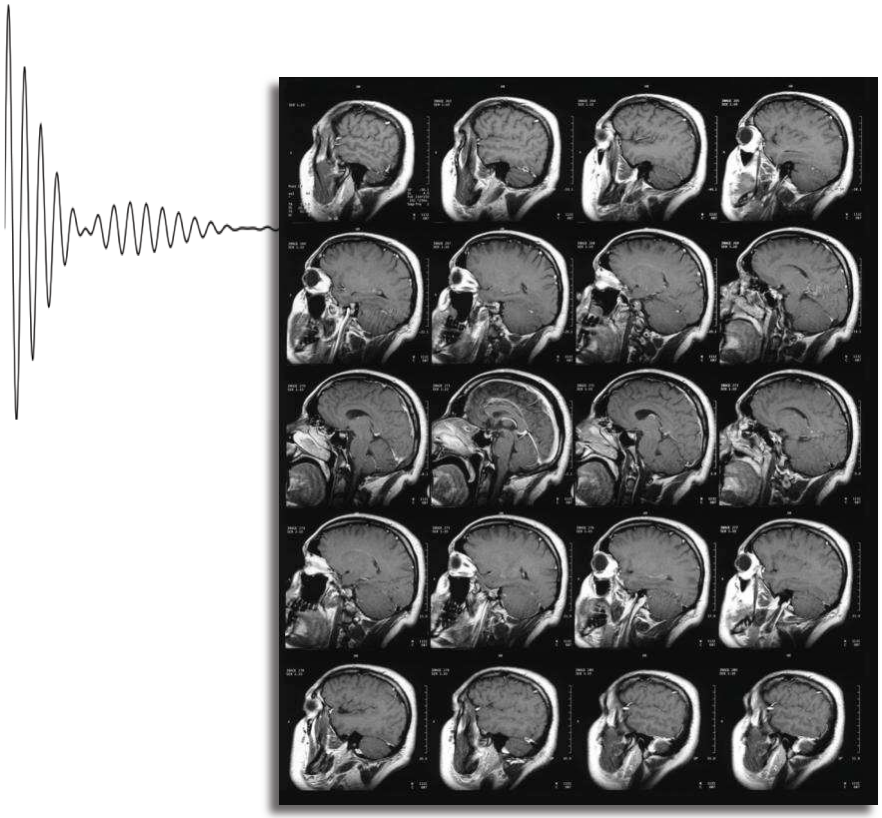
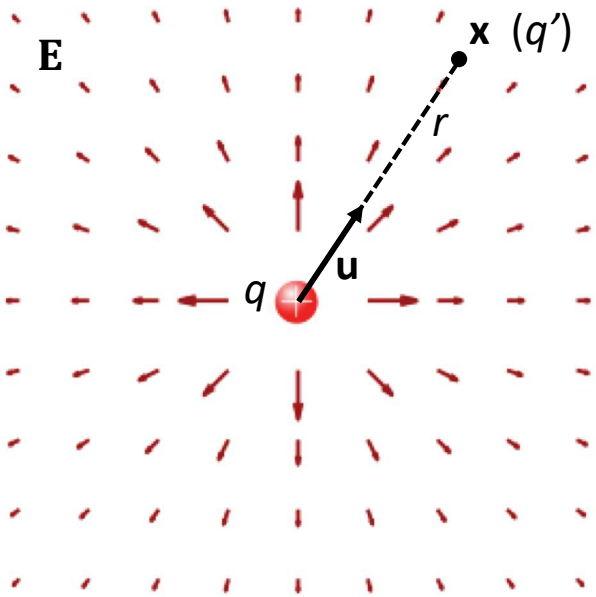


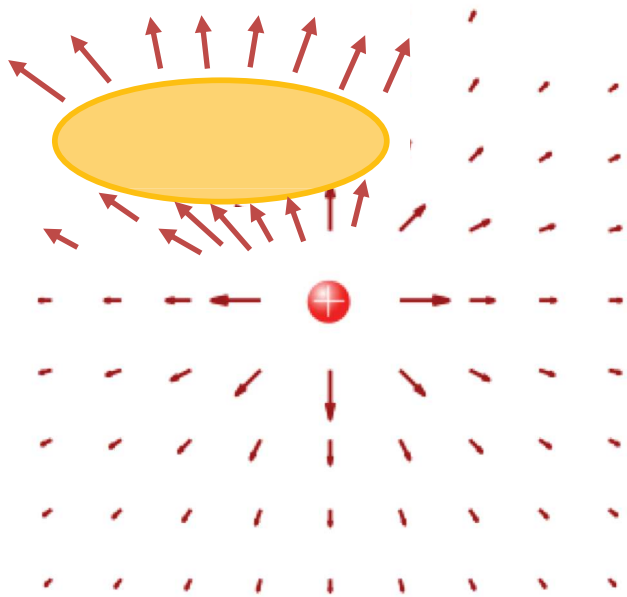
Champ & interaction électrique

Source = charge q statique (monopôle électrique)

Champ : $\mathbf{E}(\mathbf{x}) = \frac{q}{4\pi\varepsilon_0\, r^2}\mathbf{u}$ [Vm^{-1}]

ε_0= permittivité diélectrique du vide





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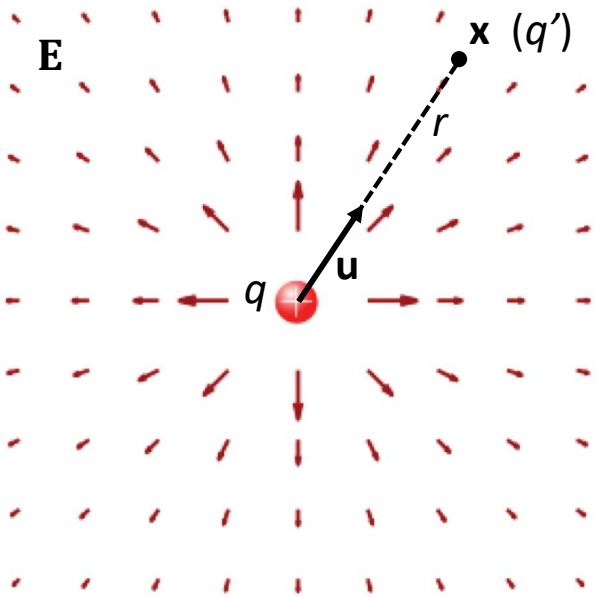
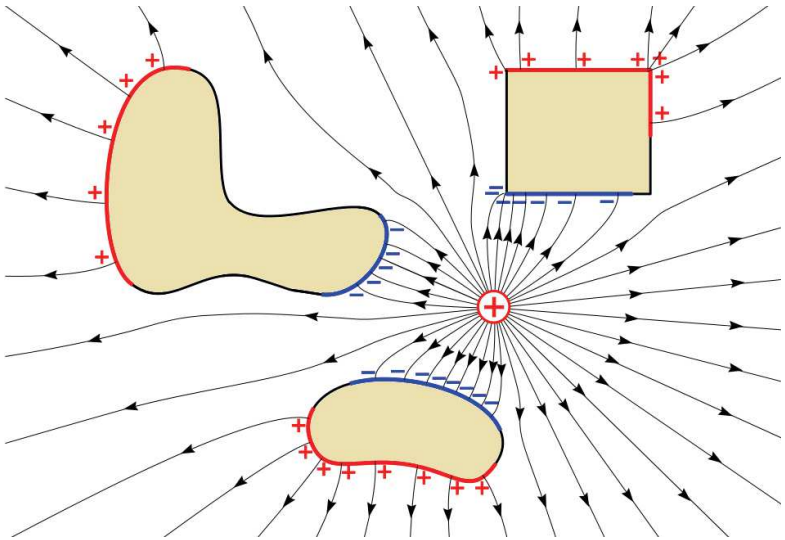
ε_0= permittivité diélectrique du **vide**





Dans un matériau: $\varepsilon = (1+\chi)\varepsilon_0 > \varepsilon_0$

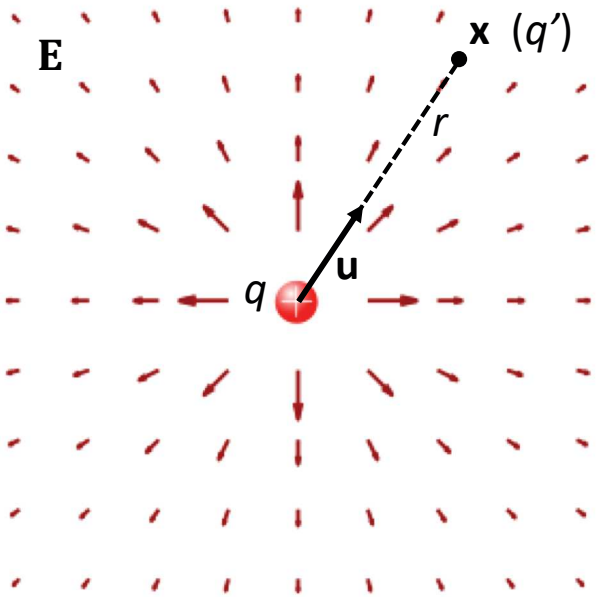
Champ & interaction électrique

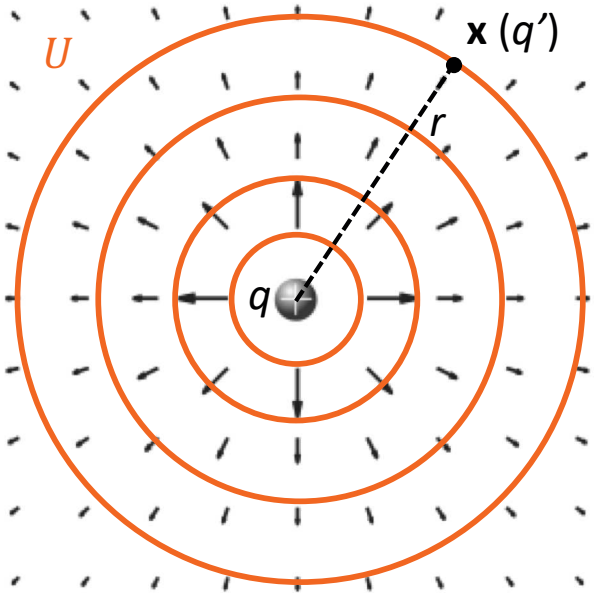
Source = charge q statique (monopôle électrique)

Champ : $\mathbf{E}(\mathbf{x}) = \frac{q}{4\pi\varepsilon_0\, r^2}\mathbf{u}$ [Vm^{-1}]

Potentiel : $V(\mathbf{x}) = \frac{q}{4\pi\varepsilon_0\, r}$ [V]

Energie : $U(\mathbf{x}) = q'V(\mathbf{x}) = \frac{q\, q'}{4\pi\varepsilon_0\, r}$ [J]



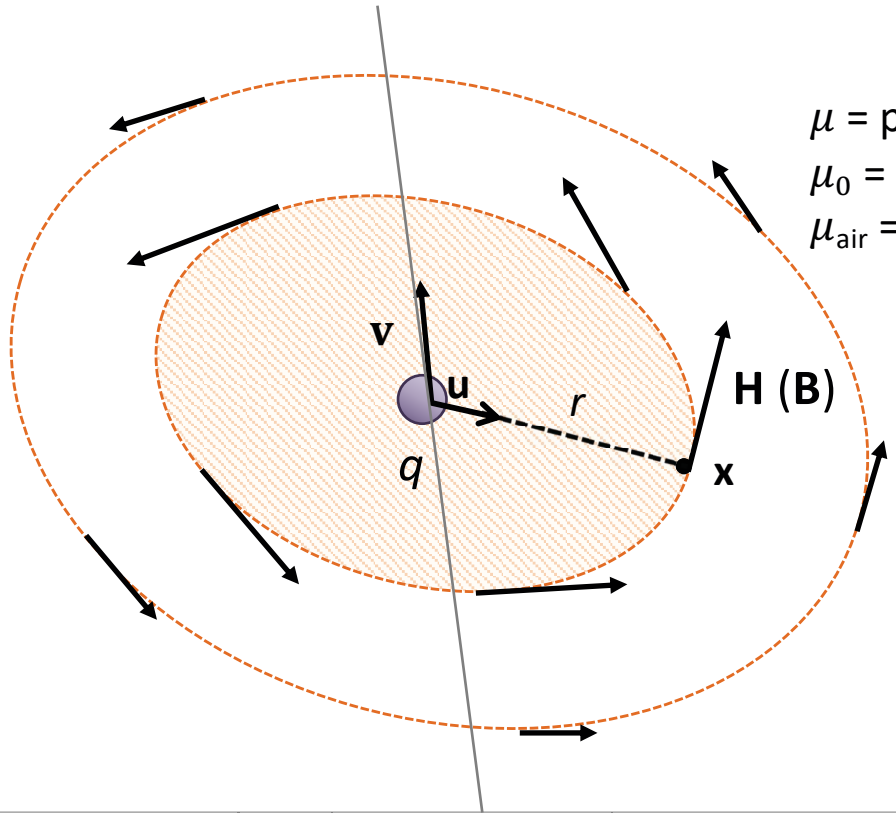


Champ & interaction magnétique

Source = charge <u>en mouvement</u> (q, **v**)

Champ magnétique : $\mathbf{H}(\mathbf{x}) = \frac{q}{4\pi\, r^2}\, \mathbf{v} \wedge \mathbf{u}$ [Am^{-1}]

Champ (d'induction) magnétique: $\mathbf{B}(\mathbf{x}) = \mu\, \mathbf{H}(\mathbf{x}) = \frac{\mu\, q}{4\pi\, r^2}\, \mathbf{v} \wedge \mathbf{u}$ [T Tesla]



μ = perméabilité magnétique de la matière
μ_0 = perméabilité magnétique du vide
$\mu_{air} = \mu_0$

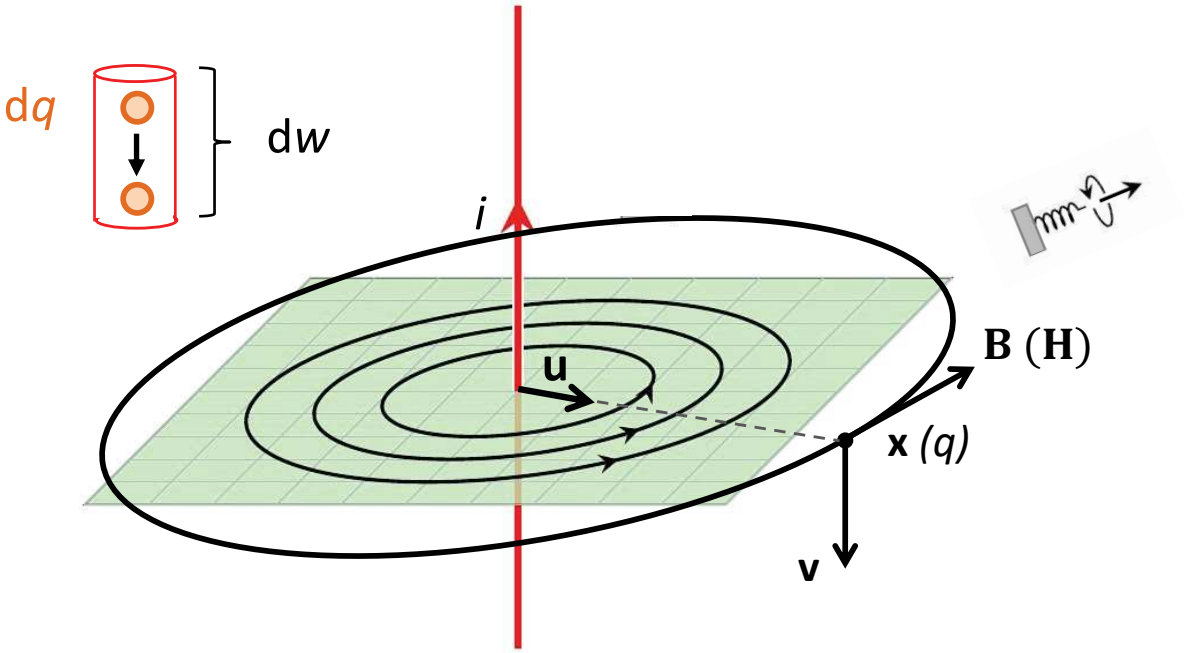
Champ & interaction magnétique

Source : courant passant dans un fil électrique droit : $i = \mathrm{d}q/\mathrm{d}t$

Vitesse : $\mathbf{v} = \mathrm{d}w/\mathrm{d}t$

Champ magnétique : $\mathbf{H}(\mathbf{x}) = \int_{fil} \frac{i}{4\pi\, r^2}\, \mathrm{d}\mathbf{w} \wedge \mathbf{u} = \frac{i}{2\pi\, r}\, \mathbf{w} \wedge \mathbf{u}$ [Am^{-1}]

Champ (d'induction) magnétique: $\mathbf{B}(\mathbf{x}) = \mu\, \mathbf{H}(\mathbf{x}) = \frac{\mu\, i}{2\pi\, r}\, \mathbf{w} \wedge \mathbf{u}$ [T Tesla]



Fil électrique rectiligne

Champ & interaction magnétique

Source : courant passant dans une boucle fermée: courant *i*

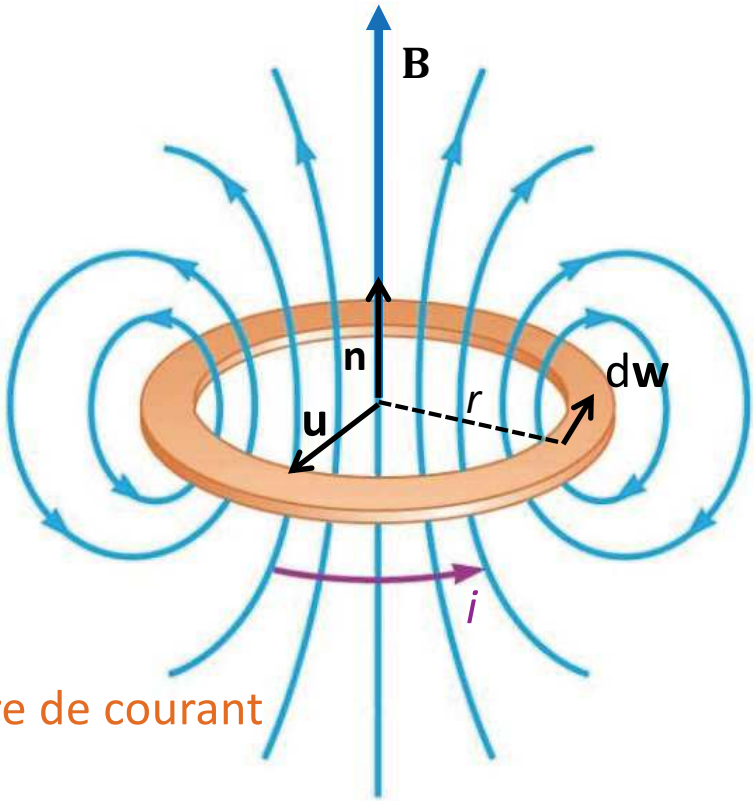
Champ : chaque élément d'une boucle de courant d**w** produit le champ magnétique élémentaire d**B**

$$\mathrm{d}\mathbf{B} = \frac{\mu i}{4\pi\, r^2}\, \mathrm{d}\mathbf{w} \wedge \mathbf{u}$$

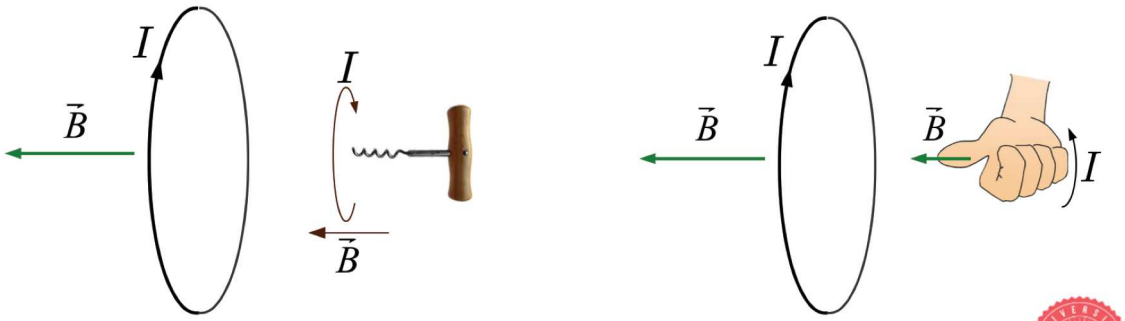
$$\mathrm{d}\mathbf{B} \perp \mathrm{d}\mathbf{w}$$

$$\mathrm{d}\mathbf{B} \perp \mathbf{u}$$

$$\mathbf{B} = \int_{spire} \frac{\mu i}{4\pi\, r^2}\, \mathrm{d}\mathbf{w} \wedge \mathbf{u} = \frac{\mu i}{2\, r}\, \mathbf{n}$$



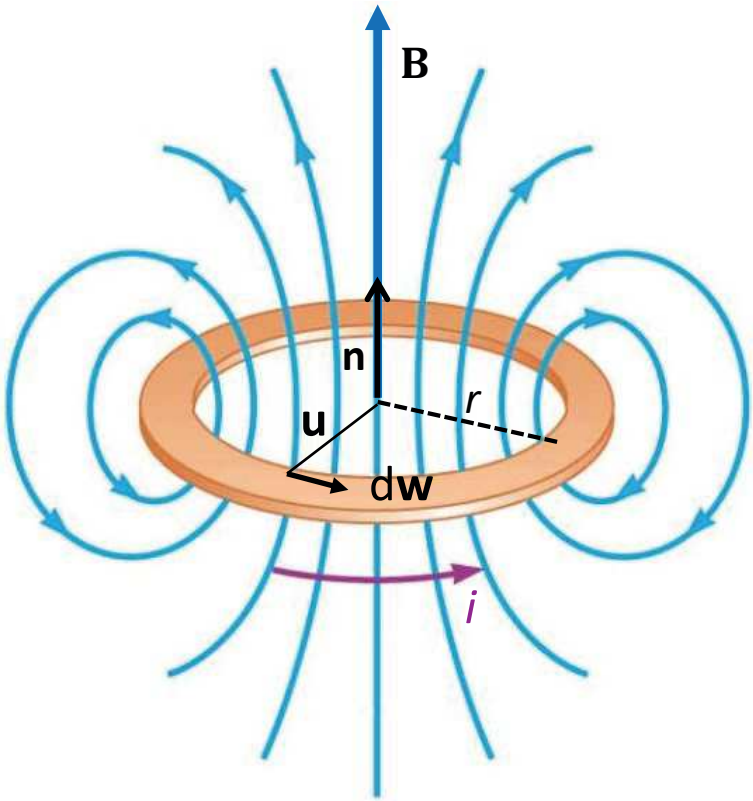
Spire de courant



Champ & interaction magnétique

Source = boucle de courant *i*

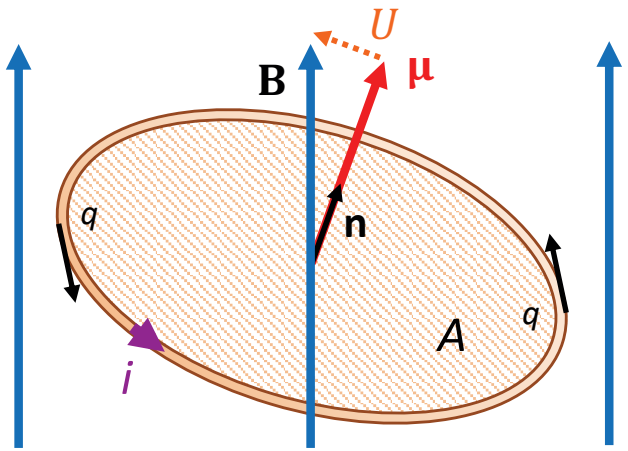
Champ : $\mathbf{B} = \frac{\mu\, i}{2r}\mathbf{n}$



Cible = dipôle magnétique

Moment magnétique : $\boldsymbol{\mu} = A\, i\, \mathbf{n}$ [Am²] ou [JT⁻¹]

Energie : $U = -\boldsymbol{\mu} \cdot \mathbf{B}$ [J]



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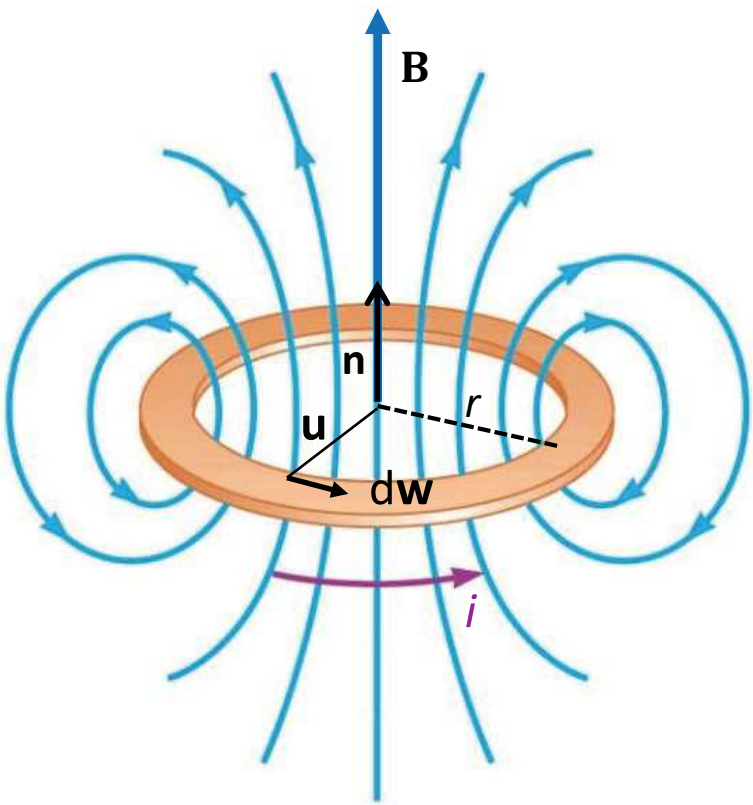

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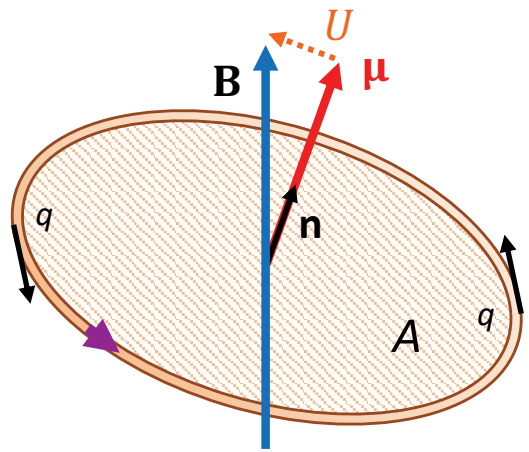
$U = 0$ si $\boldsymbol{\mu} \perp \mathbf{B}$
$U < 0$ si $\boldsymbol{\mu}$ aligné sur $\mathbf{B}$
$U > 0$ si $\boldsymbol{\mu}$ opposé sur $\mathbf{B}$



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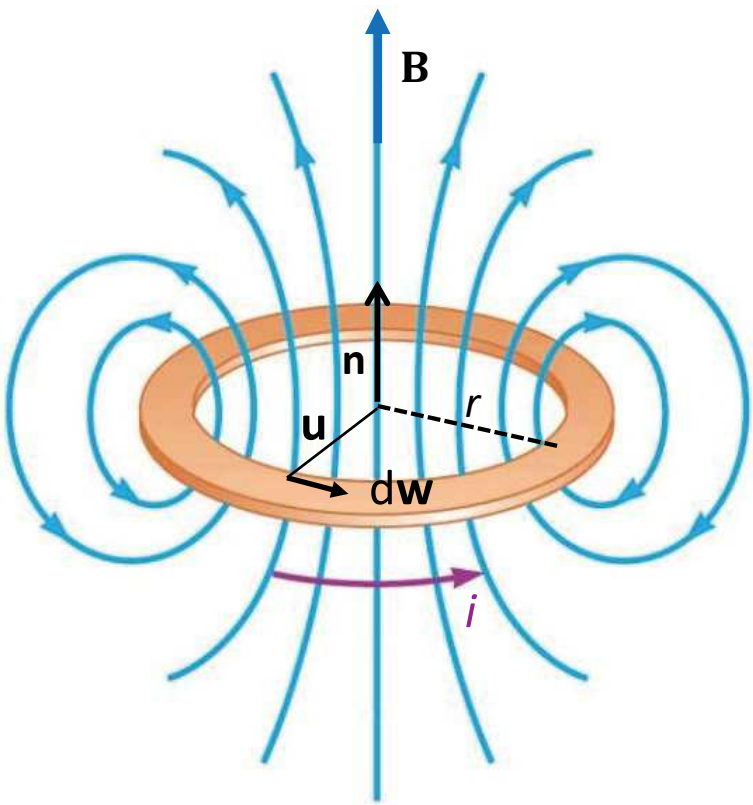

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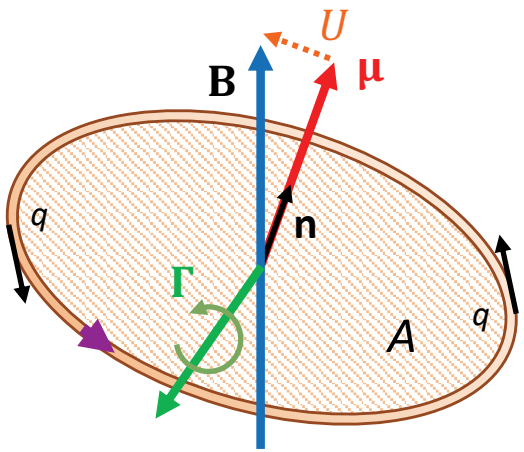
Energie : $U = -\boldsymbol{\mu} \cdot \mathbf{B}$ [J]

Couple : $\boldsymbol{\Gamma} = \boldsymbol{\mu} \wedge \mathbf{B}$ [Nm]



MAGNETISME & RMN

Plan du cours

Contexte

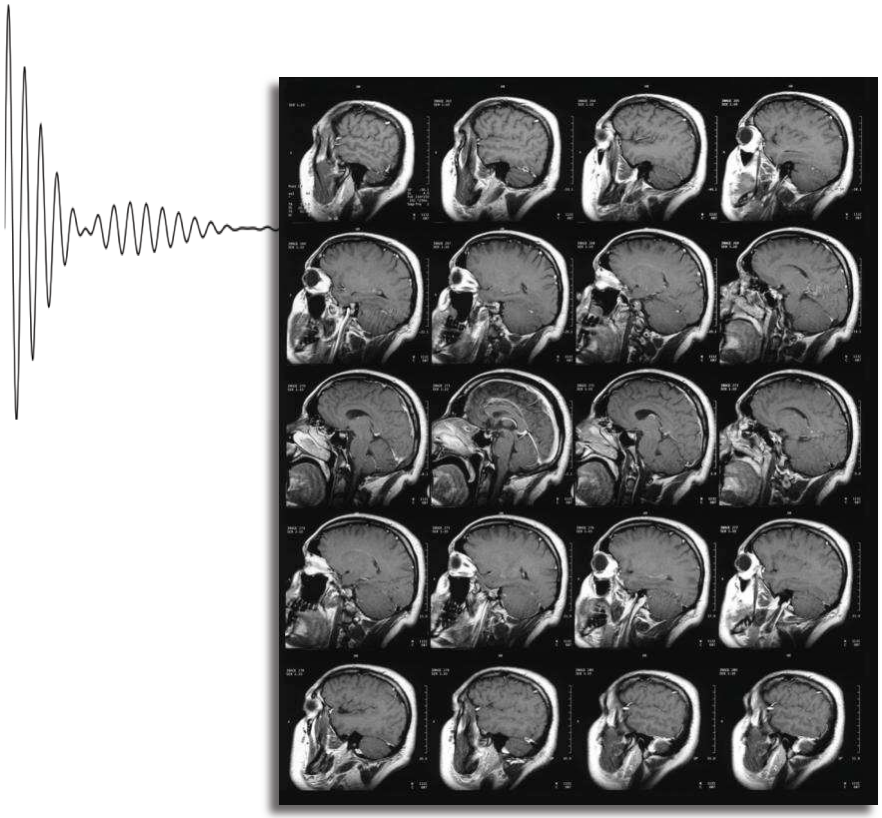
Champ et interaction électrique
Champ et interaction magnétique

Electron atomique
Mécanique quantique

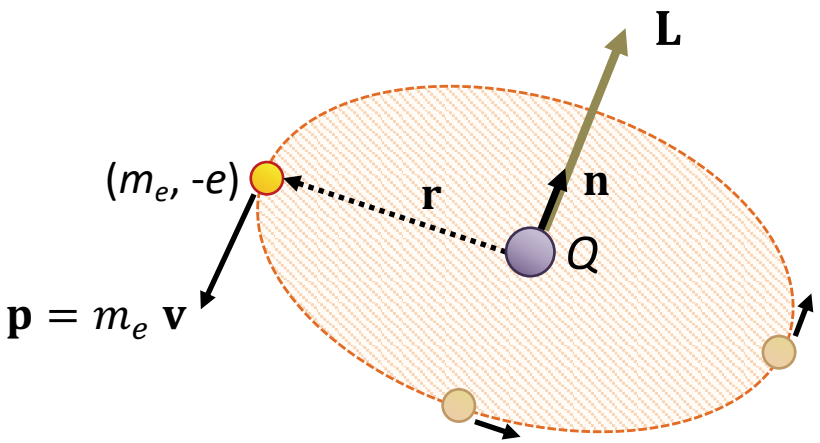
Magnétisme dans la matière
Magnétisme nucléaire

RMN – principe
RMN – séquence
RMN – contraste

Applications : IRM & SRM
Illustrations



Cas de l'électron atomique



$$i = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{-e}{T} = \frac{-e}{2\pi r/v} = \frac{-e\,v}{2\pi r}$$

ONDES OEM PROPAGATION DUALITE ONDE CORPUSCULE ATOME

MODELE ATOMIQUE DE BOHR

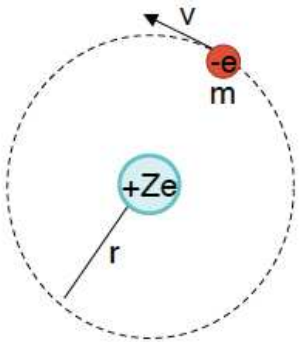
- Dualité onde-électron ⇒ quantification
- Quantification : $\left.\begin{array}{l}\lambda = \dfrac{h}{mv} \\ 2\pi\, r = n\lambda\end{array}\right\} \Rightarrow 2\pi\, r = \dfrac{nh}{mv}$

  $\Rightarrow mv\,r = n\hbar$ où $\hbar = \dfrac{h}{2\pi}$

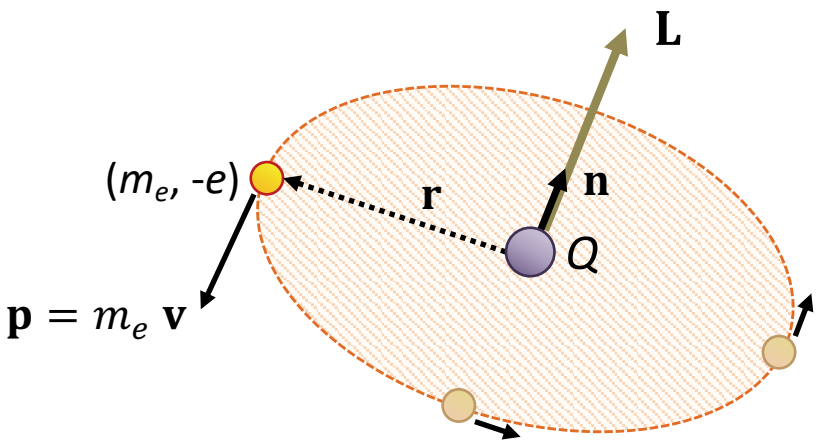
- D'où la quantification du M^t cinétique de l'e$^-$:

$$\|\vec{L}\| = \|\vec{p} \wedge \vec{r}\| = mv\,r = n\hbar \quad \text{avec } n = 1, 2, \ldots$$

hydrogénoïde : 1e$^-$



Cas de l'électron atomique

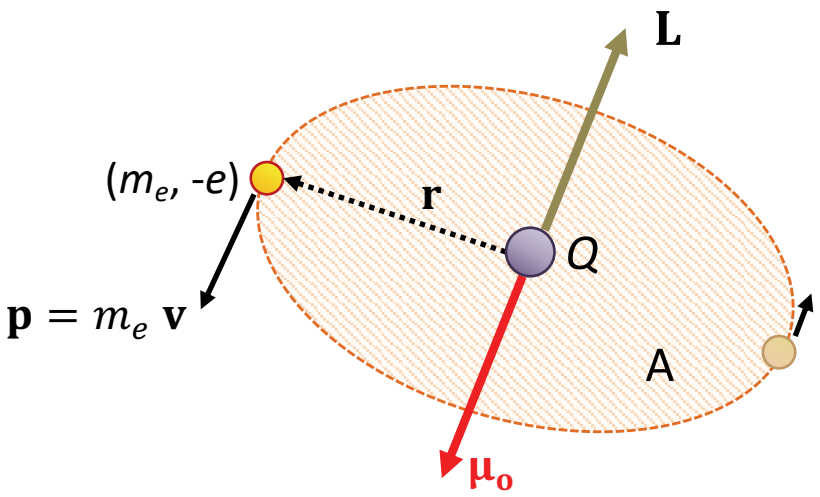


$$i = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{-e}{T} = \frac{-e}{2\pi r/v} = \frac{-e\,v}{2\pi r}$$

Moment cinétique : orbital $\mathbf{L} = \mathbf{r} \wedge \mathbf{p} = r\,m_e v\,\mathbf{n}$

Quantité de mouvement: $\mathbf{p} = m_e\mathbf{v}$

Cas de l'électron atomique



$$i = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{-e}{T} = \frac{-e}{2\pi r/v} = \frac{-e\,v}{2\pi r}$$

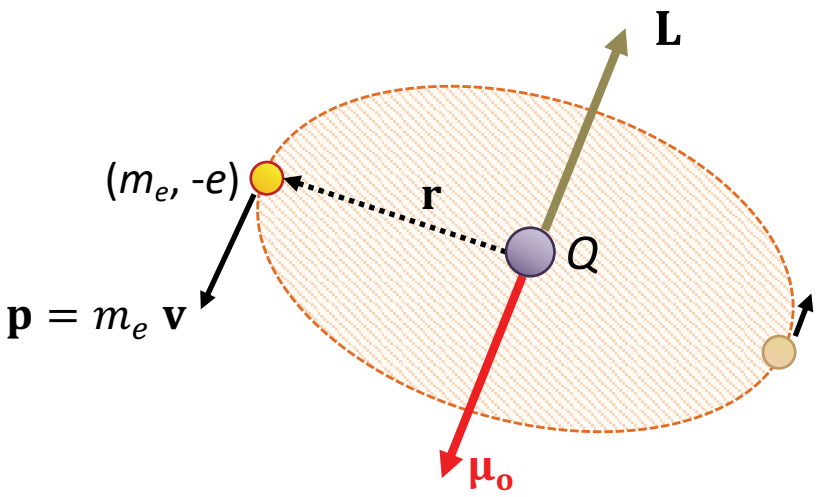
Moment cinétique : orbital $\quad \mathbf{L} = \mathbf{r} \wedge \mathbf{p} = r\,m_e v\,\mathbf{n}$

Moment magnétique : orbital $\quad \boldsymbol{\mu}_o = A\,i\,\mathbf{n} = -\,\pi r^2\,\dfrac{e\,v}{2\pi r}\,\mathbf{n} = -\,\dfrac{r\,e\,v}{2}\,\mathbf{n}$

$$\frac{\boldsymbol{\mu}_o}{\boldsymbol{L}} = -\frac{e}{2\,m_e}$$

$\boldsymbol{\mu}_o$ et **L** sont de sens opposé

Cas de l'électron atomique



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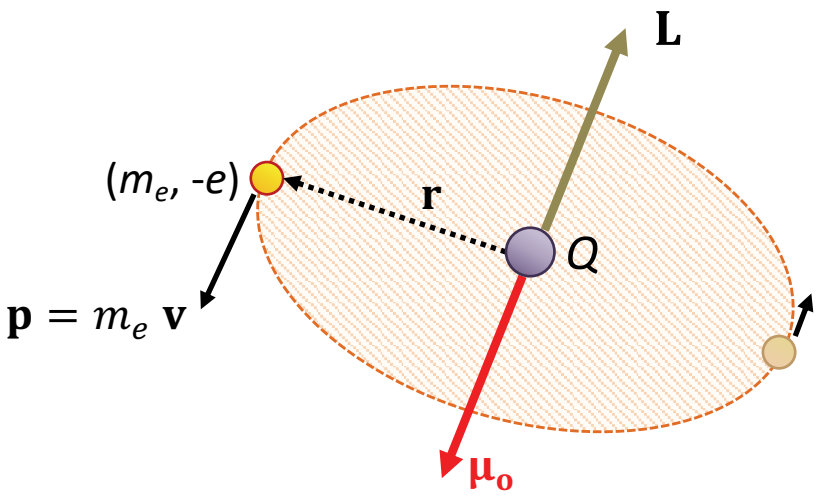
$$\boldsymbol{\mu}_o = -\frac{e}{2\,m_e}\,\mathbf{L} = \gamma_o\,\mathbf{L}$$

$$\gamma_o = -\frac{e}{2\,m_e}$$

Rapport gyromagnétique [C kg^{-1}] *ou* [rad s^{-1} T^{-1}]

Cas de l'électron atomique



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Moment magnétique : orbital $\quad \boldsymbol{\mu}_o = -\dfrac{e}{2\,m_e}\,\mathbf{L} = \gamma_o\,\mathbf{L}$

$$\boxed{\gamma_o = -\frac{e}{2\,m_e}}$$

Rapport gyromagnétique $\quad$ [C kg^{-1}] *ou* [Hz T^{-1}]

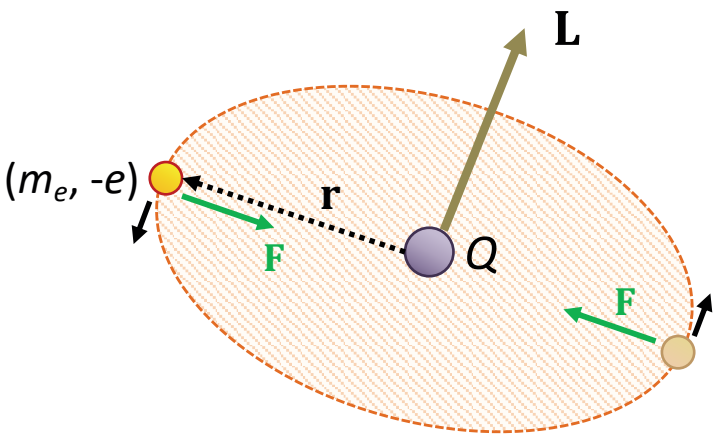
Dans le cas de l'atome de Bohr: $\quad \boldsymbol{L} = n.\hbar \qquad \gamma_o n.\hbar = -\dfrac{e}{2\,m_e}.\mathrm{n}.\hbar$

$$-\frac{e\hbar}{2\,m_e}.\mathrm{n} = -\mu_B.n$$

magnéton de Bohr

$$\boxed{\mu_\mathrm{B} = \frac{e\,\hbar}{2\,m_e}}$$

Cas de l'électron atomique



$$\frac{d\boldsymbol{p}}{dt} = \mathbf{F} \qquad \frac{d\boldsymbol{L}}{dt} = \boldsymbol{\Gamma}$$

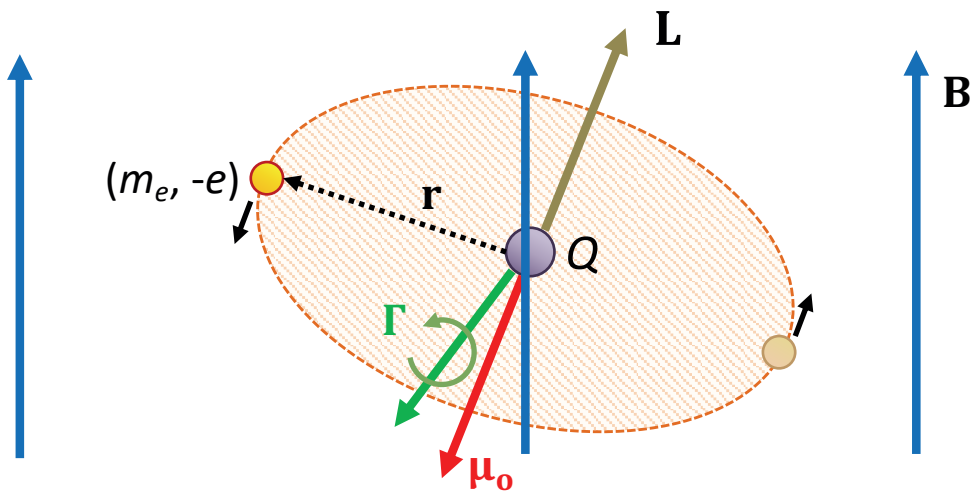
Dans le champ électrostatique du noyau

Energie : $$U = \frac{-Q\ e}{4\pi\varepsilon_0\ r}$$

Couple : $$\boldsymbol{\Gamma} = \mathbf{r} \wedge \mathbf{F} = \mathbf{0}$$ **(Force centrale F)**

Evolution : $$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \boldsymbol{\Gamma} \Rightarrow \quad \mathbf{L} = \mathbf{cste}$$

Cas de l'électron atomique



$$\frac{d\boldsymbol{p}}{dt} = \mathbf{F} \qquad \frac{d\boldsymbol{L}}{dt} = \boldsymbol{\Gamma}$$

Dans le champ électrostatique du noyau + champ magnétique externe

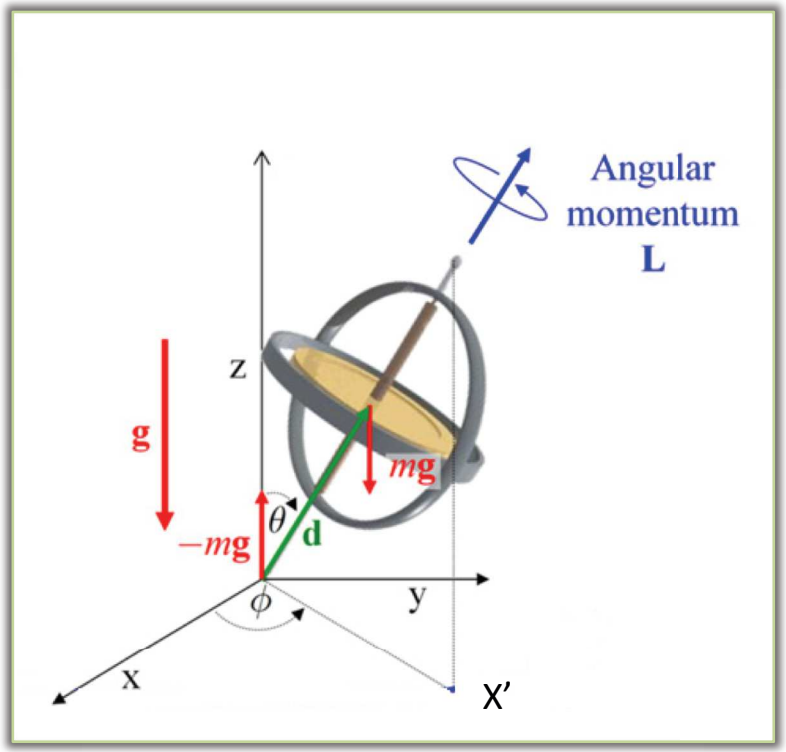
Energie : $$U = \frac{-Q\,e}{4\pi\varepsilon_0\,r} - \boldsymbol{\mu}_o \cdot \mathbf{B}$$

Couple : $$\boldsymbol{\Gamma} = \boldsymbol{\mu}_o \wedge \mathbf{B}$$

Evolution : $$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \boldsymbol{\mu}_o \wedge \mathbf{B} = \gamma_o\,\mathbf{L} \wedge \mathbf{B}$$

Cas de l'électron atomique

Analogie de la toupie

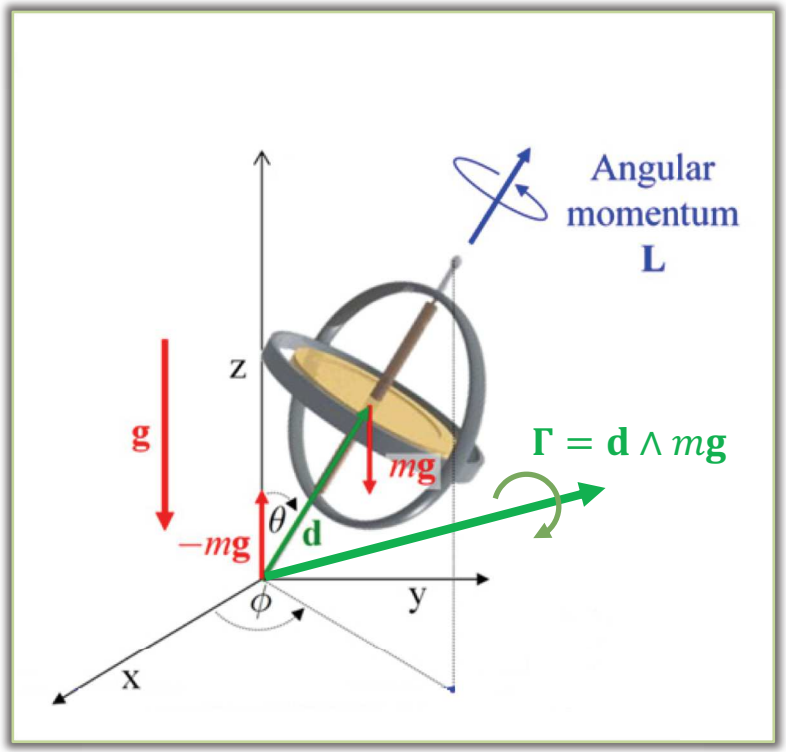


→moment cinétique

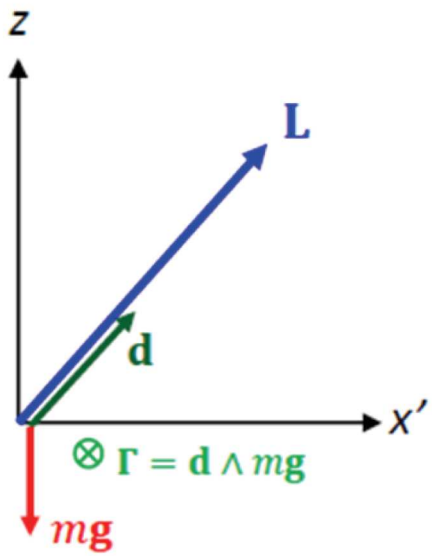
Axe de la toupie (moment cinétique) dans le plan (zOx')

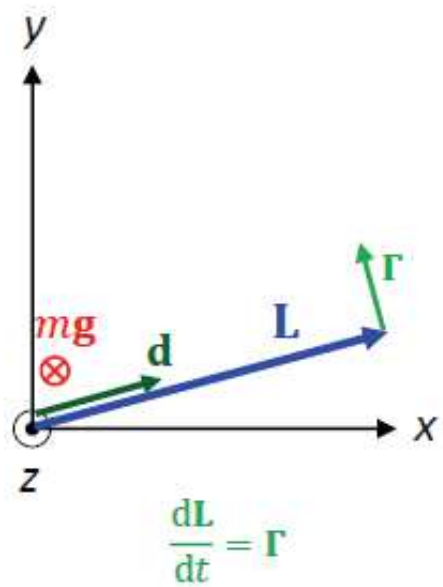
Cas de l'électron atomique

Analogie de la toupie



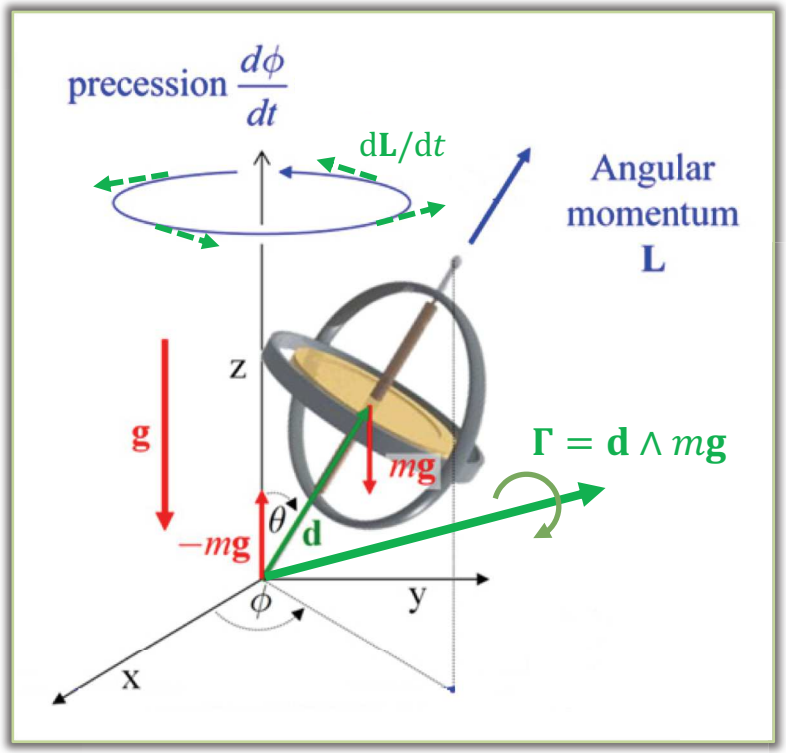
→moment cinétique



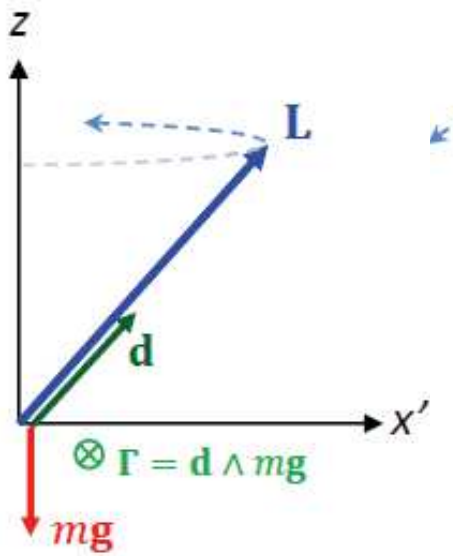


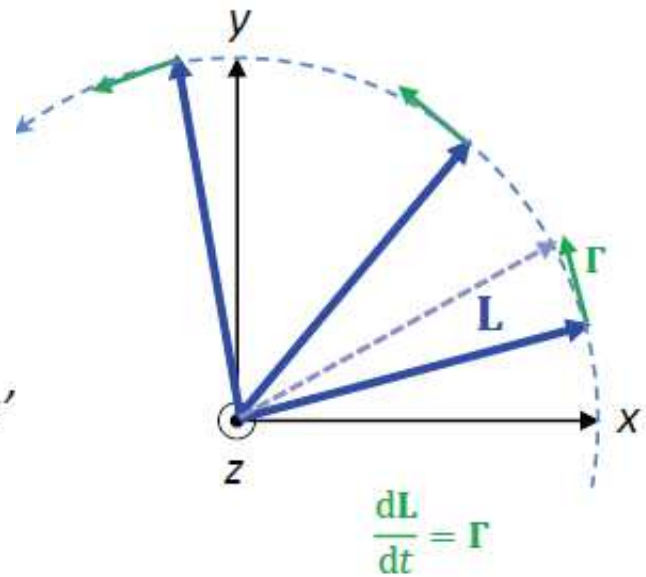
Cas de l'électron atomique

Analogie de la toupie



→moment cinétique

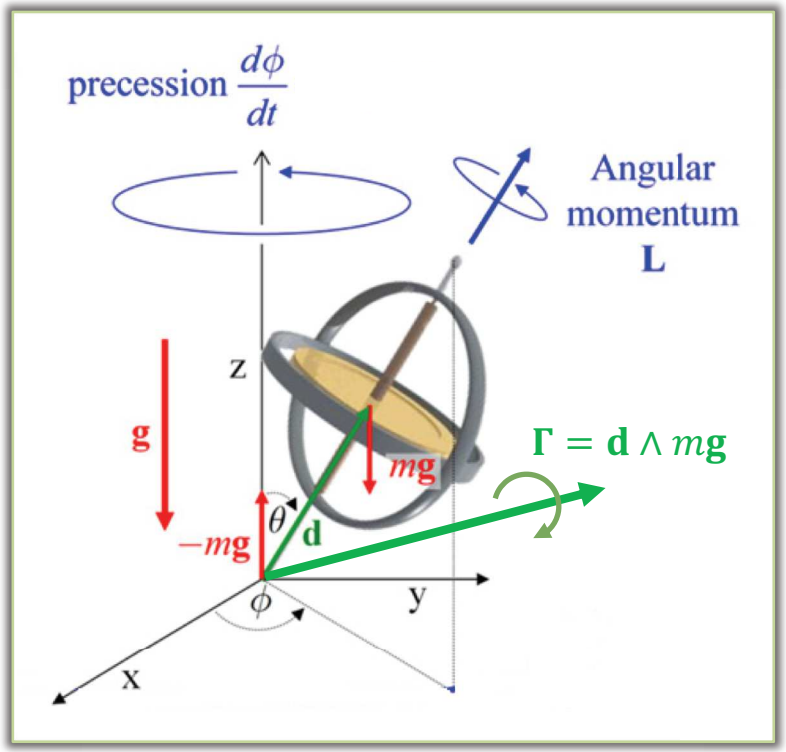


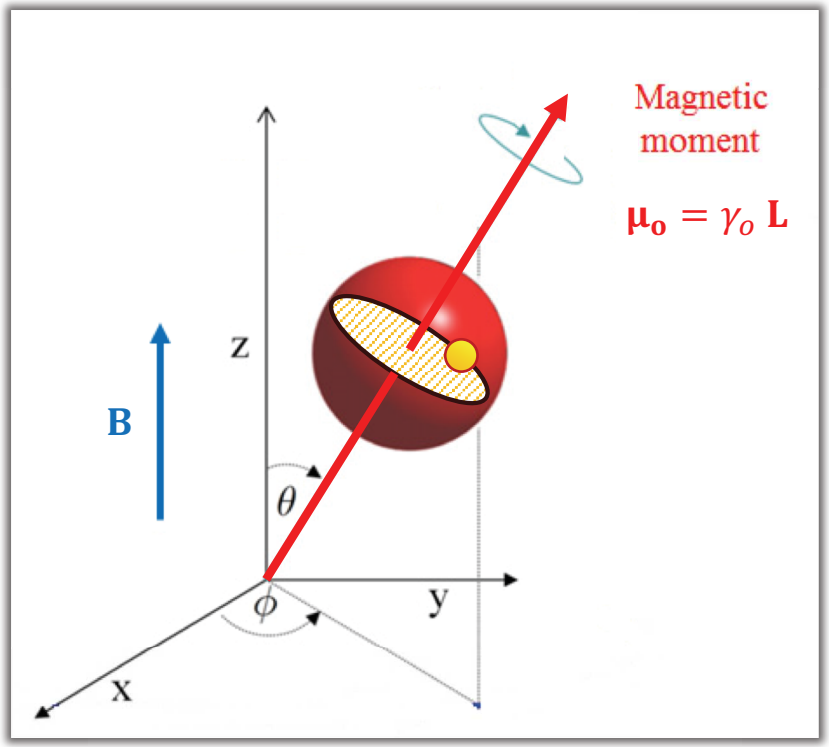


Evolution : $$\frac{d\mathbf{L}}{dt} = \mathbf{\Gamma}$$

Cas de l'électron atomique

Analogie de la toupie





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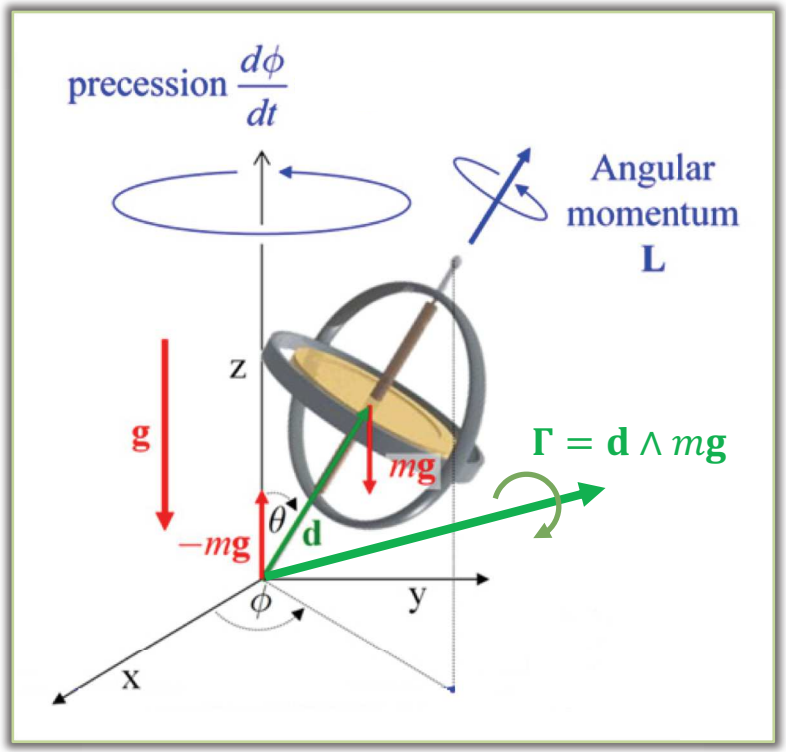
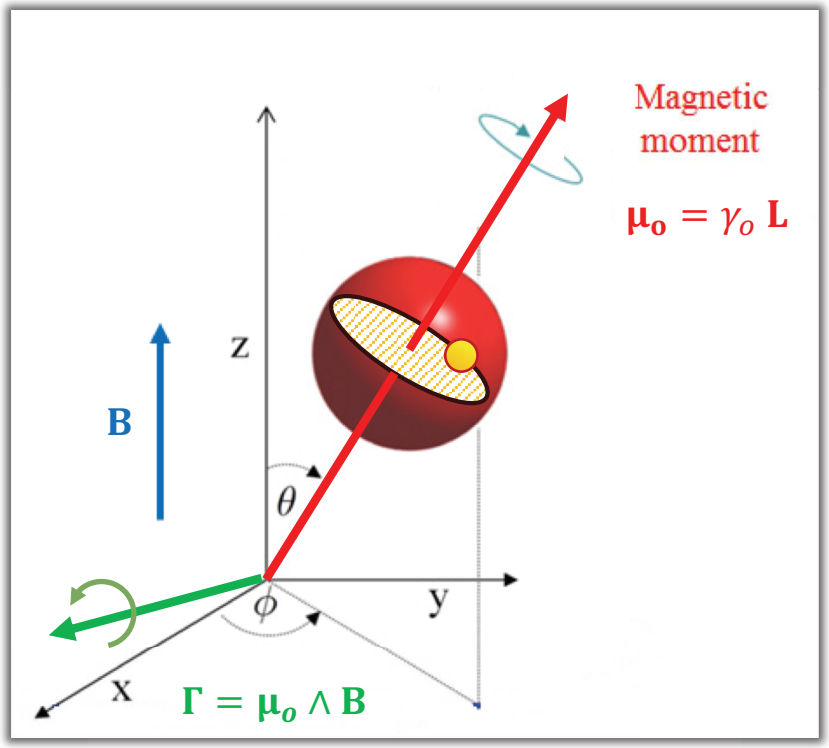
Cas de l'électron atomique

Analogie de la toupie



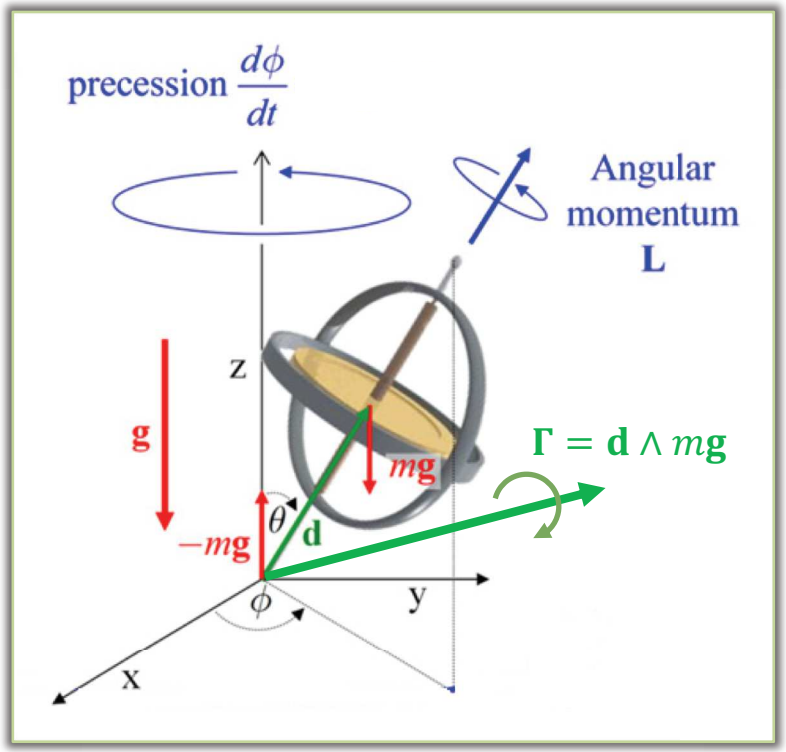


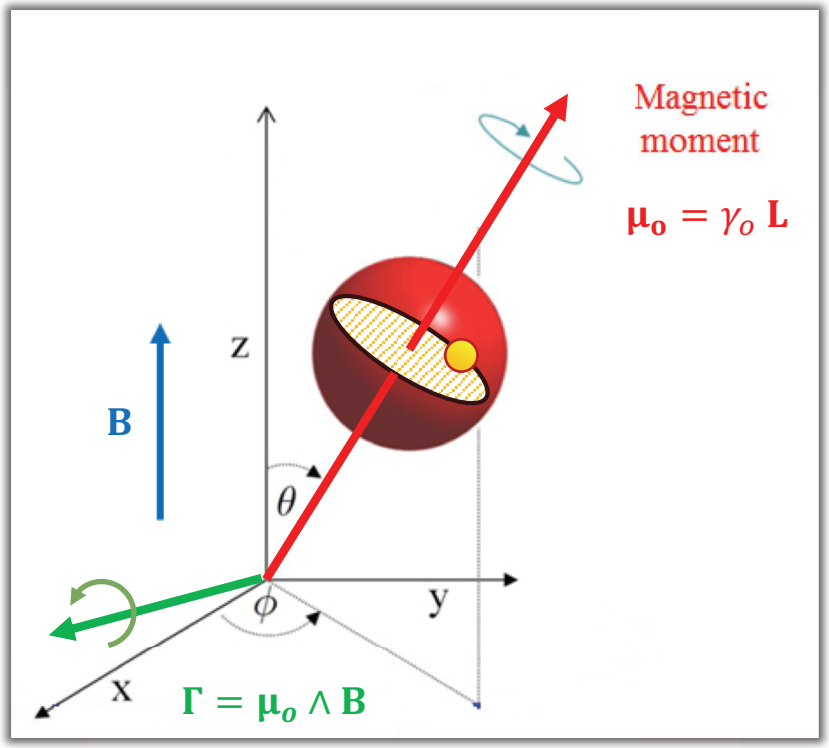
Evolution : $$\frac{d\mathbf{L}}{dt} = \Gamma$$

Γ dans le plan horizontal, perpendiculaire à μ_o

Cas de l'électron atomique

Analogie de la toupie





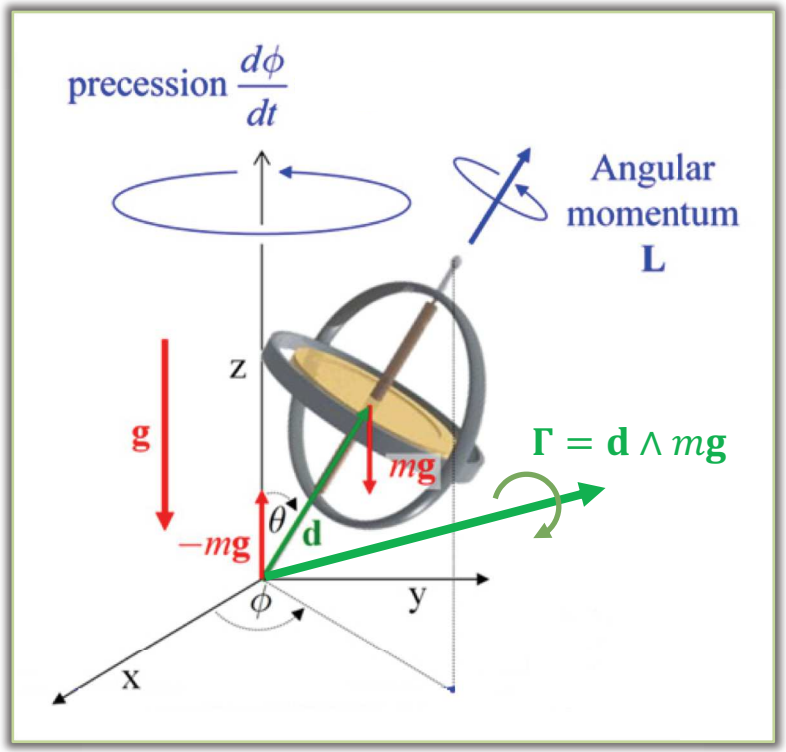
Evolution : $$\frac{d\mathbf{L}}{dt} = \mathbf{\Gamma}$$

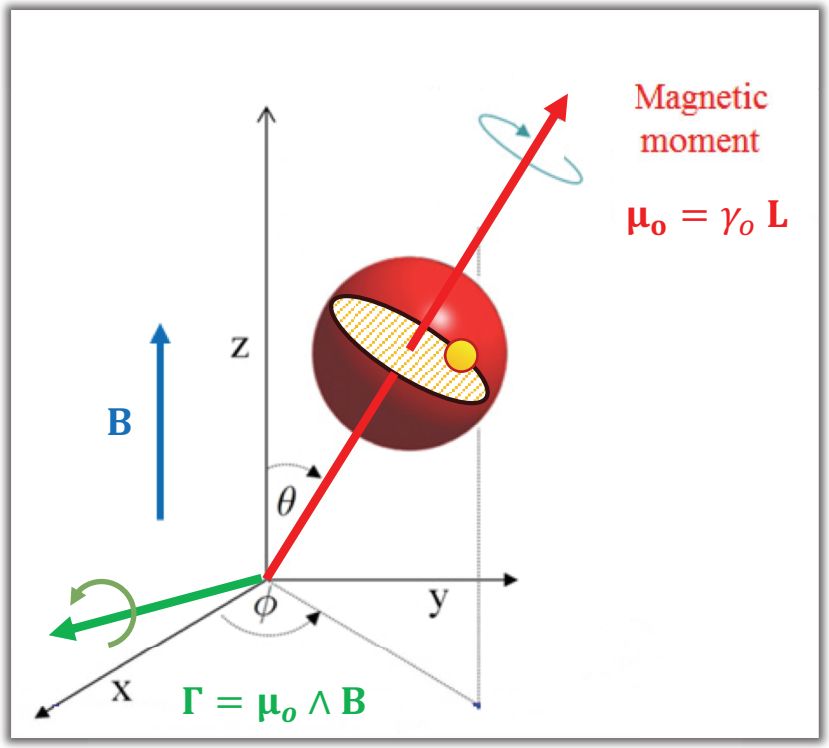
$$\frac{d\mathbf{L}}{dt} = \boldsymbol{\mu}_o \wedge \mathbf{B}$$

$$\boldsymbol{\mu}_o = \gamma_o \mathbf{L}$$

Cas de l'électron atomique

Analogie de la toupie



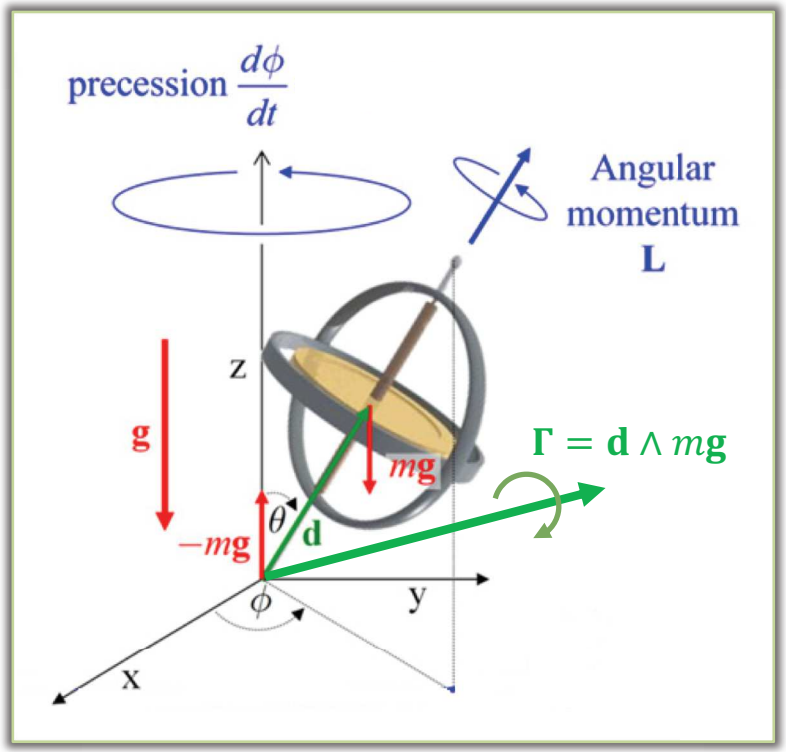


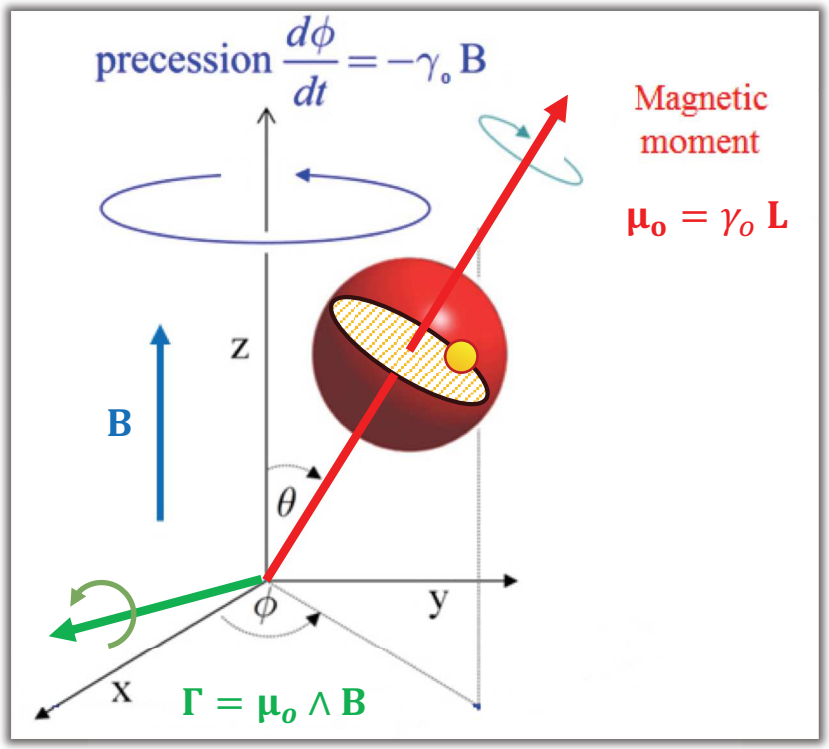
Evolution : $$\frac{d\mathbf{L}}{dt} = \boldsymbol{\Gamma}$$

$$\left.\begin{aligned} \frac{d\mathbf{L}}{dt} &= \boldsymbol{\mu}_o \wedge \mathbf{B} \\ \boldsymbol{\mu}_o &= \gamma_o \mathbf{L} \end{aligned}\right] \quad \frac{1}{\gamma_o}\frac{d\boldsymbol{\mu}_o}{dt} = \boldsymbol{\mu}_o \wedge \mathbf{B}$$

Cas de l'électron atomique

Analogie de la toupie



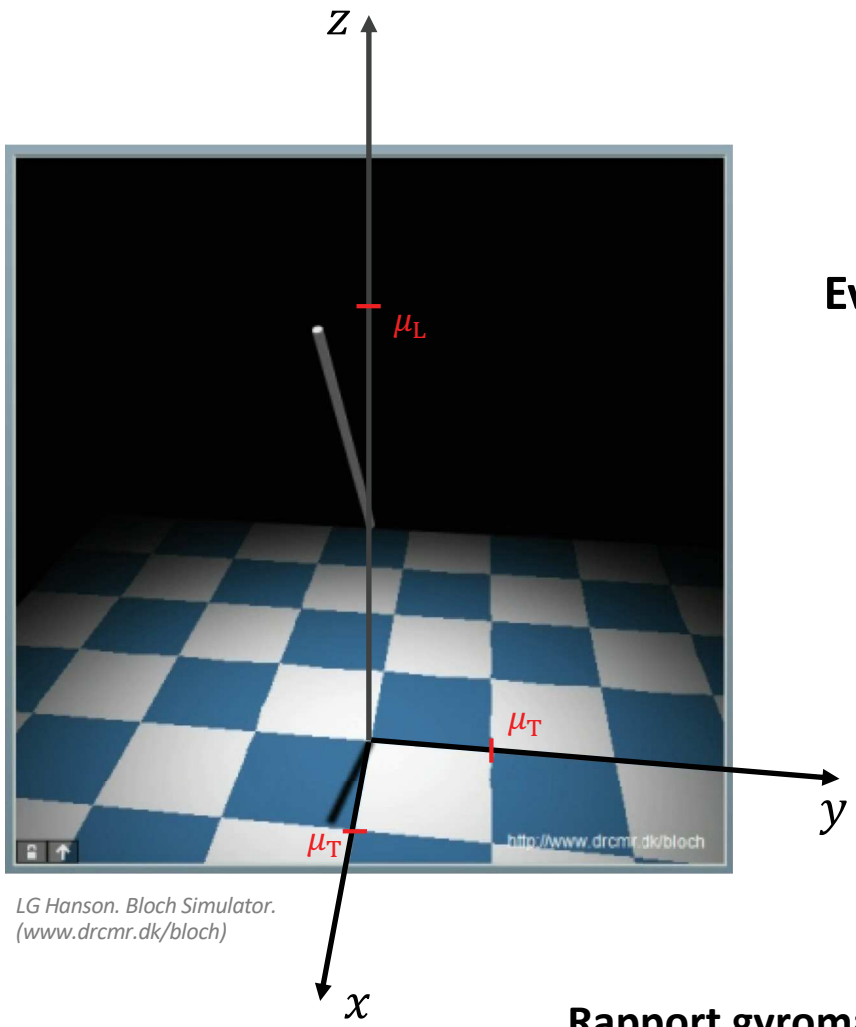


Evolution : $\frac{d\mathbf{L}}{dt} = \boldsymbol{\Gamma}$

$$\frac{d\mathbf{L}}{dt} = \boldsymbol{\mu}_o \wedge \mathbf{B} \qquad \frac{d\boldsymbol{\mu}_o}{dt} = \gamma_o \boldsymbol{\mu}_o \wedge \mathbf{B}$$

Mouvement de **précession** $\boxed{\frac{d\phi}{dt} = -\gamma_o \mathbf{B}}$

Cas de l'électron atomique



LG Hanson. Bloch Simulator. (www.drcmr.dk/bloch)

Evolution : $$\frac{d\boldsymbol{\mu}_o}{dt} = \gamma_o \boldsymbol{\mu}_o \wedge \mathbf{B}$$

$$\begin{cases} \mu_{ox} = \mu_T \cos(\omega t) \\ \mu_{oy} = \mu_T \sin(\omega t) \\ \mu_{oz} = \mu_L \end{cases}$$

Précession de Larmor : $\boldsymbol{\omega} = -\gamma_o \mathbf{B}$ [rad.s^{-1}]

$\boldsymbol{\nu} = -\bar{\gamma}_o \mathbf{B}$ [Hz]

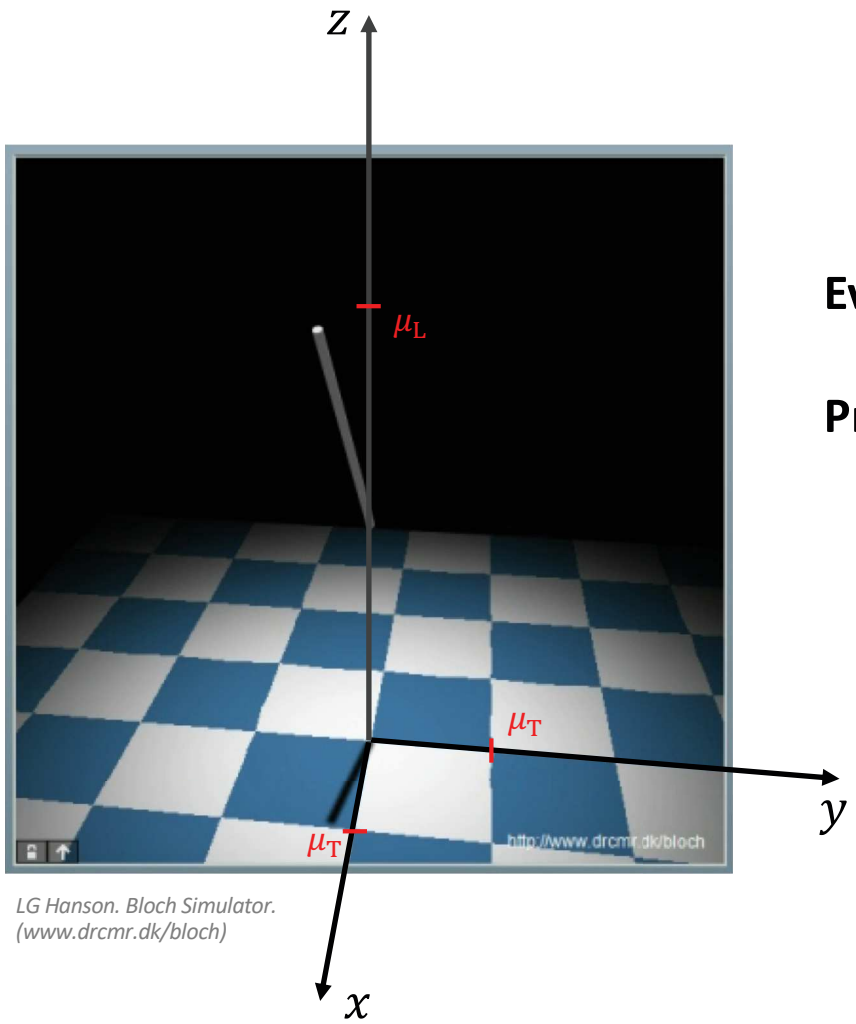
Rapport gyromagnétique $\gamma_o = -\dfrac{e}{2\, m_e}$ [C kg^{-1}] *ou* [rad Hz T^{-1}]

$\bar{\gamma}_o \simeq 14\ \mathrm{GHz\ T^{-1}}$

$\bar{\gamma} = \dfrac{\gamma}{2\pi}$ [Hz T^{-1}]

Dans un champ de **1 T** : $\nu = 14\ \mathrm{GHz}$ (micro-onde)

Cas de l'électron atomique



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Evolution : $$\frac{d\boldsymbol{\mu}_o}{dt} = \gamma_o \boldsymbol{\mu}_o \wedge \mathbf{B}$$

Précession de Larmor : $$\boldsymbol{\omega} = -\gamma_o\, \mathbf{B}$$

$$\begin{cases} \mu_{ox} = \mu_T \cos(\omega t) \\ \mu_{oy} = \mu_T \sin(\omega t) \\ \mu_{oz} = \mu_L \end{cases}$$

$$\boldsymbol{\mu}_o = \begin{pmatrix} \mu_T \cos(\omega t) \\ \mu_T \sin(\omega t) \\ \mu_L \end{pmatrix} \qquad \frac{d\boldsymbol{\mu}_o}{dt} = \omega \begin{pmatrix} -\mu_T \sin(\omega t) \\ \mu_T \cos(\omega t) \\ 0 \end{pmatrix}$$

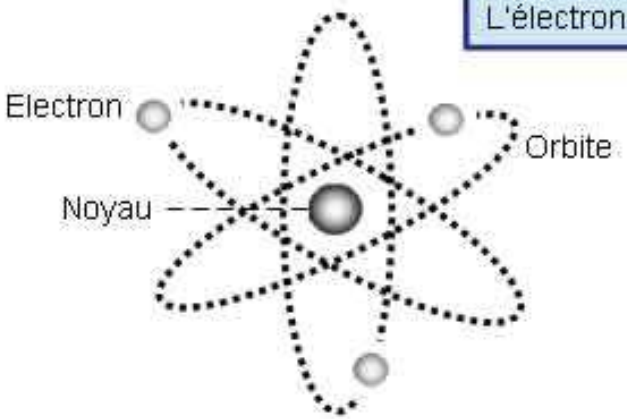
$$\gamma_o \boldsymbol{\mu}_o \wedge \mathbf{B} = \begin{pmatrix} \mu_T \cos(\omega t) \\ \mu_T \sin(\omega t) \\ \mu_L \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix} = \gamma_o B \begin{pmatrix} \mu_T \sin(\omega t) \\ -\mu_T \cos(\omega t) \\ 0 \end{pmatrix}$$

Ce que dit la mécanique quantique

1913 – Modèle de Bohr

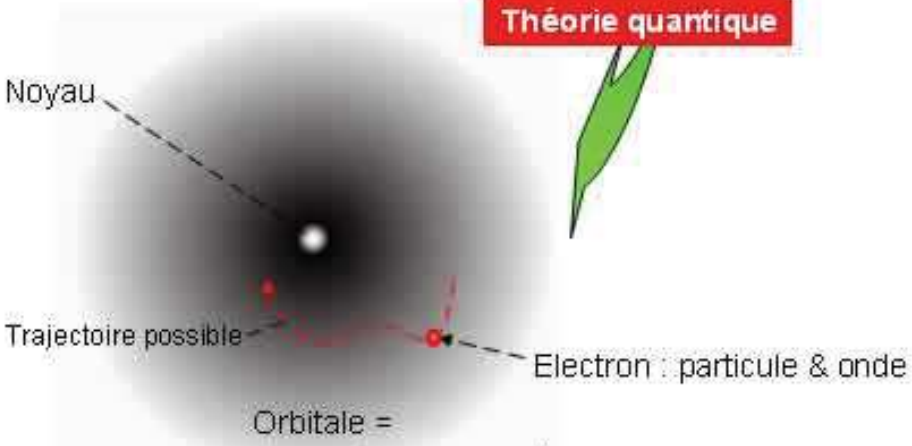
~~L'électron suit une trajectoire circulaire autour du noyau~~

Pas conforme à la réalité



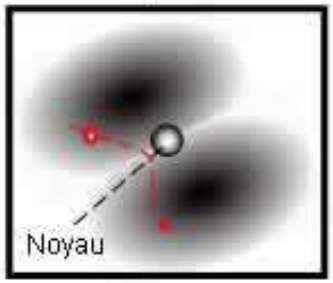
1926 – Modèle de Schrödinger

Théorie quantique



Orbitale =
zone de probabilité de présence d'un électron
(Exemple d'orbitale sphérique)

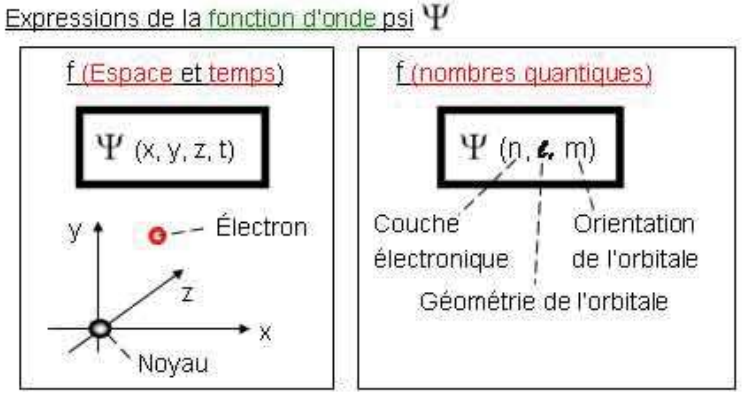
Autre exemple d'orbitale



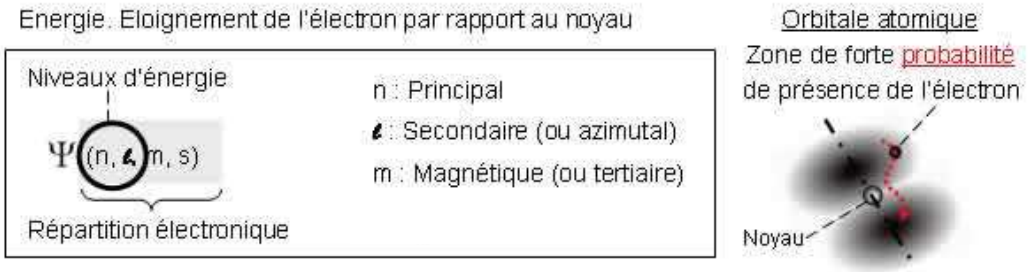
Ce que dit la mécanique quantique

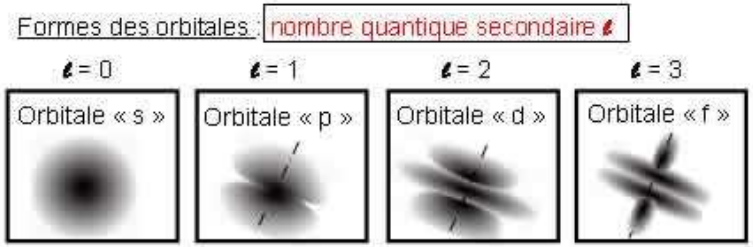
L'électron est décrit par une onde : $\Psi(\mathbf{x}, t) : \mathbb{R}^3 \times \mathbb{R}^+ \rightarrow \mathbb{C}$

La description complète de la fonction d'onde de l'électron atomique fait apparaître 3 nombres quantiques



Le 4ème nombre, « s » est caché





Ce que dit la mécanique quantique

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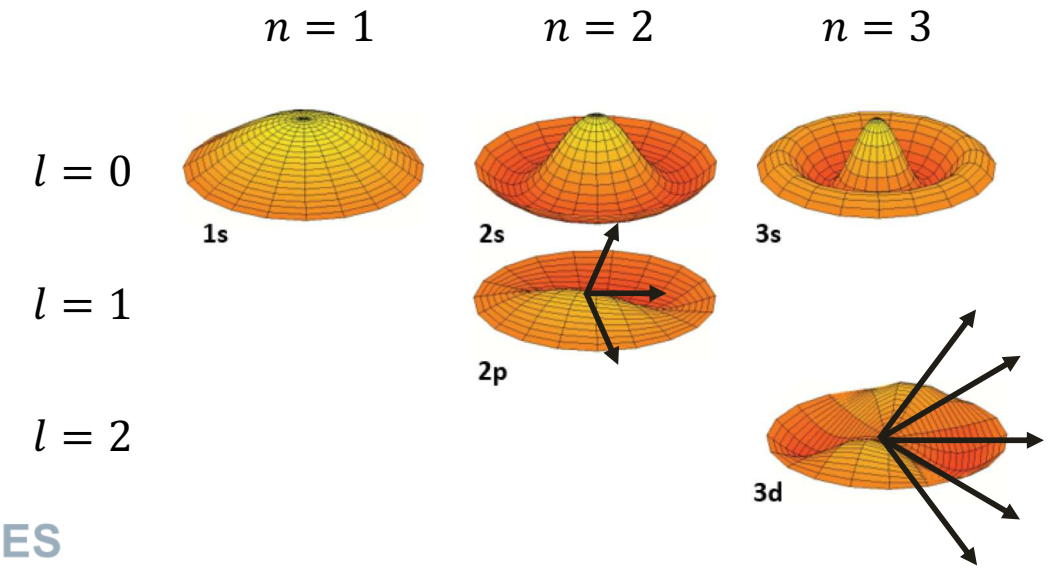
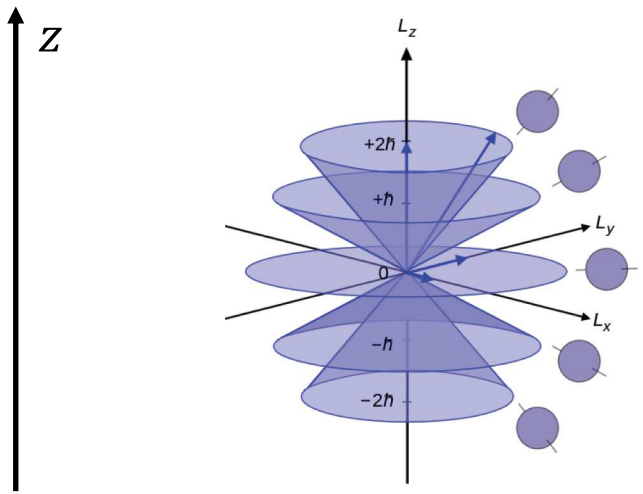
La description complète de la fonction d'onde de l'électron atomique fait apparaître 3 nombres quantiques

n : quantification de l'énergie E

l : quantification du moment cinétique $|\mathbf{L}| = \hbar\sqrt{l(l+1)}$ (l entier)

m : quantification de $L_z = m\hbar = -l\hbar \ \ldots \ 0 \ \ldots \ l\hbar$





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n entier positif >0

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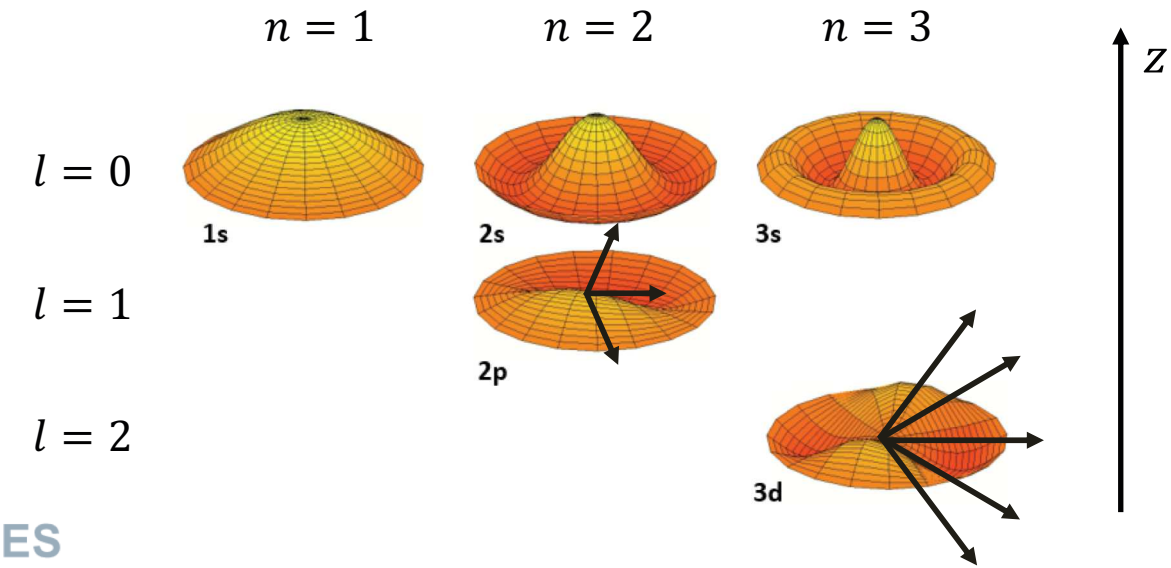
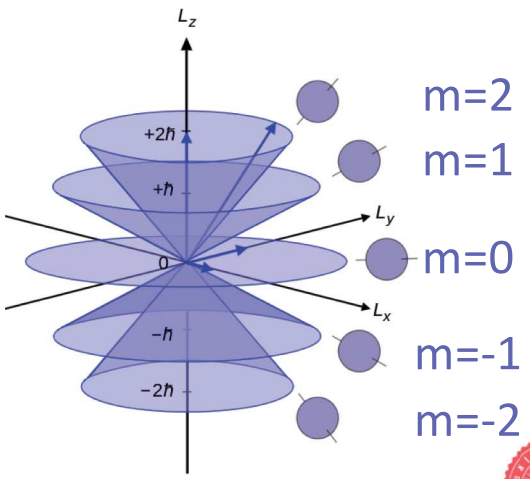
$0 \leq l \leq n-1$

m : quantification de $L_z = m\hbar = -l\hbar \ \ldots \ 0 \ \ldots \ l\hbar$

$-l \leq m \leq +l$





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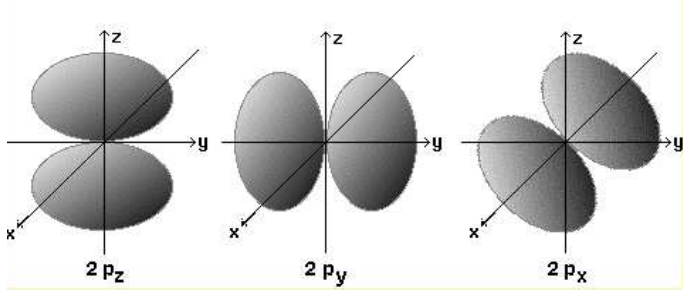

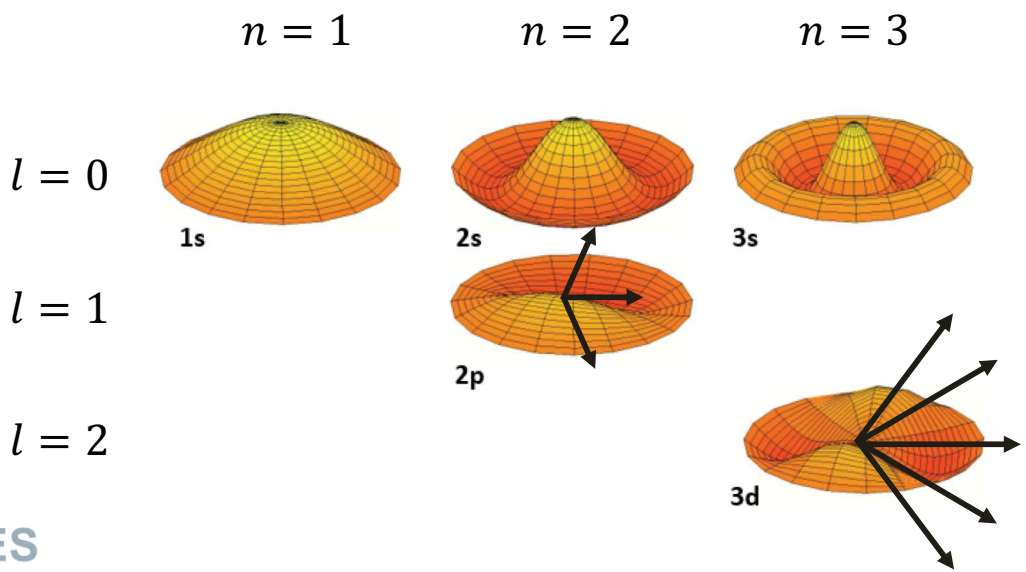

Orbitale	N. quant.			Formes des orbitales
	n	ℓ	m_ℓ	$\Psi(\vec{r})$
$2p_0$	2	1	0	
$2p_{-1/+1}$	2	1	±1	

Ce que dit la mécanique quantique

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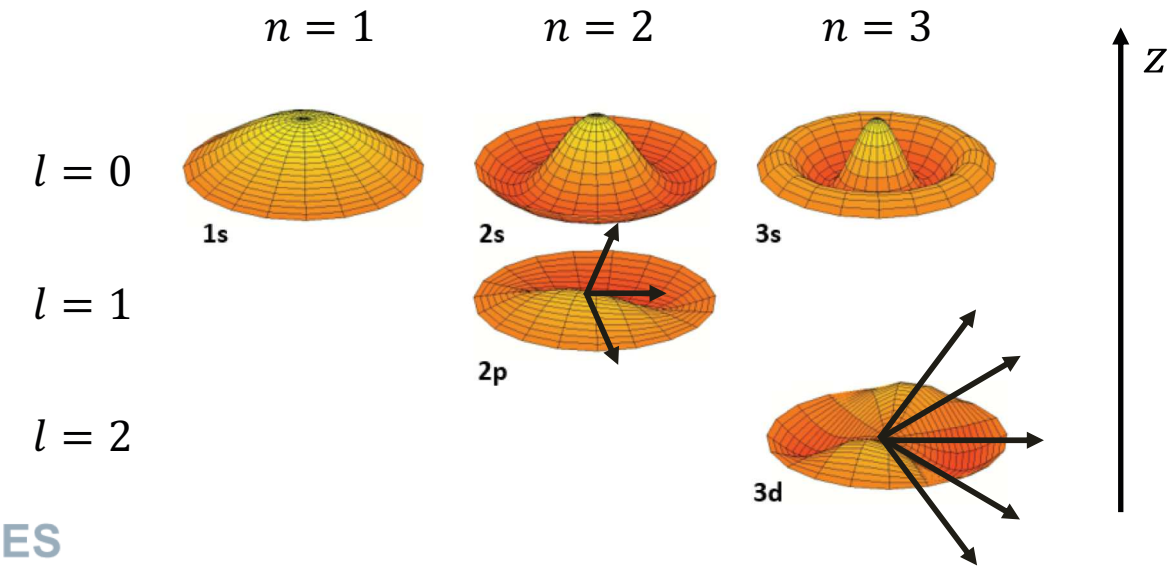
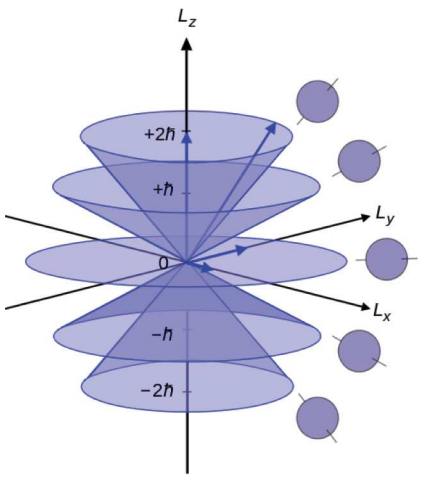
$$\boldsymbol{\mu}_\mathbf{o} = \gamma_o \, \mathbf{L}$$

$$|\boldsymbol{\mu}_\mathbf{o}| = \gamma_o \hbar \sqrt{l(l+1)}$$

$$\mu_{oz} = \gamma_o \, m \, \hbar$$





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(n, l, m)

… MAIS

$L_z = m\hbar = -l\hbar \ \ldots \ 0 \ \ldots \ l\hbar$ *2l +1 valeurs*

→ nombre discret impair de projections du moment cinétique sur (Oz)

Ce que dit la mécanique quantique

L'électron est décrit par une onde : $\Psi(\mathbf{x}, t) : \mathbb{R}^3 \times \mathbb{R}^+ \rightarrow \mathbb{C}$

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(n, l, m)

… MAIS

$L_z = m\hbar = -l\hbar \ \ldots \ 0 \ \ldots \ l\hbar$

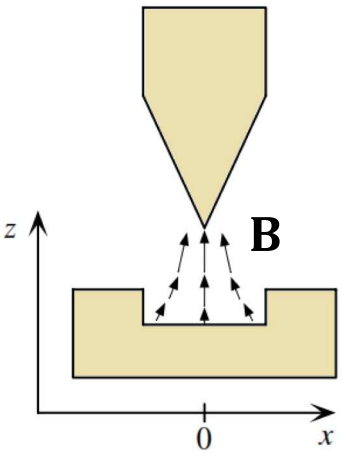
2l +1 valeurs

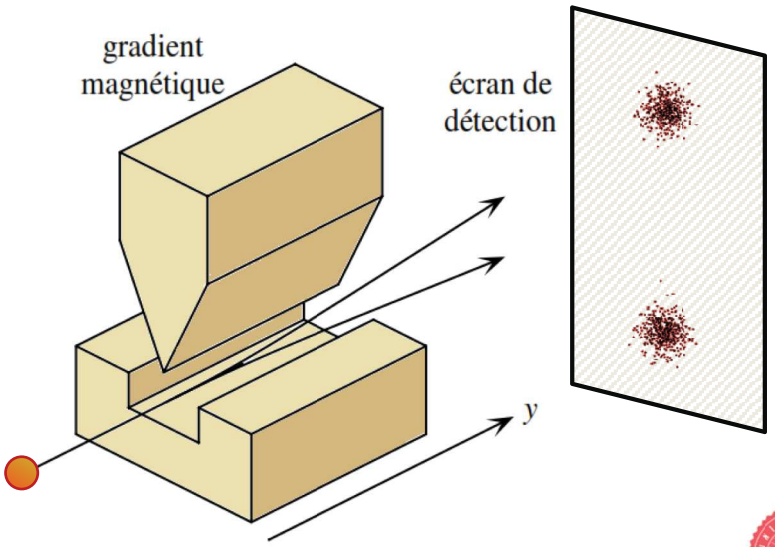
Particules dans $\partial \mathbf{B}/\partial z$ déviées en fonction de L_z

Nombre <u>pair</u> de taches

Il existe un moment cinétique demi-entier intrinsèque

SPIN





Ce que dit la mécanique quantique

L'électron est décrit par une onde : $\Psi\,(\mathbf{x}, t, \sigma)\,:\ \mathbb{R}^3 \times \mathbb{R}^+ \times \ \binom{+}{-} \rightarrow\ \mathbb{C}$

La description complète fait apparaître 4 nombres quantiques

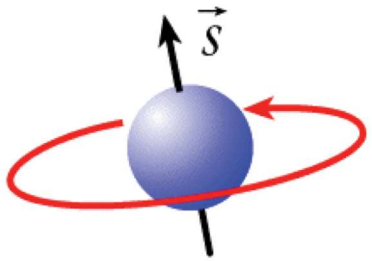
(n, l, m, σ)

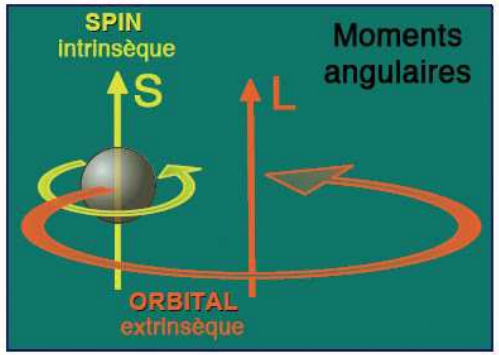
W. Pauli

moment cinétique intrinsèque = SPIN S (s'ajoute à **L**)

tel que : $|\mathbf{S}| = \hbar\sqrt{s(s+1)}$ (s entier **OU** demi-entier)

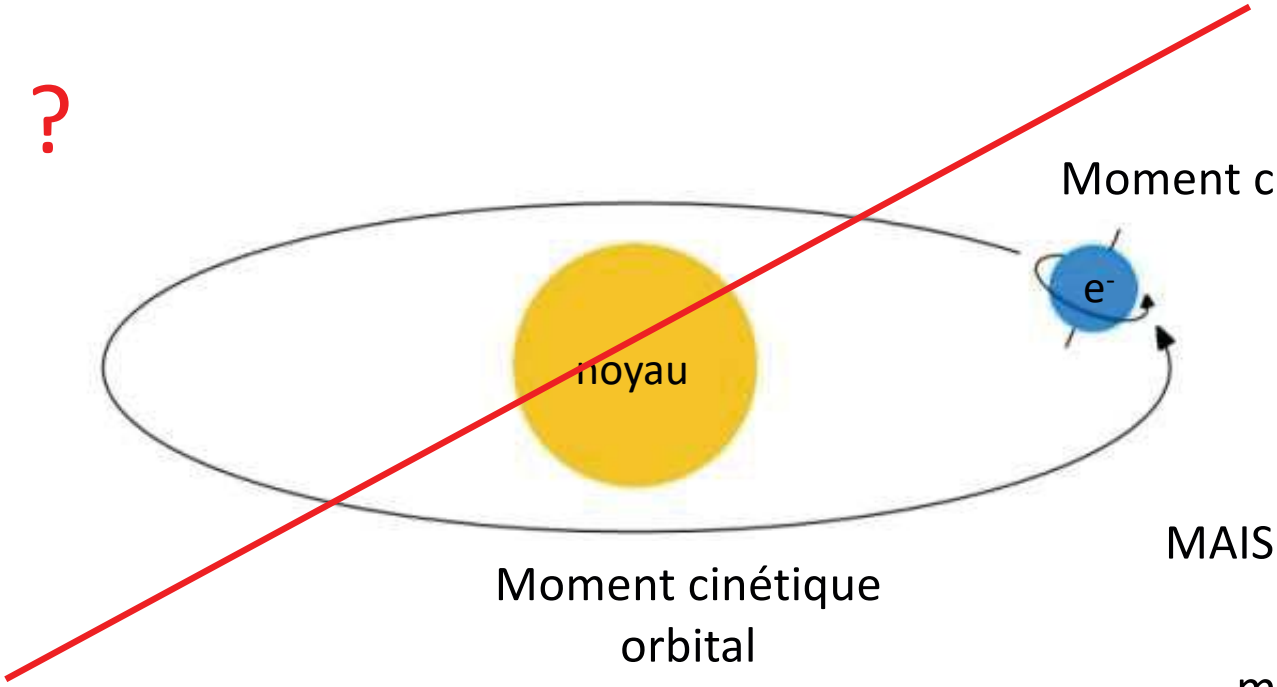
$$S_z = \sigma\hbar = -s\hbar \ldots s\hbar$$





Ce que dit la mécanique quantique

Modèle atomique de Rutheford, le *moment cinétique orbital* → mouvement orbital de l'électron autour de son noyau.



Moment cinétique de spin

Moment cinétique orbital

MAIS e^- = particule chargée

↓

moment magnétique

Ce que dit la mécanique quantique

L'électron est décrit par une onde : $\Psi\,(\mathbf{x}, t, \sigma)\,:\ \mathbb{R}^3 \times \mathbb{R}^+ \times \begin{pmatrix} + \\ - \end{pmatrix} \to \mathbb{C}$

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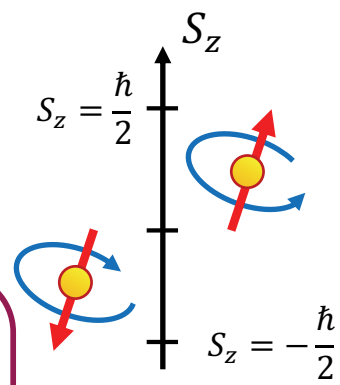
W. Pauli

SPIN S

tel que : $|\mathbf{S}| = \hbar\sqrt{s(s+1)}$ (s entier **OU** demi-entier)

$$S_z = \sigma\hbar = -s\hbar \ldots s\hbar$$

pour l'électron : $s = \frac{1}{2}$ $\quad \sigma = \pm\frac{1}{2}$



Nota:

- proton et le neutron présentent un *spin*
- noyau atomique également (*spin* résultant = vecteur somme des *spins* de ses constituants)
- valeur de ces *spins* dépend donc du nombre de masse et du numéro atomique de l'élément.

Ce que dit la mécanique quantique

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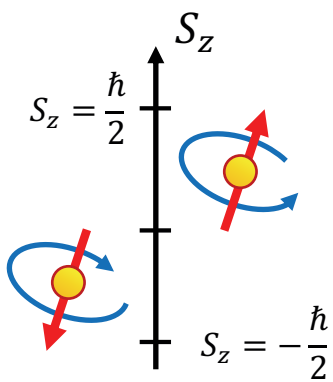

il s'y associe un moment magnétique de spin :

$$\boxed{\boldsymbol{\mu}_s = \gamma_s\,\mathbf{S} = g\,\gamma_o\,\mathbf{S}}$$

$g \simeq 2$

facteur de Landé de spin électronique
(électron libre)

γ_s= rapport gyromagnétique de spin $\quad \bar{\gamma}_s \quad \simeq 28\ \mathrm{GHz\ T^{-1}}$

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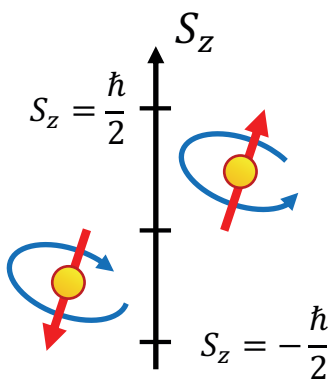

il s'y associe un moment magnétique de spin :

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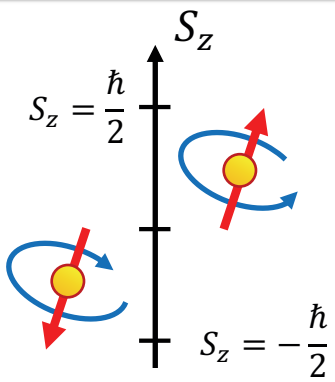
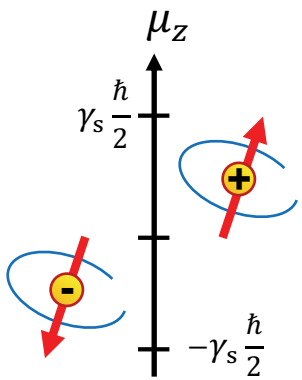
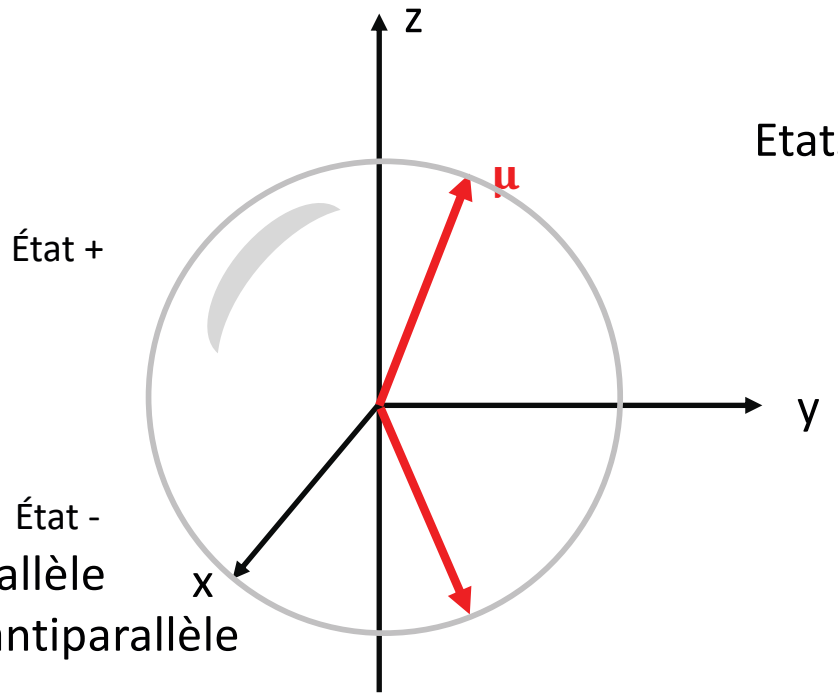
Nota: le noyau atomique se ramenant à une distribution de charges, un *moment cinétique de spin net* est associé à un *moment magnétique de spin*

Ce que dit la mécanique quantique



Projection du moment magnétique de spin $\boldsymbol{\mu}_s = \gamma_s \mathbf{S}$ → $\mu_z = \gamma_s S_z$



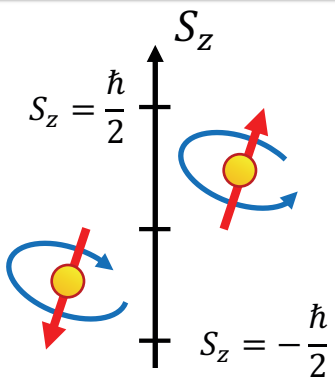


Etats superposés

+: état « up », état pseudo-parallèle

-: état « down », état pseudo-antiparallèle

Ce que dit la mécanique quantique



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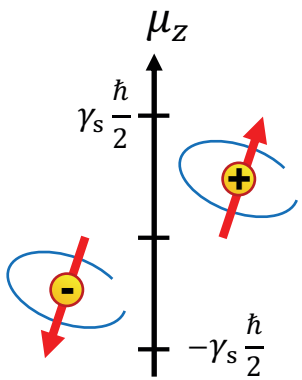
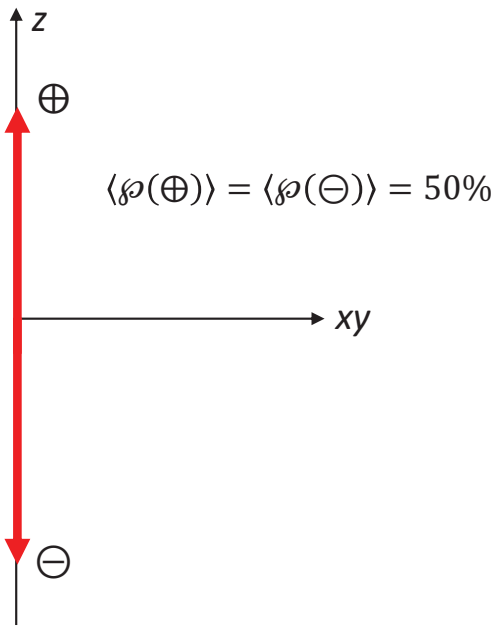
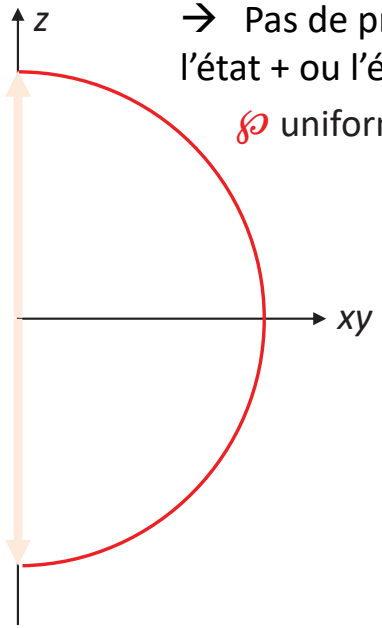


Mesures individuelles accessibles



$\langle \wp(\oplus) \rangle = \langle \wp(\ominus) \rangle = 50\%$

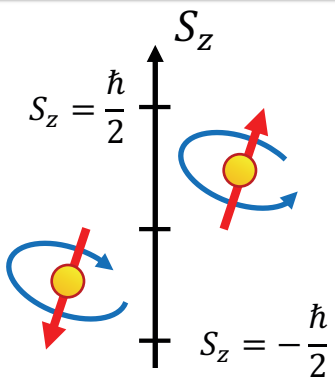
Etats superposés



→ Pas de préférence pour l'état + ou l'état -

$\wp$ uniforme

Ce que dit la mécanique quantique



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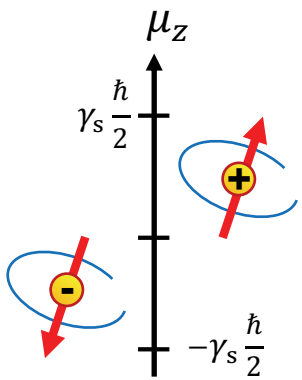

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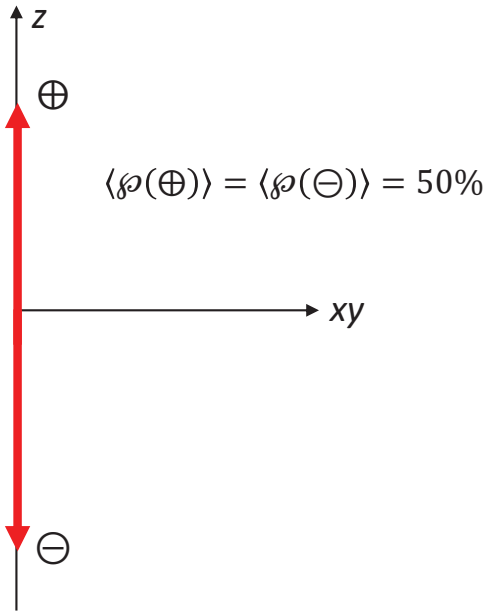

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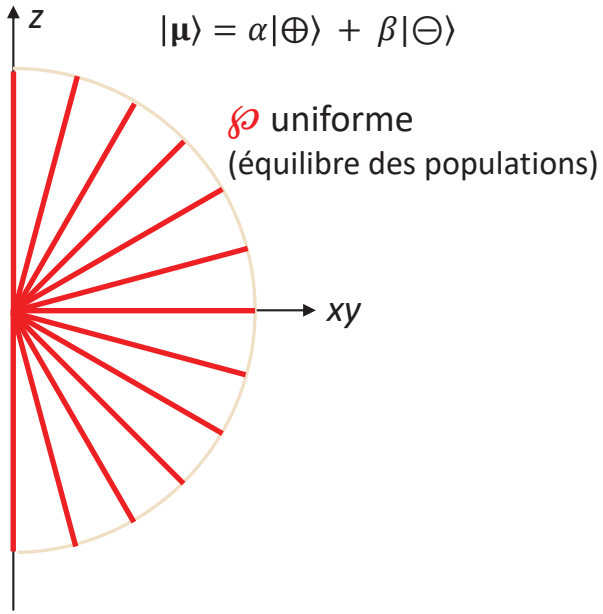
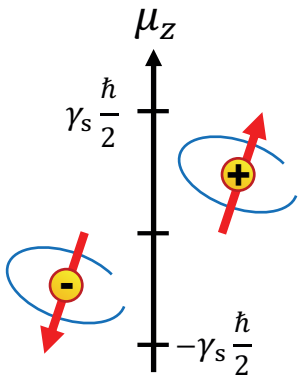


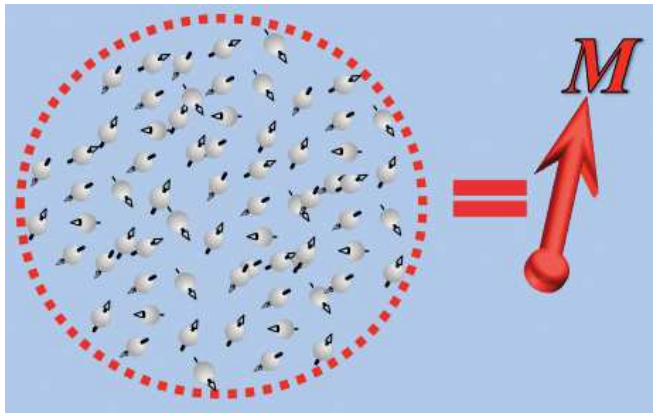
Ce que dit la mécanique quantique

Considérations énergétiques

$\mathbf{B} = 0$



$$E_+ = E_- = \hat{E}$$



$$M = \sum \mu_z$$

Ce que dit la mécanique quantique

Considérations énergétiques

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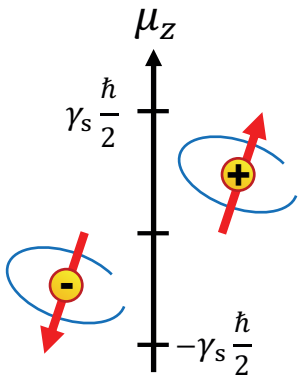


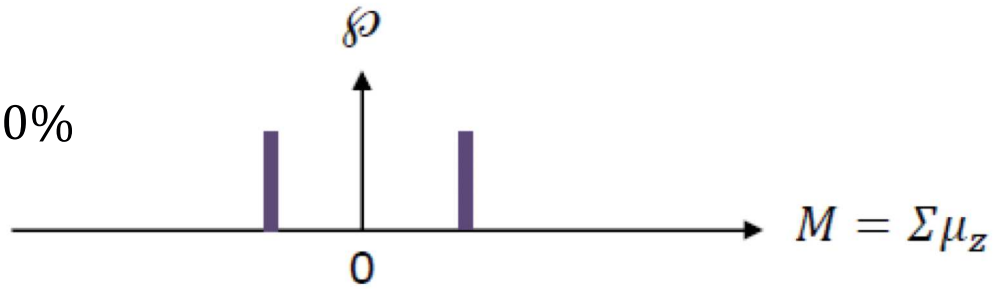
$E_+ = E_- = \hat{E}$

$M = \sum \mu_z$

Principe de superposition: 1 dipôle

dipôle 1

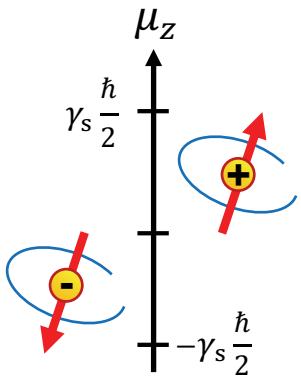
$\wp(\oplus) = \wp(\ominus) = 50\%$



Ce que dit la mécanique quantique

Considérations énergétiques

$\mathbf{B} = 0$



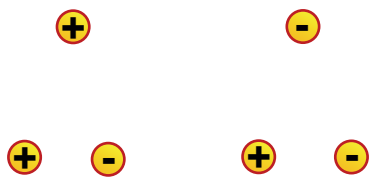
$E_+ = E_- = \hat{E}$

$$M = \sum \mu_z$$

Principe de superposition: 2 dipôles

dipôle 1

dipôle 2



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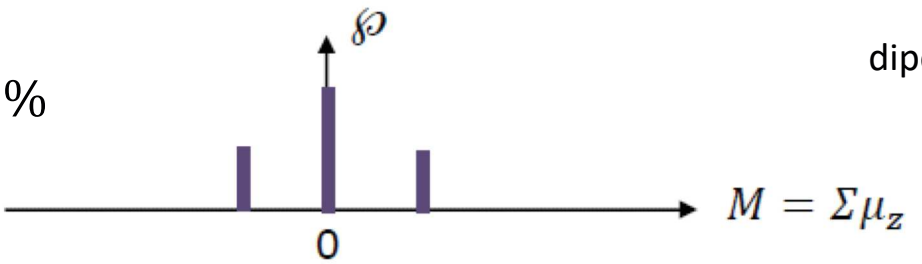
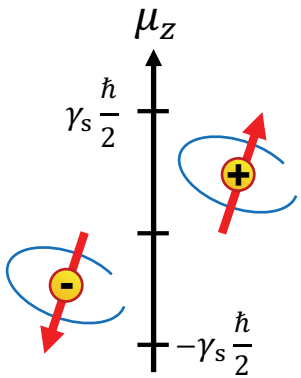


Ce que dit la mécanique quantique

Considérations énergétiques

$\mathbf{B} = 0$

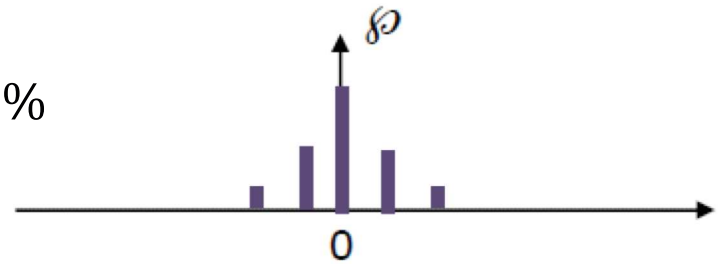


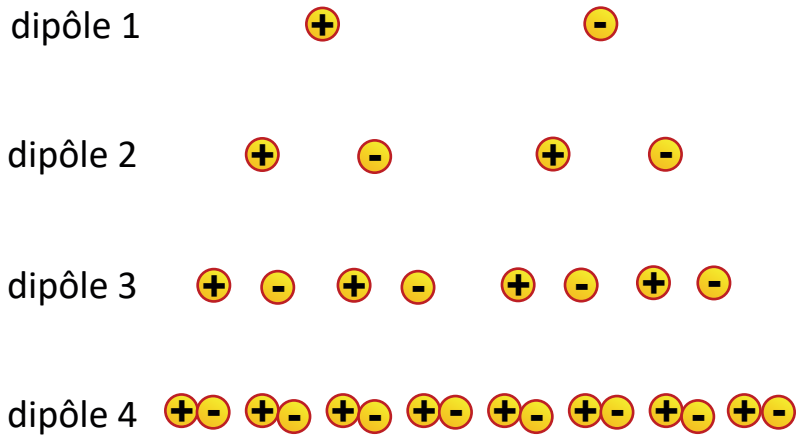
$E_+ = E_- = \hat{E}$

$$M = \sum \mu_z$$

Principe de superposition: 4 dipôles

$$\wp(\oplus) = \wp(\ominus) = 50\%$$



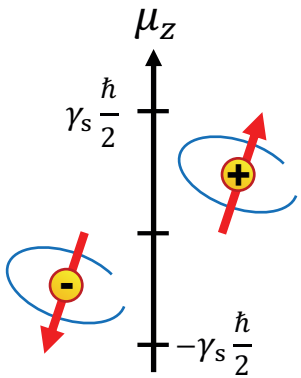


$M = \Sigma \mu_z$

Ce que dit la mécanique quantique

Considérations énergétiques

$\mathbf{B} = 0$

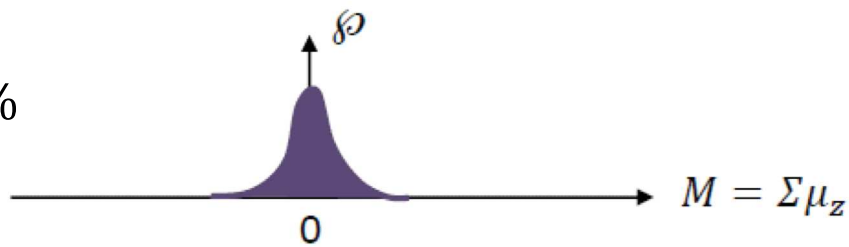


$E_+ = E_- = \hat{E}$

$$M = \sum \mu_z$$

Principe de superposition: 1000 dipôles

$$\wp(\oplus) = \wp(\ominus) = 50\%$$



Ce que dit la mécanique quantique

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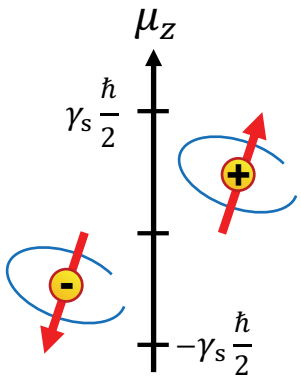

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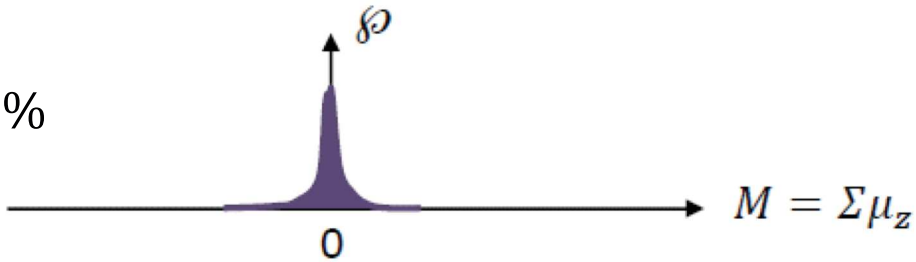

Ce que dit la mécanique quantique

Considérations énergétiques

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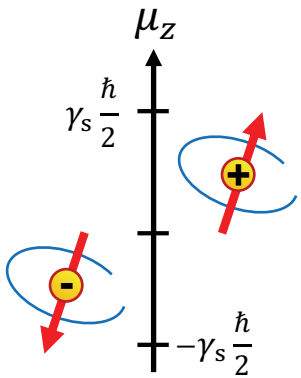

$E_+ = E_- = \hat{E}$

Aimantation résultante nulle quand **B** = 0

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Principe de superposition: 10^{20} dipôles

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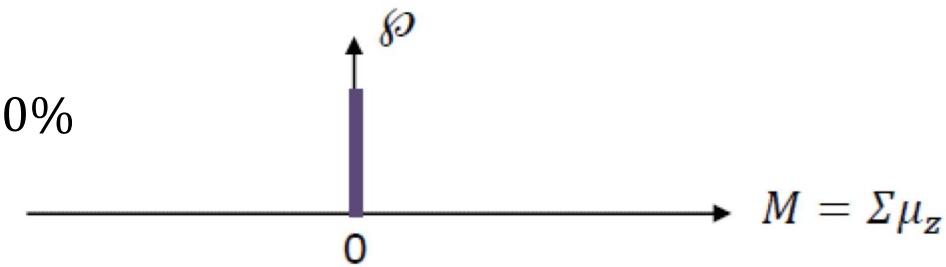


Ce que dit la mécanique quantique

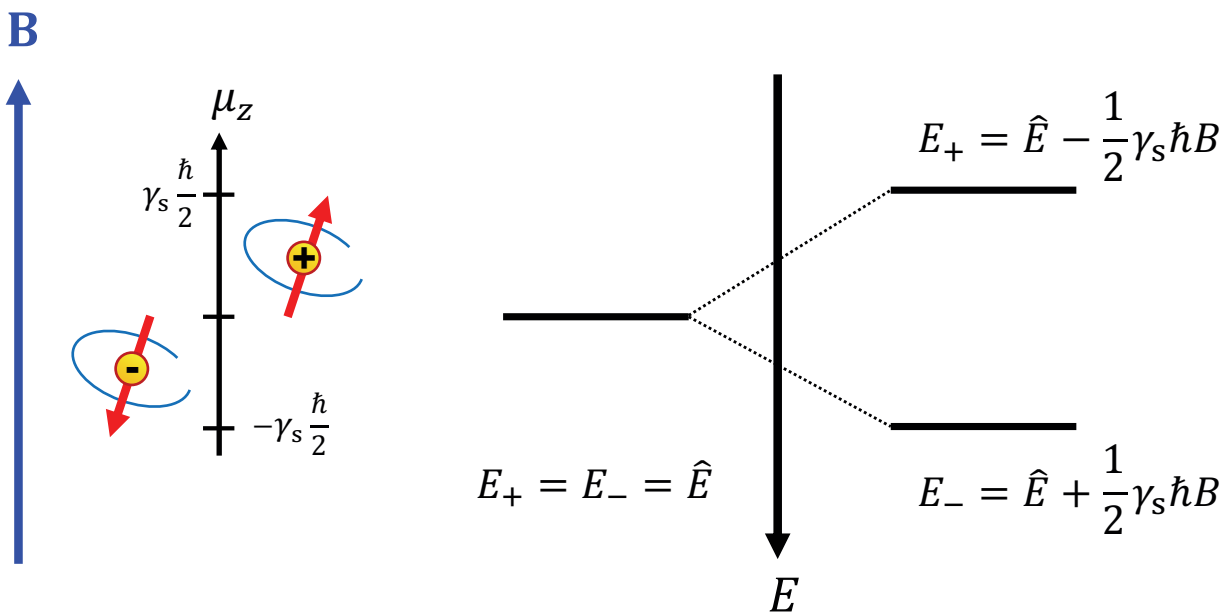
Considérations énergétiques

$\mathbf{B} = 0$

$$U = \frac{-Q\,e}{4\pi\varepsilon_0\,r}$$

$\mathbf{B} > 0$

$$U = \frac{-Q\,e}{4\pi\varepsilon_0\,r} - \boldsymbol{\mu}_s \cdot \mathbf{B} = \frac{-Q\,e}{4\pi\varepsilon_0\,r} - \gamma_s S_z B$$



+: état « up », état pseudo-parallèle

-: état « down », état pseudo-antiparallèle

Ce que dit la mécanique quantique

Considérations énergétiques

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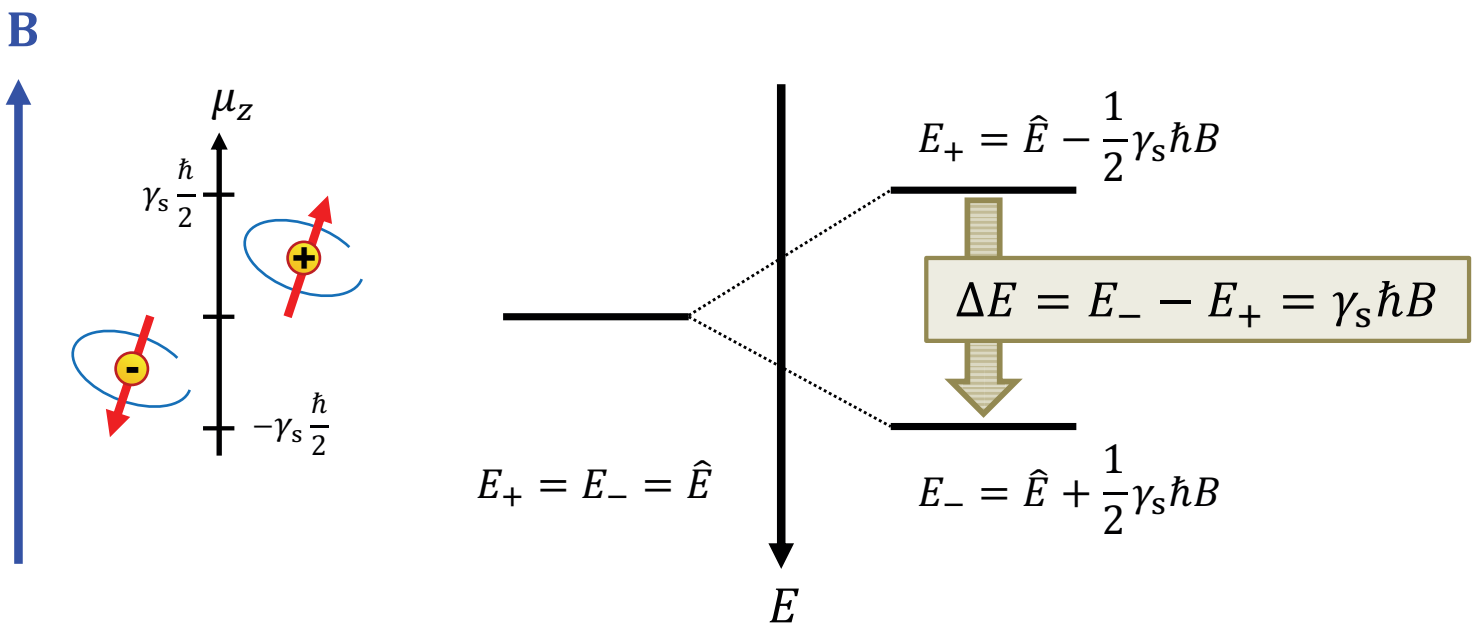


$$\Delta E = E_- - E_+ = \gamma_s \hbar B$$

(levée de dégénérescence)

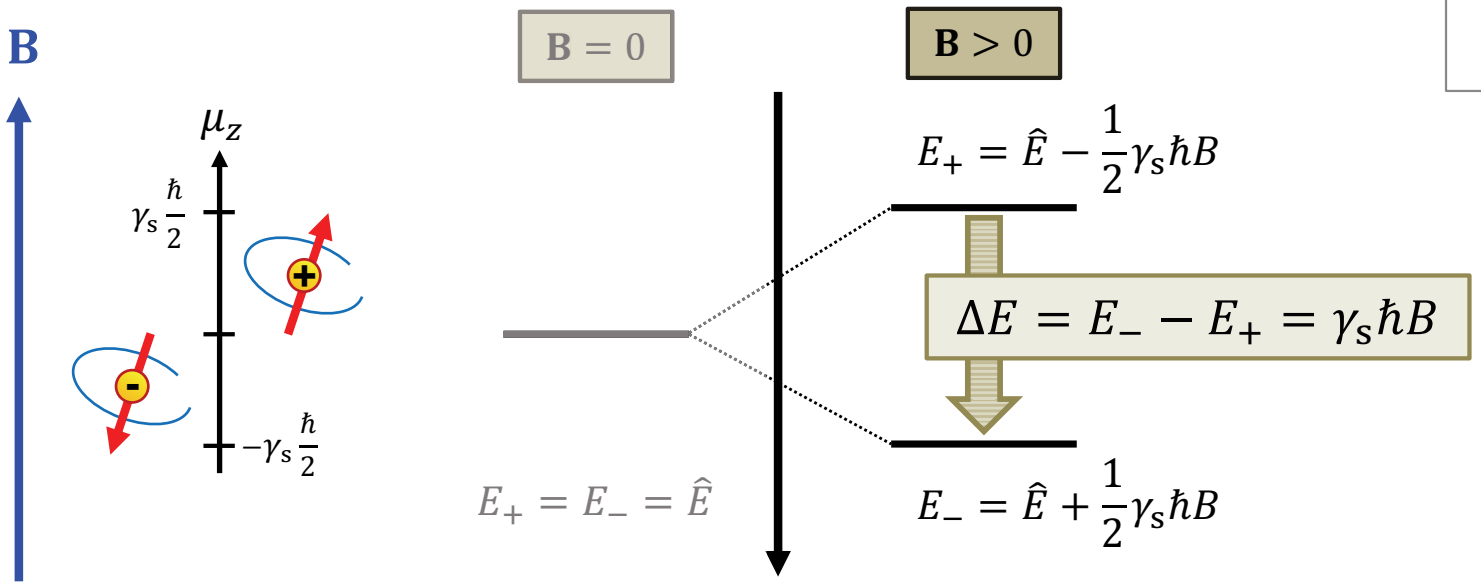
EFFET ZEEMAN

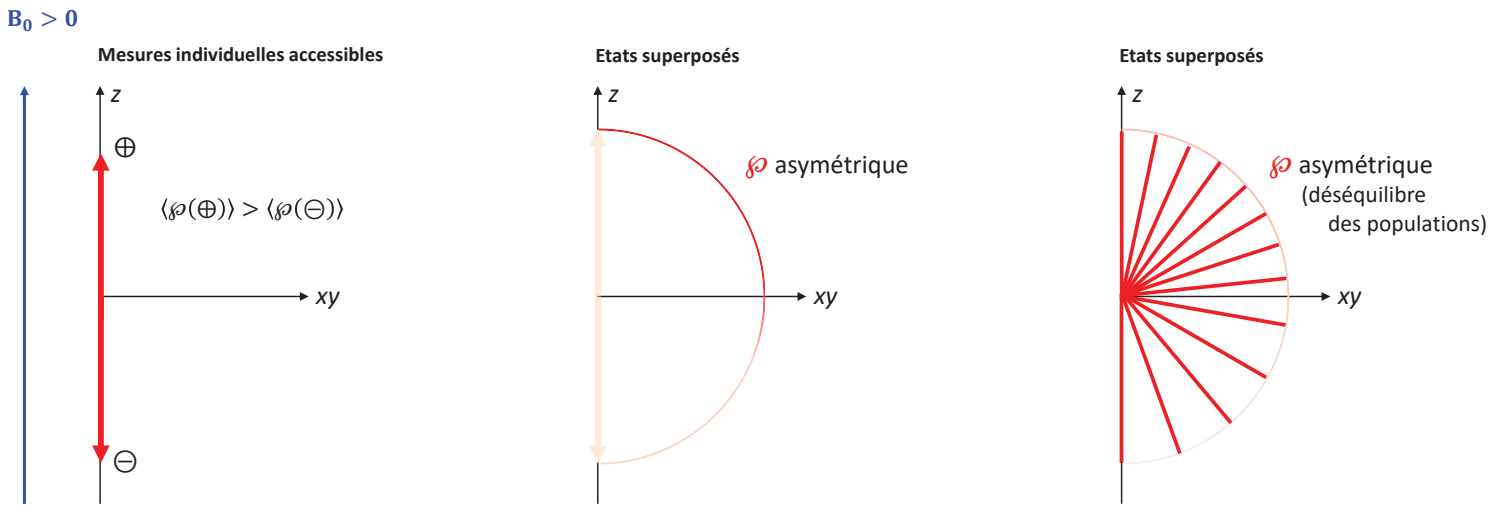
+: état « up », état pseudo-parallèle

-: état « down », état pseudo-antiparallèle

Ce que dit la mécanique quantique

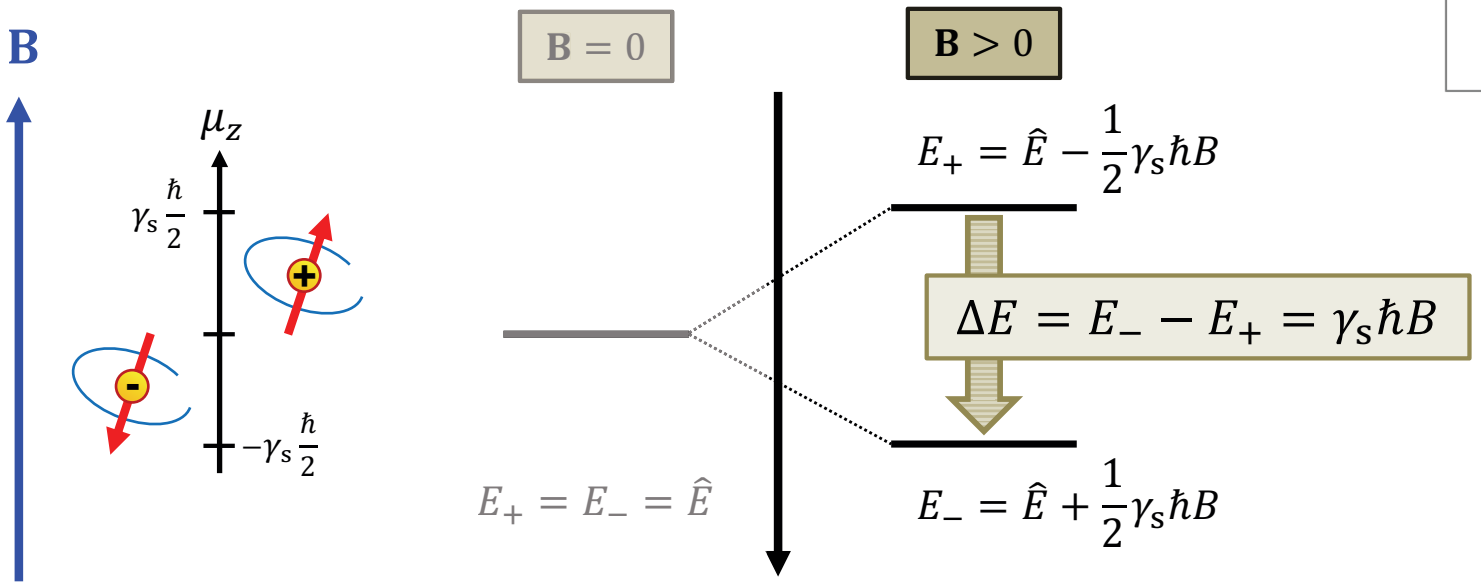
Considérations énergétiques





Ce que dit la mécanique quantique

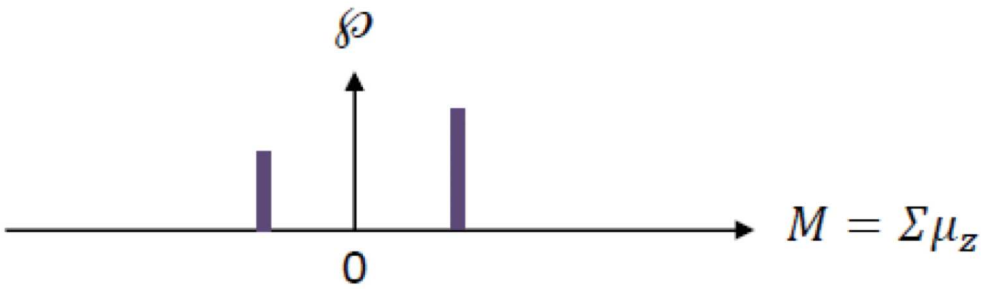
Considérations énergétiques



$$M = \sum \mu_z$$

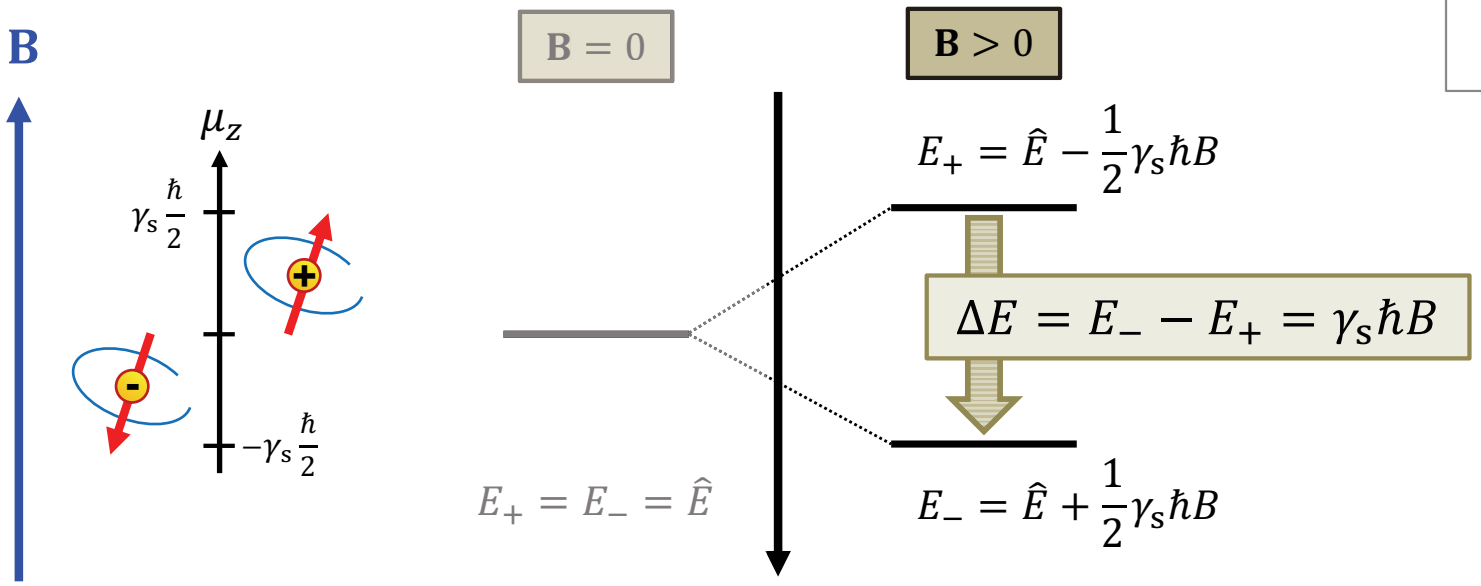
Principe de superposition: 1 dipôle

$$\wp(\oplus) > \wp(\ominus)$$



Ce que dit la mécanique quantique

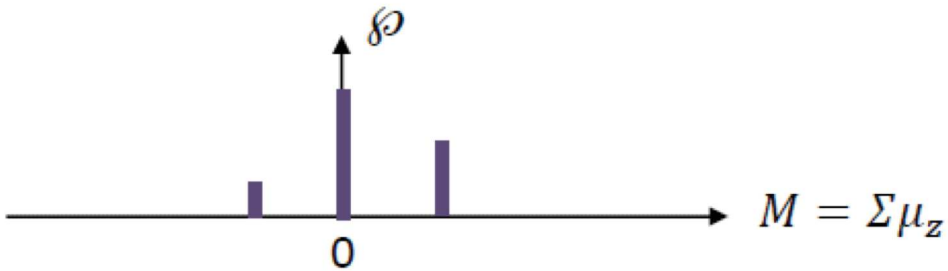
Considérations énergétiques



$$M = \sum \mu_z$$

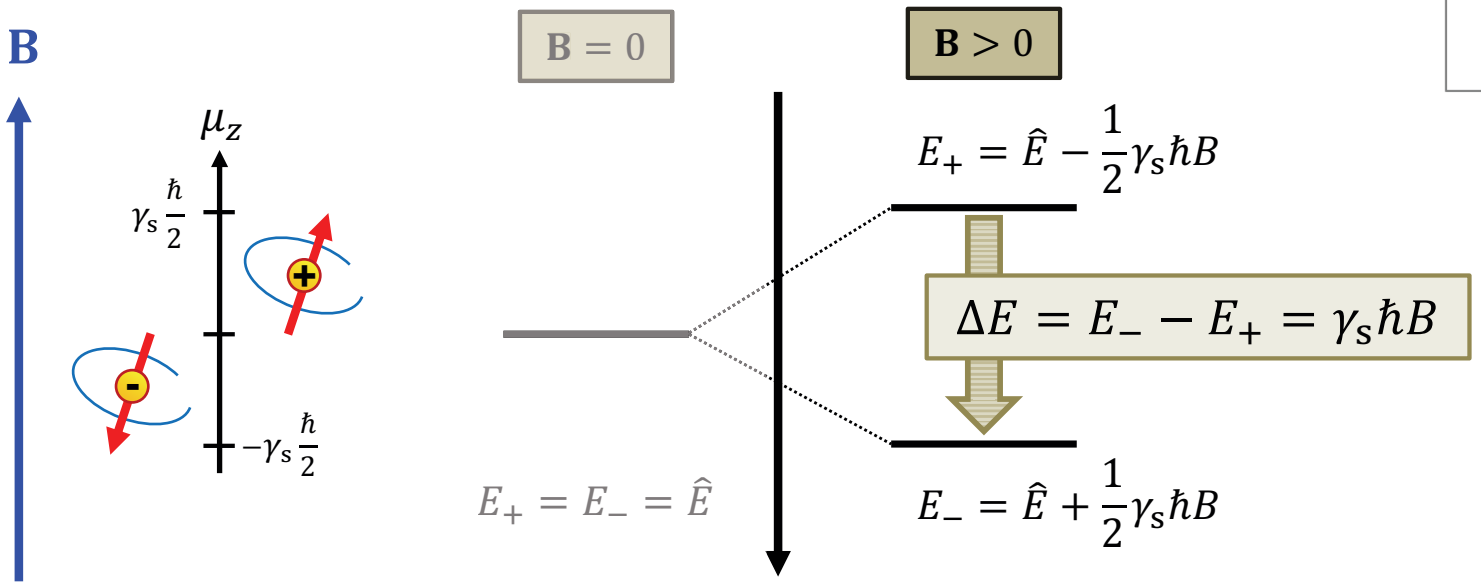
Principe de superposition: 2 dipôles

$$\wp(\oplus) > \wp(\ominus)$$



Ce que dit la mécanique quantique

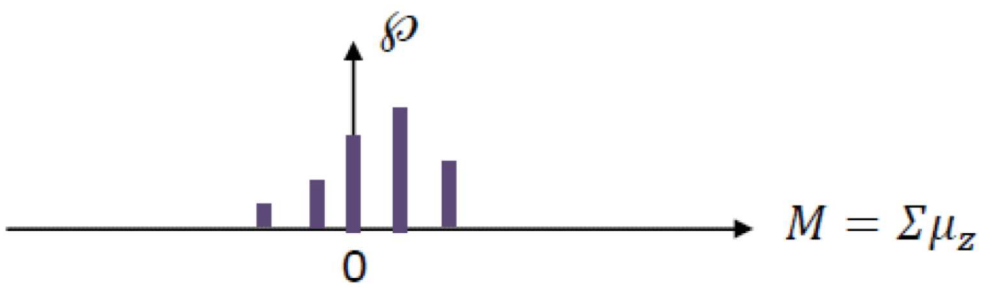
Considérations énergétiques



$$M = \sum \mu_z$$

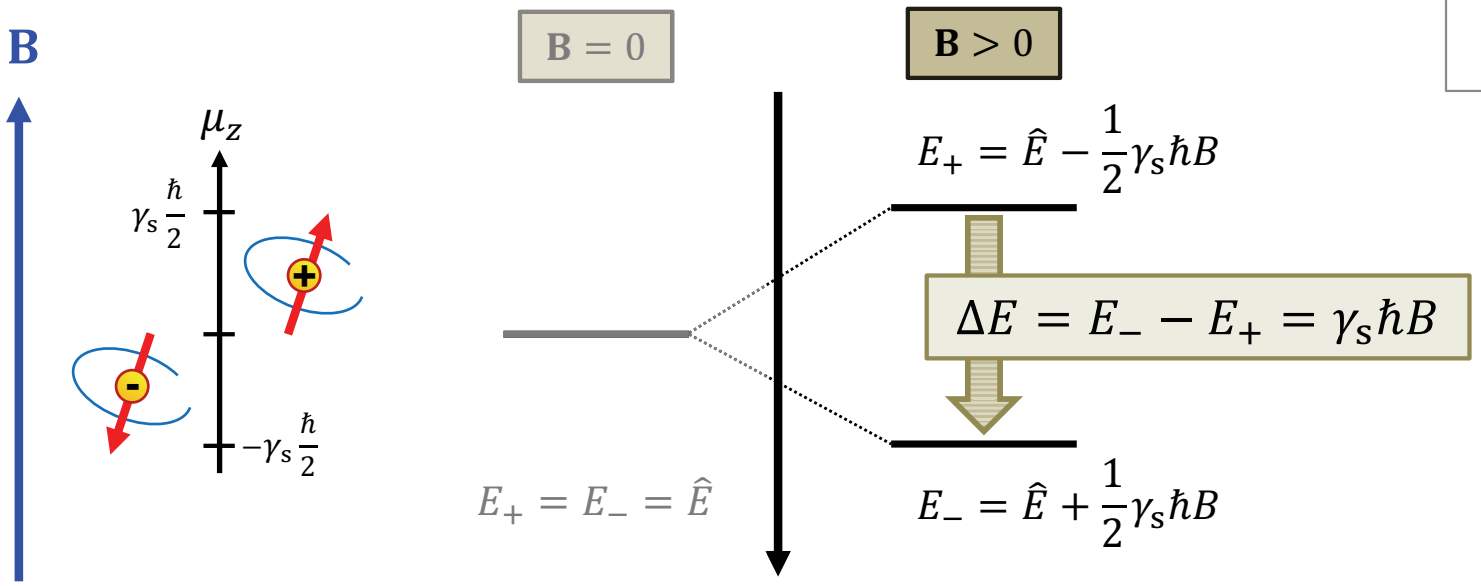
Principe de superposition: 4 dipôles

$$\wp(\oplus) > \wp(\ominus)$$



Ce que dit la mécanique quantique

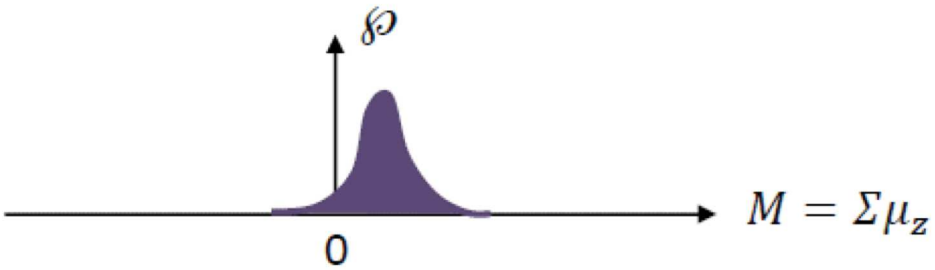
Considérations énergétiques



$$M = \sum \mu_z$$

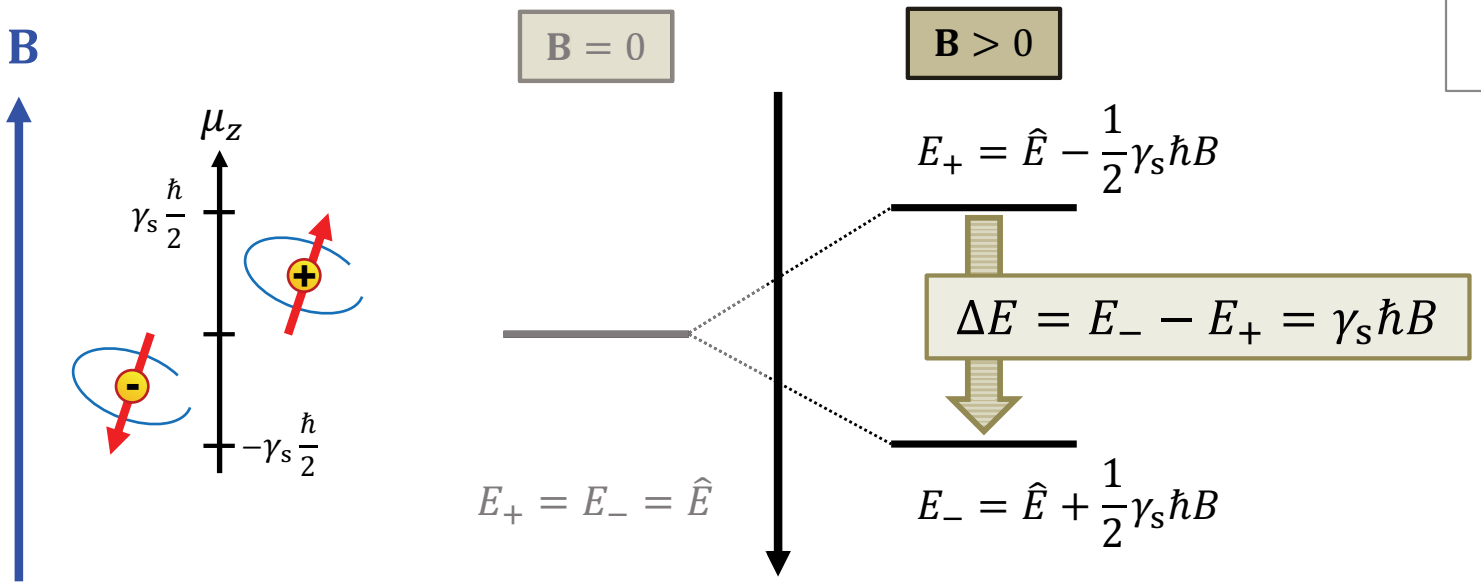
Principe de superposition: 1000 dipôles

$$\wp(\oplus) > \wp(\ominus)$$



Ce que dit la mécanique quantique

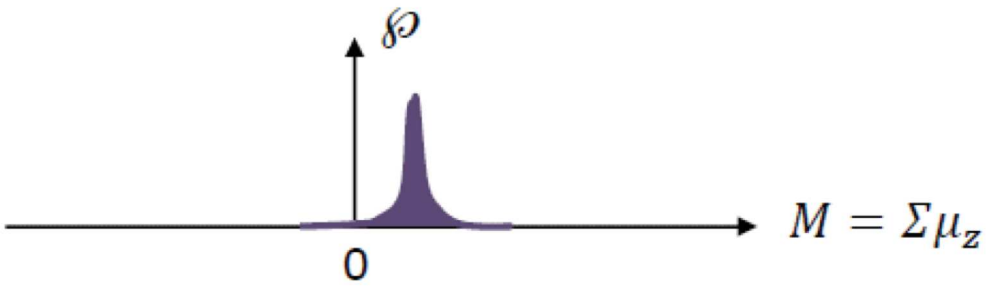
Considérations énergétiques



$$M = \sum \mu_z$$

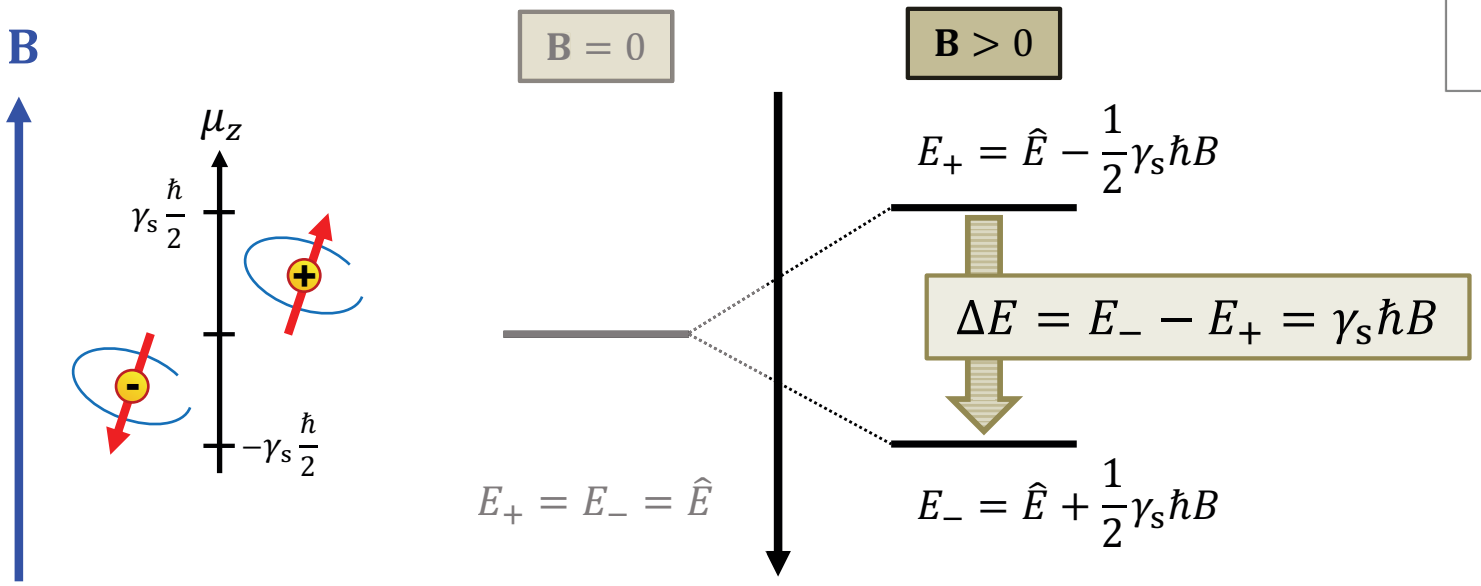
Principe de superposition: 10^6 dipôles

$$\wp(\oplus) > \wp(\ominus)$$



Ce que dit la mécanique quantique

Considérations énergétiques



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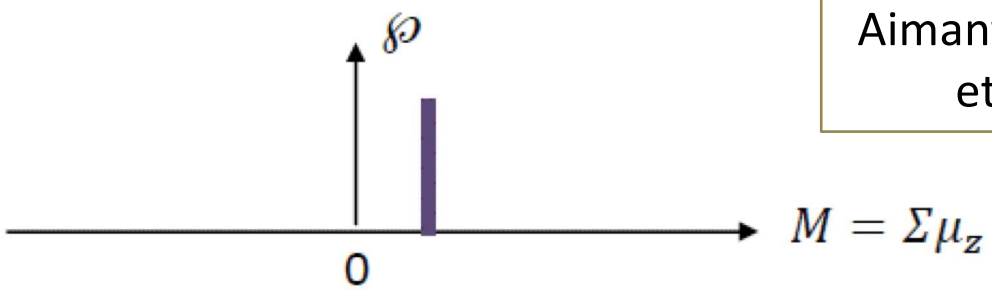
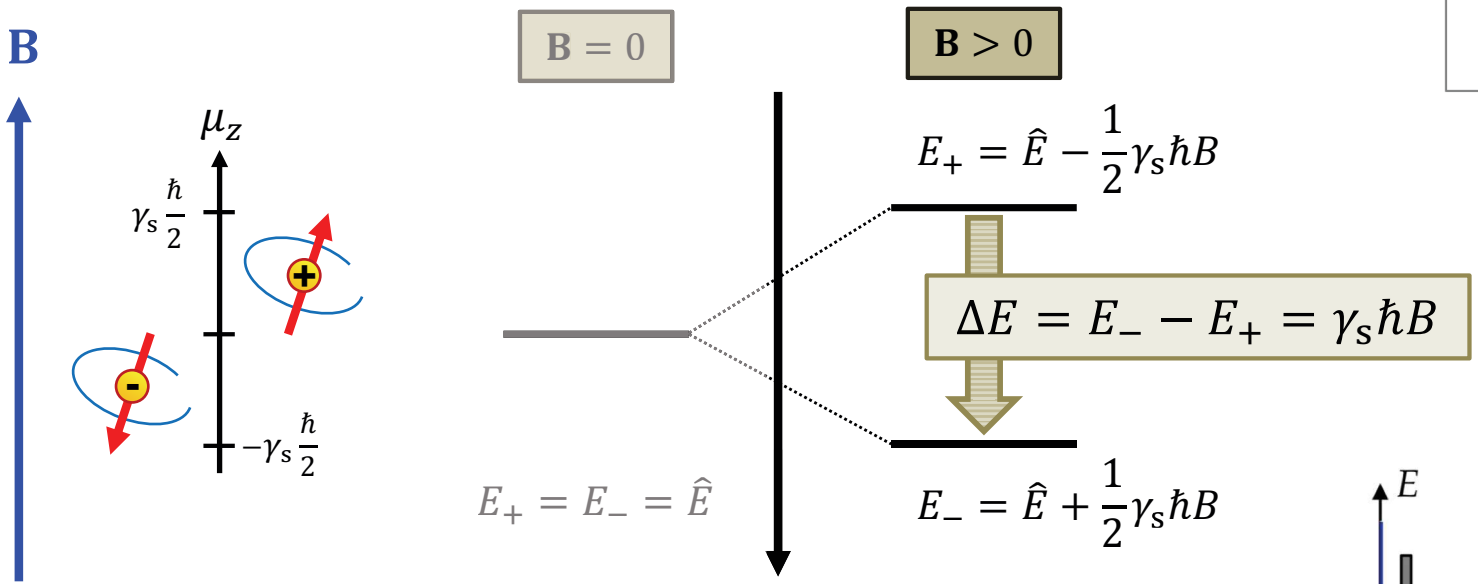


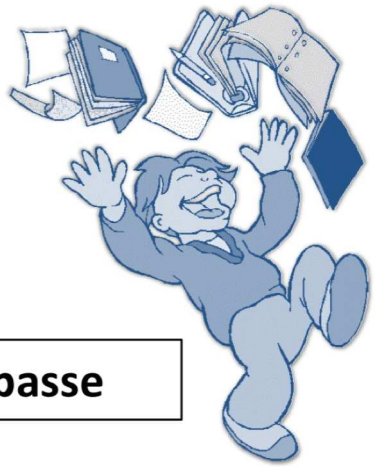
Aimantation résultante positive et mesurable quand **B** ∃

Ce que dit la mécanique quantique

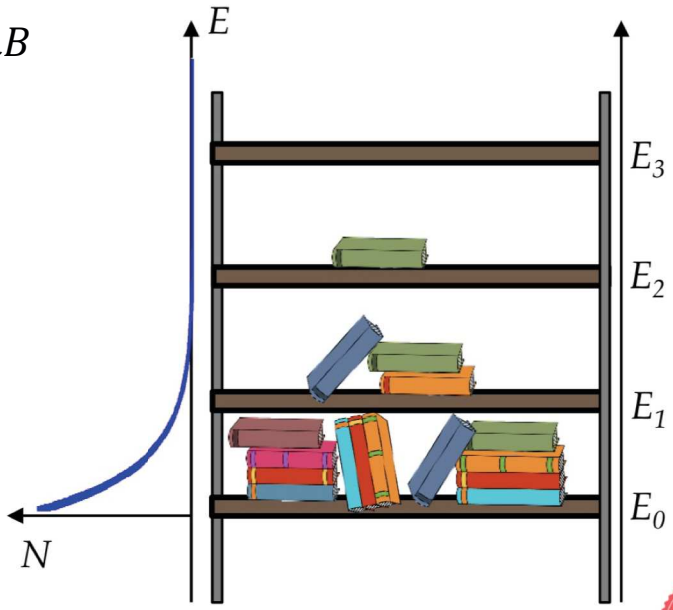
Considérations énergétiques



Comment se distribuent les spins des particules sur des états énergétiques différents ?

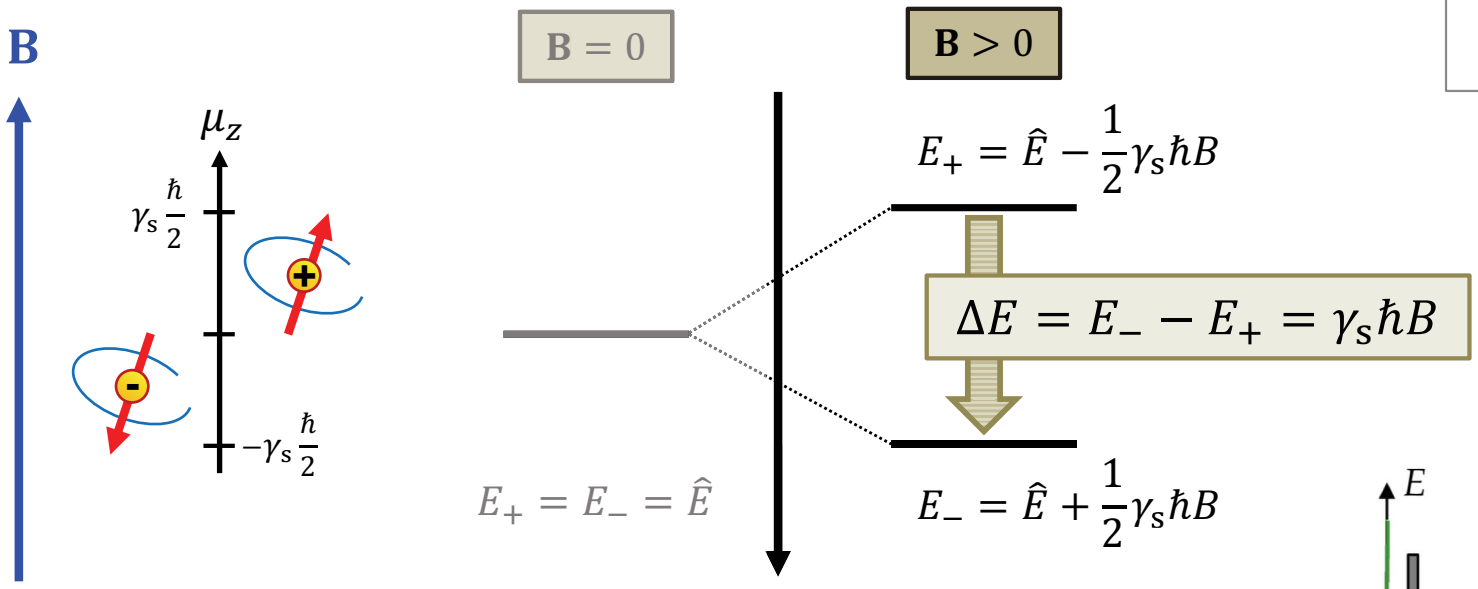


T° basse



Ce que dit la mécanique quantique

Considérations énergétiques



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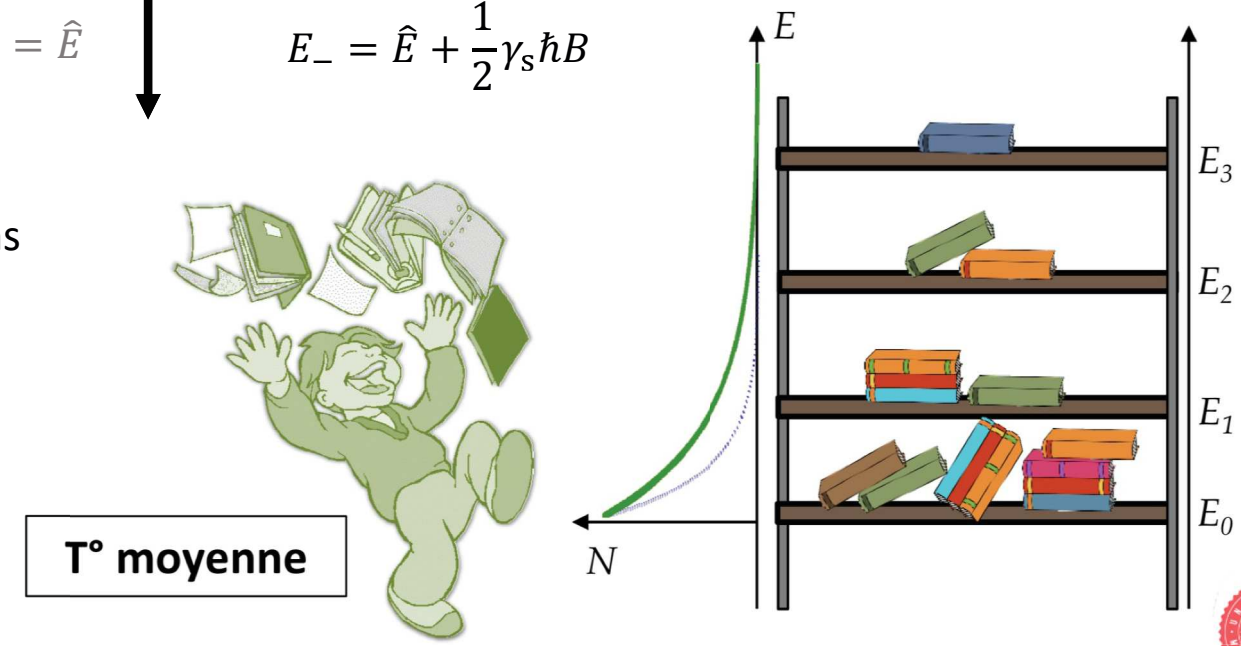

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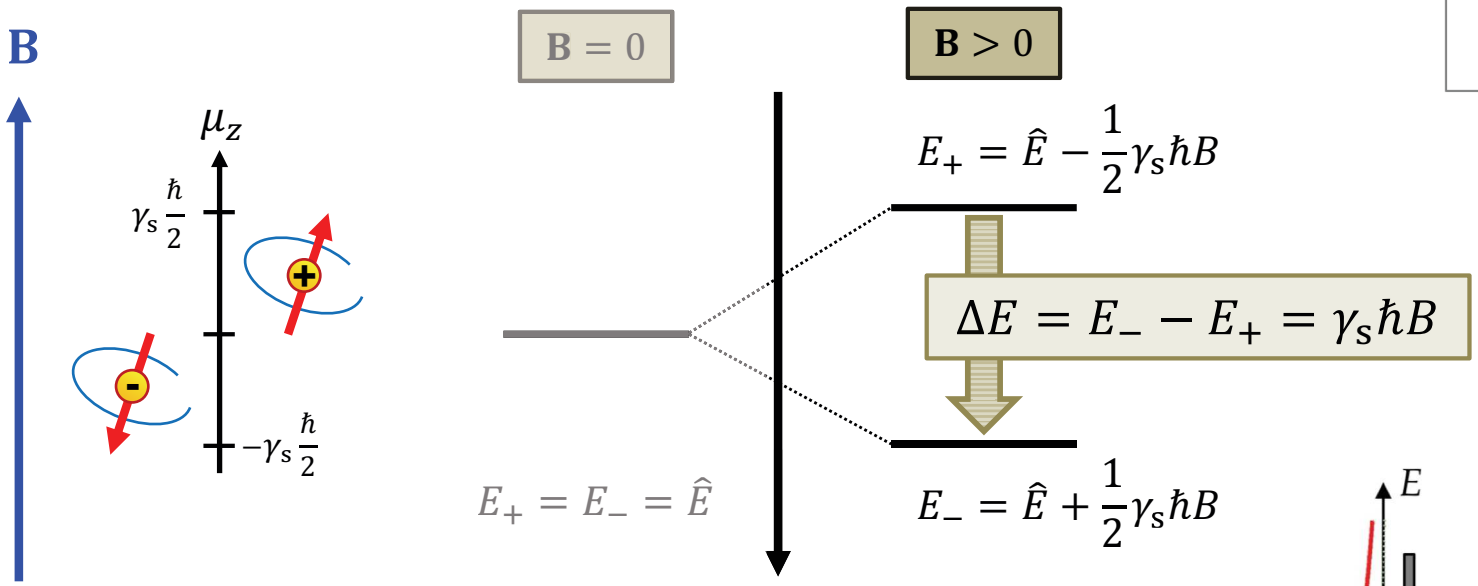

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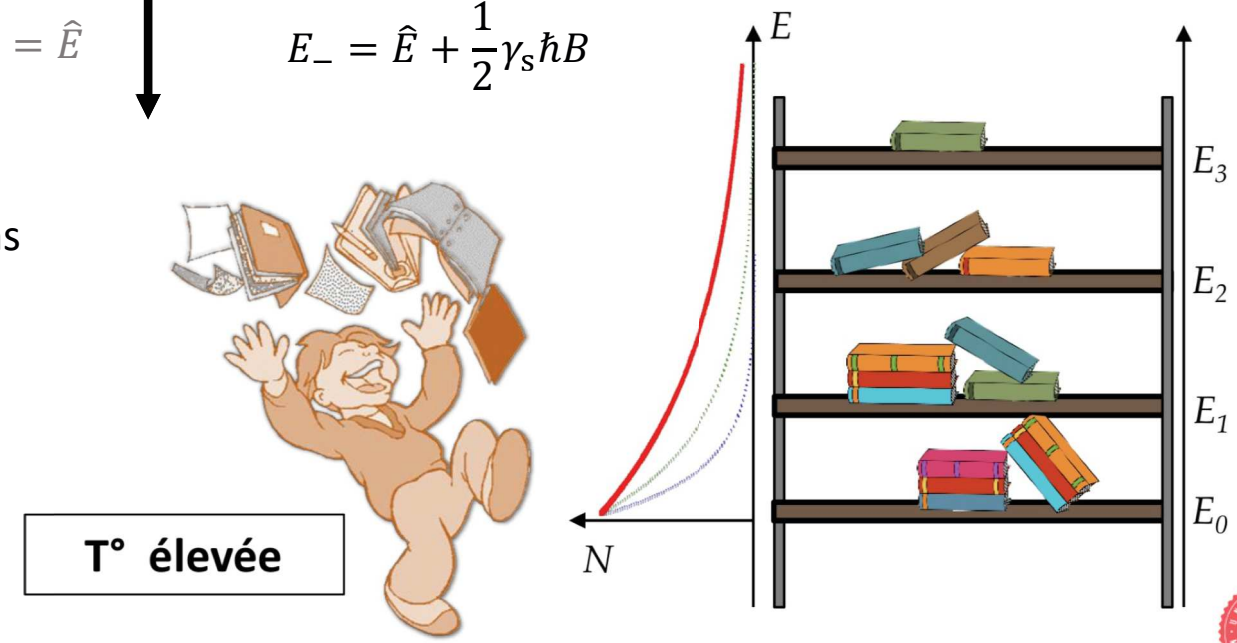
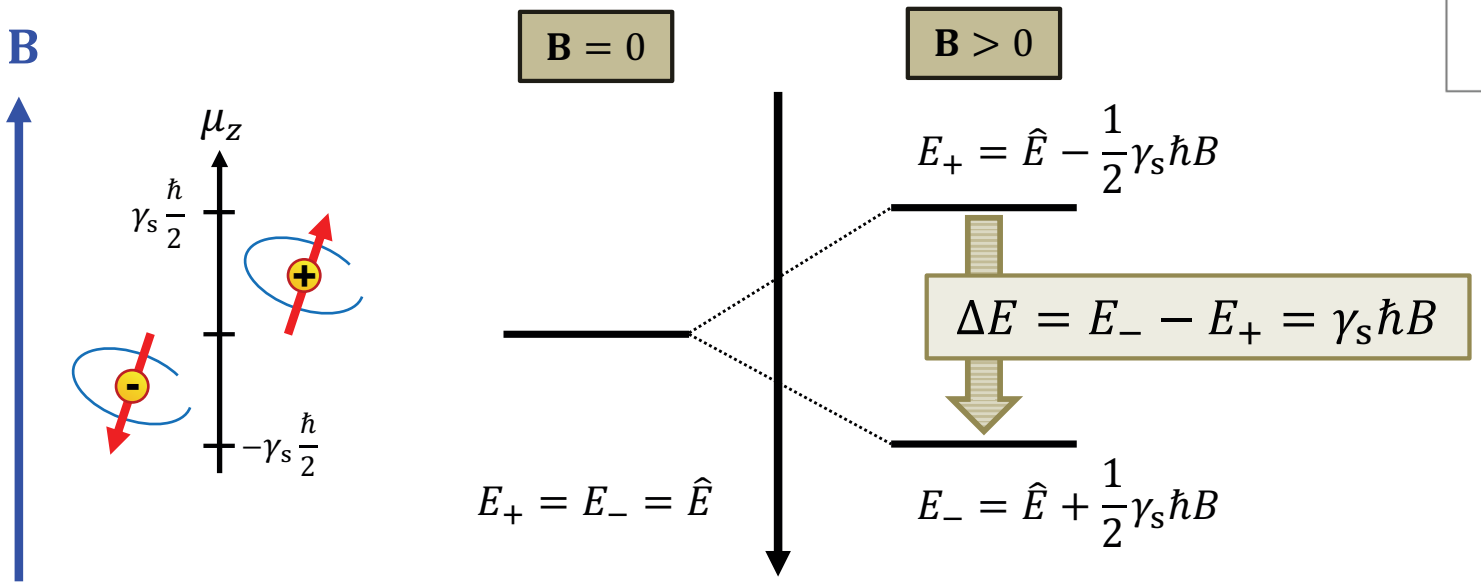


Ce que dit la mécanique quantique

Considérations énergétiques



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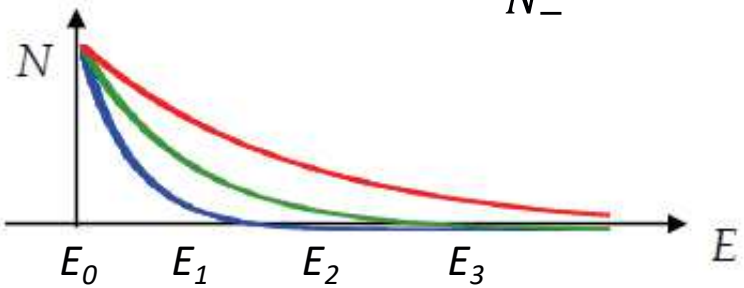
Particules indépendantes
Nombre de spins dans chaque état énergétique N_i
Etats i d'énergie E_i
Température suffisante
Equilibre thermique
Constante de Boltzmann k

$$N_i \propto e^{-\frac{E_i}{kT}}$$

$$\frac{N_+}{N_-} = e^{-\frac{E_+ - E_-}{kT}} = e^{\frac{\Delta E}{kT}}$$



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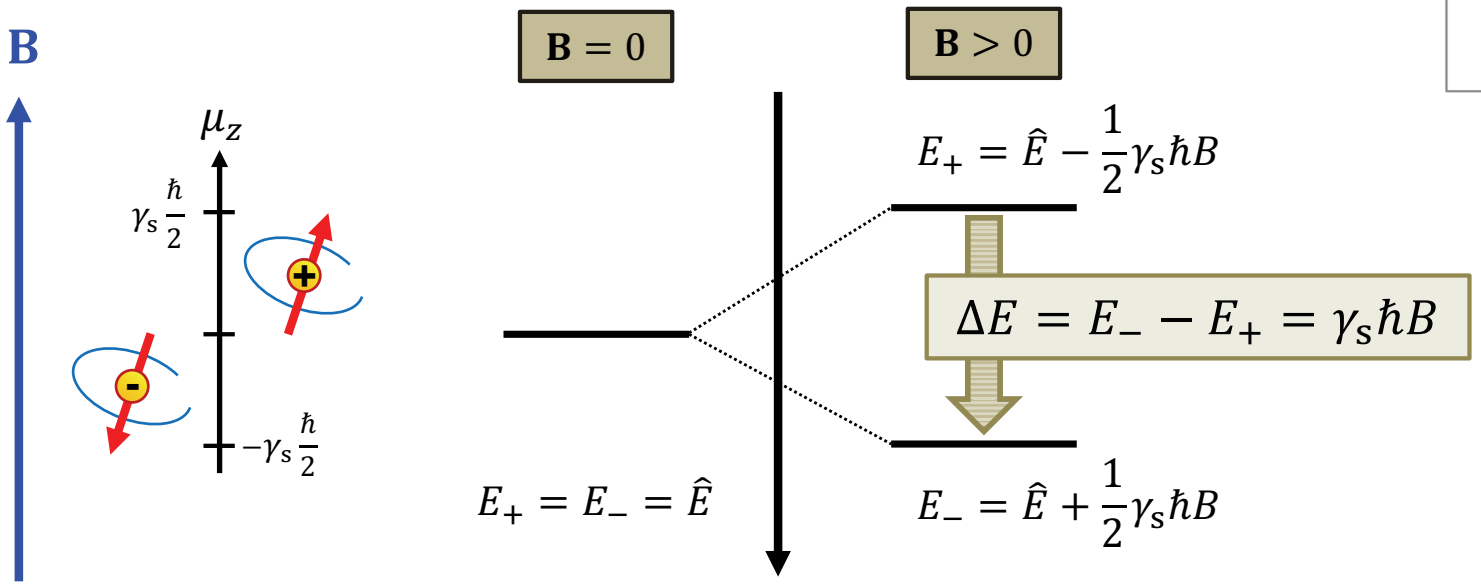

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$$N_i \propto e^{-\frac{E_i}{k\,T}}$$

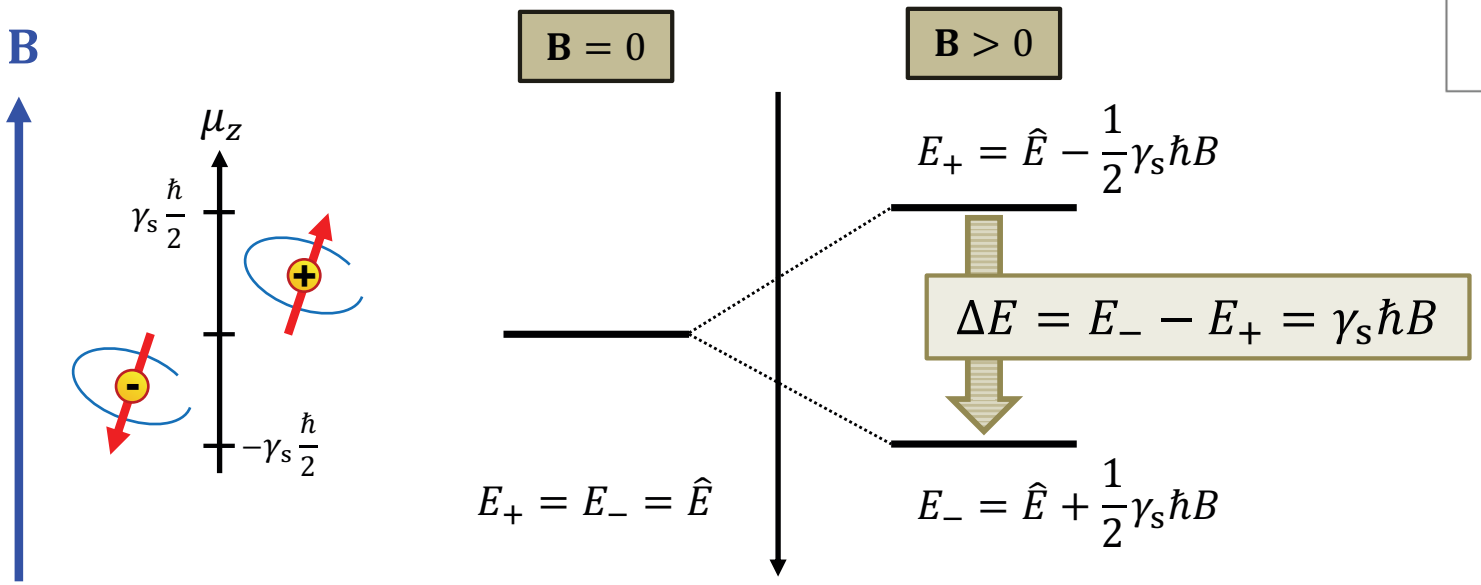
$$\frac{\Delta N}{N} = \frac{N_+ - N_-}{N_+ + N_-} = \frac{\frac{N_+}{N_-} - 1}{\frac{N_+}{N_-} + 1}$$

$$= \frac{e^{\frac{\Delta E}{k\,T}} - 1}{e^{\frac{\Delta E}{k\,T}} + 1} \simeq \frac{\frac{\Delta E}{k\,T}}{\frac{\Delta E}{k\,T} + 2}$$

$$\sim \frac{\Delta E}{2\,k\,T} \sim \frac{1}{1000}$$

Ce que dit la mécanique quantique

Considérations énergétiques



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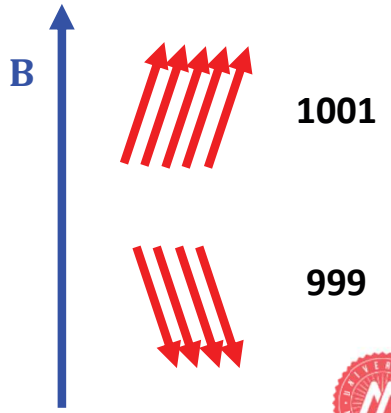


Ce que dit la mécanique quantique

Composition des moments cinétiques

Moment cinétique total $\mathbf{J}$

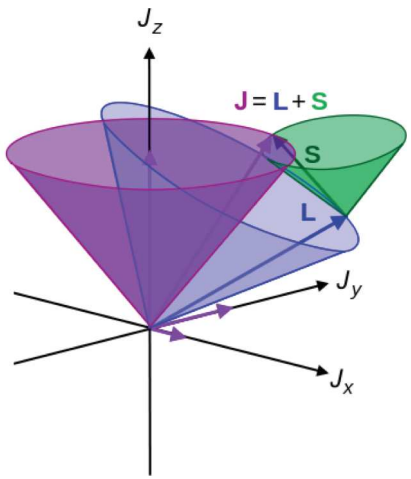
$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

$$|\mathbf{J}| = \hbar\sqrt{j(j+1)}$$

$$j = |l - s|, \dots, l + s$$

Electron : $j = l - \frac{1}{2}, l + \frac{1}{2}$

$$J_z = m_j\hbar = -j\hbar \ \dots \ 0 \ \dots j\hbar$$



Etat complet de l'électron (n, l, j, m_j)

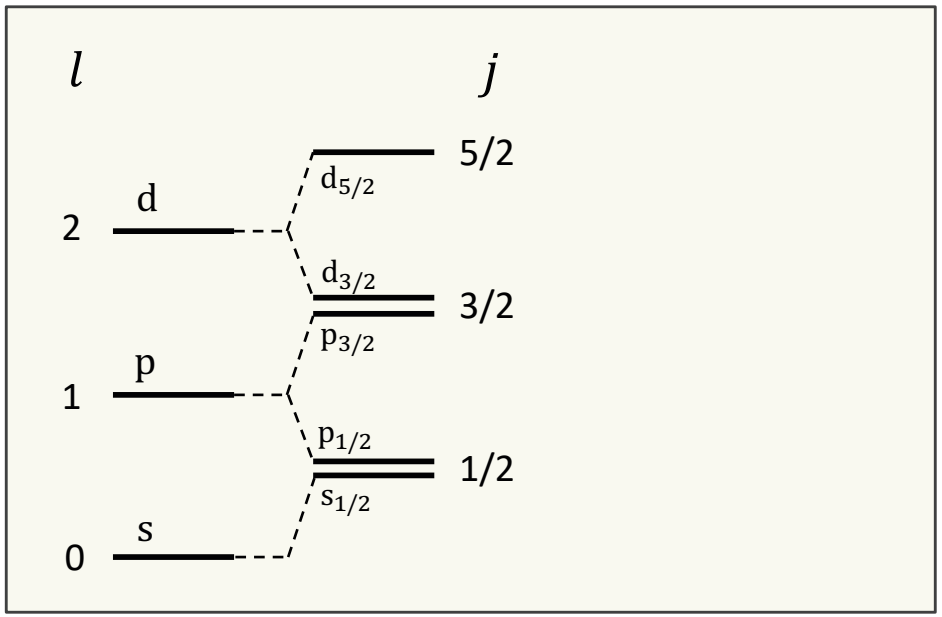
Il s'y associe un moment magnétique :

$$\boxed{\boldsymbol{\mu} = \gamma_e\,\mathbf{J} = g\,\gamma_o\,\mathbf{J}}$$

γ_e rapport gyromagnétique global
g facteur de Landé électronique

Nota: pour un électron libre $\gamma_e = \gamma_s$

$$g = \frac{\gamma_e}{\gamma_o}$$



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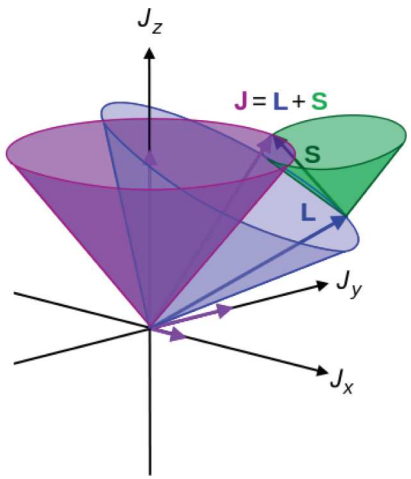

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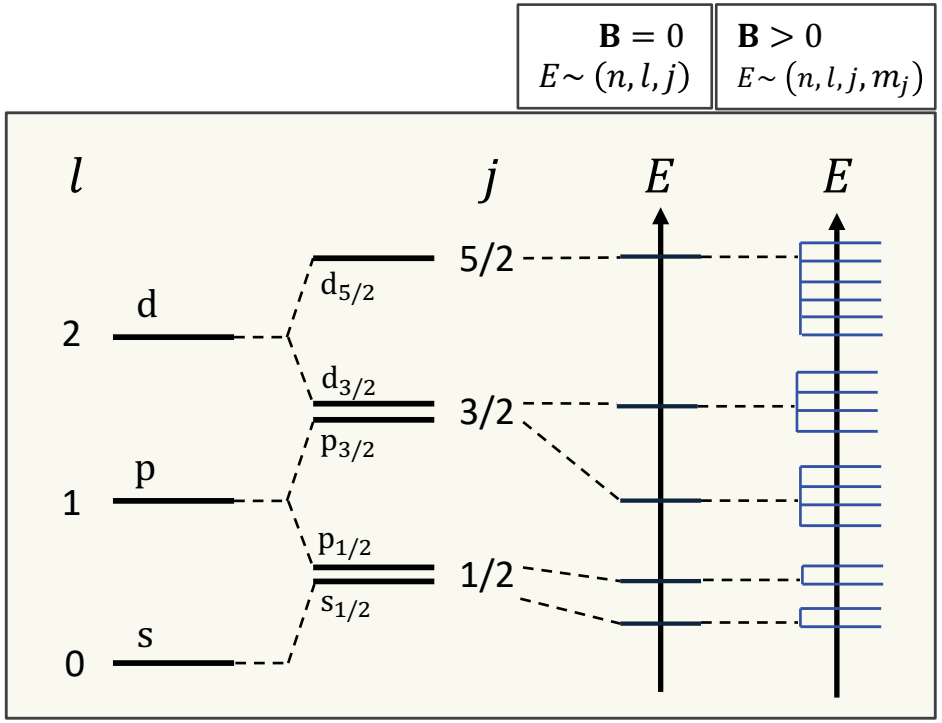

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Electron : $j = l - \frac{1}{2}, l + \frac{1}{2}$

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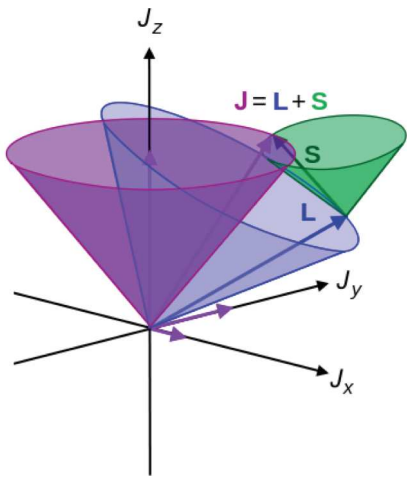

Etat complet de l'électron (n, l, j, m_j)

Il s'y associe un moment magnétique :

$$\boxed{\boldsymbol{\mu} = \gamma_e\,\mathbf{J} = g\,\gamma_o\,\mathbf{J}}$$

γ_e rapport gyromagnétique global
g facteur de Landé électronique

$$\Delta E = \Delta\mu_z\,B$$

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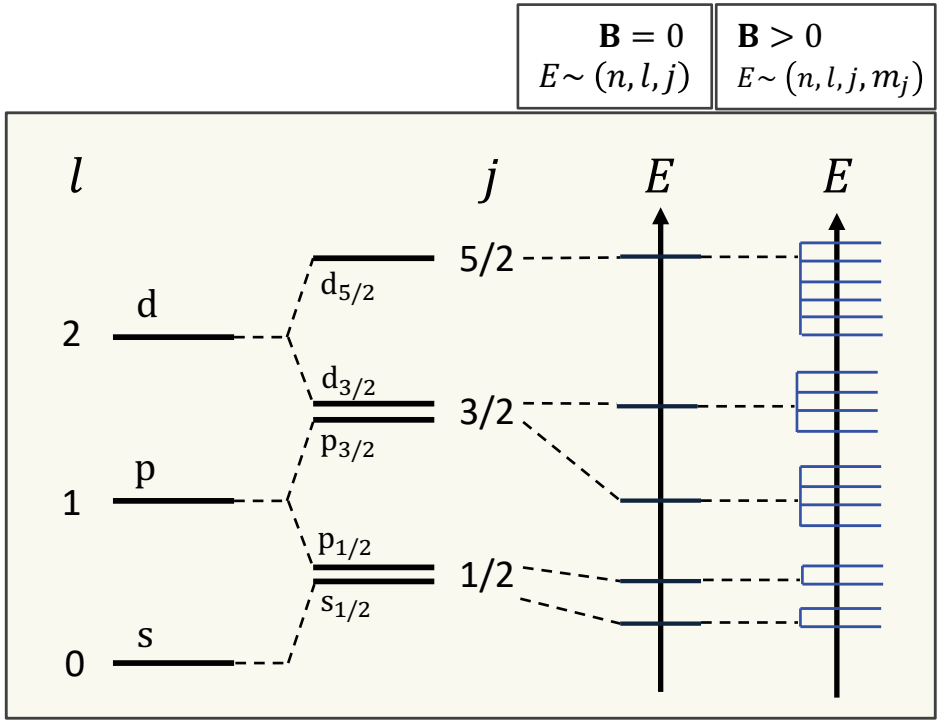

magnéton de Bohr $\mu_B = \dfrac{e\,\hbar}{2\,m_e}$

Même dimension qu'un moment magnétique
[Am^2] ou [JT^{-1}]

Ce que dit la mécanique quantique

Composition des moments cinétiques

Moment cinétique total $\mathbf{J}$

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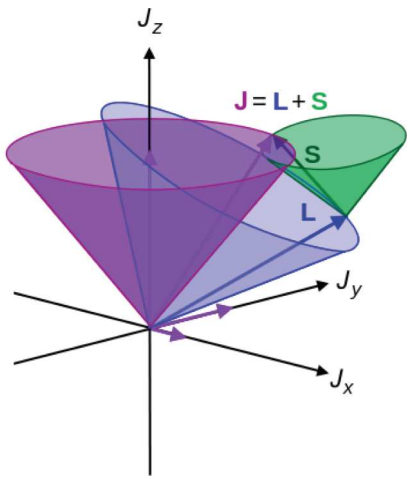

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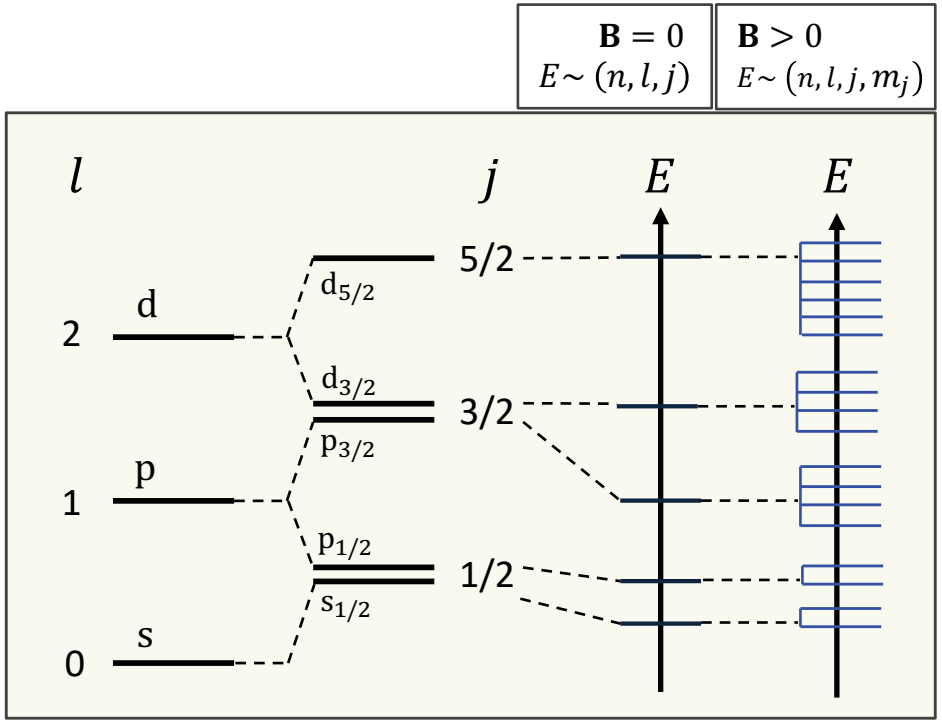

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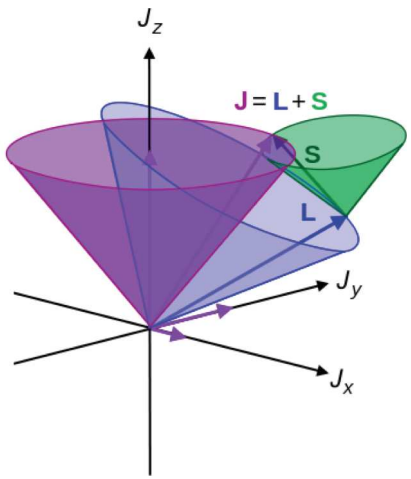

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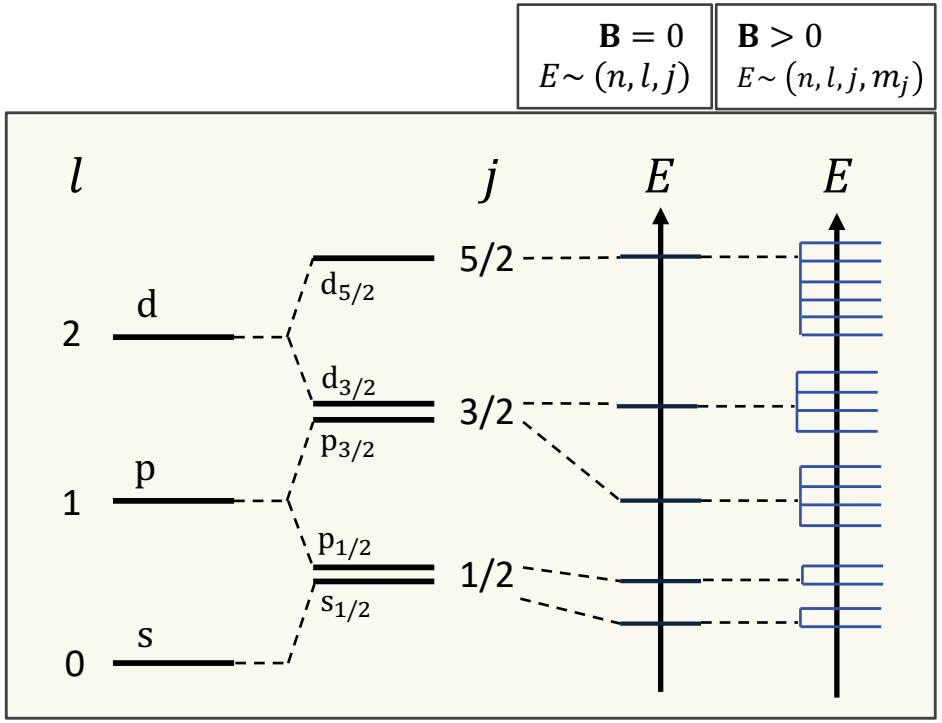
or $\Delta E = \omega\,\hbar$

$$\boxed{g = \frac{\omega\,\hbar}{\mu_B\,B}}$$



MAGNETISME & RMN

Plan du cours

Contexte

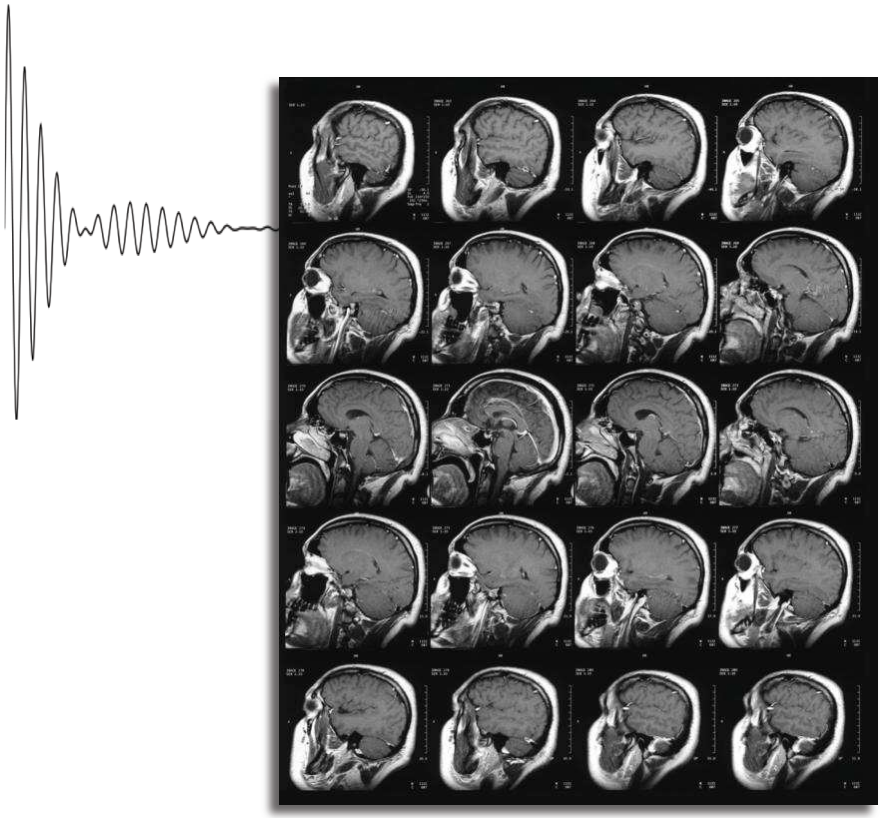
- **Champ et interaction électrique**
- **Champ et interaction magnétique**

Electron atomique
Mécanique quantique

- **Magnétisme dans la matière**
- **Magnétisme nucléaire**

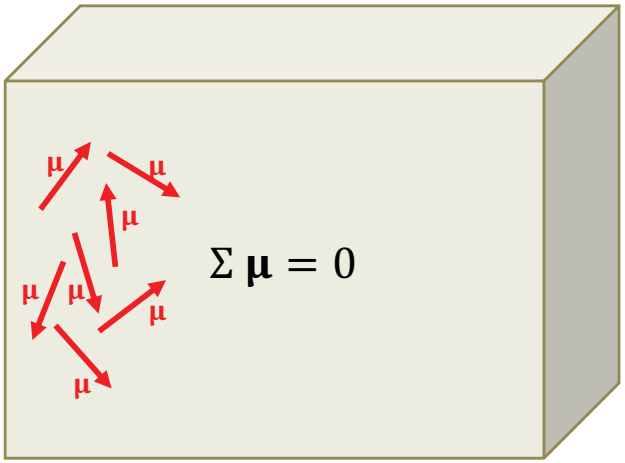
RMN – principe
RMN – séquence
RMN – contraste

- **Applications : IRM & SRM**
- **Illustrations**



Magnétisme dans la matière

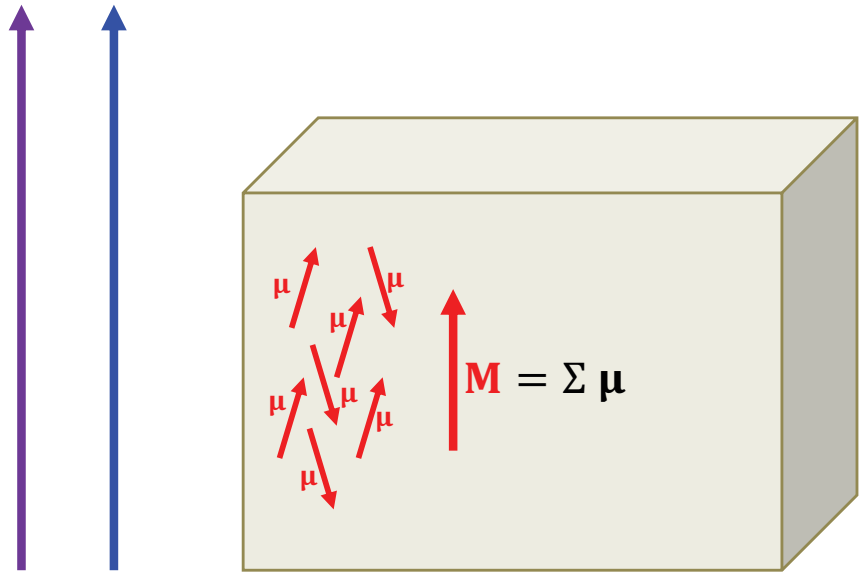
(dû aux électrons)



Magnétisme dans la matière

(dû aux électrons)

$\mathbf{H}$ $\quad \mathbf{B}_0 = \mu_0 \mathbf{H}$ $\quad$ μ_0 : perméabilité magnétique du vide



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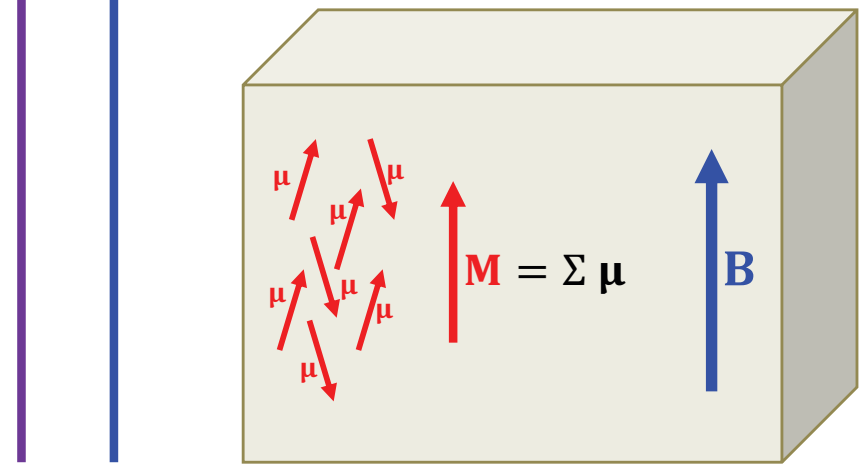


$$\mathbf{B} = \mu\, \mathbf{H}$$

$$= \mu_0\, (1 + \chi)\, \mathbf{H}$$

$$= \mu_0\, (\mathbf{H} + \mathbf{H}')$$

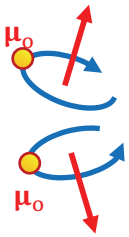
μ : perméabilité magnétique du matériau

χ : susceptibilité magnétique

$\mathbf{H'} = \frac{\mathrm{d}\mathbf{M}}{\mathrm{d}V}$: aimantation volumique

Diamagnétisme

Doublets d'électrons (tous les électrons sont appariés)
Absence de moment permanent $\mathbf{J} = 0$



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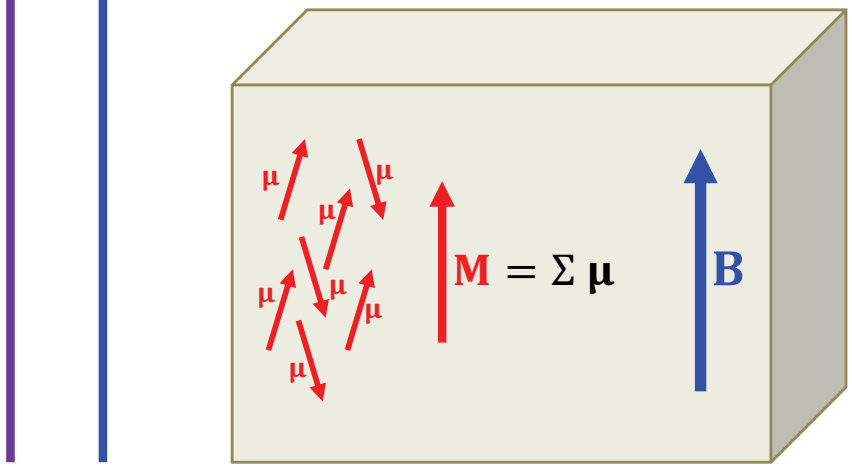

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μ : perméabilité magnétique du matériau

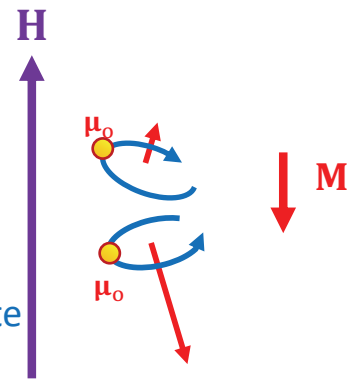
χ : susceptibilité magnétique

$\mathbf{J} = \frac{d\mathbf{M}}{dV}$: aimantation volumique

Diamagnétisme

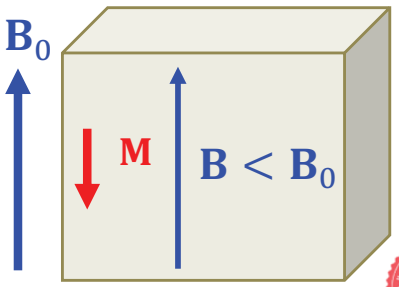
Doublets d'électrons
Absence de moment permanent $\mathbf{J} = 0$
Modification du mouvement orbital

Cuivre, plomb, eau et tissus biologiques, graphite



$\chi < 0$

$\chi \sim 10^{-5}$



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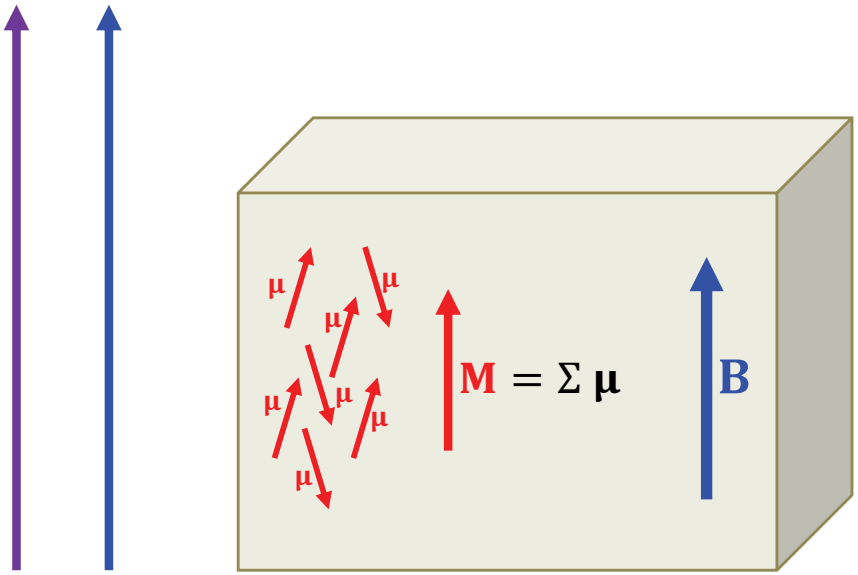

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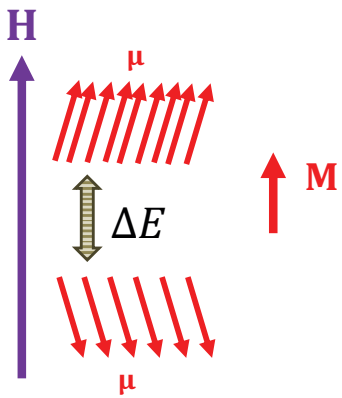
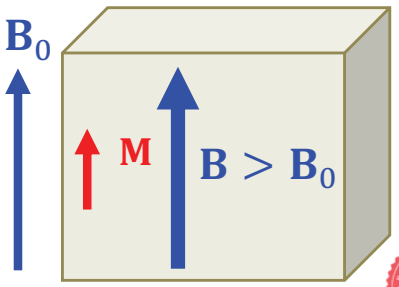
Paramagnétisme

Electrons de valence non appariés
Moment permanent $\mathbf{J}$
Alignement sur $\mathbf{B_0}$
Température dépendant
Ions métalliques, radicaux libres, certains produits de contraste



$\chi > 0$

$\chi \sim 10^{-3}$



Magnétisme nucléaire

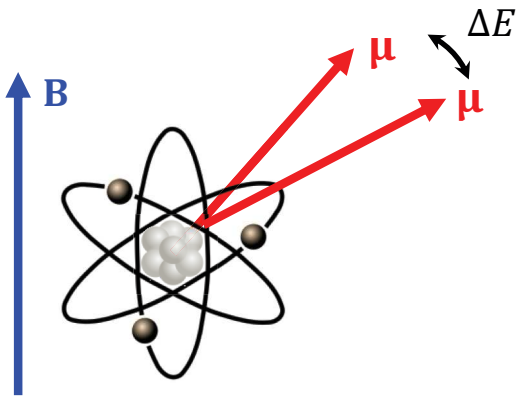
→ **Magnétisme atomique essentiellement électronique**

Electrons atomiques:

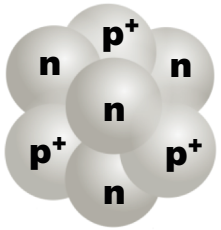
- Moment cinétique total $\mathbf{J}$
- Moment magnétique permanent $\boldsymbol{\mu}$

$$\boldsymbol{\mu} = \gamma_{\mathrm{e}}\, \mathbf{J} \qquad \overline{\gamma_e} \sim \mathrm{GHz\ T^{-1}}$$

$$\Delta E = \Delta\mu_z\, B = g\, B\, \mu_{\mathrm{B}}$$



Noyau = protons + neutrons



- Moment cinétique total du noyau $\mathbf{J}$: « SPIN »
- Moment magnétique permanent du noyau $\boldsymbol{\mu}$: « SPIN »

$$\boldsymbol{\mu} = \gamma_{\mathrm{N}}\, \mathbf{J} \qquad \overline{\gamma_N} \sim \mathrm{MHz\ T^{-1}}$$

$$\Delta E = \Delta\mu_z\, B = g_N\, \mu_{\mathrm{N}}\, B = \gamma_N \hbar B$$

$$\boxed{\gamma_N = g_N \frac{e}{2m_p}} \qquad m_p \gg m_e$$

de l'ordre de 2000 fois

Proton: $g = 5{,}6$

Neutron: $g = -3{,}8$

magnéton nucléaire $\mu_{\mathrm{N}} = \dfrac{e\,\hbar}{2\,m_p} \ll \mu_{\mathrm{B}}$

Magnétisme nucléaire

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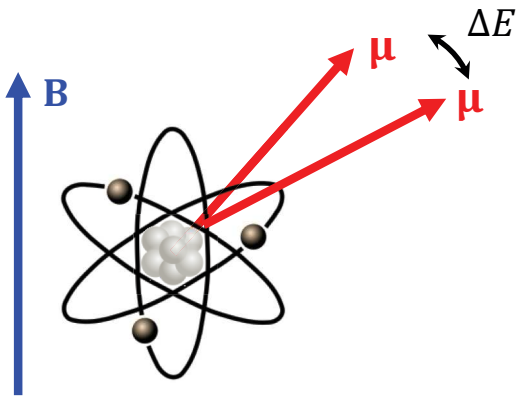

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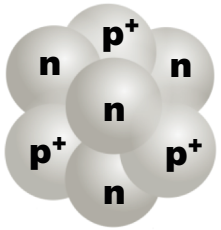

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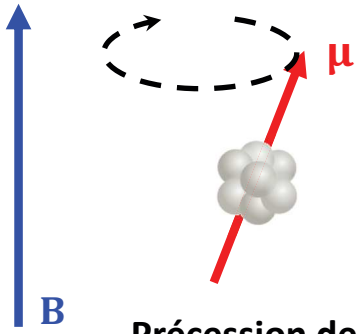


Précession de Larmor

$$\boldsymbol{\omega} = -\gamma_{\mathrm{N}}\, \mathbf{B}$$

Magnétisme nucléaire

Noyau ^{1}H = 1 proton

- Moment cinétique total **J** : « SPIN »
- Moment magnétique permanent **μ** : « SPIN »
- j = ½
- 2j+1 orientations

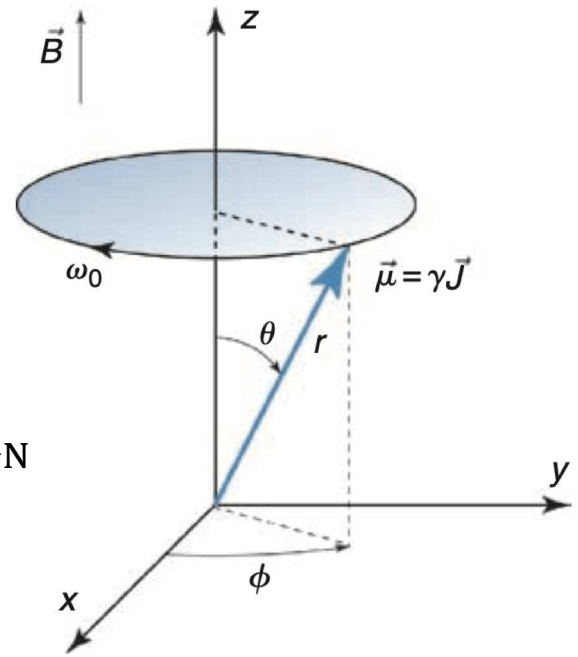
$$\boldsymbol{\mu} = \gamma_{\mathrm{p}}\, \mathbf{J}$$

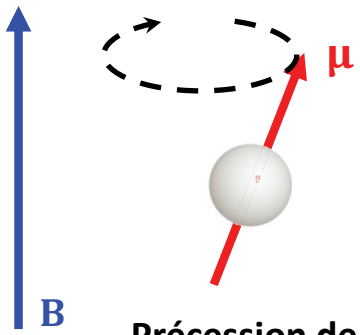
$$\mu_z = \gamma_{\mathrm{p}}\, J_z = \gamma_{\mathrm{p}}\, m_j\, \hbar$$

Proton: $g = 5{,}6$ $\quad \Delta E = \Delta\mu_z\, B = g\, B\, \mu_{\mathrm{N}}$

$$\Delta E = \Delta\mu_z\, B = \gamma_p \hbar B$$

$$\bar{\gamma_p} = 42{,}6\ \mathrm{MHz}\ T^{-1}$$





Précession de Larmor

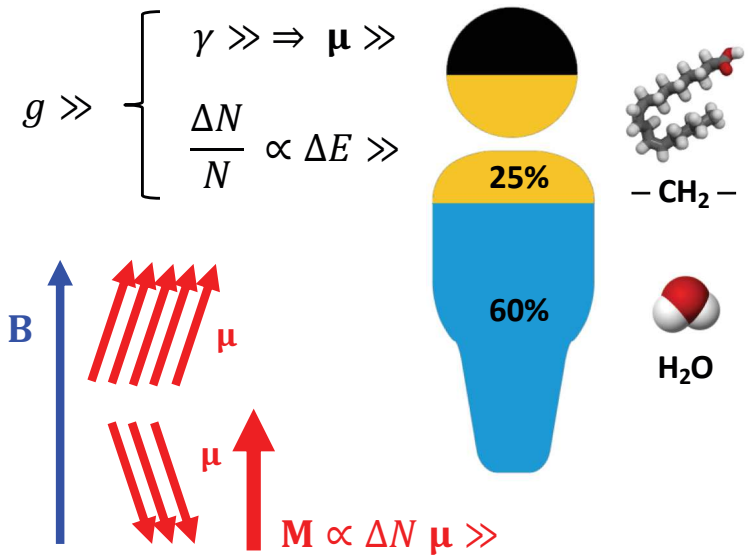
$B = 1{,}5\ \mathrm{T}$: $\nu = 64\ \mathrm{MHz}$
$B = 3\ \mathrm{T}$: $\nu = 128\ \mathrm{MHz}$
ondes radio

Magnétisme nucléaire

Nucleus	*Spin = j*	*Landé factor*
proton p	1/2	5.5883
neutron n	1/2	−3.8263
deuteron $^{2}_{1}D$	1	0.85742
$^{3}_{2}He$	1/2	−4.255
$^{4}_{2}He$	0	—
$^{12}_{6}C$	0	—
$^{16}_{8}O$	0	—
$^{39}_{19}K$	3/2	0.2609
$^{67}_{30}Zn$	5/2	0.35028
$^{85}_{37}Rb$	5/2	0.54108
$^{129}_{54}Xe$	1/2	−1.5536
$^{133}_{55}Cs$	7/2	0.7369
$^{199}_{80}Hg$	1/2	1.0054
$^{201}_{80}Hg$	3/2	−0.37113

- **Spin dépend de la composition du noyau**
- **Pas de spin ⇔ Pas de magnétisme**
- **$^{1}H = p^{+}$: magnétisme maximal**

$$g \gg \begin{cases} \gamma \gg \Rightarrow \boldsymbol{\mu} \gg \\ \dfrac{\Delta N}{N} \propto \Delta E \gg \end{cases}$$



Magnétisme nucléaire ou électronique?

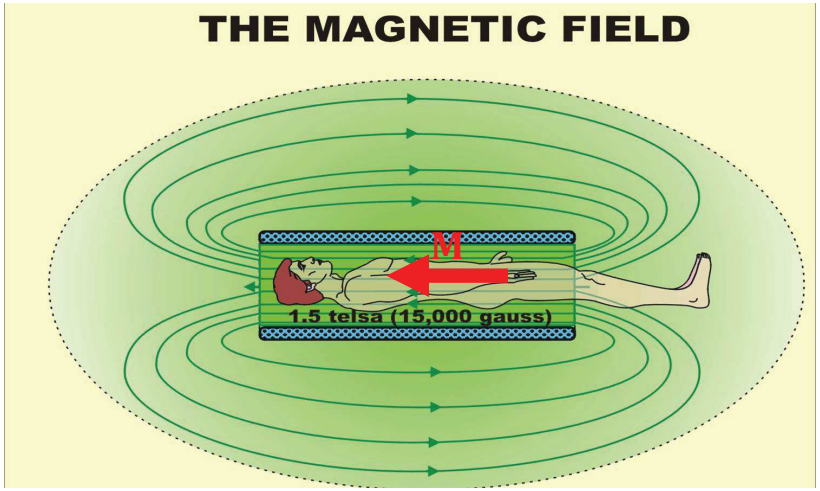
Magnétisme atomique essentiellement électronique

Eléments avec symétrie moléculaire insuffisante ou électrons célibataires:

$\boldsymbol{\mu}_e$ permanent $B_{matériau} \approx 1$ mT pour B = 1T dû au magnétique électronique

Le noyau d'^{1}H:

$\gamma \ll \gamma_e$ $\frac{\Delta N}{N} \ll$ $\boldsymbol{\mu} \ll$ $B_{matériau} \approx 1$ nT pour B = 1T dû au magnétique nucléaire



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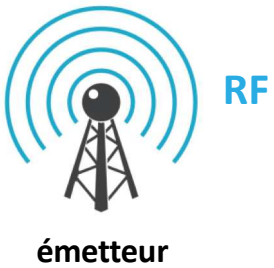
Toutes les particules qui portent un spin peuvent entrer en résonnance dans un champ magnétique

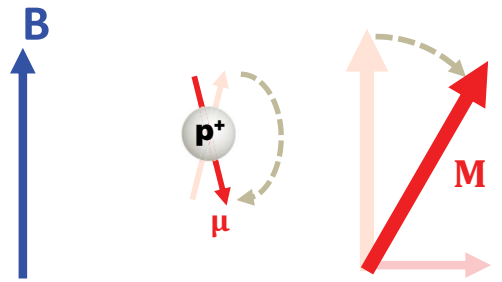
Précession de Larmor :

$\boldsymbol{\omega} = -\gamma\, \mathbf{B}$ [rad.s^{-1}]

$\boldsymbol{\nu} = -\bar{\gamma}\, \mathbf{B}$ [Hz]



RF à ν donnée



Magnétisme nucléaire ou électronique?

Magnétisme atomique essentiellement électronique

Eléments avec symétrie moléculaire insuffisante ou électrons célibataires:

B = 1 T : $\nu \sim \mathrm{GHz}$ (micro-onde) = Résonnance paramagnétique électronique

Le noyau d'^{1}H:

$B = 1\ \mathrm{T} : \nu = 42{,}6\ \mathrm{MHz}$ = Résonnance magnétique nucléaire

Toutes les particules qui portent un spin peuvent entrer en résonnance dans un champ magnétique

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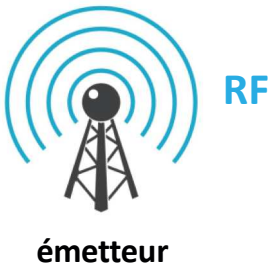
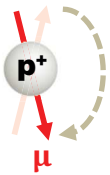
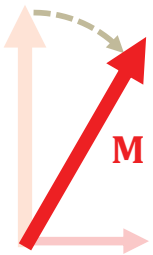


RF à ν donnée





MAGNETISME & RMN

Plan du cours

Contexte

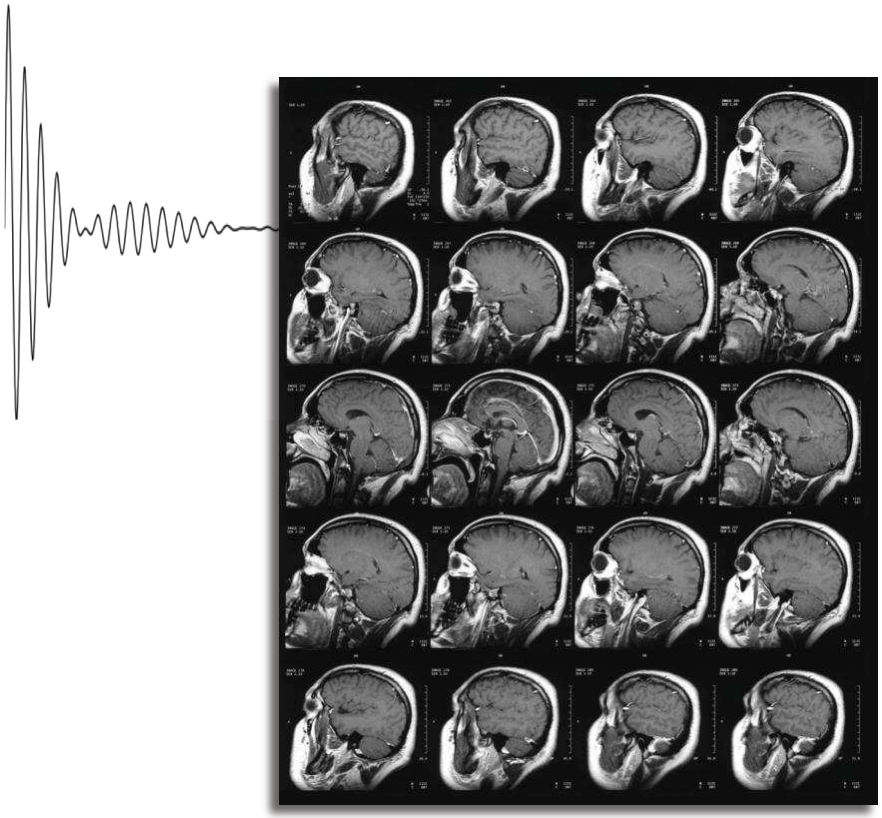
- **Champ et interaction électrique**
- **Champ et interaction magnétique**

Electron atomique
Mécanique quantique

- **Magnétisme dans la matière**
- **Magnétisme nucléaire**

RMN – principe
RMN – séquence
RMN – contraste

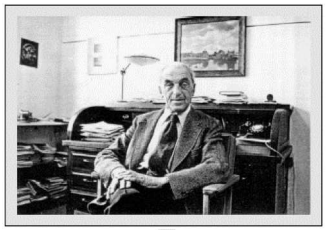
- Applications : IRM & SRM
- Illustrations



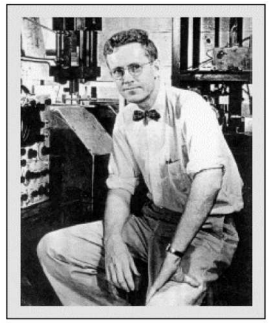
RMN – principe

Prix Nobel

Lauréats	*Année*	*Discipline*	*Travaux*
O. Stern	1943	Physique	Moment magnétique du proton (1933)
I. Rabi	1944	Physique	RMN
F. Bloch & E. Purcell	1952	Physique	RMN
R. Ernst	1991	Chimie	Spectroscopie RMN haute résolution
P. Lauterbur & P. Mansfield	2003	Médecine	IRM



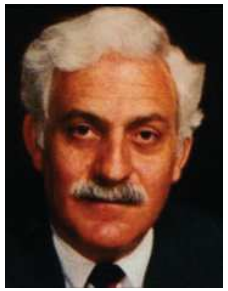
F. Bloch (Stanford)

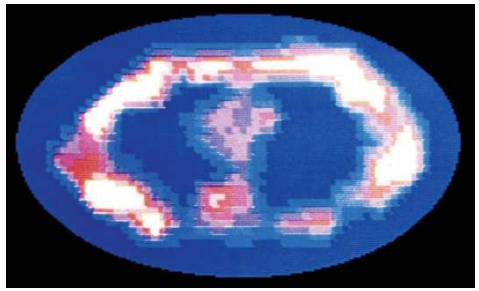


E. Purcell (Harvard)

et bien d'autres…

R. DAMADIAN

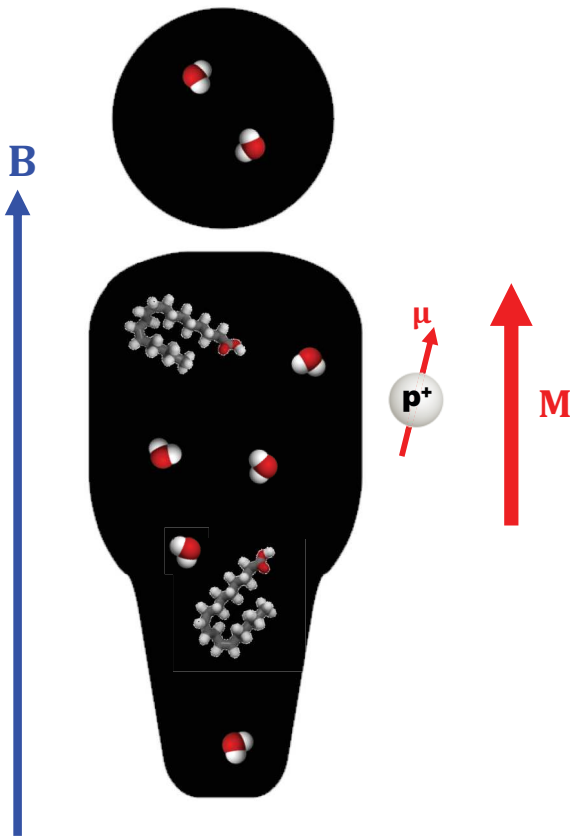




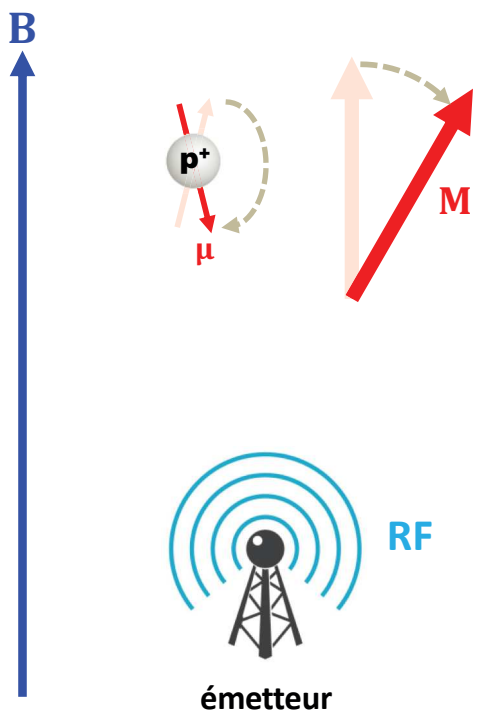
RMN – principe

Une séquence en 3 étapes:

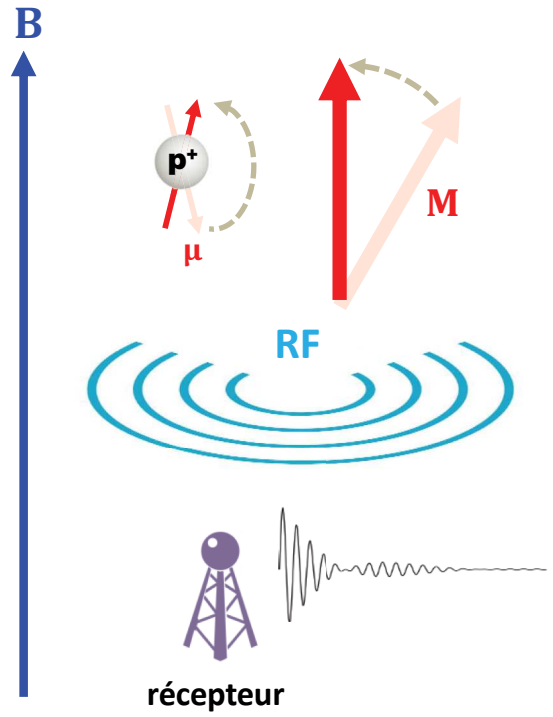
1. Préparation (aimantation)



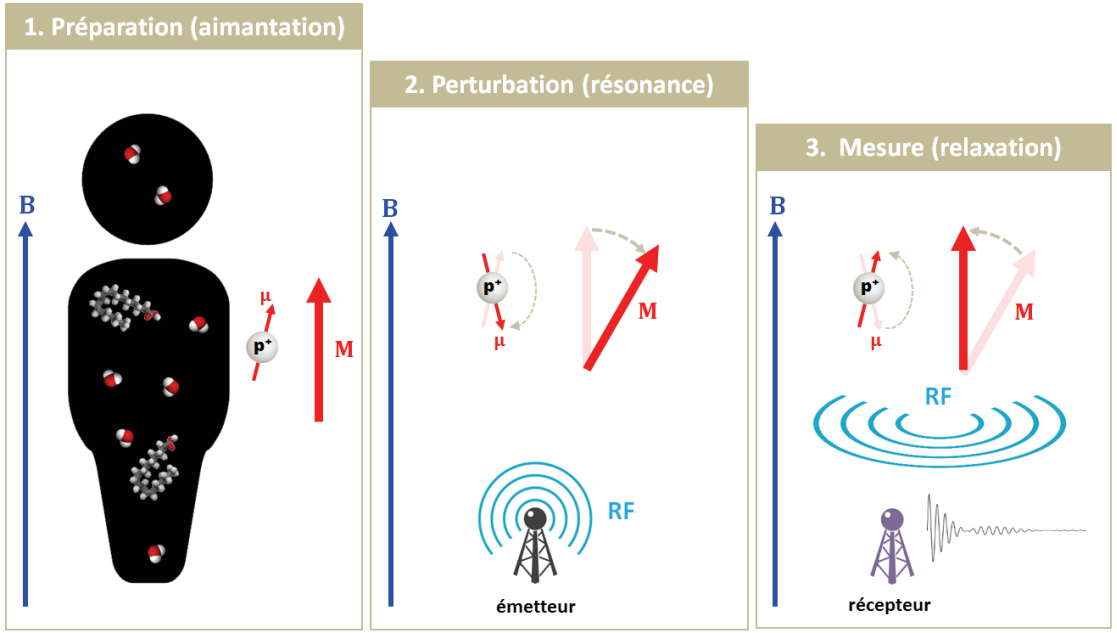
2. Perturbation (résonance)



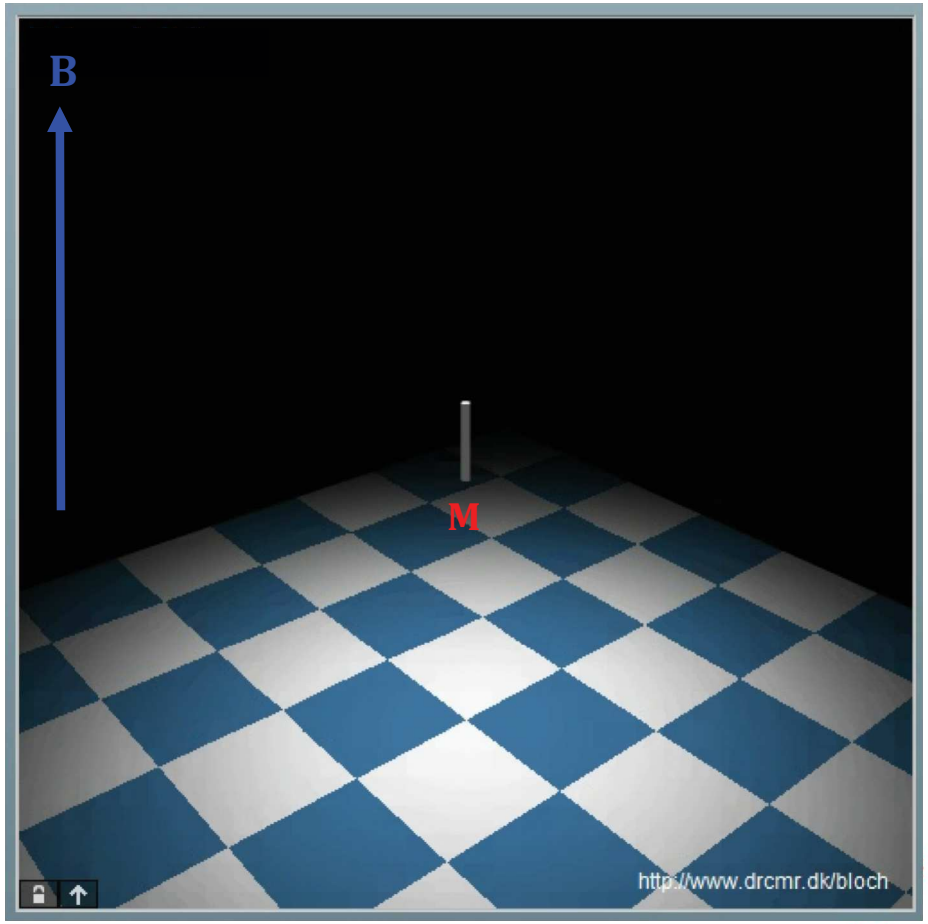
3. Mesure (relaxation)



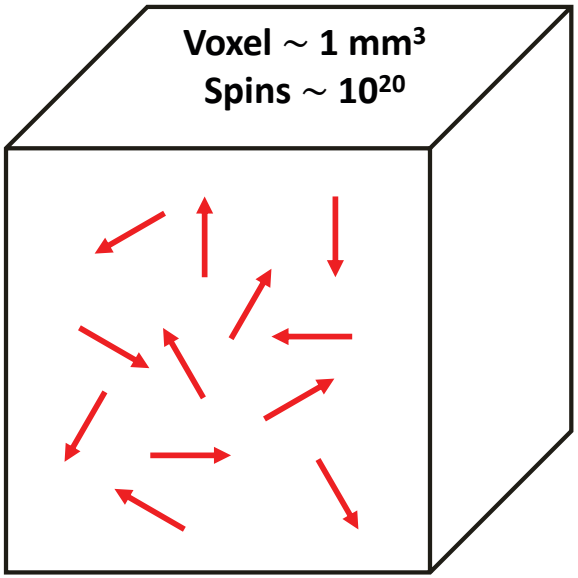
RMN – principe



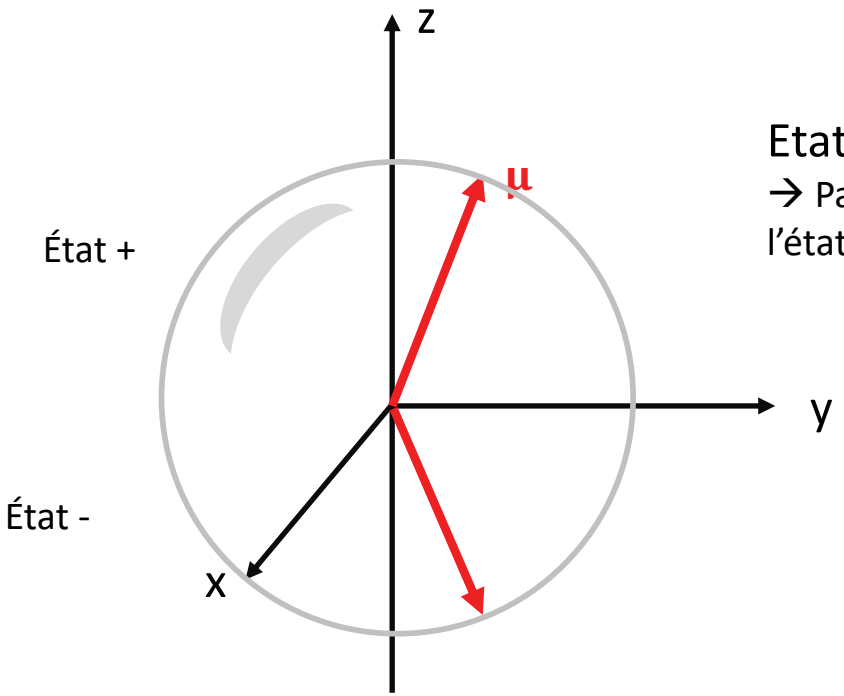
LG Hanson. Bloch Simulator. (www.drcmr.dk/bloch)



RMN – principe

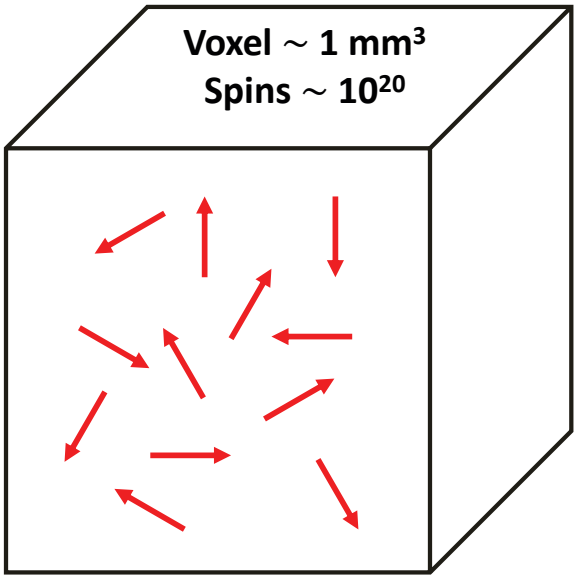


$B_0 \sim 0$ (champ magnétique terrestre = 47 µT)



Etats superposés
→ Pas de préférence pour l'état + ou l'état -

RMN – principe

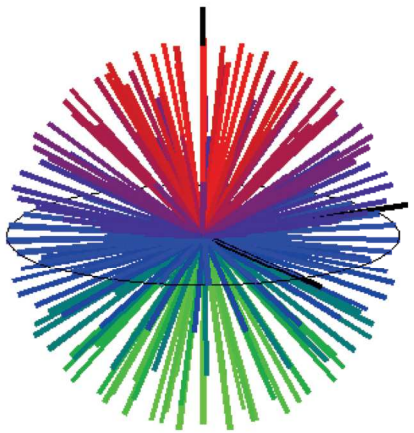


$\mathbf{B_0} \sim 0$ (champ magnétique terrestre = 47 µT)

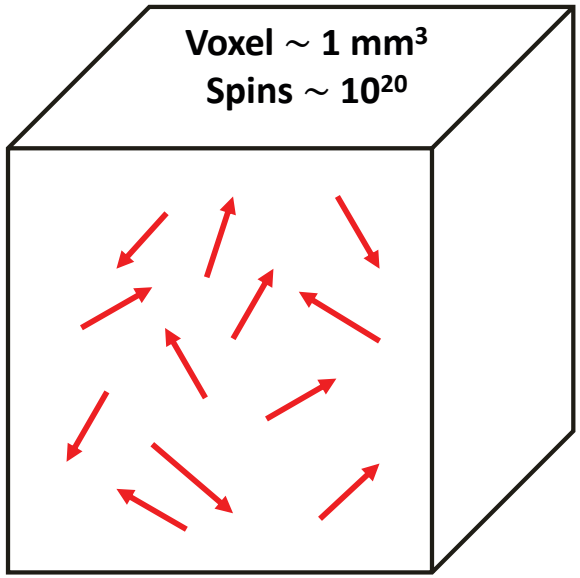
Agitation thermique >> effets magnétiques

$$\mathbf{M} = \sum \boldsymbol{\mu}$$

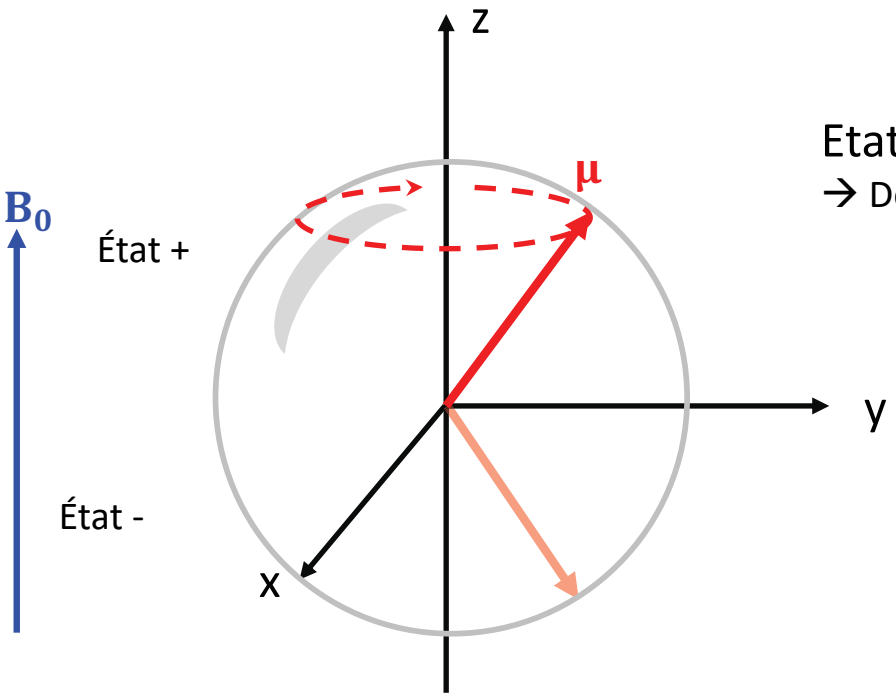
$$\mathbf{M} = 0$$



RMN – principe



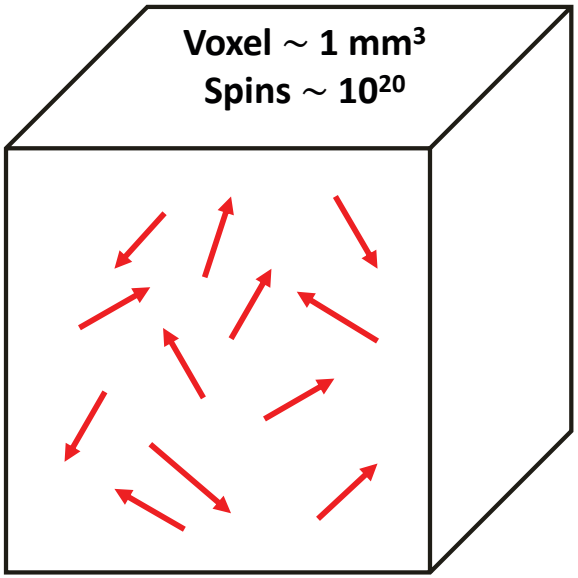
$B_0 > 0$



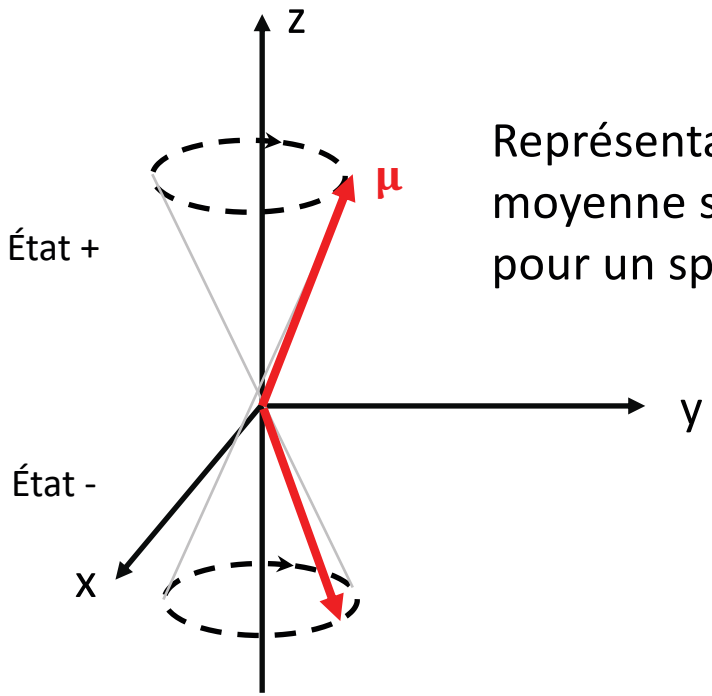
Etats superposés
→ Déséquilibre des populations

Dipôles en mouvement de précession autour du champ

RMN – principe



$B_0 > 0$



Représentation par la moyenne statistique de μ pour un spin donné

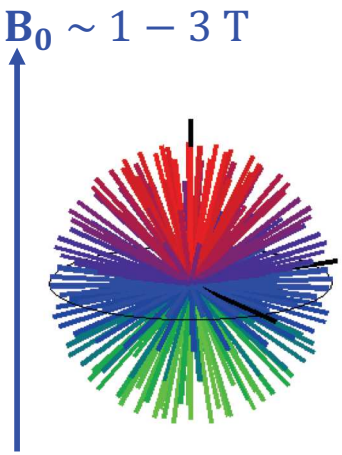
RMN – principe

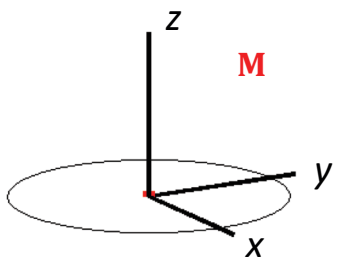
1. Préparation (aimantation)

Précession de Larmor : $\omega_0 = \gamma B_0$

$$\mathbf{M} = \sum \boldsymbol{\mu}$$

$$\mathbf{M} > 0$$



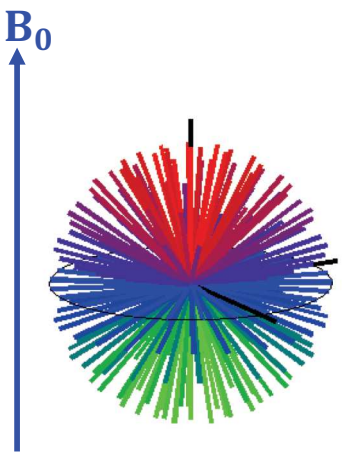


RMN – principe

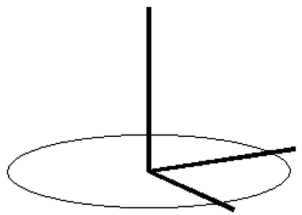
1. Préparation (aimantation)

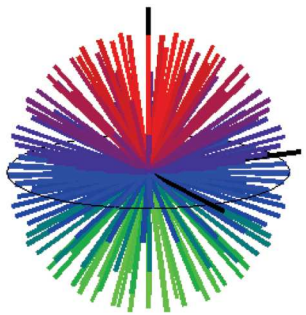
Précession de Larmor : $\omega_0 = \gamma B_0$

$\mathbf{M} > 0$

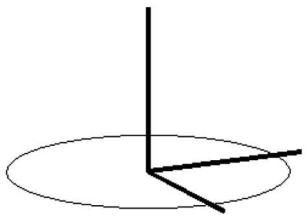


référentiel fixe





référentiel tournant
" $\mathbf{B_0} = \mathbf{0}$ "



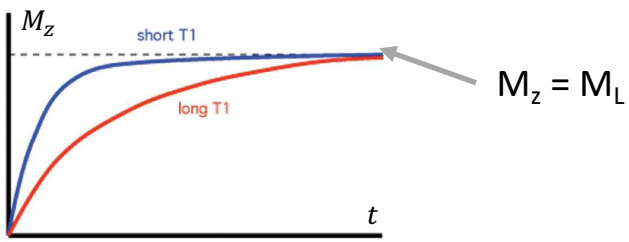
RMN – principe

1. Préparation (aimantation)

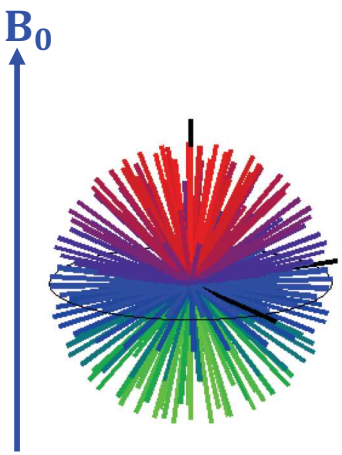
Précession de Larmor : $\omega_0 = \gamma B_0$

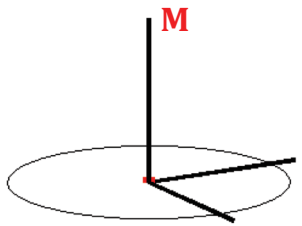
Alignement sur le champ

- **Aimantation longitudinale M_z**
- **Cinétique exponentielle T_1**

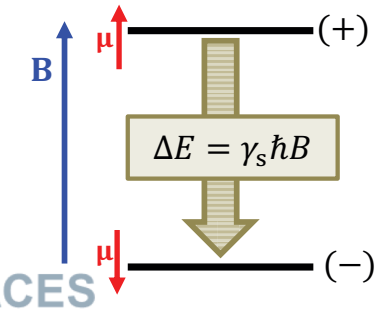


$$M_z(t) = M_L\left(1 - e^{-t/T_1}\right)$$

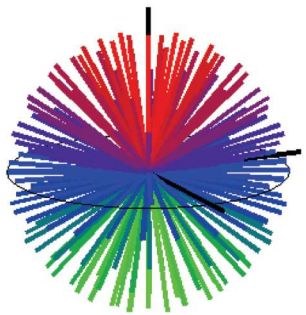


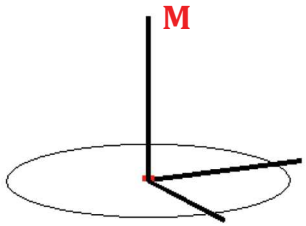


- **Echanges énergétiques :** $E = -\boldsymbol{\mu} \cdot \mathbf{B}$



$$proba(+) > proba(-)$$





RMN – principe

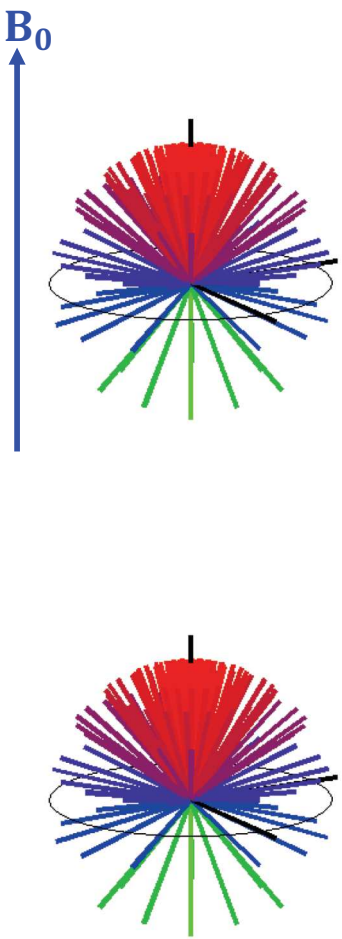
2. Perturbation (résonance)

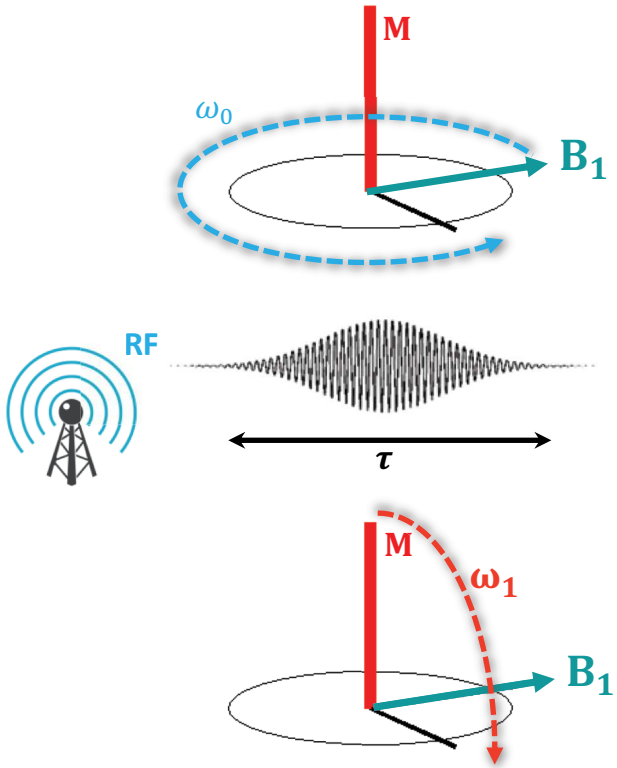
Application d'un champ tournant B_1

- de fréquence $2\pi\omega_0$
- pendant une durée τ (~ms)
- dans le plan horizontal

Précession de M autour de B_1

- Nutation à $\omega_1 = \gamma B_1$
- Bascule d'un angle $\eta = \tau\,\omega_1$





RMN – principe

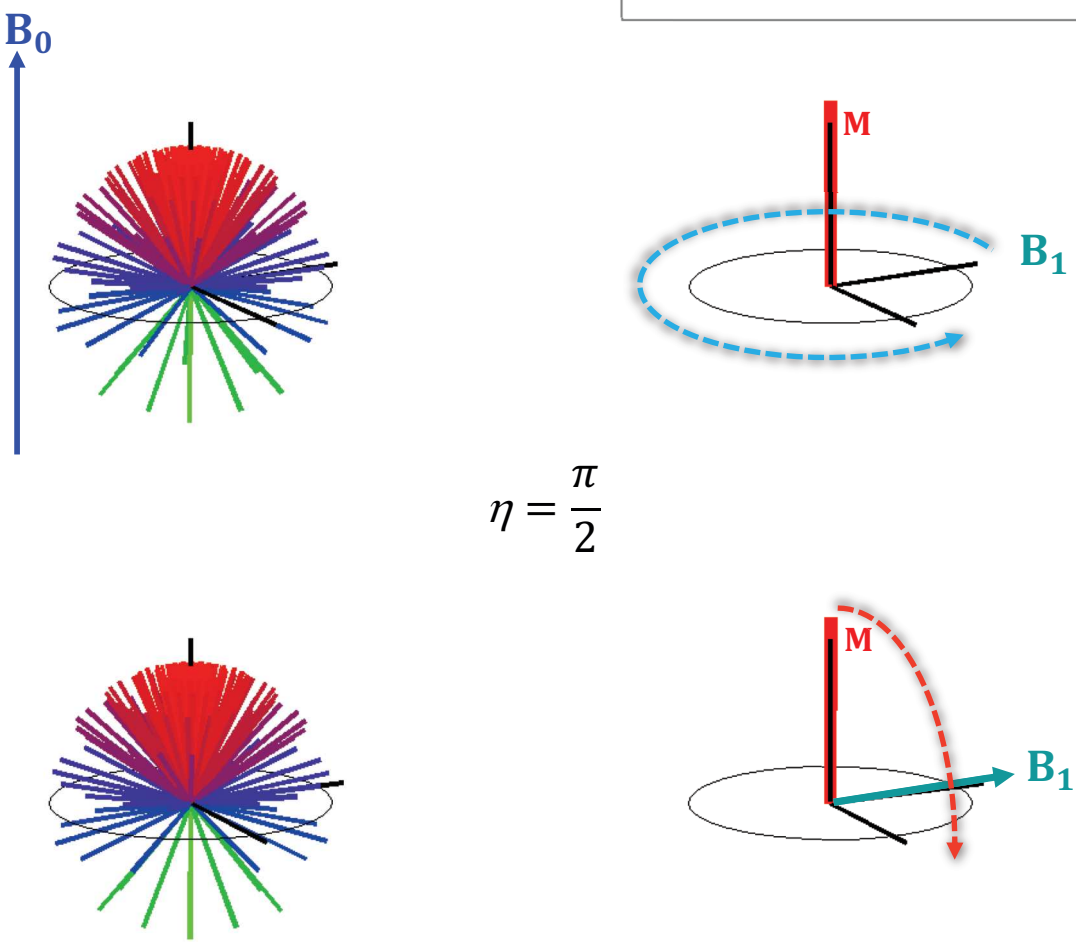
2. Perturbation (résonance)

Application d'un champ tournant B_1

- de $\boxed{\text{fréquence } 2\pi\omega_0}$
- pendant une durée τ (~ms)
- dans le plan horizontal

Précession de M autour de B_1

- Nutation à $\omega_1 = \gamma B_1$
- Bascule d'un angle $\eta = \tau\,\omega_1$



$$\eta = \frac{\pi}{2}$$

RMN – principe

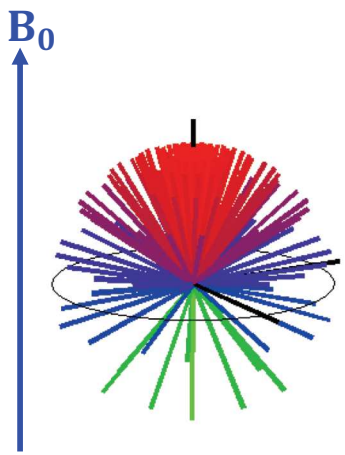
2. Perturbation (résonance)

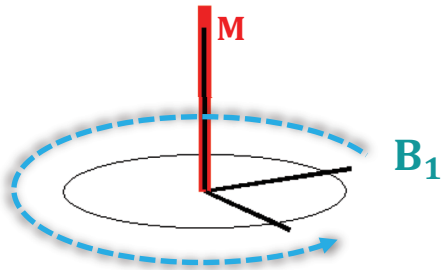
Application d'un champ tournant B_1

- de **fréquence $2\pi\omega_0$**
- **pendant une durée τ (~ms)**
- **dans le plan horizontal**

Précession de M autour de B_1

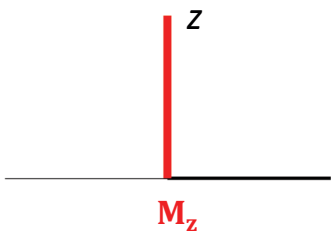
- **Nutation à** $\omega_1 = \gamma B_1$
- **Bascule d'un angle** $\eta = \tau\,\omega_1$

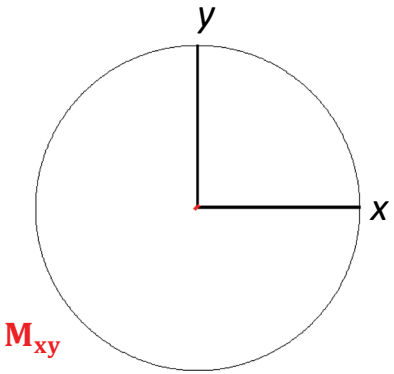


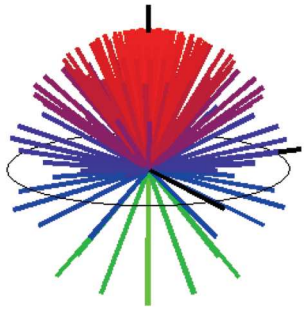


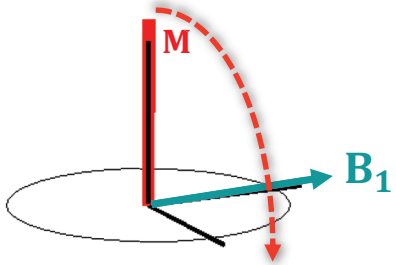
$$\eta = \frac{\pi}{2}$$

Disparition de M_z
Apparition de M_{xy} (aimantation transverse)







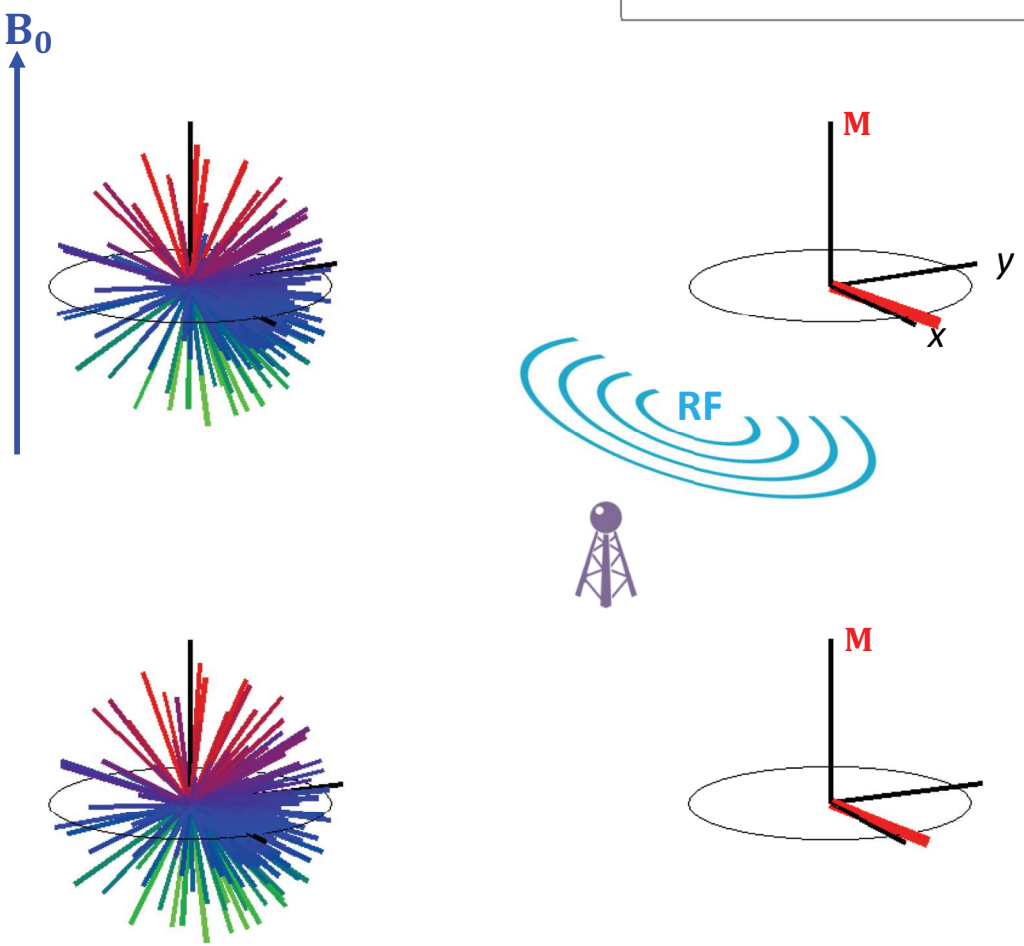


RMN – principe

3. Mesure

Arrêt de la stimulation RF B_1

Signal = M_x

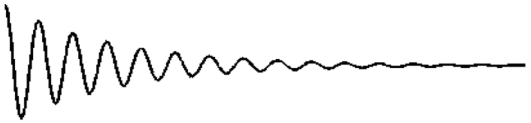


RMN – principe

3. Mesure (relaxation)

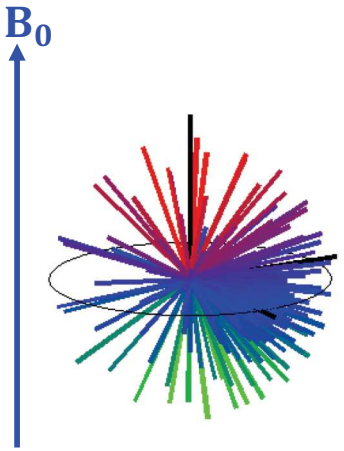
Arrêt de la stimulation RF B_1

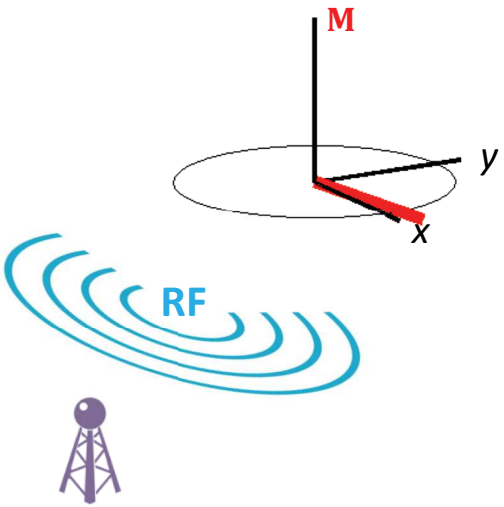
Signal = M_x

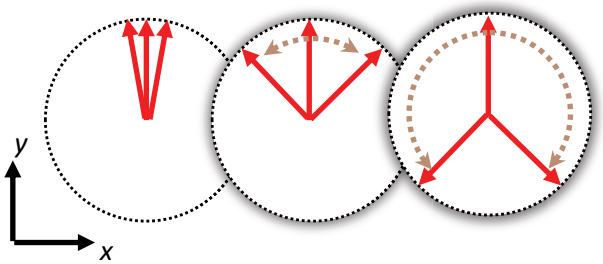


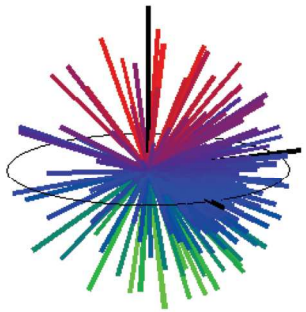
RELAXATION T_2

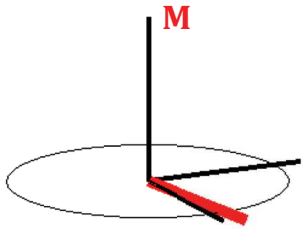
- **Déphasage progressif**
- **Inhomogénéités de champ**
- **Phénomène entropique**









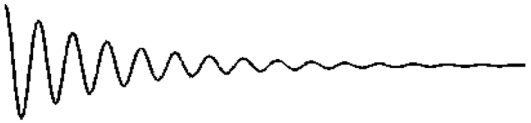


RMN – principe

3. Mesure (relaxation)

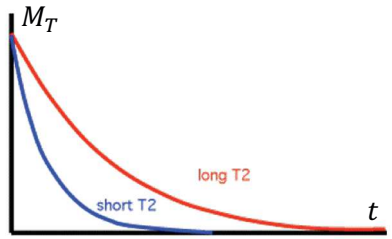
Arrêt de la stimulation RF B_1

Signal = M_x

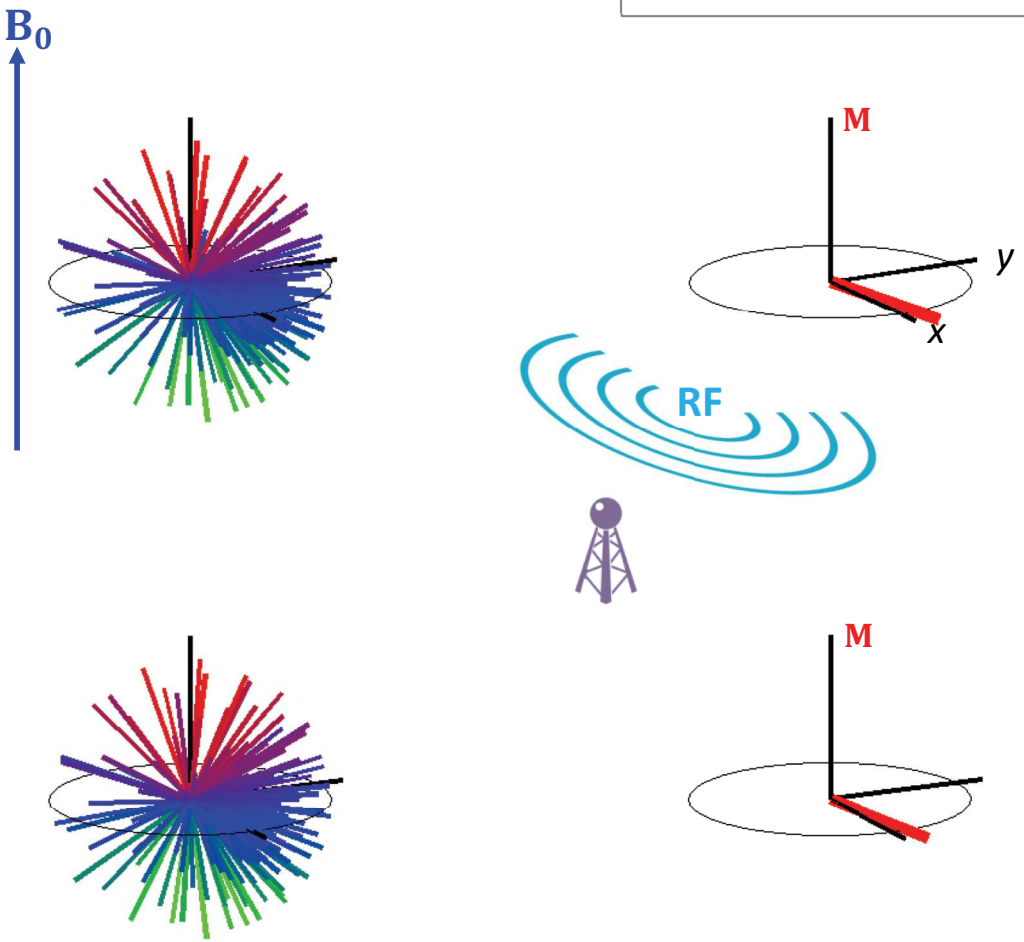


RELAXATION T_2

- **Déphasage progressif**
- **Inhomogénéités de champ**
- **Phénomène entropique**
- **Cinétique T_2**



$$M_{xy}(t) = M_T\, e^{-t/T_2}$$



RMN – principe

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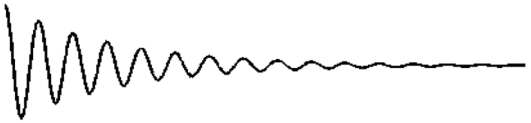
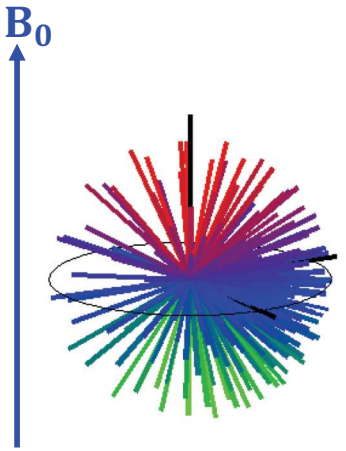
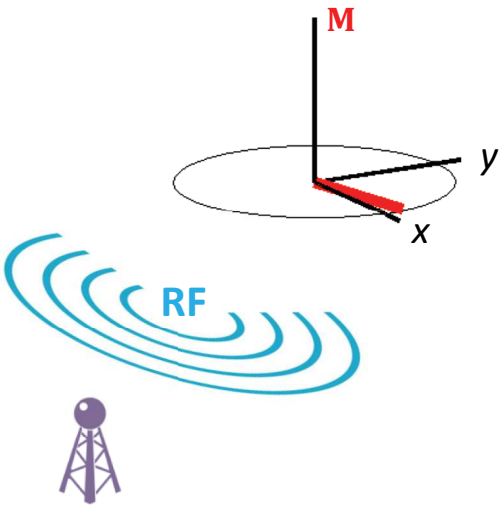
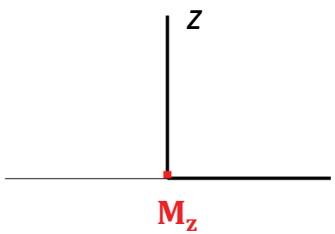


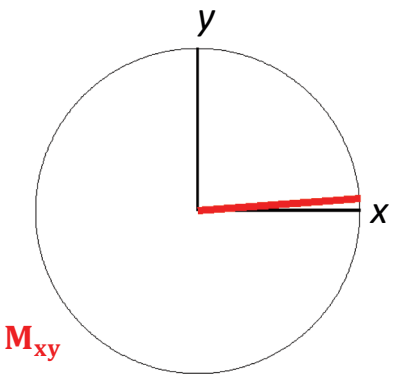
RELAXATION T_2: Disparition de M_{xy}

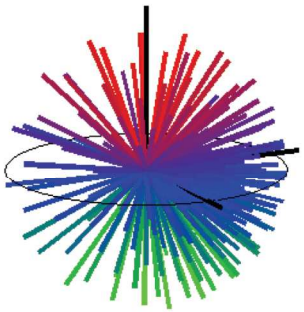
RELAXATION T_1: Repousse de M_z

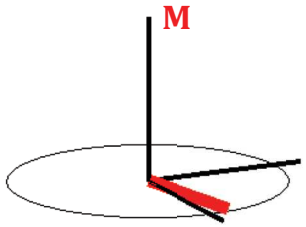










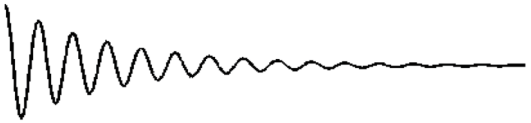


RMN – principe

3. Mesure (relaxation)

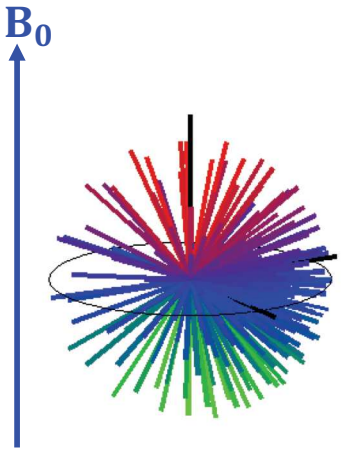
Arrêt de la stimulation RF B_1

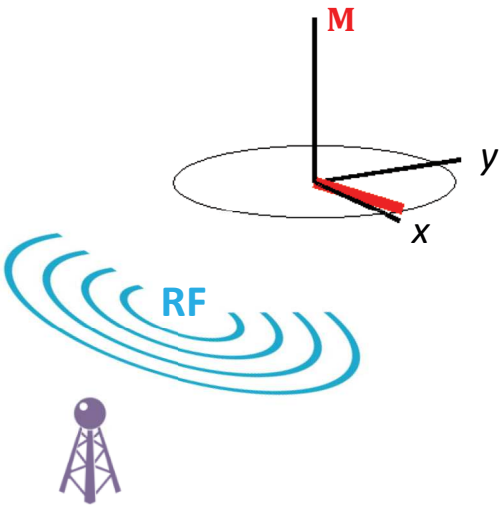
Signal = M_x



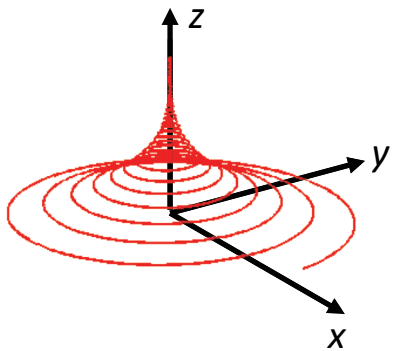
RELAXATION T_2: Disparition de M_{xy}

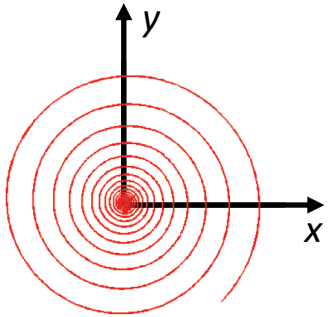
RELAXATION T_1: Repousse de M_z

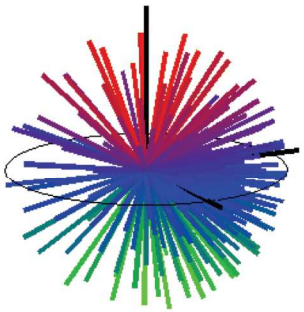


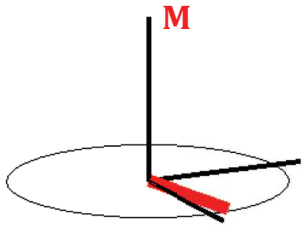


Dans les tissus biologiques : $T_1 \sim 10\ T_2$





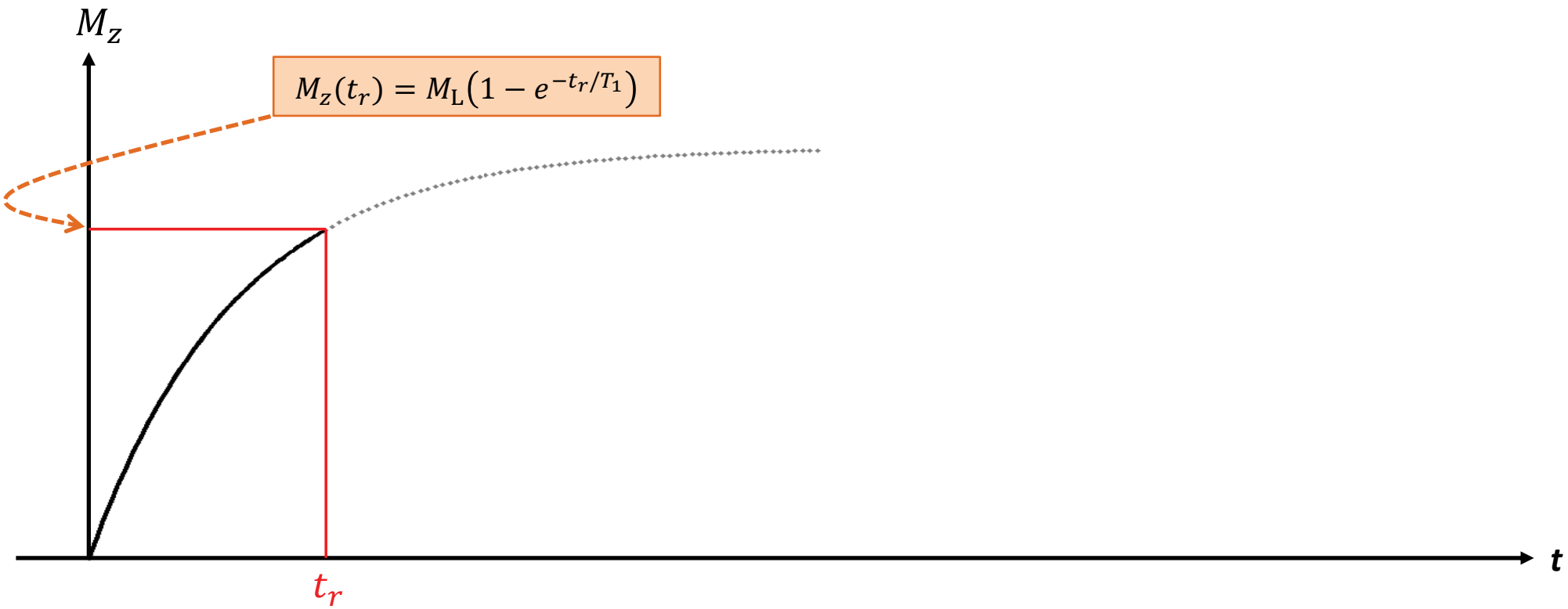




RMN – séquence

Préparation

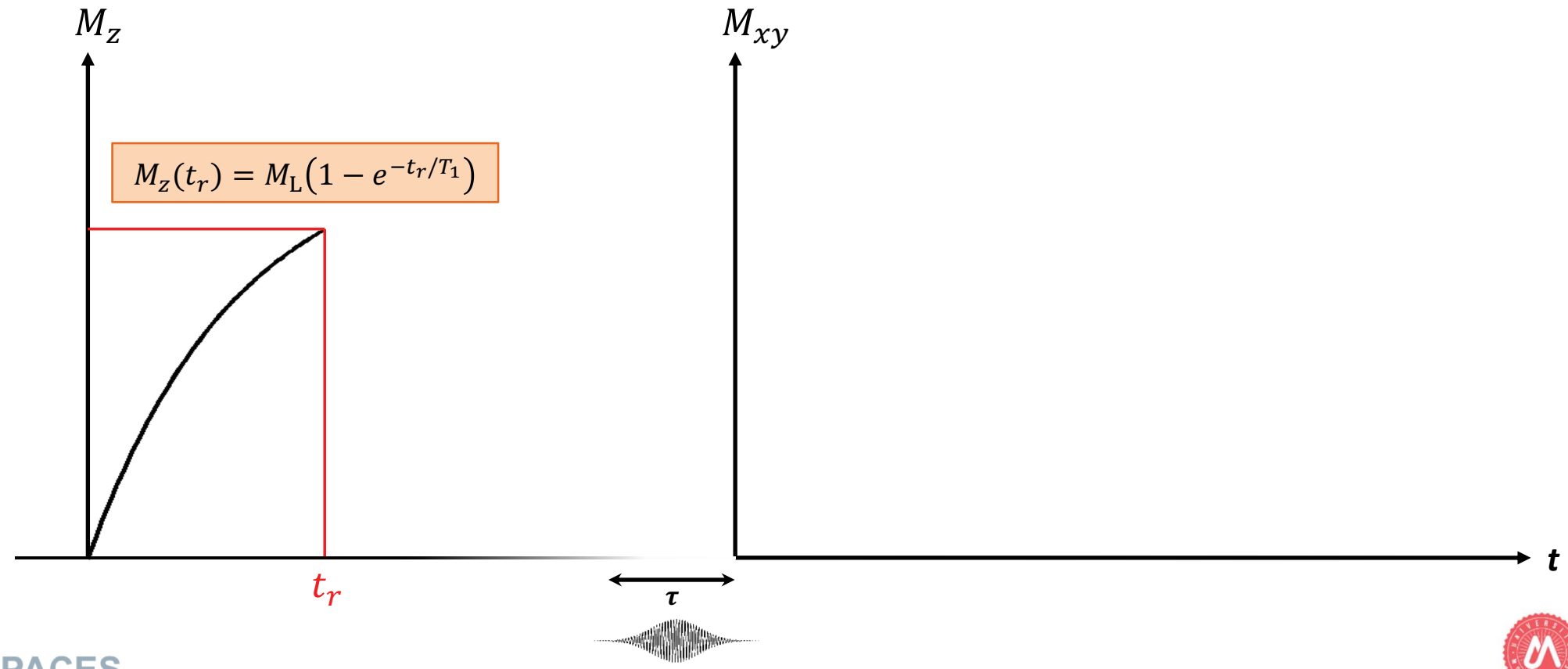
Aimantation longitudinale dans B_0
Relaxation T_1
Durant un temps t_r (temps de répétition)



RMN – séquence

Préparation **RF**

Stimulation RF durant un temps τ
Bascule de l'aimantation d'un angle η
Apparition d'une aimantation transverse

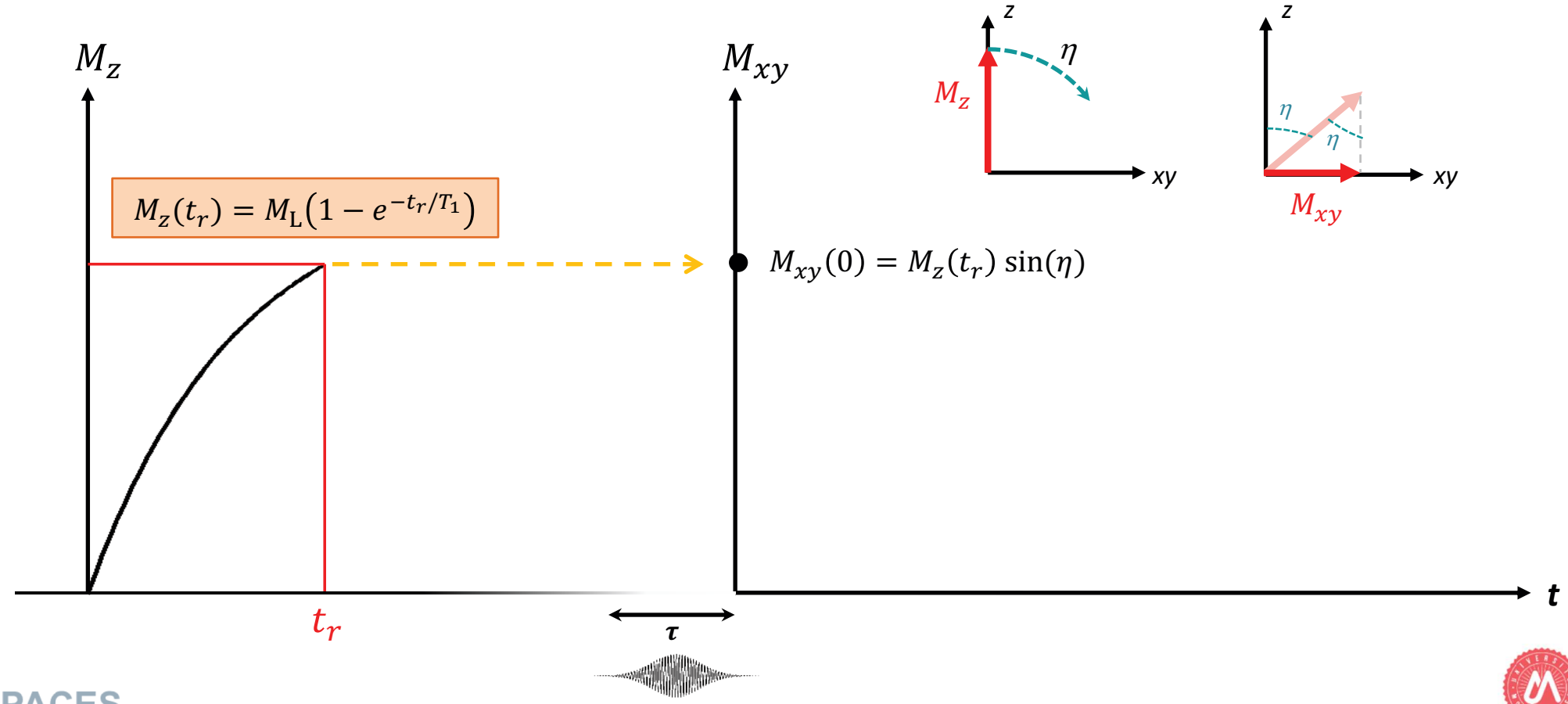


RMN – séquence

Préparation

RF

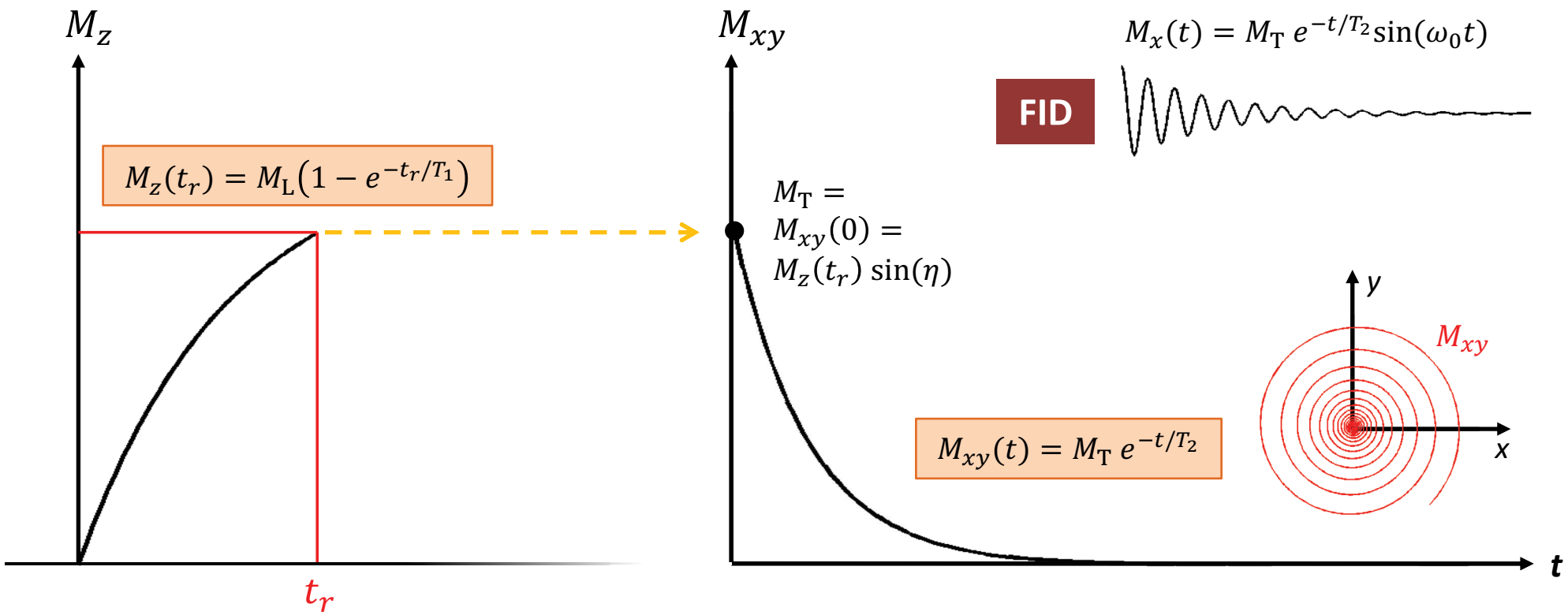
Stimulation RF durant un temps τ
Bascule de l'aimantation d'un angle η
Apparition d'une aimantation transverse



RMN – séquence

Préparation | RF | Mesure

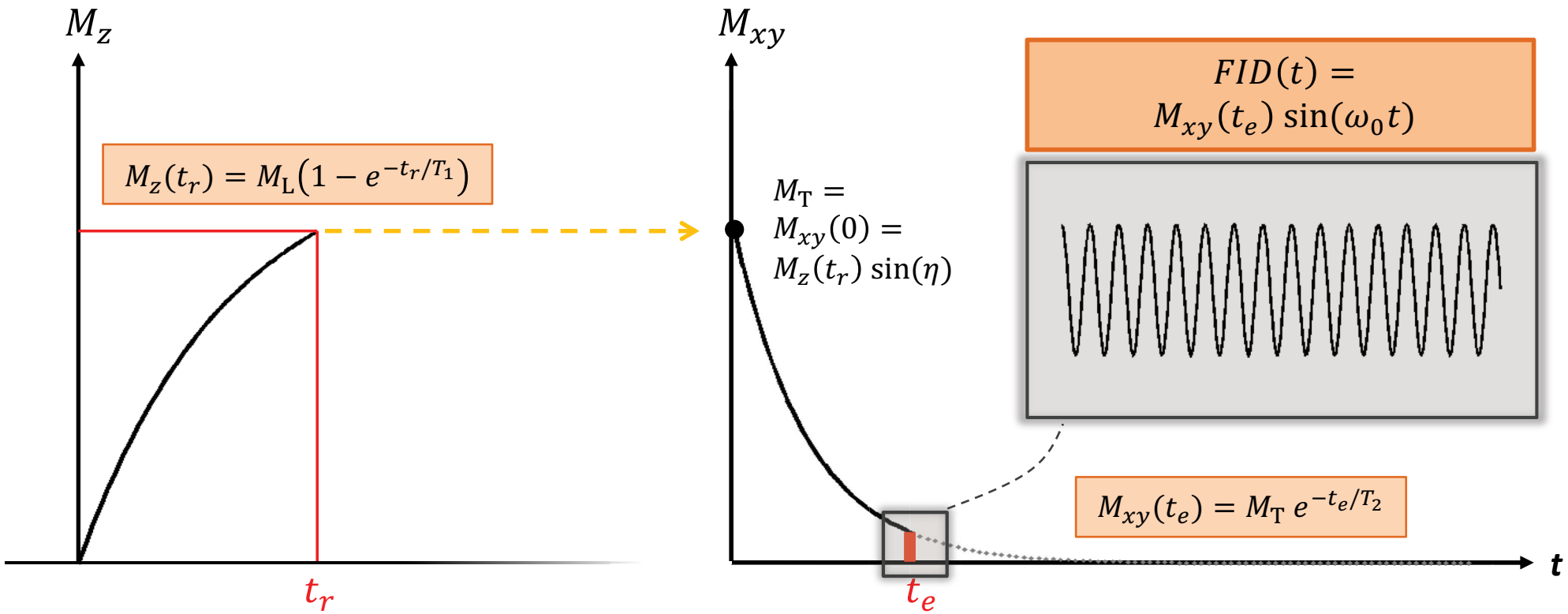
Retour à l'équilibre
Relaxation T_2



RMN – séquence

Préparation | RF | Mesure

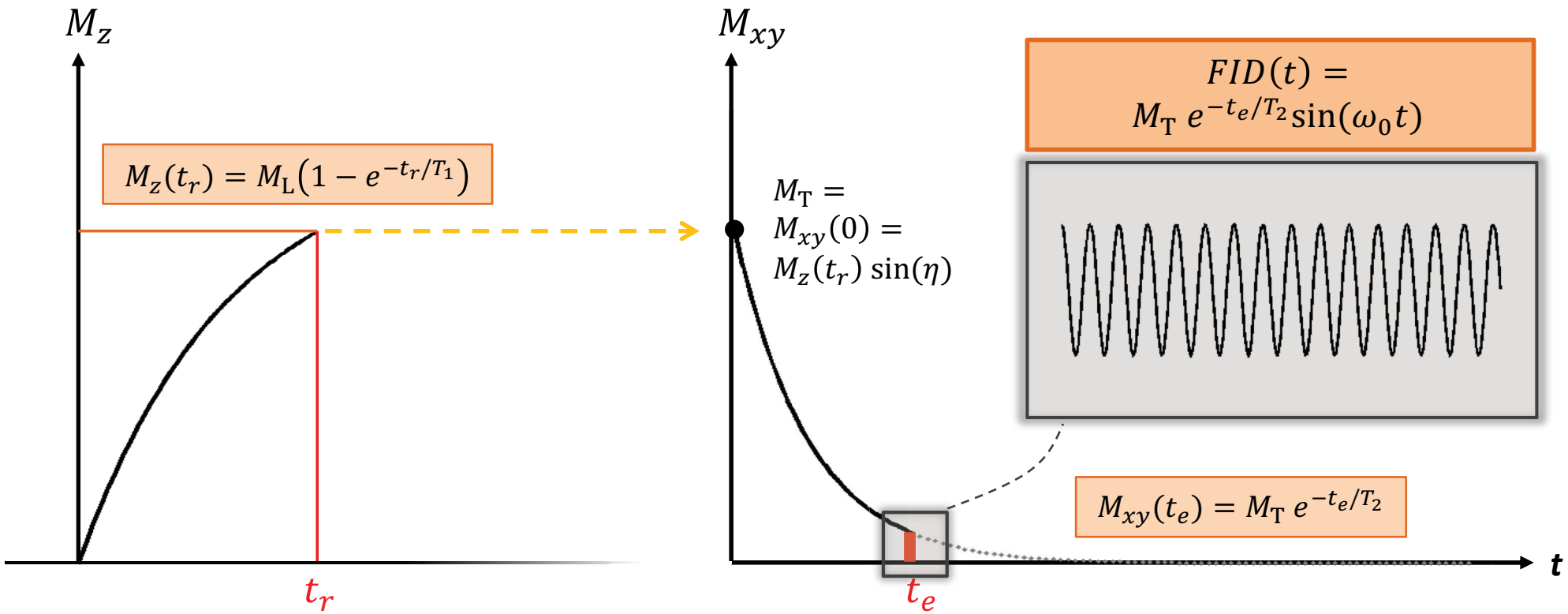
Retour à l'équilibre
Relaxation T_2
Recueil du signal **(FID)** au temps t_e
(temps d'écho)



RMN – séquence

Préparation | RF | Mesure

Retour à l'équilibre
Relaxation T_2
Recueil du signal **(FID)** au temps t_e
(temps d'écho)



RMN – séquence

Préparation | RF | Mesure

Retour à l'équilibre
Relaxation T_2
Recueil du signal **(FID)** au temps t_e
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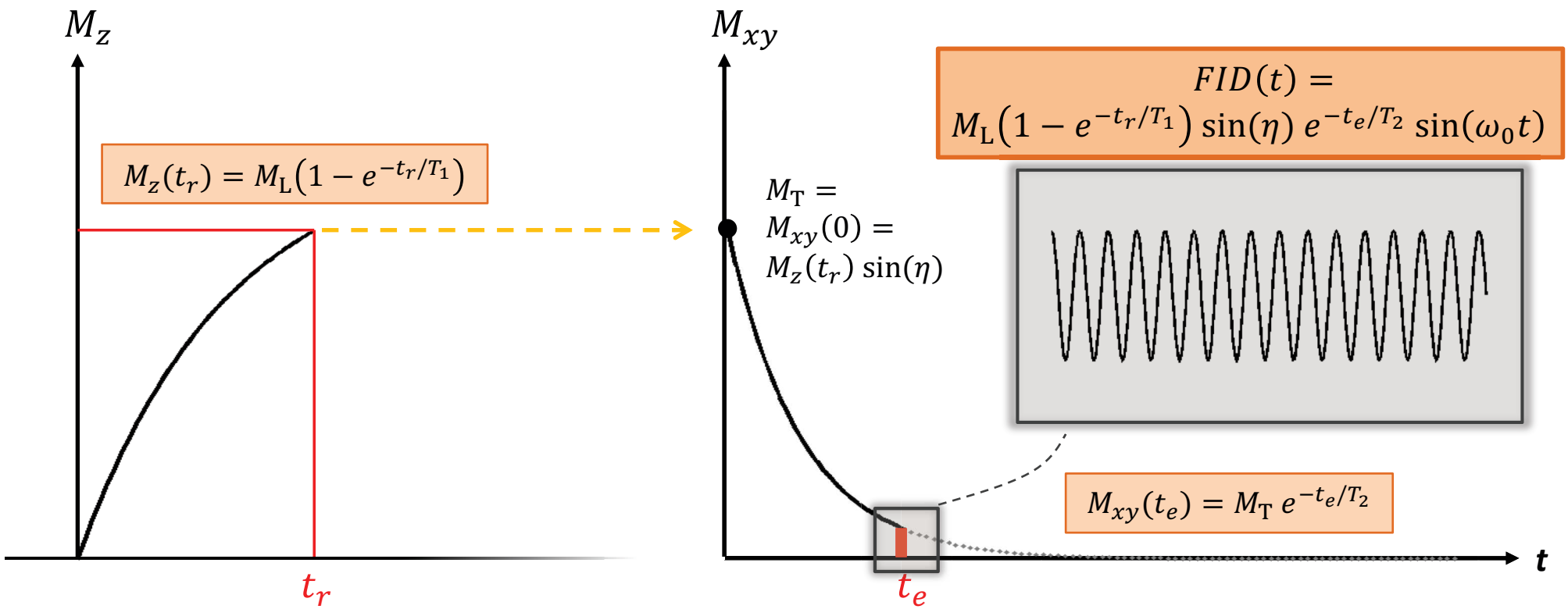
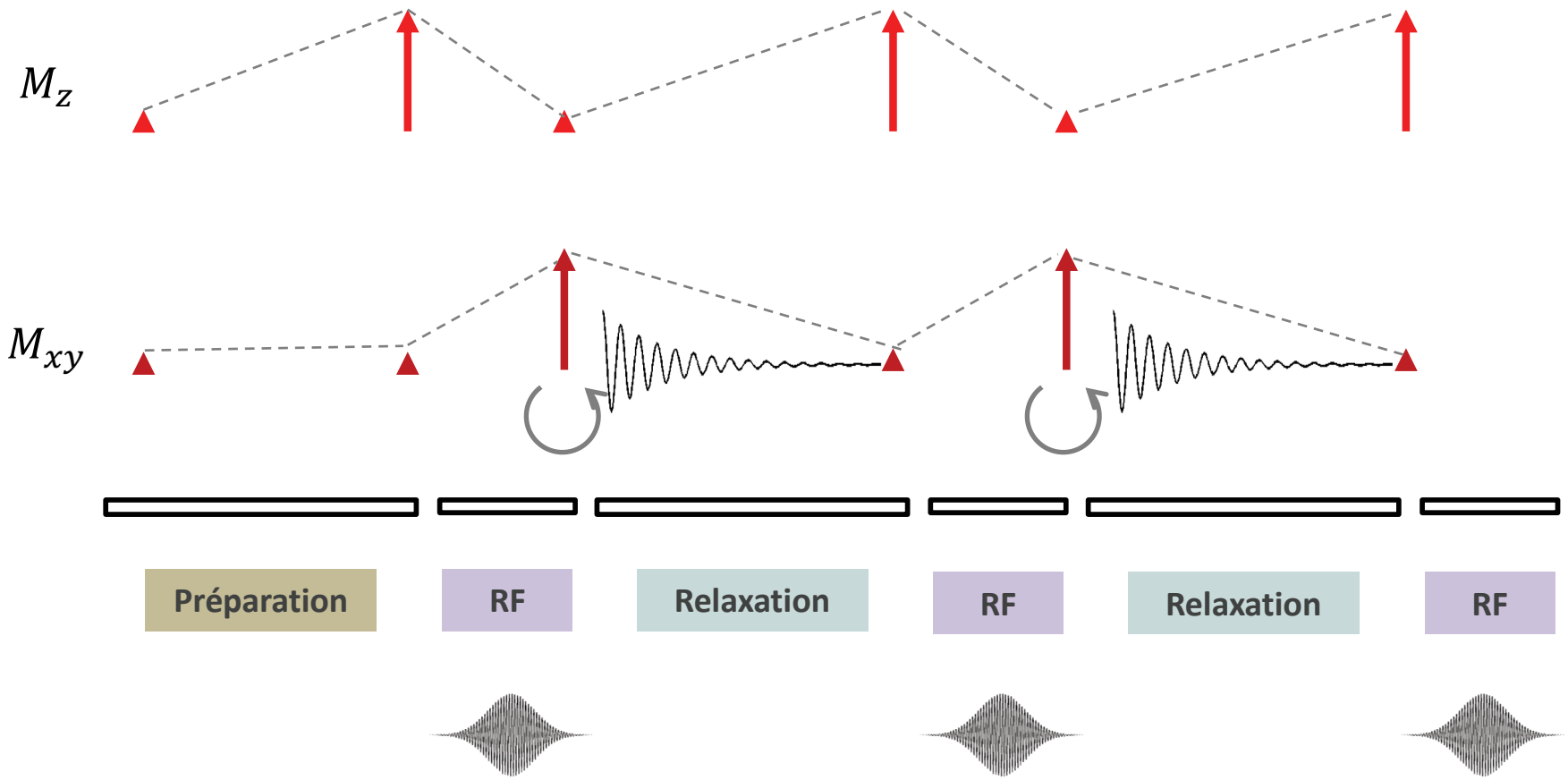
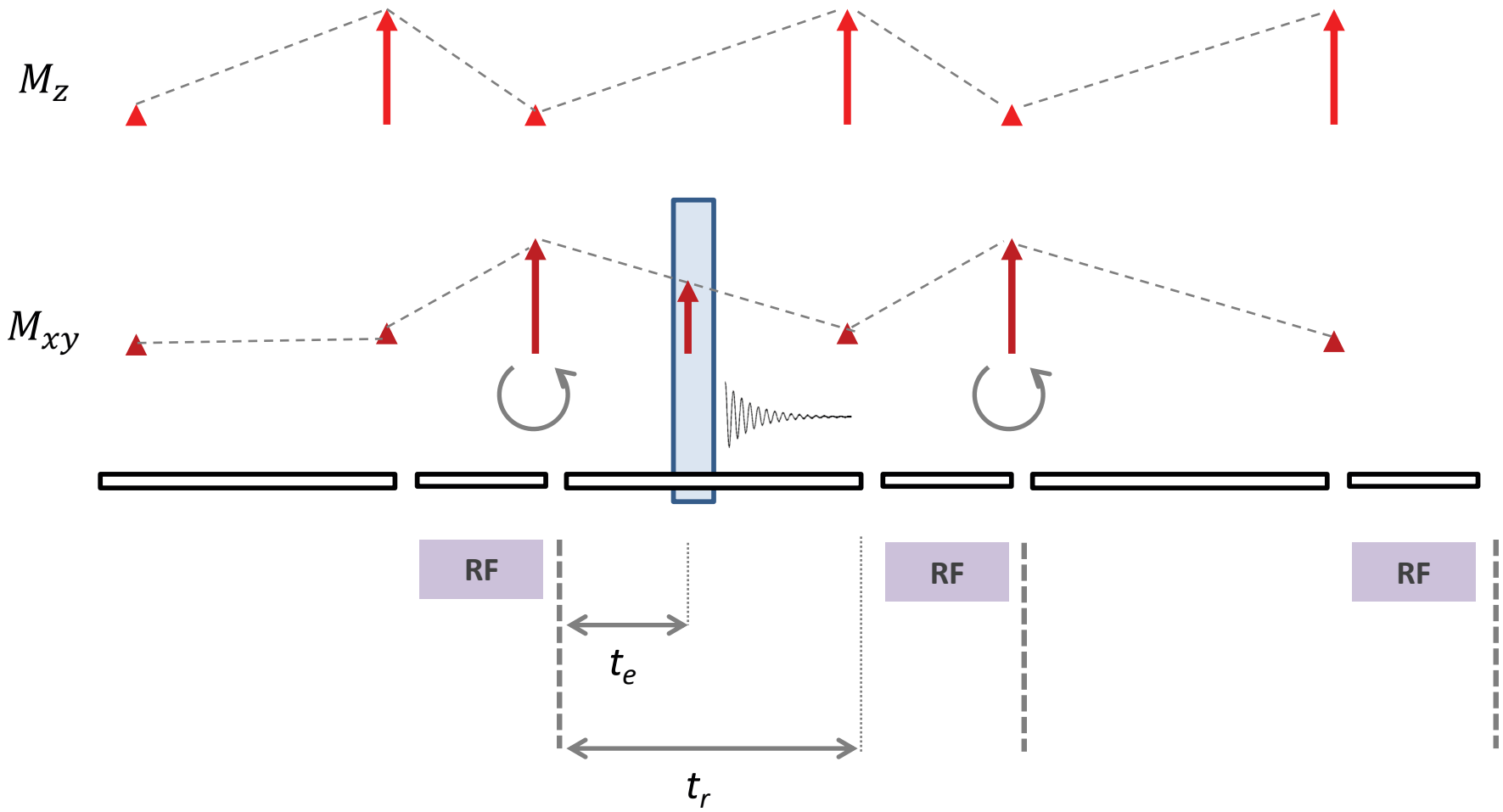


RMN – séquence: synthèse



RMN – séquence: synthèse



RMN – contraste

$$FID(t) = M_L\left(1 - e^{-t_r/T_1}\right)\sin(\eta)\, e^{-t_e/T_2}\sin(\omega_0 t)$$

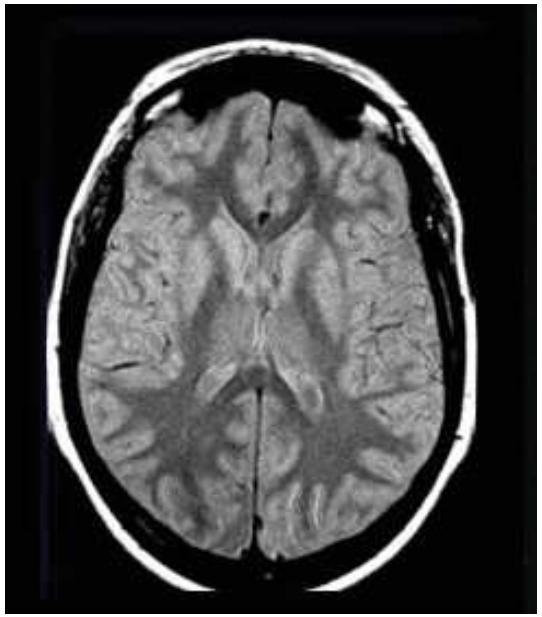
Déterminants du signal :

$$M_L = \sum \mu_z \quad \begin{cases} \propto \boldsymbol{\mu} \propto \gamma & \textit{intrinsèque} \\ \propto \rho\left({}^1\mathrm{H}\right) & \textit{intrinsèque} \\ \propto \dfrac{\Delta N}{N} \propto \dfrac{B_0}{T} & \textit{extrinsèque} \end{cases}$$

T_1, T_2 *intrinsèque*

t_r, t_e, η *extrinsèque*

Densité de protons ρ :
Degré d'hydratation / lipidation



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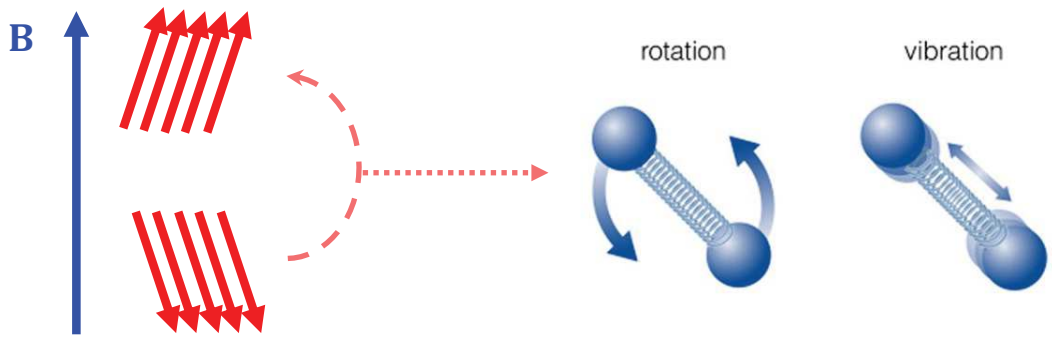
T_1, T_2 *intrinsèque*

t_r, t_e, η *extrinsèque*

Relaxation T_1 : échanges énergétiques avec le réseau



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Déterminants du signal :

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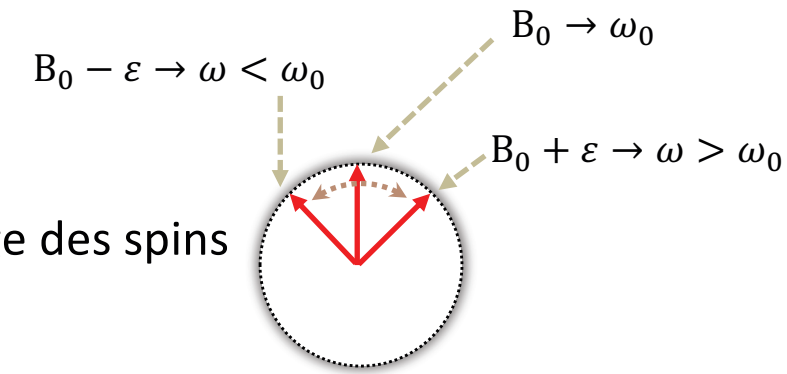
T_1, T_2 *intrinsèque*

Relaxation T_2 : déphasage des spins



t_r, t_e, η *extrinsèque*

- Inhomogénéités technologiques de champ B_0
- Fluctuations locales dues à l'environnement chimique (T_2^*)

RMN – contraste

$$FID(t) = M_L\left(1 - e^{-t_r/T_1}\right)\sin(\eta)\, e^{-t_e/T_2}\sin(\omega_0 t)$$

Déterminants du signal :

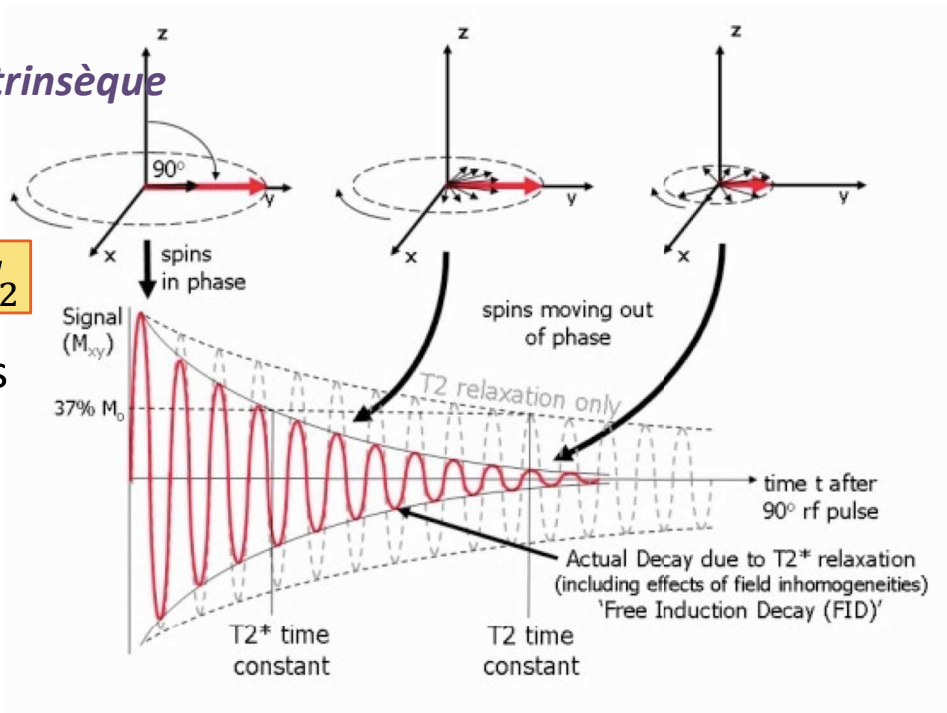
$$M_L = \sum \mu_z \quad \begin{cases} \propto \boldsymbol{\mu} \propto \gamma & \textit{intrinsèque} \\ \propto \rho(\,^1\mathrm{H}) & \textit{intrinsèque} \\ \propto \dfrac{\Delta N}{N} \propto \dfrac{B_0}{T} & \textit{extrinsèque} \end{cases}$$

T_1, T_2 *intrinsèque*

t_r, t_e, η *extrinsèque*

Relaxation T_2

déphasage des spins



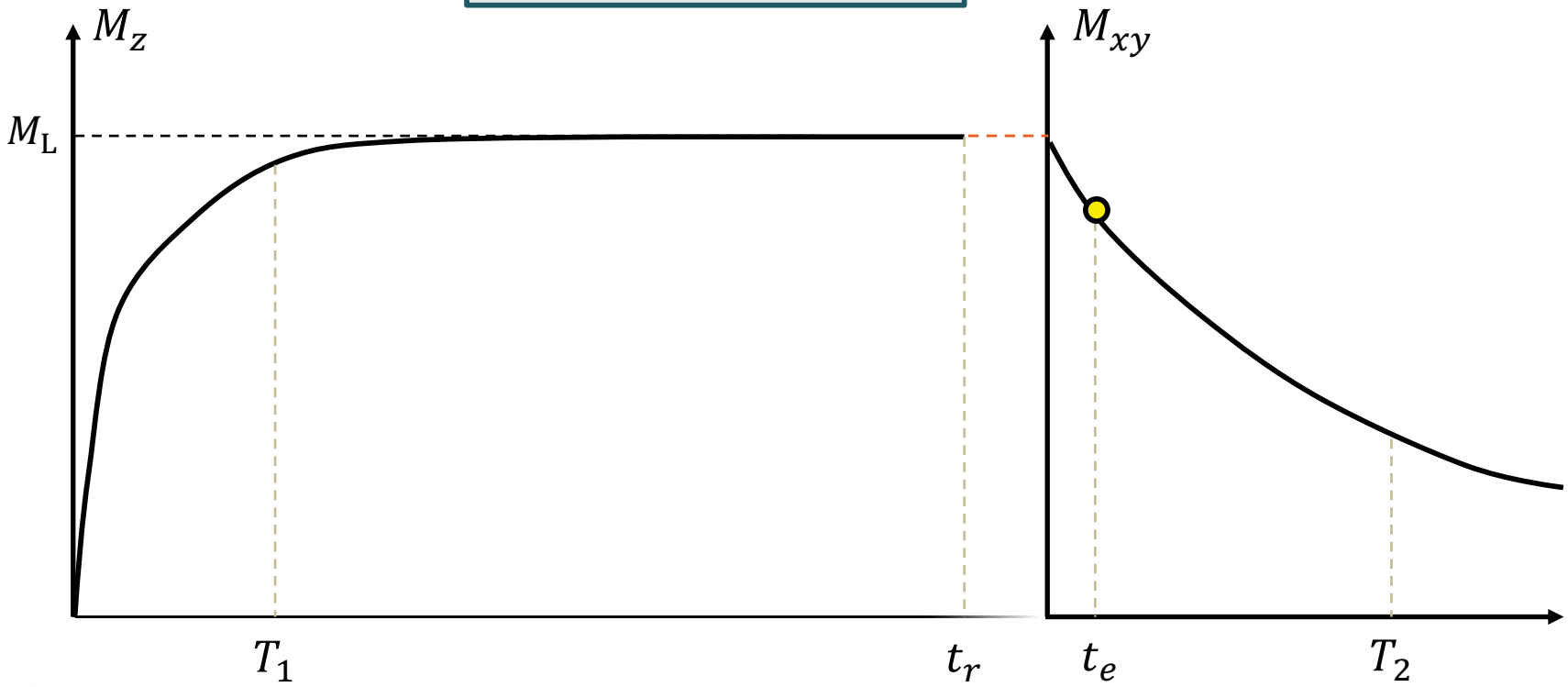
RMN – contraste

Pondération ρ (densité de protons)

$t_r \gg T_1$
$t_e \ll T_2$

$$FID(t) = M_L\left(1 - e^{-t_r/T_1}\right)\sin(\eta)\, e^{-t_e/T_2}\sin(\omega_0 t)$$

Signal $\propto M_L \propto \rho(\,^1\mathrm{H})$



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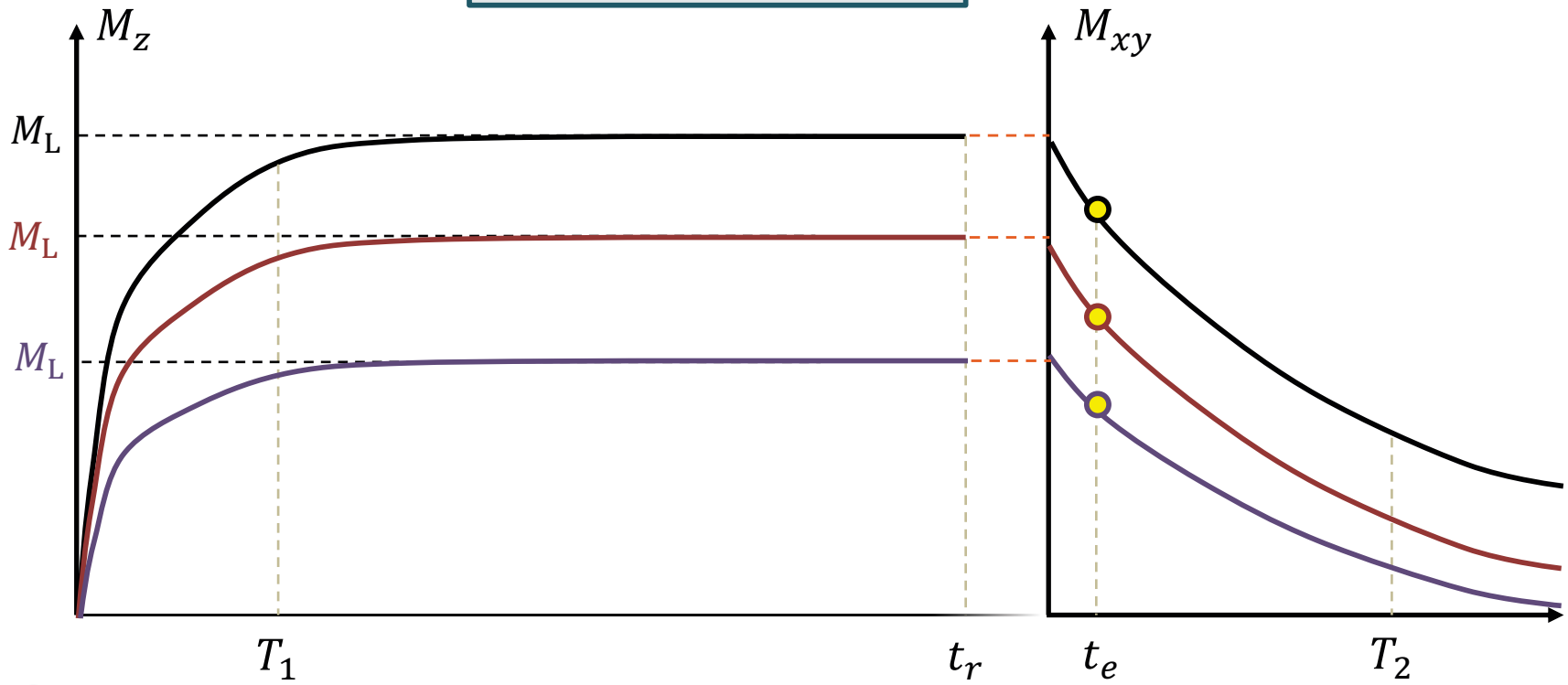


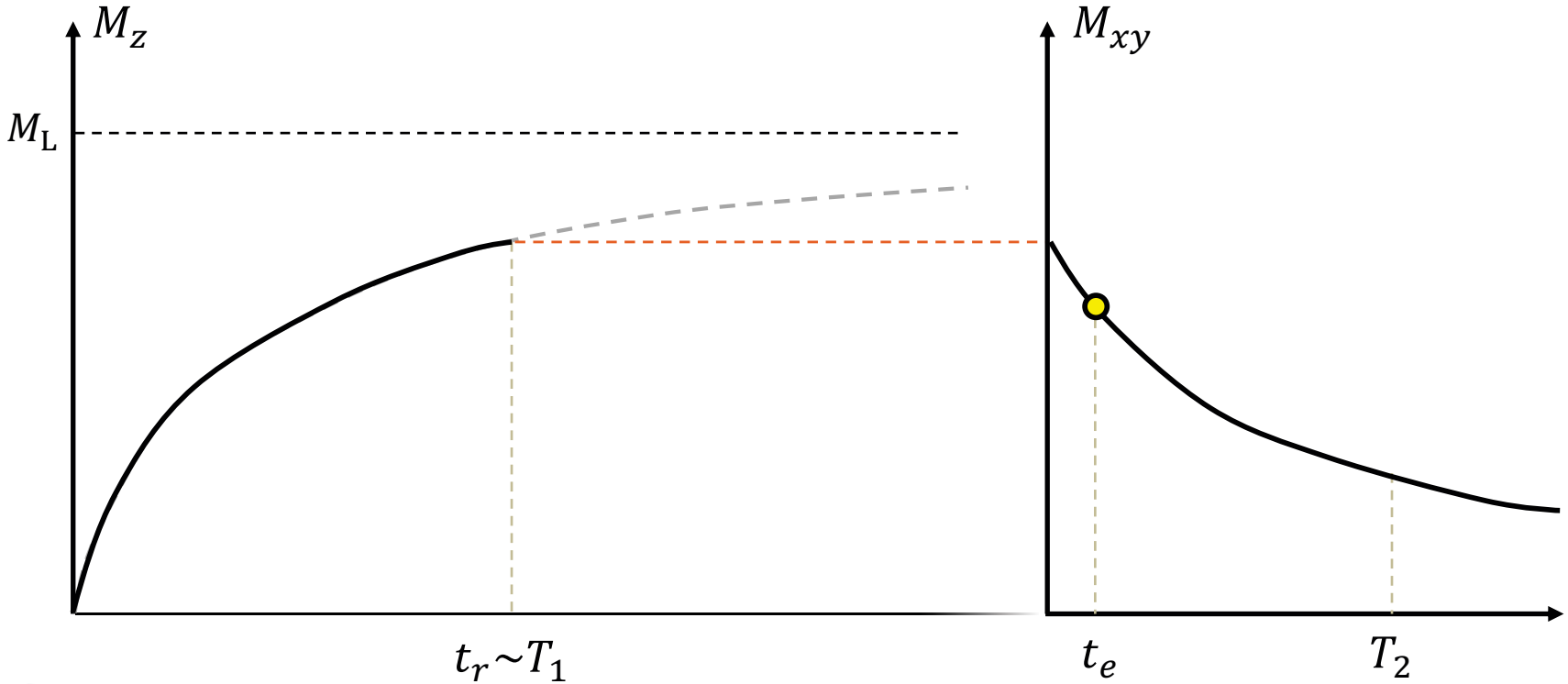
RMN – contraste

Pondération T_1

$t_r \sim T_1$
$t_e \ll T_2$

$$FID(t) = M_L\left(1 - e^{-t_r/T_1}\right)\sin(\eta)\, e^{-t_e/T_2}\sin(\omega_0 t)$$

Signal $\propto M_L(1 - e^{-t_r/T_1})$



RMN – contraste

Pondération T_1

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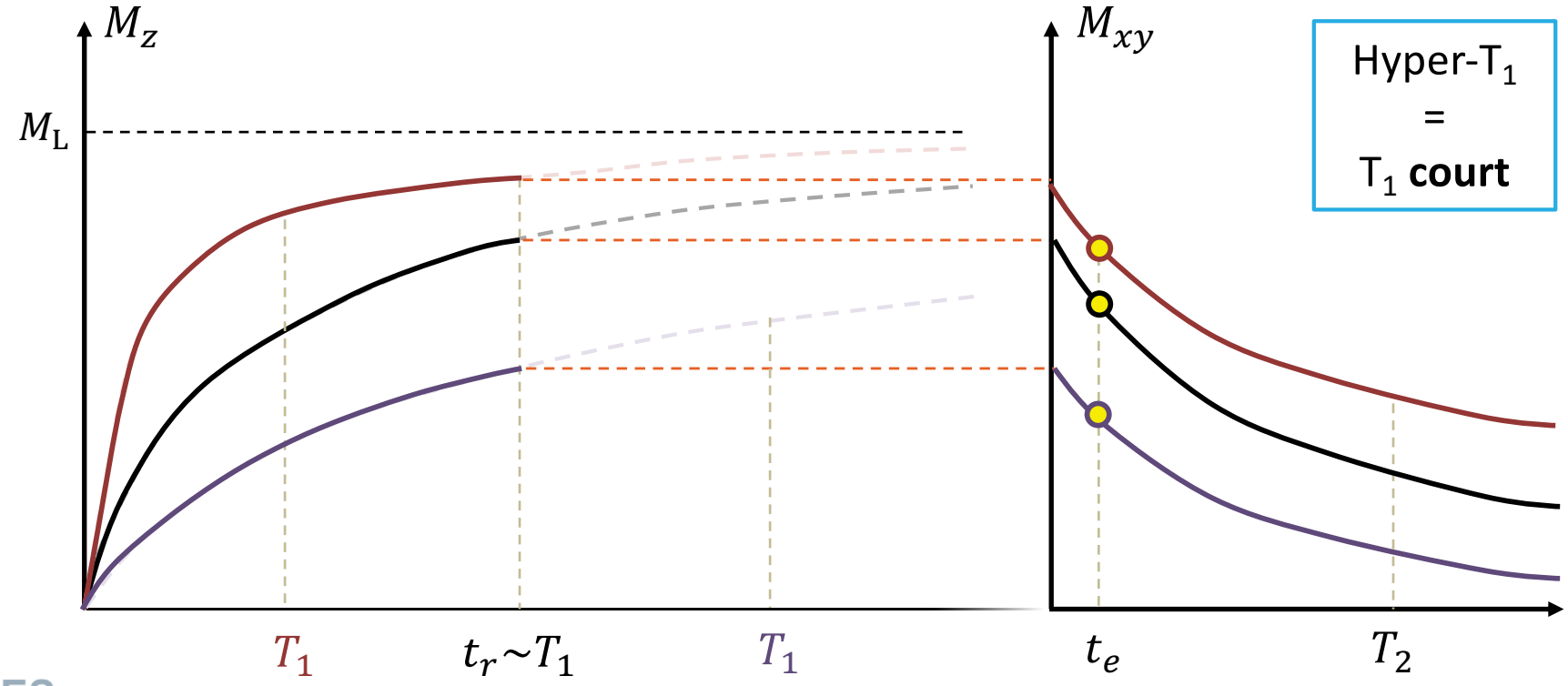
Signal $\propto M_L(1 - e^{-t_r/T_1})$



Hyper-T_1
=
T_1 **court**

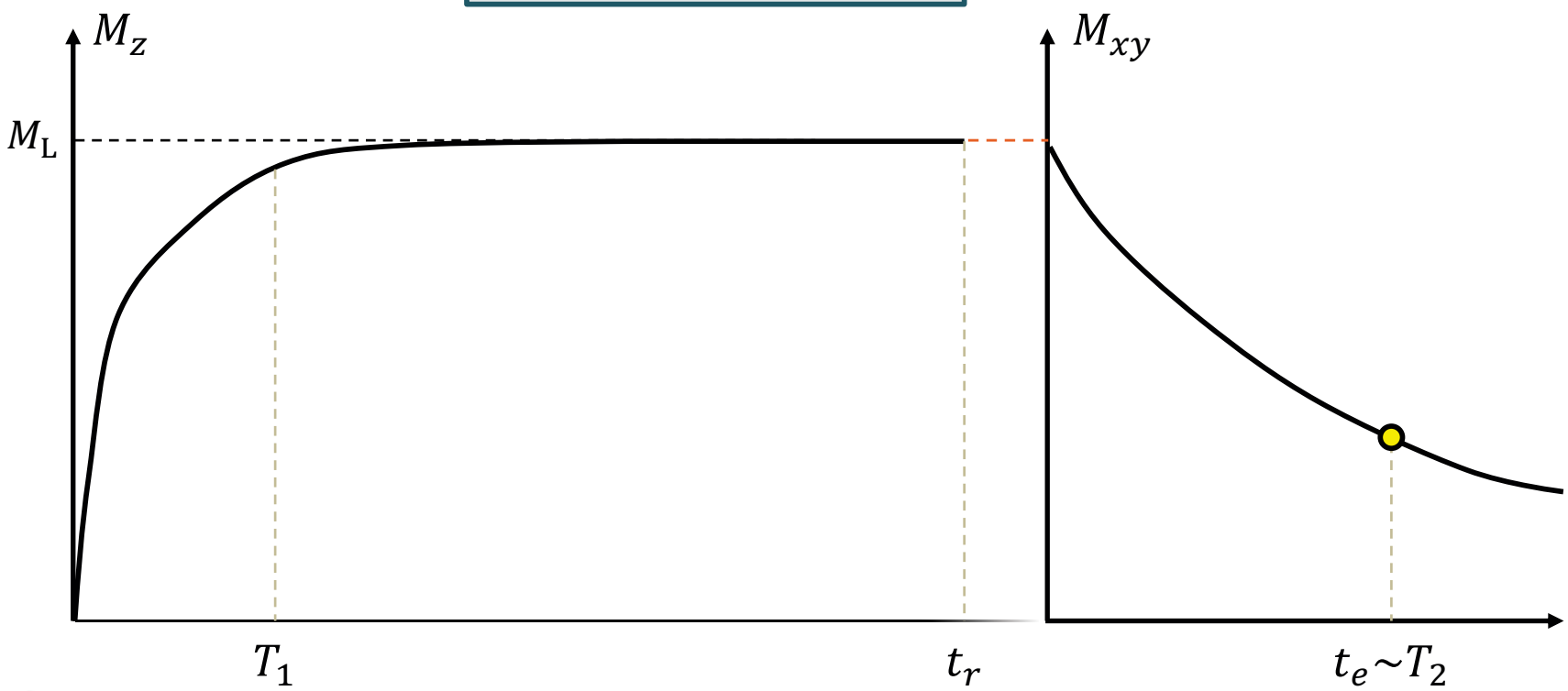
RMN – contraste

Pondération T_2

$t_r \gg T_1$
$t_e \sim T_2$

$$FID(t) = M_L\left(1 - e^{-t_r/T_1}\right)\sin(\eta)\, e^{-t_e/T_2}\sin(\omega_0 t)$$

Signal $\propto M_L(\, e^{-t_e/T_2})$



RMN – contraste

Pondération T_2

$t_r \gg T_1$
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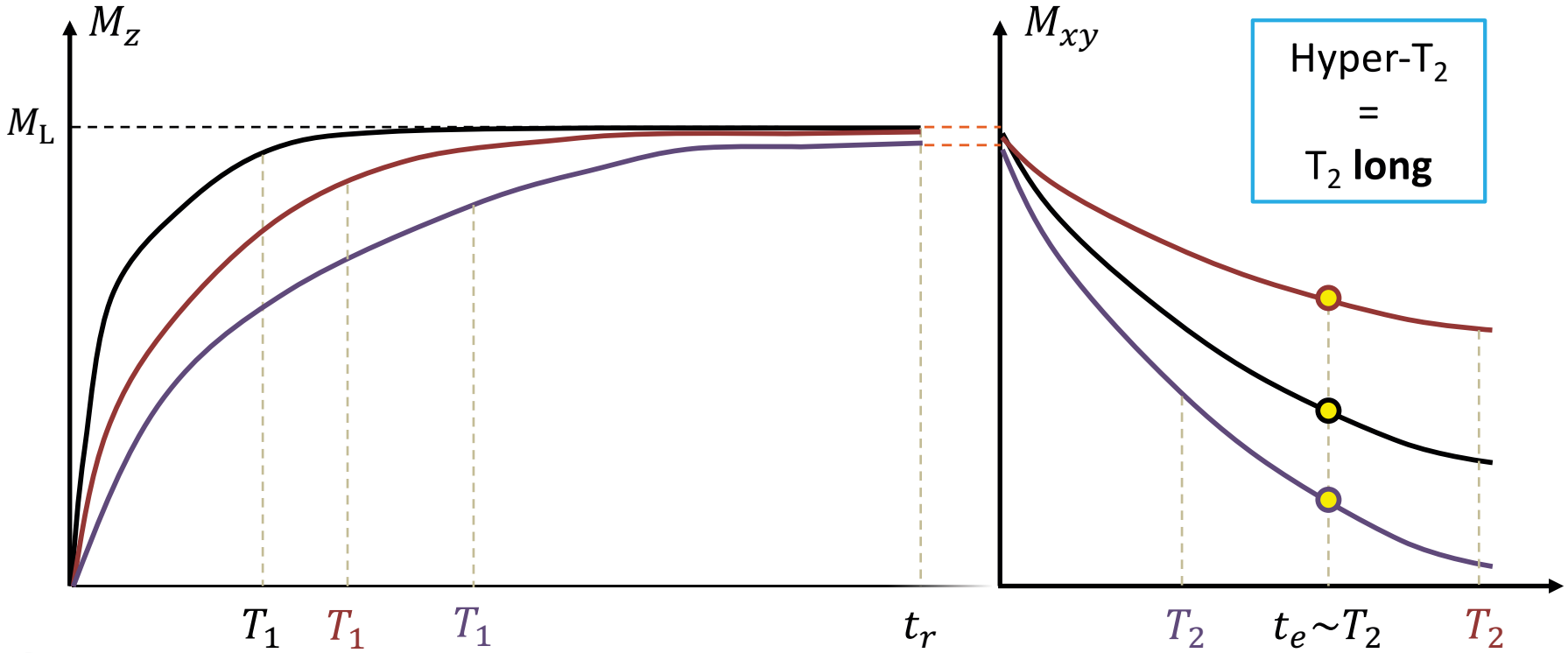
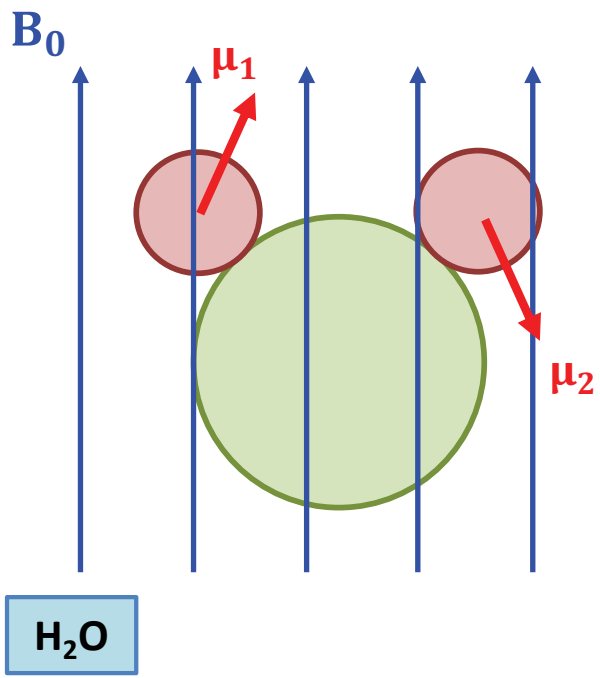
Signal $\propto M_L(\, e^{-t_e/T_2})$



RMN – contraste

Relaxation :

- **Couplage dipolaire:** interaction magnétique dipôle-dipôle
- **Mouvements moléculaires**



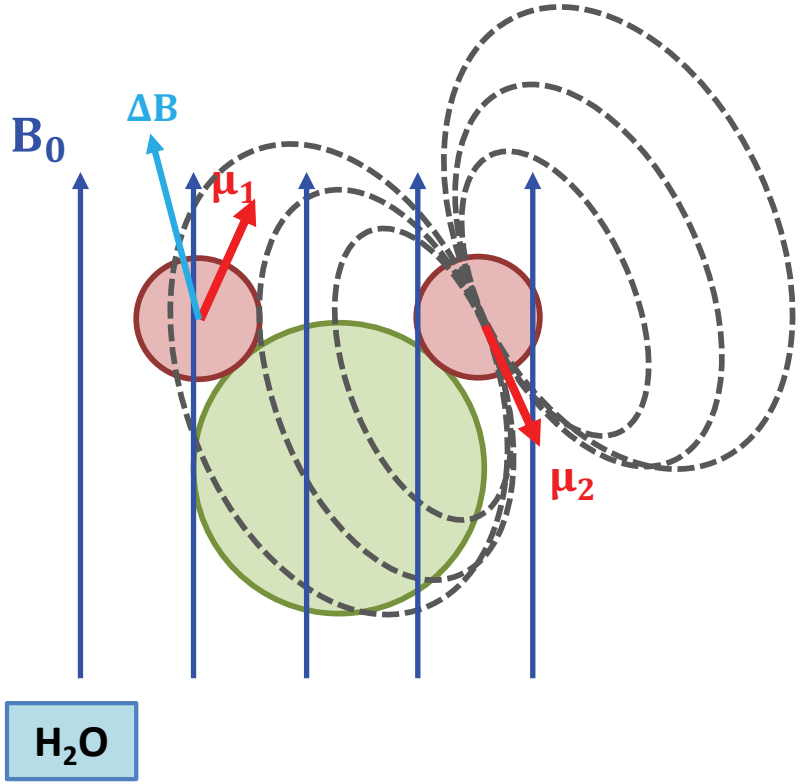
Couplage entre μ_1 et μ_2

μ_1 perçoit B_0 et sa perturbation par μ_2

RMN – contraste

Relaxation :

- **Couplage dipolaire:** interaction magnétique dipôle-dipôle
- **Mouvements moléculaires**



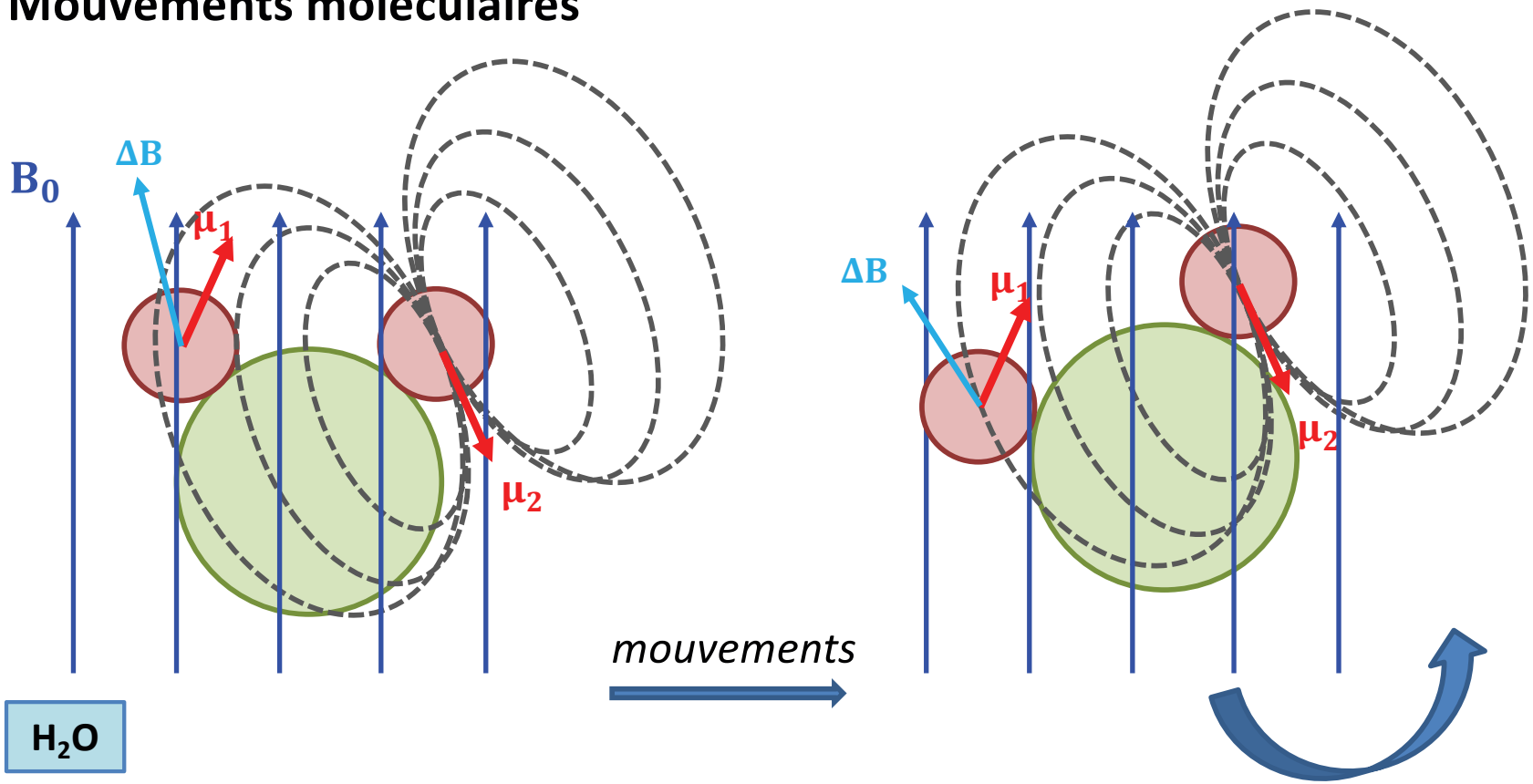
$B_0 \sim 1 - 3T$

$\Delta B_0 \sim 10^{-3} T$

RMN – contraste

Relaxation :

- **Couplage dipolaire:** interaction magnétique dipôle-dipôle
- **Mouvements moléculaires**

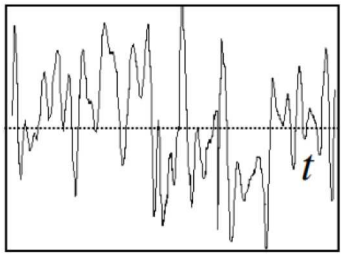


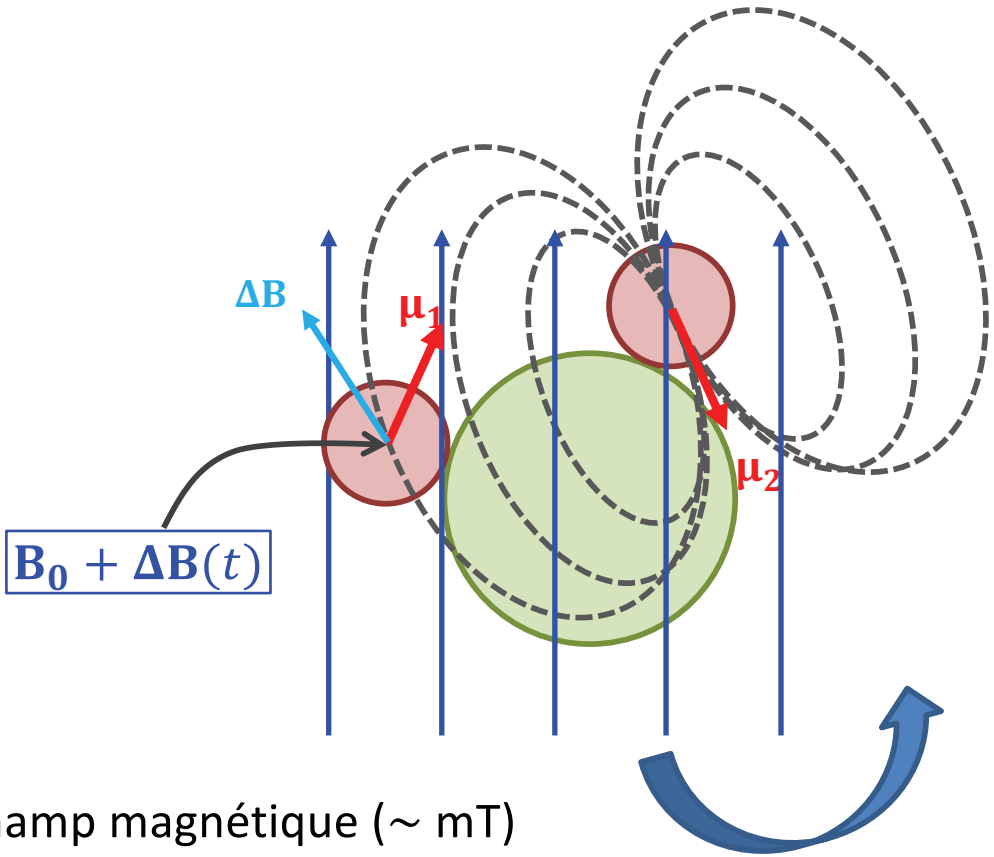
RMN – contraste

Relaxation :

- **Couplage dipolaire:** interaction magnétique dipôle-dipôle
- **Mouvements moléculaires**

$\mathbf{\Delta B}(t)$





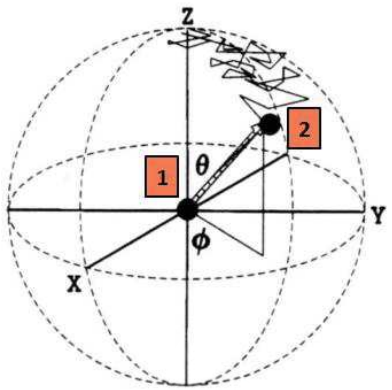
$\mathbf{\Delta B}$: bruit de moyenne nulle

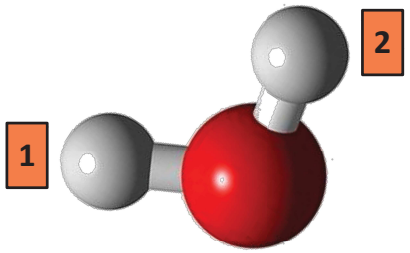
ΔB^2: amplitude de variation du champ magnétique (~ mT)

RMN – contraste

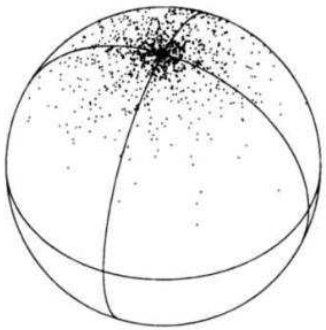
Relaxation :

- **Couplage dipolaire**
- **Mouvements moléculaires**

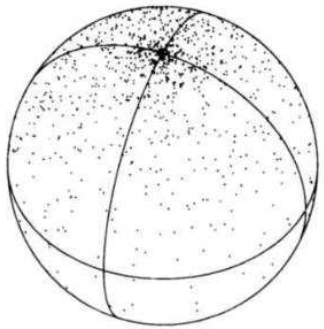




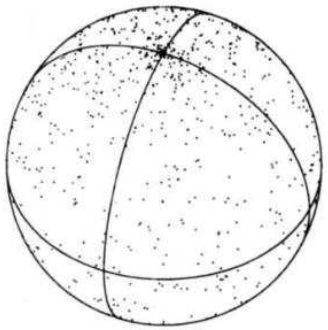
τ_C long



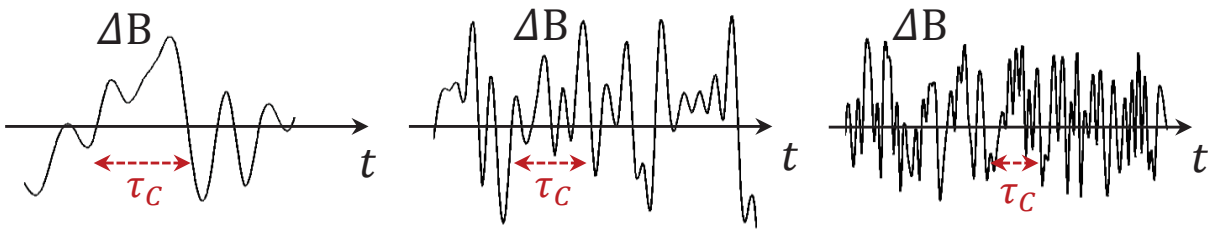
τ_C moyen



τ_C court



Temps d'autocorrélation τ_C



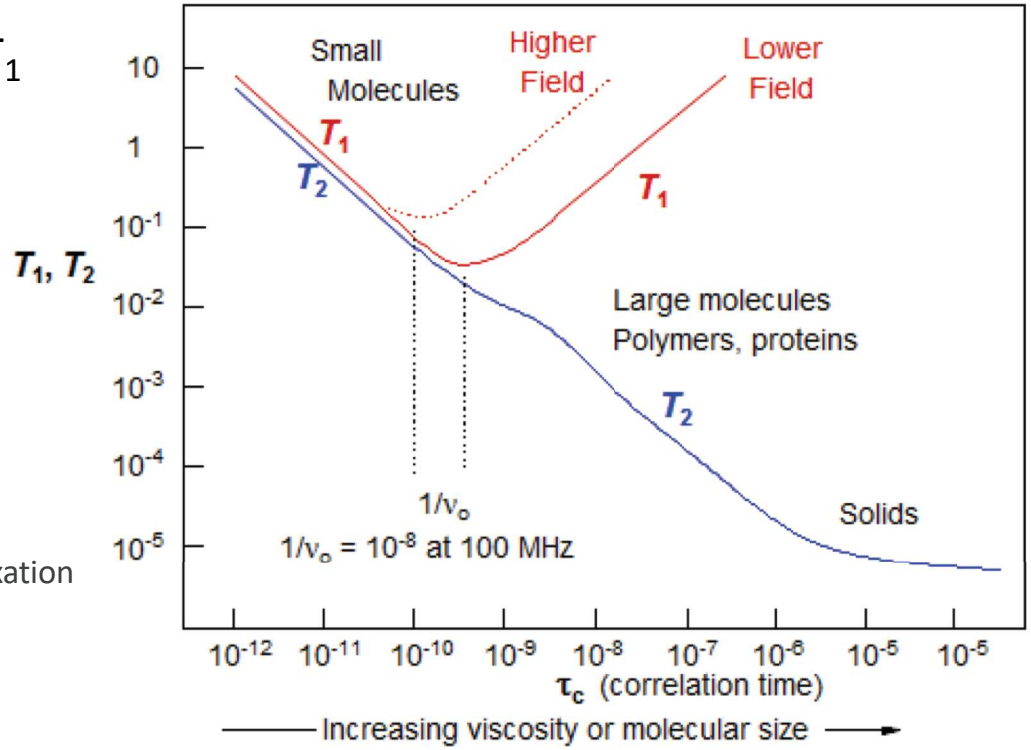
$$\tau_c \propto \frac{V\,\eta}{kT}$$

V volume moléculaire
η viscosité

RMN – contraste

Relaxation :

- Echanges énergétiques avec le réseau
- Déphasage des spins
- Relaxation liée à τ_c
- MW → rayon molécule → volume molécule → τ_c
- Plus la molécule est petite, plus elle tourne vite
- $T_2 \leq T_1$
- Très petites molecules: $T_2 = T_1$
- Macromolécules: $T_2 < T_1$

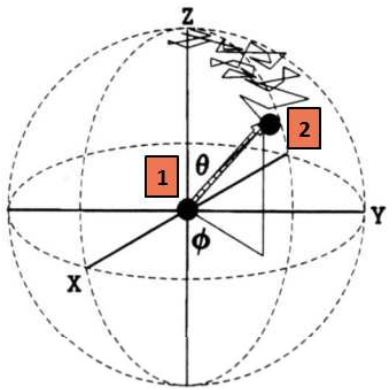


Bloembergen, E.M. Purcell, R.V. Pound "Relaxation Effects in Nuclear Magnetic Resonance Absorption" *Physical Review* **1948**,

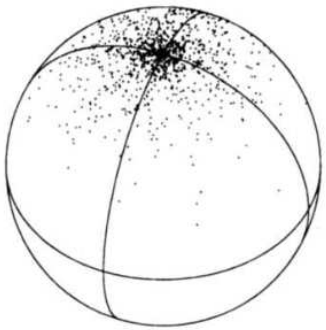
RMN – contraste

Relaxation :

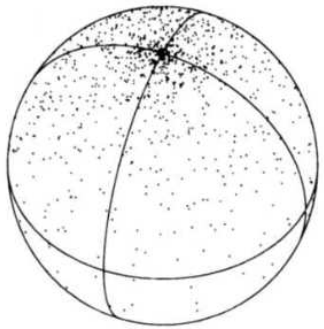
- **Couplage dipolaire**
- **Mouvements moléculaires**



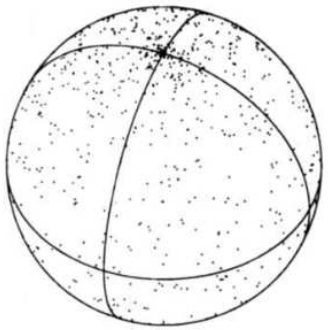
τ_c long

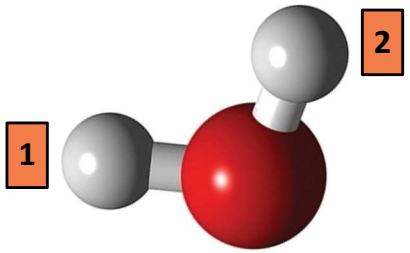


τ_c moyen



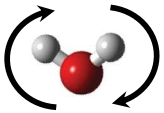
τ_c court



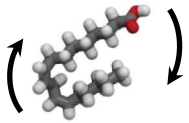


Temps d'autocorrélation τ_c

Eau libre : mouvement rapide : τ_C court



Eau liée / lipides : τ_C moyen

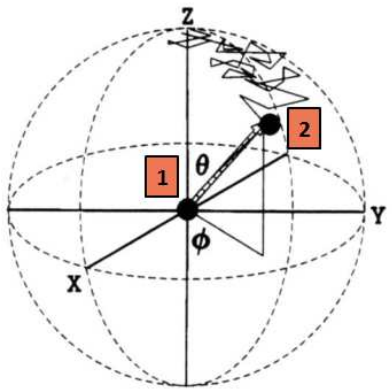


Macromolécules : mouvement lent : τ_C long

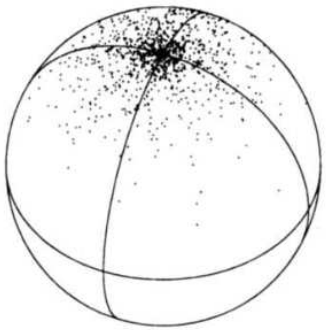
RMN – contraste

Relaxation :

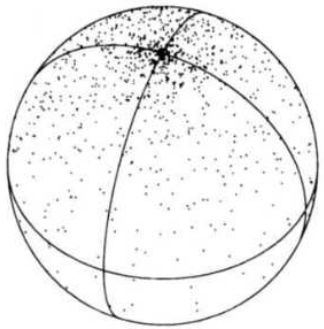
- **Couplage dipolaire**
- **Mouvements moléculaires**



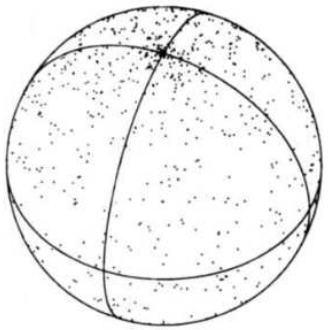
τ_c long



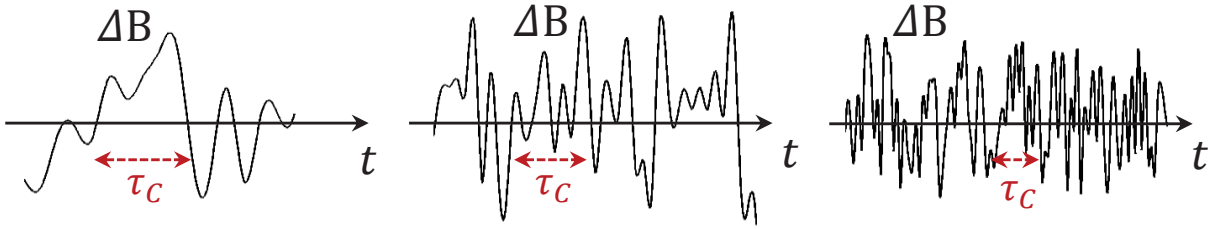
τ_c moyen



τ_c court

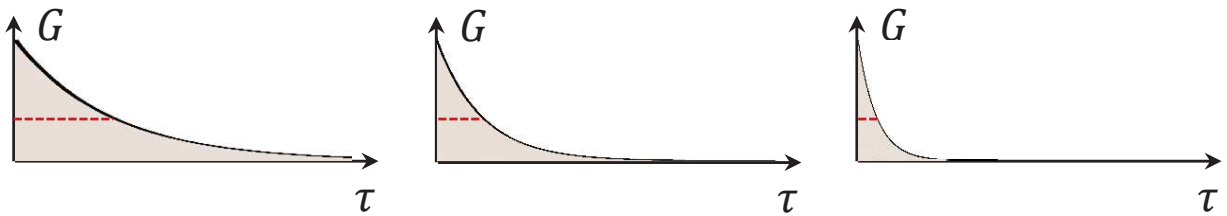


Temps d'autocorrélation τ_c



Fonction d'autocorrélation

$$G(\tau) \propto e^{-\tau/\tau_c}$$

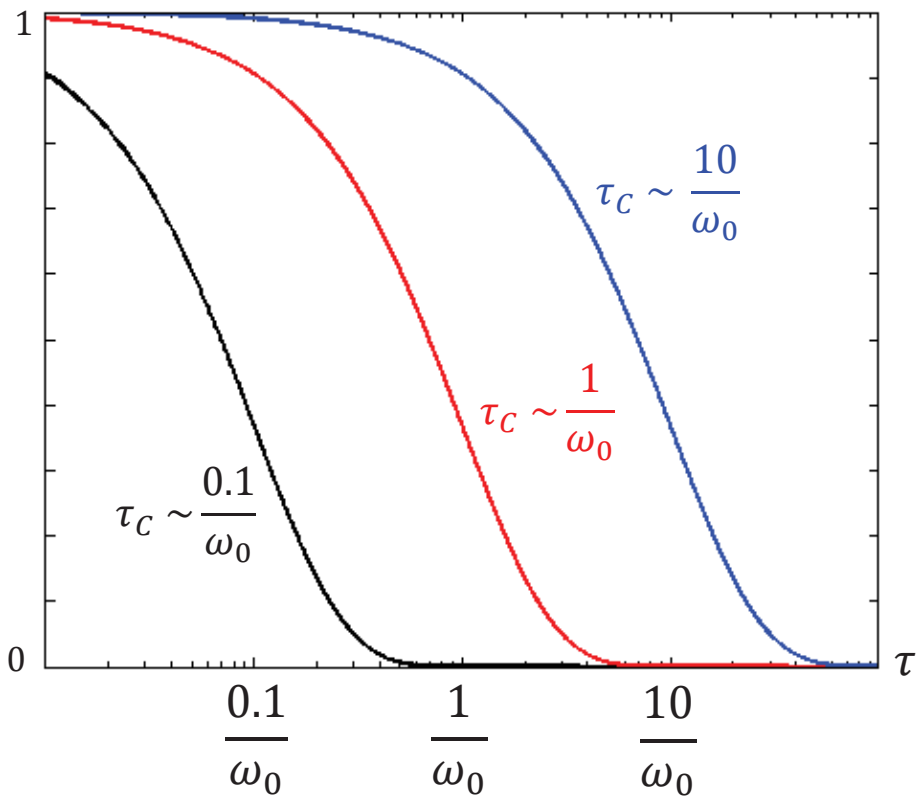


RMN – contraste

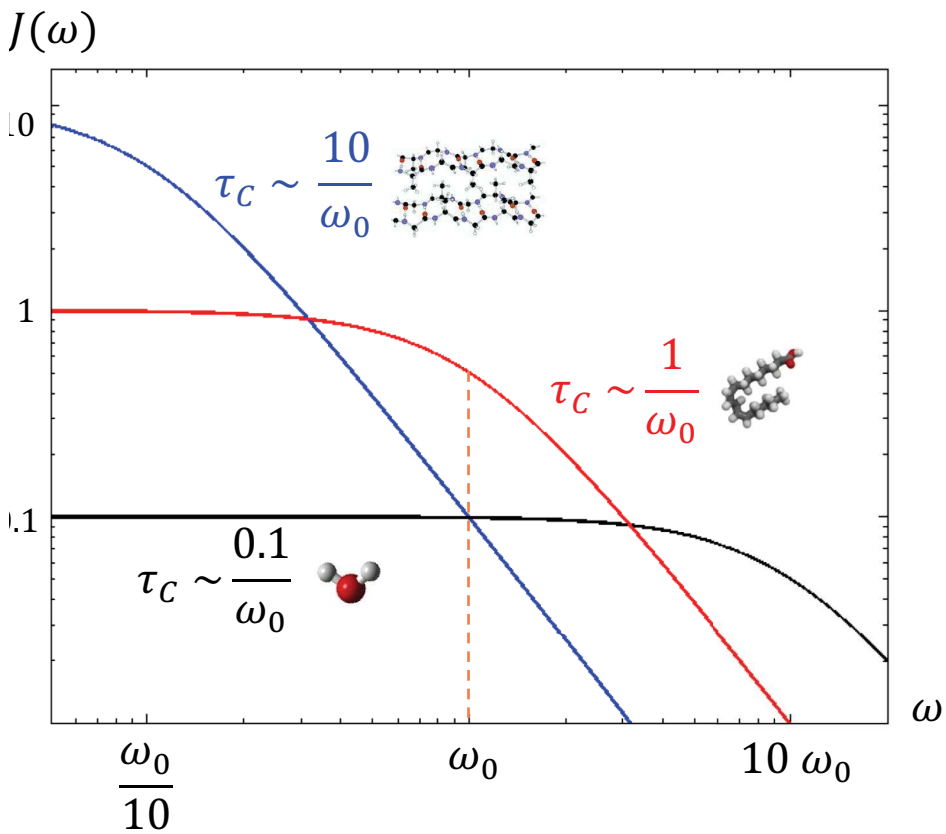
Relaxation :

Fonction d'autocorrélation $G(\tau)$

$$G(\tau) \propto e^{-\tau/\tau_c}$$



$J(\omega)$: spectre de $G(\tau)$



RMN – contraste

Relaxation :

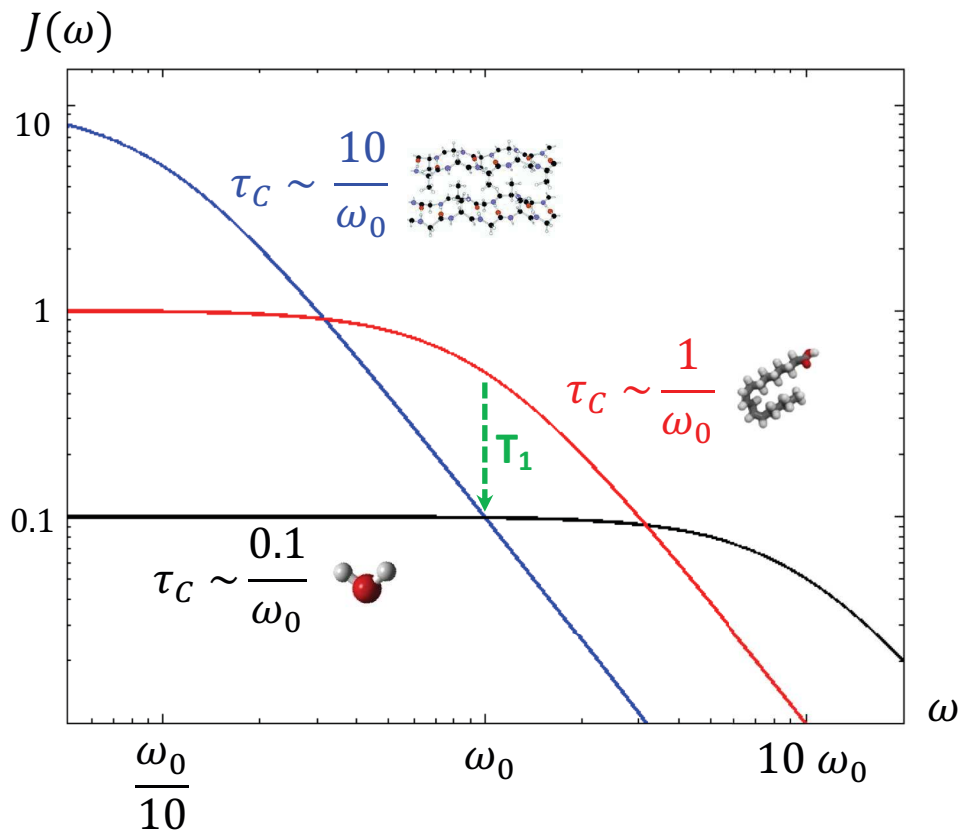
T_1

Fluctuations transverses
→ Échanges d'énergie

$$\frac{1}{T_1} = \gamma^2 \left(\langle \Delta B_x^2 \rangle + \langle \Delta B_y^2 \rangle\right) J(\omega_0)$$
$$= 2\gamma^2 \langle \Delta B^2 \rangle J(\omega_0)$$

Intensité du champ fluctuant

Proportion de fluctuation à la fréquence de Larmor



RMN – contraste

Relaxation :

T_1

Fluctuations transverses
→ Échanges d'énergie

$$\frac{1}{T_1} = \gamma^2 \left(\langle \Delta B_x^2 \rangle + \langle \Delta B_y^2 \rangle\right) J(\omega_0)$$
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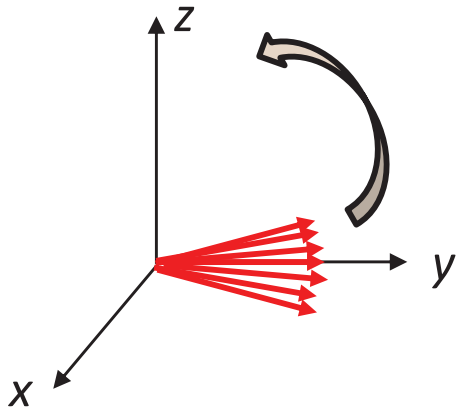
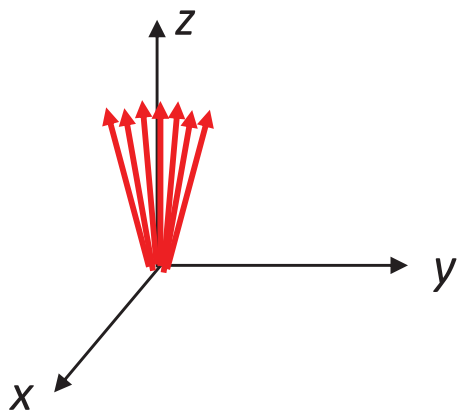
Intensité du champ fluctuant

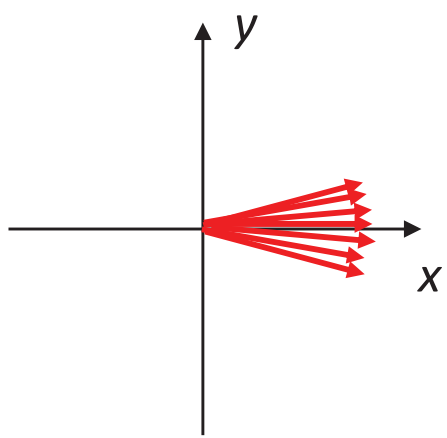
Proportion de fluctuation à la fréquence de Larmor

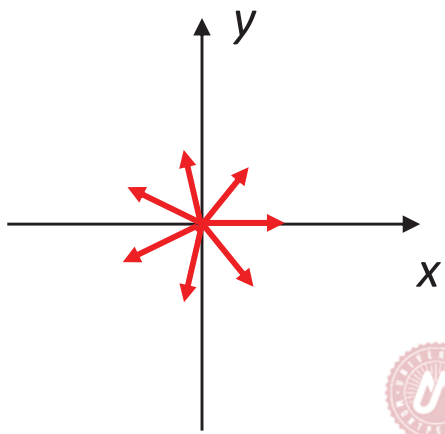




T_2

$$\frac{1}{T_2} = \frac{1}{2T_1} + \gamma^2 \langle \Delta B_z^2 \rangle J(0)$$
$$= \gamma^2 \langle \Delta B^2 \rangle \left(J(0) + J(\omega_0)\right)$$





RMN – contraste

Relaxation :

T_1

Fluctuations transverses
→ Échanges d'énergie

$$\frac{1}{T_1} = \gamma^2 \left(\langle \Delta B_x^2 \rangle + \langle \Delta B_y^2 \rangle\right) J(\omega_0)$$
$$= 2\gamma^2 \langle \Delta B^2 \rangle J(\omega_0)$$

Intensité du champ fluctuant

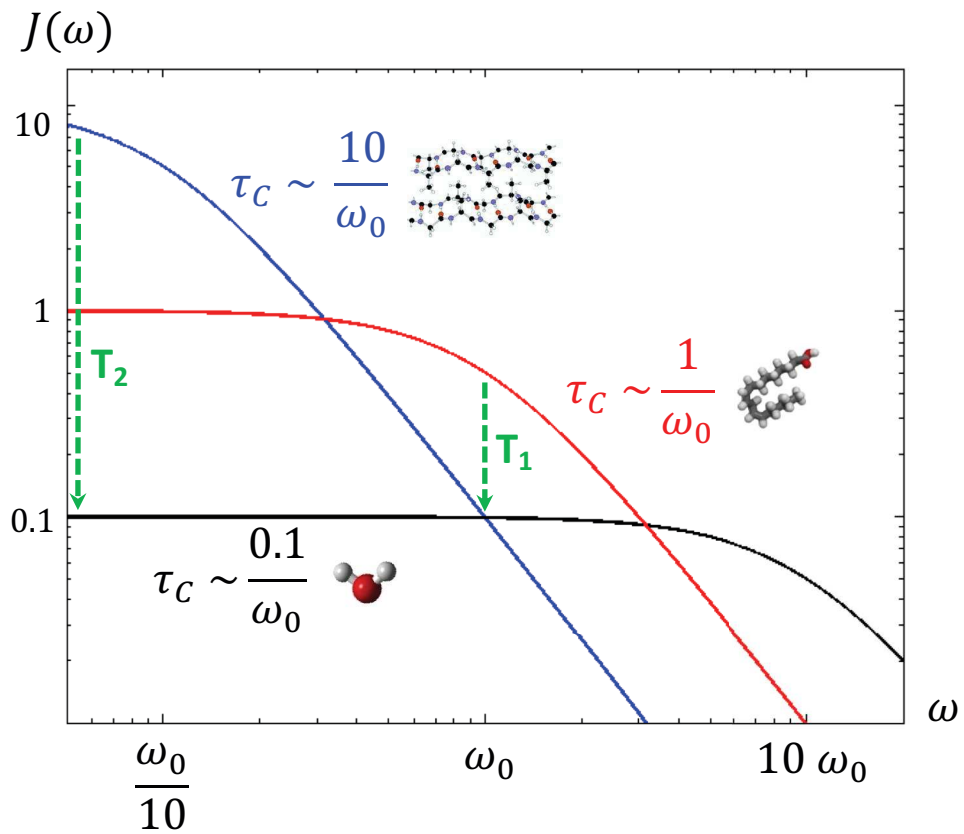
Proportion de fluctuation à la fréquence de Larmor

T_2

Fluctuations longitudinales
→ Déphasage des spins

$$\frac{1}{T_2} = \frac{1}{2T_1} + \gamma^2 \langle \Delta B_z^2 \rangle J(0)$$
$$= \gamma^2 \langle \Delta B^2 \rangle \left(J(0) + J(\omega_0)\right)$$

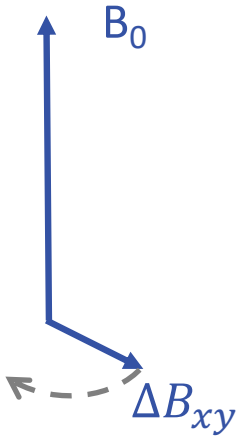
Proportion de champ statique



RMN – contraste

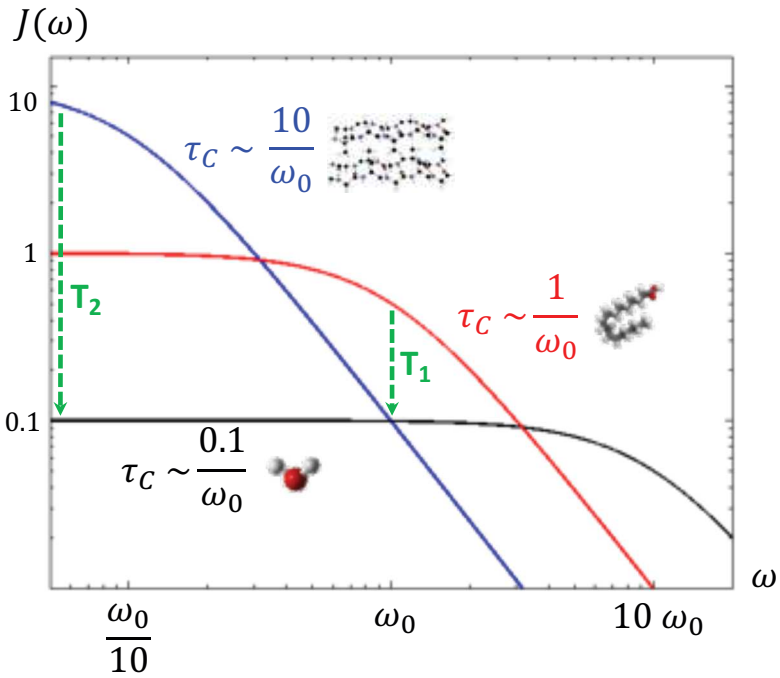
Relaxation :

T_1

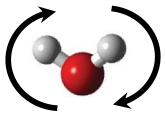


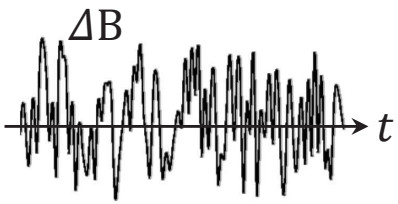
(Larmor ω_0)

>> LENT

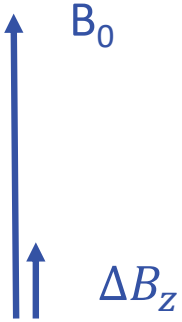


EAU LIBRE





T_2



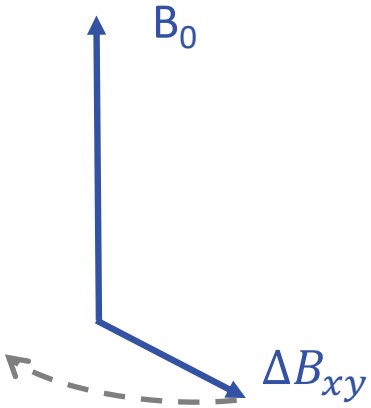
(statique)

>> LENT

RMN – contraste

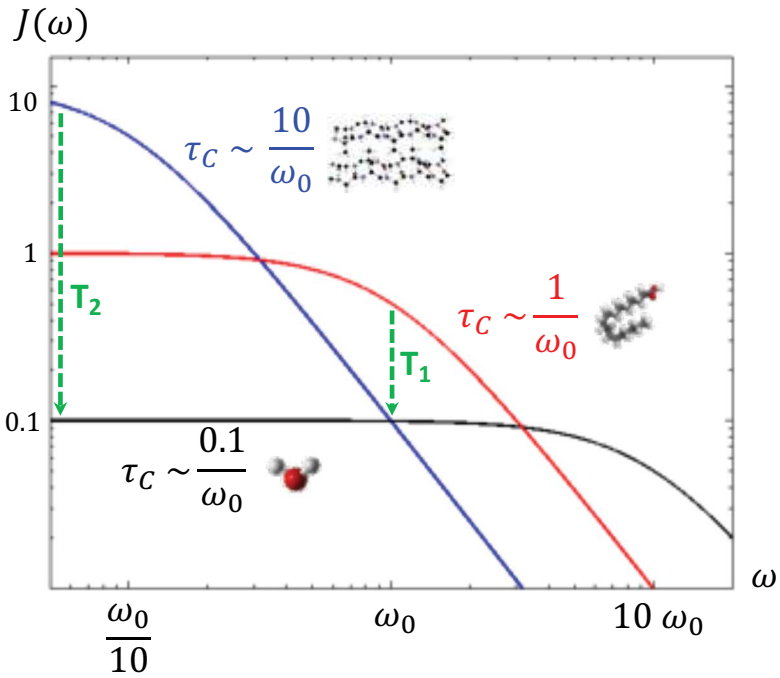
Relaxation :

T_1

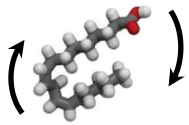


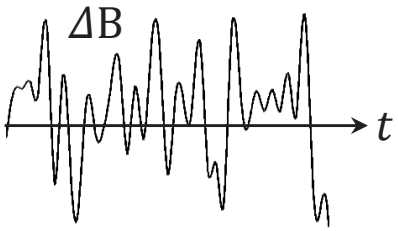
(Larmor ω_0)

>> RAPIDE

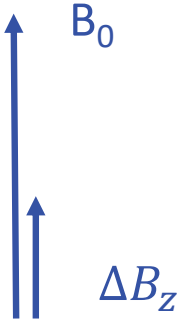


LIPIDES





T_2



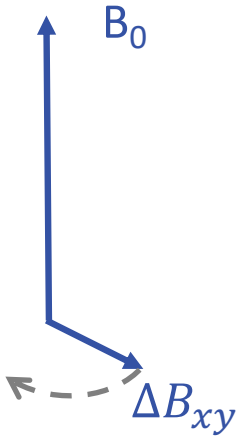
(statique)

>> +/- RAPIDE

RMN – contraste

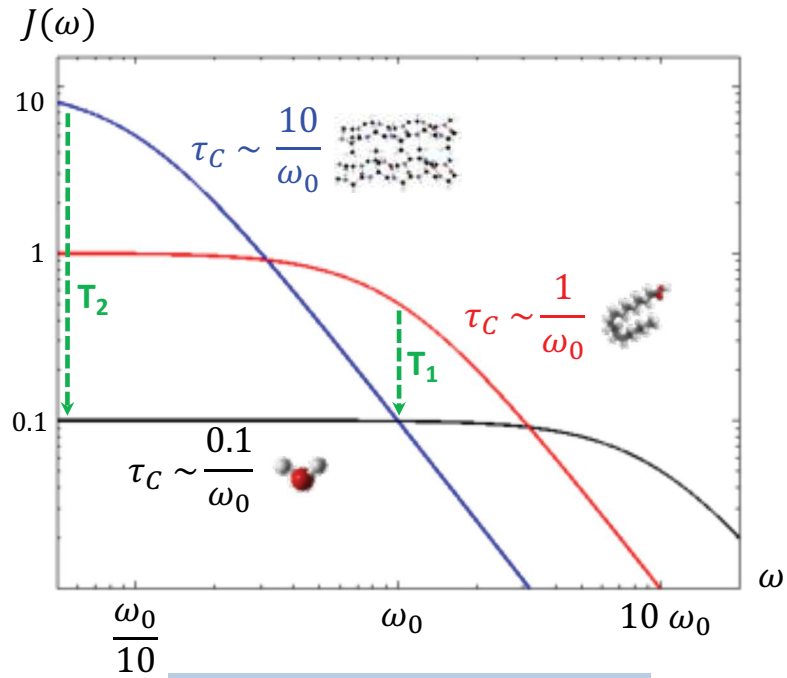
Relaxation :

T_1

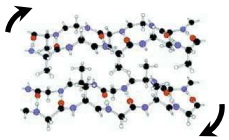


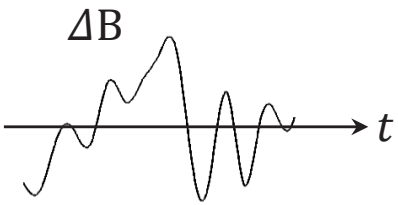
(Larmor ω_0)

>> LENT

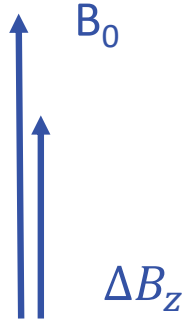


MACROMOLECULES





T_2

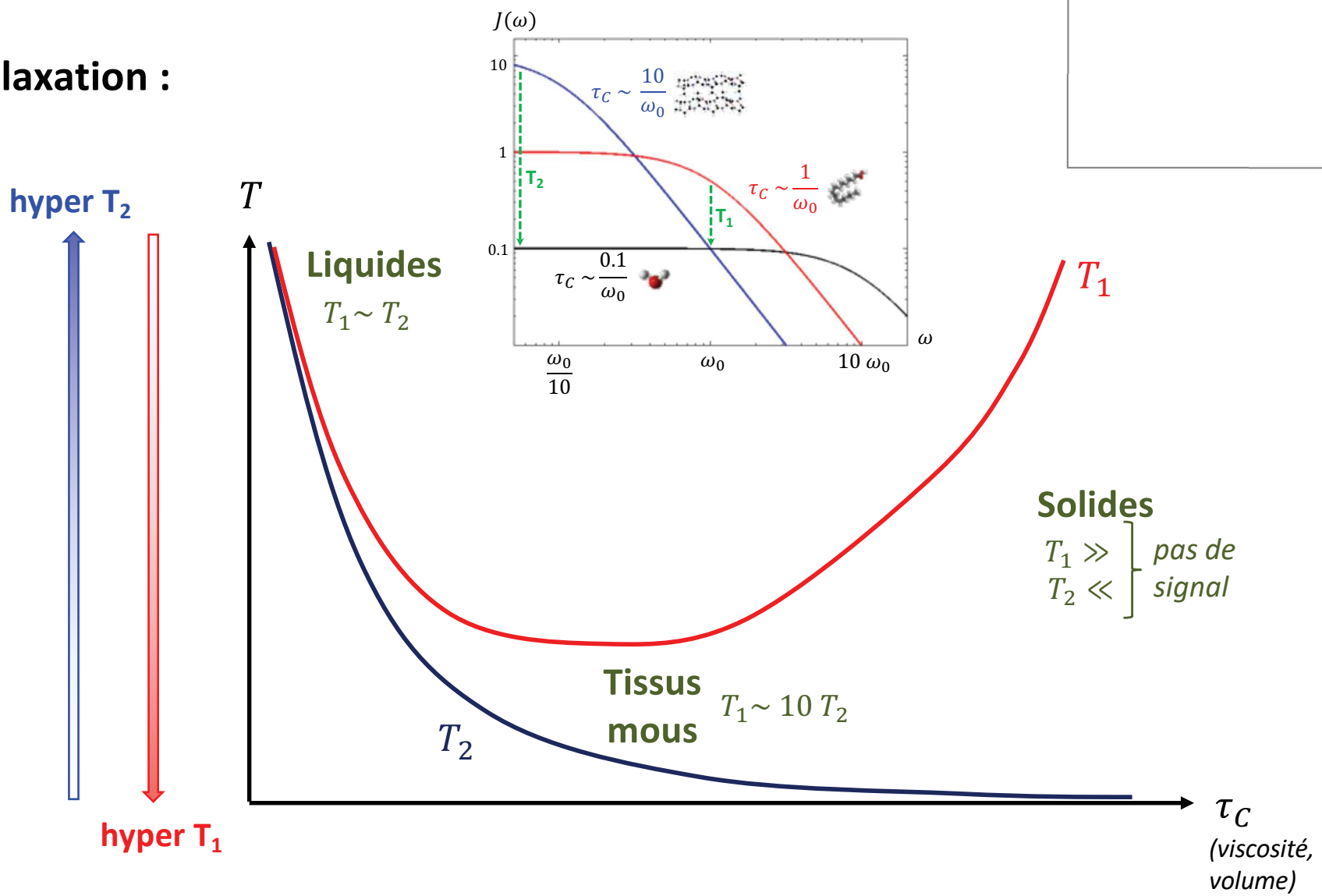


(statique)

>> RAPIDE

RMN – contraste

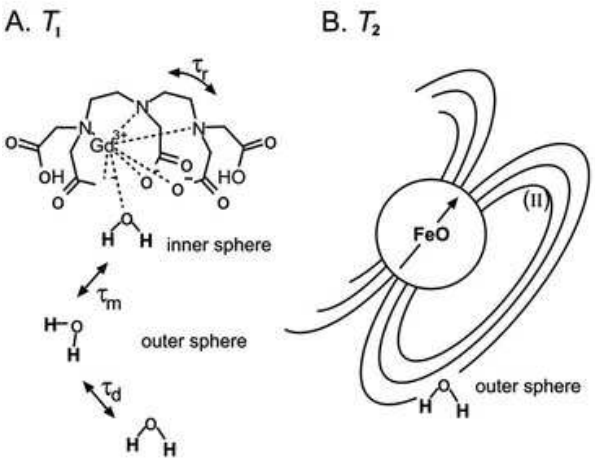
Relaxation :



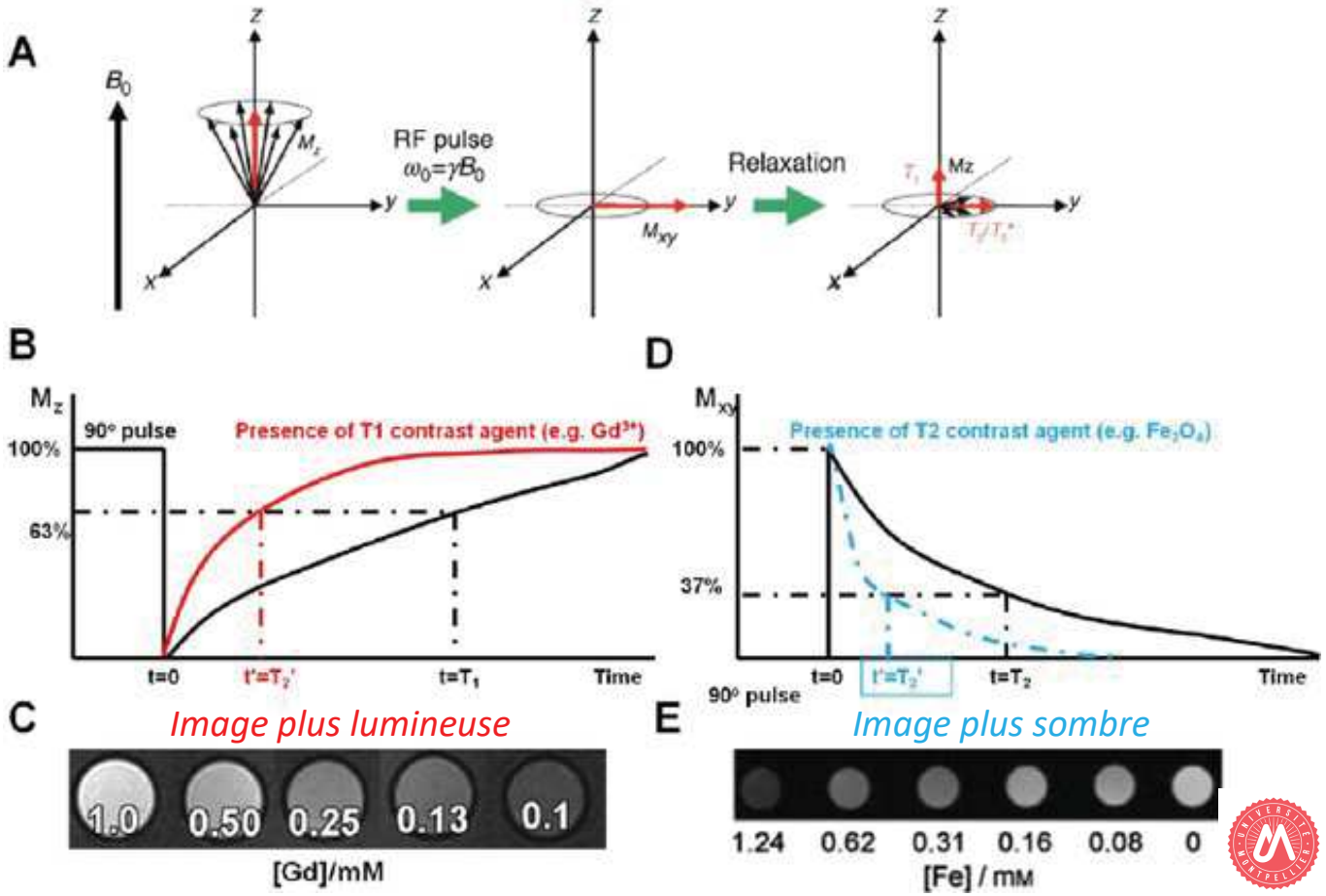
RMN – contraste

Agents de contraste :

Ions métalliques (super)paramagnétiques Gd^{3+} (Fe^{3+})
Comportement des protons des molécules d'eau avec les complexes (nanoparticules)
→ Augmentation de la relaxation par diminution des temps T_1 et T_2 des protons



G. Strijkers, 2007



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